Soil Mechanics

-Theoretical: 2 hr. /week

-Applicatory: 1 hr. /week

-Practical (Lab.): 2 hr. /week

Required Text Books

- Principles of Geotechnical Engineering (By: Braja M. Das, 7th Ed.)

-Soil Mechanics (By: R.F. Craig, 4th Ed. or higher)

-Soil Mechanics (By: T.W. Lambe and R.V. Whitman)

- مبادئ ميكانيك التربة (تأليف و إعداد: د. محمد عمر العشو)

-Experiments in Soil Mechanics

جامعة البصرة – كلية الهندسة – قسم الهندسة المدنية

Syllabus

1. Geotechnical Engineering – A Historical Perspective
2. Basic Characteristics of Soils
3. Weight-Volume Relationships (Phase Relationships)
4. Plasticity and Structure of Soil
5. Classification of Soil
6. Soil Compaction
7. Permeability
8. Seepage
9. In Situ Stresses
10. Stresses in Soil Mass
11. Compressibility of Soil
12. Shear Strength of Soil
13. Lateral Earth Pressure

Chapter 2: Basic Characteristics of Soils

2.1 Soil Formation

Soils are formed from the physical and chemical weathering of rocks. Physical weathering involves reduction of size without any change in the original composition of the parent rock. Chemical weathering causes both reduction in size and chemical alteration of the original parent rock (hydration, carbonation, and oxidation).

Soils that remain at the site of weathering are called residual soils. These soils retain many of the elements that comprise the parent rock. The transported soils may be classified into several groups depending on their mode of transportation and deposition:

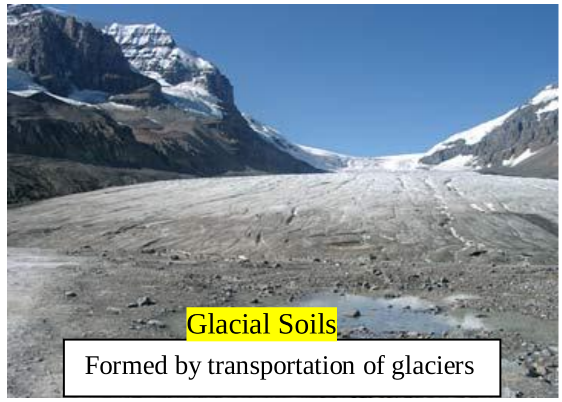
1. Alluvial Soils: transported by rivers and streams.
2. Glacial Soils: formed by transportation and deposition of glaciers.
3. Marine Soils: formed by deposition in the seas.
4. Aeolian Soils: transported and deposited by wind.
5. Gypsum: Calcium sulphate formed under heat and pressure from sediments in ocean brine.
6. Loam: mixture of sand, silt, and clay that may contain organic material.
7. Loess: a windblown, uniform fine-grained soil.
8. Mud: clay and silt mixed with water into a viscous fluid.

2.2 Clay Minerals

The basic structural units of most clay minerals consist of a silica tetrahedron and an alumina octahedron. The basic units combine to form sheet structures (Silica sheet and Alumina or Gibbsite sheet).

Alluvial Soils

Transported by rivers and streams



Glacial Soils

Formed by transportation of glaciers

Marine Soils

Formed by deposition in the seas

Aeolian Soils

Transported and deposited by wind

Gypsum

Calcium sulphate formed under heat & pressure

Loam

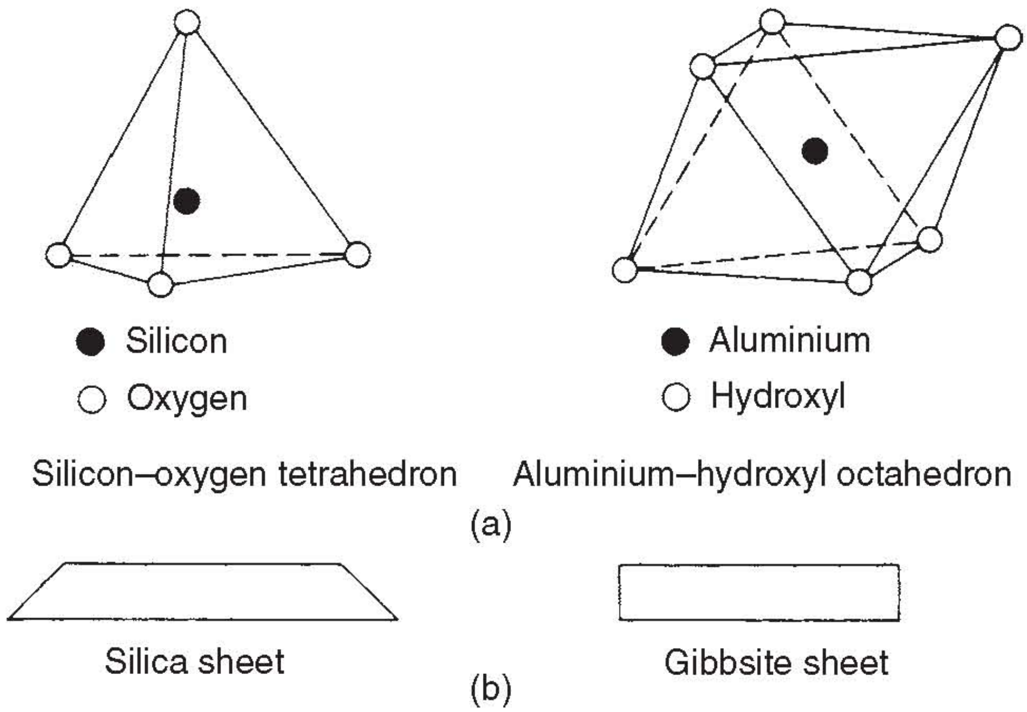
Mix. sand, silt, and clay with organic materials

Loess

Windblown, uniform fine-grained soil

Mud

Clay & silt mixed with water into viscous fluid



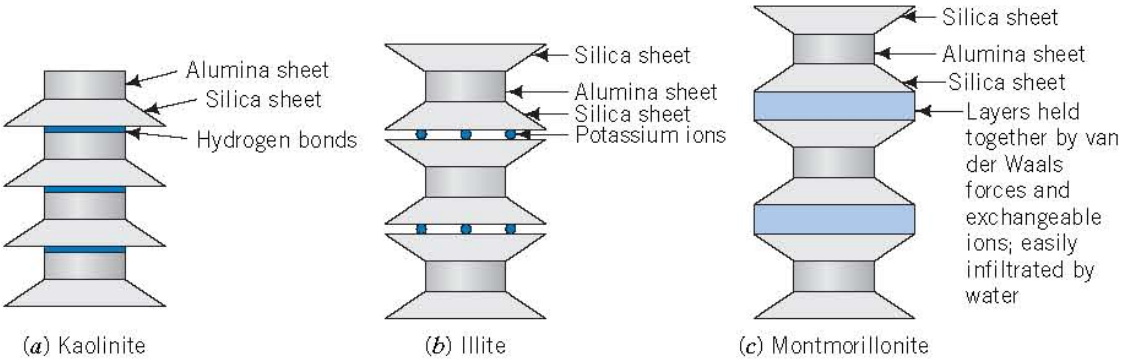
Clay minerals: basic units.

The main groups of crystalline materials that make up clays are:

Kaolinite: Consists of one silica sheet and one alumina sheet. The combined silica-alumina sheets are held together by hydrogen bonding. A kaolinite particle may consist of over one hundred stacks.

Illite: Consists of repeated layers of one alumina sheet sandwiched by two silica sheets. The combined sheets are linked together by weak bonding due to potassium ions held between them.

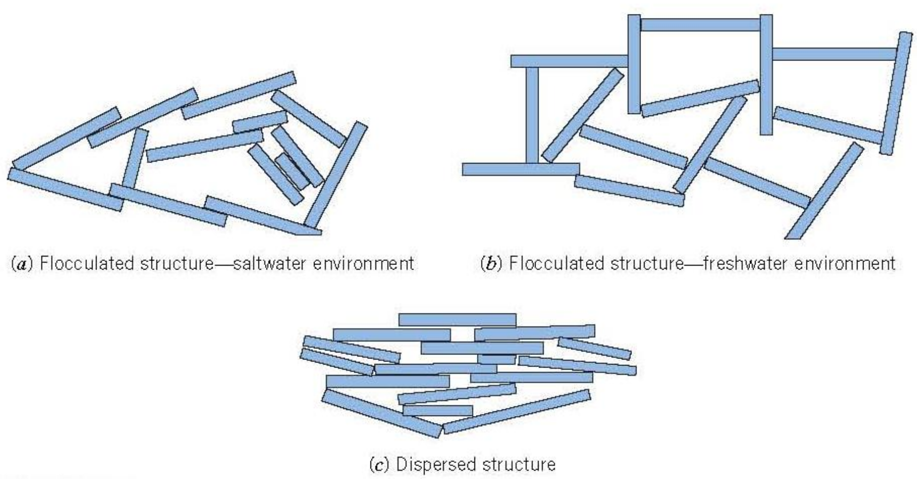
Montmorillonite: Has a structure similar to illite, but the layers are held together by weak van der Waals forces and exchangeable ions. Water can easily enter the bond and separate the layers causing swelling. Montmorillonite is often called swelling or expansive clay.



Structure of kaolinite, illite, and montmorillonite.

2.3 Soil Fabric

During deposition, the mineral particles are arranged into structural frameworks that we call soil fabric. The environment under which deposition occurs influences the structural framework that is formed. Two common types of soil fabric: flocculated and dispersed are formed during deposition of fine-grained soils. A flocculated structure, formed under a saltwater environment, results when many particles tend to orient parallel to each other. A flocculated structure, formed under a freshwater environment, results when many particles tend to orient perpendicular to each other. A dispersed structure occurs when a majority of the particles orient parallel to each other.



Soil fabric.

Chapter 3: Weight-Volume Relationships (Phase Ralationships)

Partially saturated soil (three-phase soil) is composed of solids (soil particles), liquids (usually water), and gases (usually air). The spaces between the solids are called voids. The soil water is commonly called pore water and it plays a very important role in the behavior of soils under load. If all voids are filled with water, the soil is saturated (two-phase). Otherwise, the soil is unsaturated. If all the voids are filled with air, the soil is said to be dry (two-phase).

3.1 Weight-Volume Relationships

Figure (3.1a) shows an element of soil of volume V and weight W as it would exist in a natural state. To develop the weight–volume relationships, we must separate the three phases (that is, solid, water, and air) as shown in Figure (3.1b). Thus, the total volume of a given soil sample can be expressed as

$$V = Vs + Vv = Vs + Vw + Va \tag{3.1}$$

where Vs = volume of soil solids
Vv = volume of voids
Vw = volume of water in the voids
Va =volume of air in the voids

Assuming that the weight of the air is negligible, we can give the total weight of the sample as

$$W = Ws + Ww \tag{3.2}$$

where Ws = weight of soil solids
Ww = weight of water

The *volume relationships* commonly used for the three phases in a soil element are *void ratio, porosity, and degree of saturation. Void ratio* (e) is defined as the ratio of the volume of voids to the volume of solids. Thus,

$$e = \frac{V_v}{V_s} \tag{3.3}$$

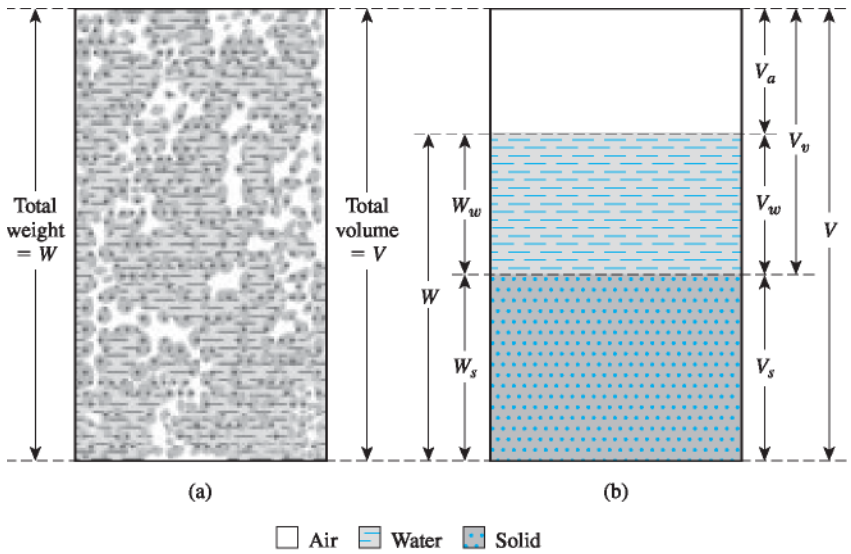


Figure (3.1a) Soil element in natural state; (b) three phases of the soil element

Porosity (n) is defined as the ratio of the volume of voids to the total volume, or

$$n = \frac{V_v}{V} \tag{3.4}$$

The degree of saturation (S) is defined as the ratio of the volume of water to the volume of voids, or

$$S = \frac{V_w}{V_v} \tag{3.5}$$

It is commonly expressed as a percentage.

The relationship between void ratio and porosity can be derived from Eqs. (3.1), (3.3), and (3.4) as follows:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{(\frac{V_v}{V})}{1-(\frac{V_v}{V})} = \frac{n}{1-n} \tag{3.6}$$

Also, from Eq. (3.6),

$$n = \frac{e}{1+e} \tag{3.7}$$

The common terms used for *weight relationships are moisture content* and *unit weight. Moisture content* (w) is also referred to as *water content* and is defined as the ratio of the weight of water to the weight of solids in a given volume of soil:

$$w = \frac{W_w}{W_s} \qquad (3.8)$$

Unit weight (γ) is the weight of soil per unit volume. Thus,

$$\gamma = \frac{W}{V} \qquad (3.9)$$

The unit weight can also be expressed in terms of the weight of soil solids, the moisture content, and the total volume. From Eqs. (3.2), (3.8), and (3.9),

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s\left[1 + \left(\frac{W_w}{W_s}\right)\right]}{V} = \frac{W_s(1+w)}{V} \qquad (3.10)$$

Soils engineers sometimes refer to the unit weight defined by Eq. (3.9) as the *moist unit weight*.

Often, to solve earthwork problems, one must know the weight per unit volume of soil, excluding water. This weight is referred to *as the dry unit weight*, γ_d. Thus,

$$\gamma_d = \frac{W_s}{V} \qquad (3.11)$$

From Eqs. (3.10) and (3.11), the relationship of unit weight, dry unit weight, and moisture content can be given as

$$\gamma_d = \frac{\gamma}{1+w} \qquad (3.12)$$

Sometimes it is convenient to express soil densities in terms of mass densities (ρ). The SI unit of mass density is kilograms cubic meter (kg/m^3). We can write the density equations [similar to Eqs. (3.9) and (3.11)] as

$$\rho = \frac{M}{V} \tag{3.13}$$

and

$$\rho_d = \frac{M_s}{V} \tag{3.14}$$

where ρ = density of soil (kg/m^3)
ρ_d = dry density of soil (kg/m^3)
M = total mass of the soil sample (kg)
Ms = mass of soil solids in the sample (kg)

The unit of total volume, V, is m^3.

The unit weight in kN/m^3 can be obtained from densities in kg/m^3 as

$$\gamma\ (\text{kN/m}^3) = \frac{g\rho(\frac{kg}{m^3})}{1000}$$

and

$$\gamma_d\ (\text{kN/m}^3) = \frac{g\rho_d(\frac{kg}{m^3})}{1000}$$

where g = acceleration due to gravity = 9.81 m/sec^2.
Note that unit weight of water (γ_w) is equal to 9.81 kN/m^3.

3.2 Relationships among Unit Weight, Void Ratio, Moisture Content, and Specific Gravity

To obtain a relationship among unit weight (or density), void ratio, and moisture content, let us consider a volume of soil in which the volume of the soil solids is one, as shown in Figure 3.2. If the volume of the soil solids is one, then the volume of voids is numerically equal to the void ratio, e [from Eq. (3.3)]. The weights of soil solids and water can be given as

$$W_s = G_s\,\gamma_w$$
$$W_w = wW_s = wG_s\,\gamma_w$$

where G_s = specific gravity of soil solids
w = moisture content
γ_w = unit weight of water

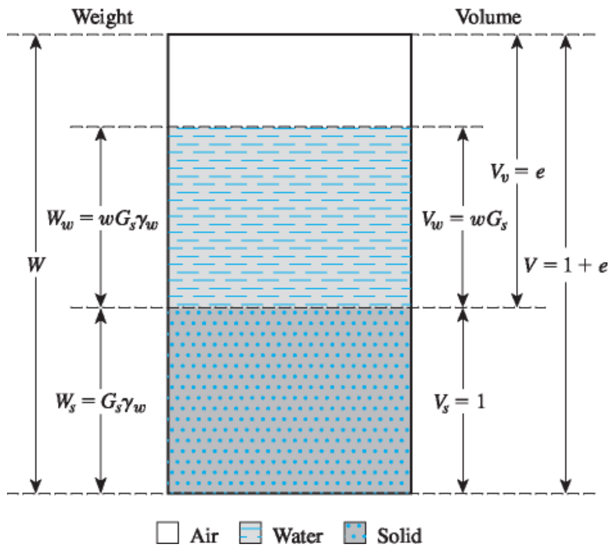


Figure 3.2 Three separate phases of a soil element with volume of soil solids equal to one

Now, using the definitions of unit weight and dry unit weight [Eqs. (3.9) and (3.11)], we can write

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + wG_s\gamma_w}{1+e} = \frac{(1+w)G_s\gamma_w}{1+e} \tag{3.15}$$

and

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w}{1+e} \tag{3.16}$$

or

$$e = \frac{G_s\gamma_w}{\gamma_d} - 1 \tag{3.17}$$

Because the weight of water for the soil element under consideration is $wG_s\gamma_w$, the volume occupied by water is

$$V_w = \frac{W_w}{\gamma_w} = \frac{wG_s\gamma_w}{\gamma_w} = wG_s$$

Hence, from the definition of degree of saturation [Eq. (3.5)],

$$S = \frac{V_w}{V_v} = \frac{wG_s}{e}$$

or

$$Se = wG_s \tag{3.18}$$

This equation is useful for solving problems involving three-phase relationships.

If the soil sample is saturated—that is, the void spaces are completely filled with water (Figure 3.3)—the relationship for saturated unit weight (γ_{sat}) can be derived in a similar manner:

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s+W_w}{V} = \frac{G_s\gamma_w+e\gamma_w}{1+e} = \frac{(G_s+e)\gamma_w}{1+e} \tag{3.19}$$

Also, from Eq. (3.18) with S=1,

$$e = wG_s \tag{3.20}$$

As mentioned before, due to the convenience of working with densities in the SI system, the following equations, similar to unit–weight relationships given in Eqs. (3.15), (3.16), and (3.19), will be useful:

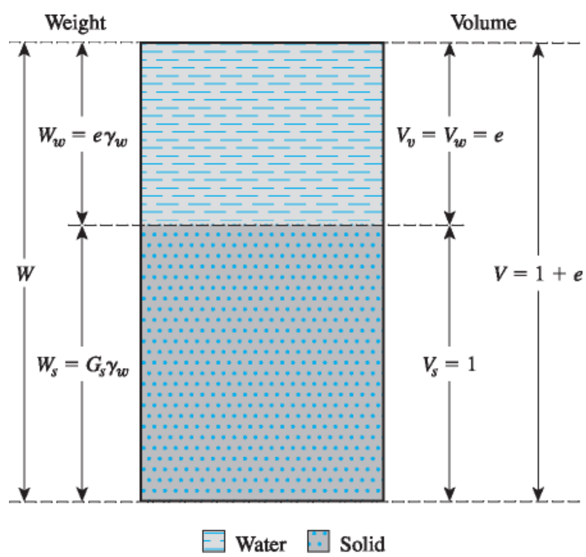


Figure (3.3) Saturated soil element with volume of soil solids equal to one

$$\text{Density} = \rho = \frac{(1+w)G_s\rho_w}{1+e} \tag{3.21}$$

$$\text{Dry density} = \rho_d = \frac{G_s\rho_w}{1+e} \tag{3.22}$$

$$\text{Saturated density} = \rho_{sat} = \frac{(G_s+e)\rho_w}{1+e} \tag{3.23}$$

where ρ_w = density of water = 1000 kg/m^3.

Equation (3.21) may be derived by referring to the soil element shown in Figure 3.4, in which the volume of soil solids is equal to 1 and the volume of voids is equal to *e*.

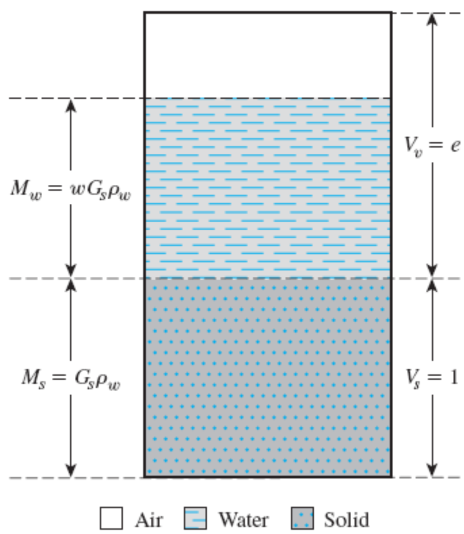


Figure (3.4) Three separate phases of a soil element showing mass–volume relationship

Hence, the mass of soil solids, M_s, is equal to $G_s\rho_w$. The moisture content has been defined in Eq. (3.8) as

$$w = \frac{W_w}{W_s} = \frac{(mass\ of\ water).g}{(mass\ of\ soild).g}$$

$$= \frac{M_w}{M_s}$$

where M_w = mass of water.

Since the mass of soil in the element is equal to $G_s\rho_w$, the mass of water

$$M_w = wM_s = wG_s\rho_w$$

From Eq. (3.13), density

$$\rho = \frac{M}{V} = \frac{M_s + M_w}{V_s + V_v} = \frac{G_s\rho_w + wG_s\rho_w}{1 + e}$$

$$= \frac{(1+w)G_s\rho_w}{1+e}$$

Equations (3.22) and (3.23) can be derived similarly.

3.3 Relationships among Unit Weight, Porosity, and Moisture Content

The relationship among *unit weight, porosity, and moisture content* can be developed in a manner similar to that presented in the preceding section. Consider a soil that has a total volume equal to one, as shown in Figure 3.5. From Eq. (3.4),

$$n = \frac{V_v}{V}$$

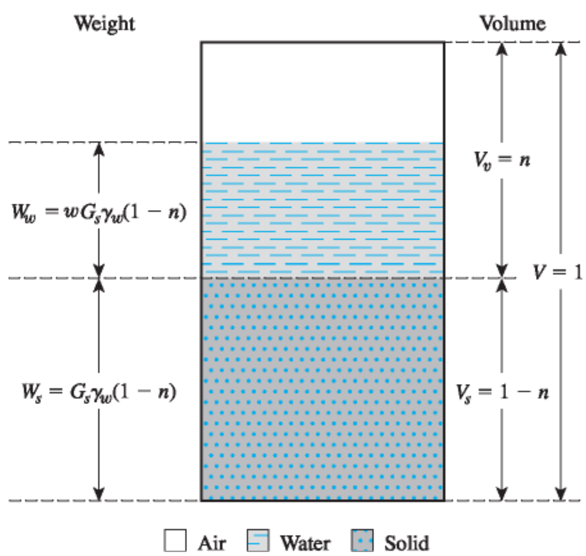


Figure (3.5) Soil element with total volume equal to one

If V is equal to 1, then V_v is equal to n, so $V_s = 1 - n$. The weight of soil solids (W_s) and the weight of water (W_w) can then be expressed as follows:

$$W_s = G_s\gamma_w(1-n) \qquad (3.24)$$

$$W_w = wW_s = wG_s\gamma_w(1-n) \qquad (3.25)$$

So, the dry unit weight equals

$$\gamma_d = \frac{W_s}{V} = \frac{G_s\gamma_w(1-n)}{1} = G_s\gamma_w(1-n) \qquad (3.26)$$

The moist unit weight equals

$$\gamma = \frac{W_s+W_w}{V} = G_s\gamma_w(1-n)(1+w) \qquad (3.27)$$

Figure (3.6) shows a soil sample that is saturated and has $V = 1$. According to this figure,

$$\gamma_{sat} = \frac{W_s+W_w}{V} = \frac{(1-n)G_s\gamma_w+n\gamma_w}{1} = [(1-n)G_s + n]\gamma_w \qquad (3.28)$$

The moisture content of a saturated soil sample can be expressed as

$$w = \frac{W_w}{W_s} = \frac{n\gamma_w}{(1-n)\gamma_w G_s} = \frac{n}{(1-n)G_s} \qquad (3.29)$$

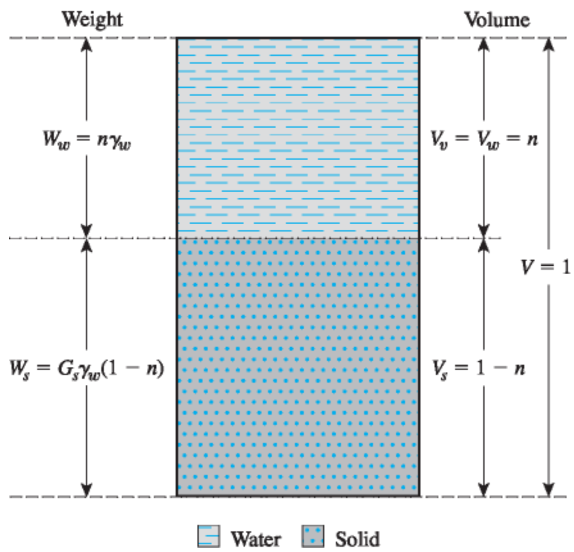


Figure (3.6) Saturated soil element with total volume equal to one

3.4 Various Unit-Weight Relationships

In Sections 3.2 and 3.3, we derived the fundamental relationships for the moist unit weight, dry unit weight, and saturated unit weight of soil. Several other forms of relationships that can be obtained for γ, γ_d, and γ_{sat} are given in Table (3.1). Some typical values of void ratio, moisture content in a saturated condition, and dry unit weight for soils in a natural state are given in Table (3.2).

Table 3.1 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\frac{\gamma}{1+w}$	G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\frac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n)+nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat}-\frac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d+\left(\frac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat}-n\gamma_w$	γ_d, n	$\gamma_d+n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat}-\gamma_w)G_s}{(G_s-1)}$	γ_d, S	$\left(1-\frac{1}{G_s}\right)\gamma_d+\gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

Table 3.2 Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d	
			lb/ft^3	kN/m^3
Loose uniform sand	0.8	30	92	14.5
Dense uniform sand	0.45	16	115	18
Loose angular-grained silty sand	0.65	25	102	16
Dense angular-grained silty sand	0.4	15	121	19
Stiff clay	0.6	21	108	17
Soft clay	0.9–1.4	30–50	73–93	11.5–14.5
Loess	0.9	25	86	13.5
Soft organic clay	2.5–3.2	90–120	38–51	6–8
Glacial till	0.3	10	134	21

Example 3.1

For a saturated soil, show that

$$\gamma_{sat} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$$

Solution

From Eqs. (3.19) and (3.20),

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} \quad \text{(a)}$$

and

$$e = wG_s$$

or

$$G_s = \frac{e}{w} \quad \text{(b)}$$

Combining Eqs. (a) and (b) gives

$$\gamma_{sat} = \frac{\left(\frac{e}{w} + e\right)\gamma_w}{1+e} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$$ ■

Example 3.2

For a moist soil sample, the following are given.

- Total volume: $V = 1.2\ \text{m}^3$
- Total mass: $M = 2350$ kg
- Moisture content: $w = 8.6\%$
- Specific gravity of soil solids: $G_s = 2.71$

Determine the following.

a. Moist density
b. Dry density
c. Void ratio
d. Porosity
e. Degree of saturation
f. Volume of water in the soil sample

Solution

Part a

From Eq. (3.13),

$$\rho = \frac{M}{V} = \frac{2350}{1.2} = \mathbf{1958.3\ kg/m^3}$$

Part b

From Eq. (3.14),

$$\rho_d = \frac{M_s}{V} = \frac{M}{(1+w)V} = \frac{2350}{\left(1+\frac{8.6}{100}\right)(1.2)} = \mathbf{1803.3\ kg/m^3}$$

Part c

From Eq. (3.22),

$$\rho_d = \frac{G_s\rho_w}{1+e}$$

$$e = \frac{G_s\rho_w}{\rho_d} - 1 = \frac{(2.71)(1000)}{1803.3} - 1 = \mathbf{0.503}$$

Part d

From Eq. (3.7),

$$n = \frac{e}{1+e} = \frac{0.503}{1+0.503} = \mathbf{0.335}$$

Part e

From Eq. (3.18),

$$S = \frac{wG_s}{e} = \frac{\left(\frac{8.6}{100}\right)(2.71)}{0.503} = 0.463 = \mathbf{46.3\%}$$

Part f

Volume of water:

$$\frac{M_w}{\rho_w} = \frac{M - M_s}{\rho_w} = \frac{M - \frac{M}{1+w}}{\rho_w} = \frac{2350 - \left(\frac{2350}{1+\frac{8.6}{100}}\right)}{1000} = \mathbf{0.186\ m^3}$$

Alternate Solution

Refer to Figure 3.7.

Part a

$$\rho = \frac{M}{V} = \frac{2350}{1.2} = \mathbf{1958.3\ kg/m^3}$$

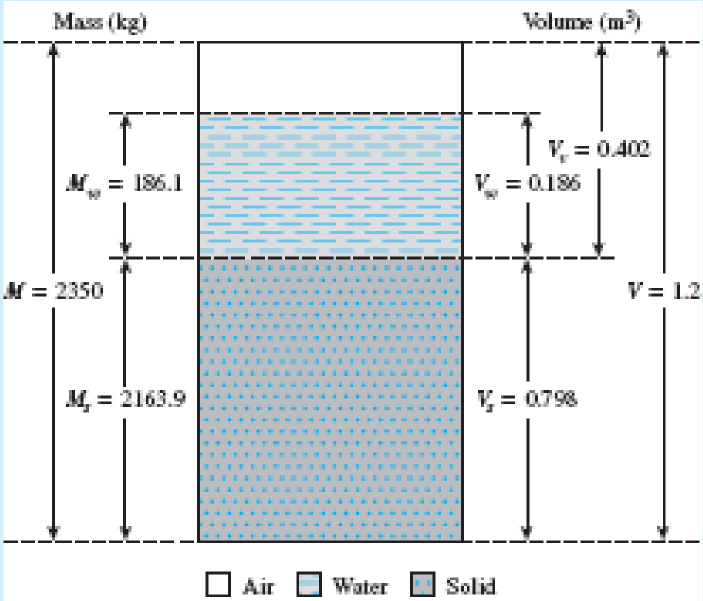


Figure 3.7

Part b

$$M_s = \frac{M}{1+w} = \frac{2350}{1+\frac{8.6}{100}} = 2163.9 \text{ kg}$$

$$\rho_d = \frac{M_s}{V} = \frac{M}{(1+w)V} = \frac{2350}{\left(1+\frac{8.6}{100}\right)(1.2)} = \mathbf{1803.3\ kg/m^3}$$

Part c

The volume of solids: $\dfrac{M_s}{G_s\rho_w} = \dfrac{2163.9}{(2.71)(1000)} = 0.798 \text{ m}^3$

The volume of voids: $V_v = V - V_s = 1.2 - 0.798 = 0.402 \text{ m}^3$

Void ratio: $e = \dfrac{V_v}{V_s} = \dfrac{0.402}{0.798} = \mathbf{0.503}$

Part d

$$\text{Porosity: } n = \frac{V_v}{V} = \frac{0.402}{1.2} = \mathbf{0.335}$$

Part e

$$S = \frac{V_w}{V_v}$$

$$\text{Volume of water: } V_w = \frac{M_w}{\rho_w} = \frac{186.1}{1000} = 0.186 \text{ m}^3$$

Hence,

$$S = \frac{0.186}{0.402} = 0.463 = \mathbf{46.3\%}$$

Part f

From Part e,

$$V_w = \mathbf{0.186\ m^3}$$

Example 3.3

The following data are given for a soil:

- Porosity: $n = 0.4$
- Specific gravity of the soil solids: $G_s = 2.68$
- Moisture content: $w = 12\%$

Determine the mass of water to be added to 10 m^3 of soil for full saturation.

Solution

Equation (3.27) can be rewritten in terms of density as

$$\rho = G_s\rho_w(1 - n)(1 + w)$$

Similarly, from Eq. (3.28)

$$\rho_{sat} = [(1 - n)G_s + n]\rho_w$$

Thus,

$$\rho = (2.68)(1000)(1 - 0.4)(1 + 0.12) = 1800.96\ kg/m^3$$

$$\rho_{sat} = [(1 - 0.4)(2.68) + 0.4]\,(1000) = 2008\ kg/m^3$$

Mass of water needed per cubic meter equals

$$\rho_{sat} - \rho = 2008 - 1800.96 = 207.04\ Kg/m^3$$

So, total mass of water to be added equals

$$207.04 \times 10 = \mathbf{2070.4\ kg}$$

Example 3.4

A saturated soil has a dry unit weight of 16.2 kN/m^3. Its moisture content is 23% Determine:

a- Saturated unit weight, γ_{sat}

b- Specific gravity, G_s

c- Void ratio, e

Solution

Part a: saturated Unit Weight

From Eq. (3.12),

$$\gamma_{sat} = \gamma_d(1+w) = (16.2)\left(1+\frac{23}{100}\right) = 19.93\ \frac{kN}{m^3}$$

Part b: Specific Gravity, G_s

From Eq. (3.16),

$$\gamma_d = \frac{G_s\gamma_w}{1+e}$$

Also from Eq(3.20) for saturated soils, $e=wG_s$. Thus,

$$\gamma_d = \frac{G_s\gamma_w}{1+wG_s}$$

So,

$$16.2 = \frac{G_s(9.81)}{1+(0.23)(G_s)}$$

or

$$16.2 + 3.726G_s = 9.81G_s$$

$$G_s = 2.66$$

Part c: Void Ratio, e

For saturated soils,

$$e = wG_s = (0.23)(2.66) = 0.61$$

3.5 Relative Density

The term relative density is commonly used to indicate the *in situ* denseness or looseness of granular soil. It is defined as

$$D_r = \frac{e_{max}-e}{e_{max}-e_{min}} \tag{3.30}$$

where D_r = relative density, usually given as a percentage

e = in situ void ratio of the soil

e_{max} = void ratio of the soil in the loosest state

e_{min} = void ratio of the soil in the densest state

The values of D_r may vary from a minimum of 0% for very loose soil to a maximum of 100% for very dense soils. Soils engineers qualitatively describe the granular soil deposits according to their relative densities, as shown in Table (3.3). In-place soils seldom have relative densities less than 20 to 30%. Compacting a granular soil to a relative density greater than about 85% is difficult.

Table 3.3 Qalitative Description of Granular Soil Deposits

Relative density (%)	Description of soil deposit
0–15	Very loose
15–50	Loose
50–70	Medium
70–85	Dense
85–100	Very dense

The relationships for relative density can also be defined in terms of porosity, or

$$e_{max} = \frac{n_{max}}{1-n_{max}} \quad (3.31)$$

$$e_{min} = \frac{n_{min}}{1-n_{min}} \quad (3.32)$$

$$e = \frac{n}{1-n} \quad (3.33)$$

where n_{max} and n_{min} = porosity of the soil in the loosest and densest conditions, respectively. Substituting Eqs. (3.31), (3.32), and (3.33) into Eq. (3.30), we obtain

$$D_r = \frac{(1-n_{min})(n_{max}-n)}{(n_{max}-n_{min})(1-n)} \quad (3.34)$$

By using the definition of dry unit weight given in Eq. (3.16), we can express relative density in terms of maximum and minimum possible dry unit weights. Thus,

$$D_r = \frac{\left[\frac{1}{\gamma_{d(\min)}}\right]-\left[\frac{1}{\gamma_d}\right]}{\left[\frac{1}{\gamma_{d(\min)}}\right]-\left[\frac{1}{\gamma_{d(\max)}}\right]} = \left[\frac{\gamma_d-\gamma_{d(\min)}}{\gamma_{d(\max)}-\gamma_{d(\min)}}\right]\left[\frac{\gamma_{d(max)}}{\gamma_d}\right] \quad (3.35)$$

where $\gamma_{d(min)}$ = dry unit weight in the loosest condition (at a void ratio of e_{max})
γ_d = in situ dry unit weight (at a void ratio of e)
$\gamma_{d(max)}$ = dry unit weight in the densest condition (at a void ratio of e_{min})

In terms of density, Eq. (3.35) can be expressed as

$$D_r = \left[\frac{\rho_d-\rho_{d(\min)}}{\rho_{d(\max)}-\rho_{d(\min)}}\right]\frac{\rho_{d(\max)}}{\rho_d} \quad (3.36)$$

ASTM Test Designations D-4253 and D-4254 (2007) provide a procedure for determining the maximum and minimum dry unit weights of granular soils so that they can be used in Eq. (3.35) to measure the relative density of compaction in the field. For sands, this procedure involves using a mold with a volume of 2830 cm^3 (0.1 ft^3). For a determination of the *minimum dry unit weight*, sand is poured loosely into the mold from a funnel with a 12.7 mm ($\frac{1}{2}$in.) diameter spout. The average height of the fall of sand into the mold is maintained at about 25.4 mm (1 in.). The value of $\gamma_{d(min)}$ then can be calculated by using the following equation

$$\gamma_{d(min)} = \frac{W_s}{V_m} \tag{3.37}$$

Where W_s = weight of sand required to fill the mold
V_m = volume of the mold

The maximum dry unit weight is determined by vibrating sand in the mold for 8 min. A surcharge of 14 kN/m^2 (2 lb/in^2) is added to the top of the sand in the mold. The mold is placed on a table that vibrates at a frequency of 3600 cycles/min and that has an amplitude of vibration of 0.635 mm (0.025 in.). The value of $\gamma_{d(max)}$ can be determined at the end of the vibrating period with knowledge of the weight and volume of the sand.

Example 3.5

For a given sandy soil, e_{max}=0.75 and e_{min}=0.4. Let G_s=2.68. In the field, the soil is compacted to a moist density of 17.63 kN/m^3 at a moisture content of 12%. Determine the relative density of compaction.

Solution

From Eq. (3.21)

$$\rho = \frac{(1+w)G_s\rho_w}{1+e}$$

or

$$e = \frac{G_s\gamma_w(1+w)}{\gamma} - 1 = \frac{(2.68)(9.81)(1+0.12)}{17.63} - 1 = 0.67$$

From Eq. (3.30),

$$D_r = \frac{e_{max}-e}{e_{max}-e_{min}} = \frac{0.75-0.67}{0.75-0.4} = 0.229 = 22.9\%$$

Problems

3.1- For a given soil, show that

$$\gamma_{sat} = \gamma_d + n\gamma_w$$

3.2- For a given soil, show that

$$\gamma_{sat} = \gamma_d + (\frac{e}{1+e})\gamma_w$$

3.3 - For a given soil, show that

$$\gamma_d = \frac{eS\gamma_w}{(1+e)w}$$

3.4 - A 0.4 m^3 moist soil sample has the following:

* Moist mass = 711.2 Kg
* Dry mass = 623.9 Kg
* Specific gravity of soil solids = 2.68

Estimate:

a-Moisture content

b-Moist density

c-Dry density

d-Void ratio

e-Porosity

Ans: (a) 13.99% (b) 1778 kg/cm^3 (c) 1559.75 kg/cm^3 (d) 0.72 (e) 0.42

3.5- The moist weight of 5600 cm^3 of a soil is 102.3N. The moisture content and the specific gravity of the soil solids are determined in the laboratory to be 11% and 2.7, respectively. Calculate the following:

a- Moist unit weight

b- Dry unit weight

c- Void ratio

d- Porosity

e- Degree of saturation (%)

f- Volume occupied by water

Ans: (a) 18.27 kN/m^3 (b) 16.46 kN/m^3 (c) 0.61 (d) 0.38 (e) 48.7% (f) 0.297

3.6 - The saturated unit weight of a soil is 19.8 kN/m^3. The moisture content of the soil is 17.1%. Determine the following:

a- Dry unit weight

b- Specific gravity of soil solids

c-Void ratio

Ans: (a) 16.91 kN/m^3 (b) 2.44 (c) 0.417

3.7 - The unit weight of a soil is 14.84 kN/m^3. The moisture content of this soil is 19.2% when the degree of saturation is 60%. Determine:

a-Void ratio

b-Specific gravity of soil solids

c-Saturated unit weight

Ans: (a) 0.68 (b) 2.13 (c) 16.41 kN/m^3

3.8 - For a given soil, the following are given: G_s = 2.67; moist unit weight, γ = 17.5 kN/m^3; and moisture content, w =10.8%. Determine:

a-Dry unit weight
b-Void ratio
c-Porosity
d- Degree of saturation

Ans: (a) 15.8 kN/m^3 (b) 0.658 (c) 0.397 (d) 44%

3.9 - The moist density of a soil is 1680 kg/m^3. Given w = 18% and G_s = 2.73, determine:

a- Dry density
b- Porosity
c- Degree of saturation
d- Mass of water, in kg/m^3, to be added to reach full saturation

Ans: (a) 1423.7 kg/cm^3 (b) 0.479 (c) 53.5% (d) 222.06 kg

3.10 - The dry density of a soil is 1780 kg/m^3. Given G_s = 2.68, what would be the moisture content of the soil when saturated?
Ans: 19%

3.11 - The porosity of a soil is 0.35. Given G_s = 2.69, calculate:

a- Saturated unit weight (kN/m^3)
b- Moisture content when moist unit weight = 17.5 kN/m^3

Ans: (a) 20.59 kN/m^3 (b) 2%

3.12 - A saturated soil has w = 23% and G_s = 2.62. Determine its saturated and dry densities in kg/m^3.
Ans: ρ_{sat} = 2012.5 kg/m^3 , ρ_d = 1637.5 kg/m^3

3.13 - A soil has w = 18.2%, G_s =2.67, and S = 80%. Determine the moist and dry unit weights of the soil.
Ans: 19.23 kN/m^3 , 16.27 kN/m^3

3.14 - The moist unit weight of a soil is 17.55 kN/m^3 at a moisture content of 10%. Given G_s = 2.7, determine:

a- e
b- Saturated unit weight

Ans: (a) 0.66 (b) 19.86 kN/m^3

3.15 - For a given sand, the maximum and minimum void ratios are 0.78 and 0.43, respectively. Given G_s =2.67, determine the dry unit weight of the soil in kN/m^3 when the relative density is 65%.
Ans: 16.87 kN/m^3

3.16 - For a given sandy soil, e_{max} = 0.75, e_{min} = 0.46, and G_s = 2.68. What will be the moist unit weight of compaction (kN/m^3) in the field if D_r = 78% and w = 9%?
Ans: 18.8 kN/m^3

3.17 - The moisture content of a soil sample is 18.4%, and its dry unit weight is 15.63 kN/m^3. Assuming that the specific gravity of solids is 2.65,

a- Calculate the degree of saturation.

b- What is the maximum dry unit weight to which this soil can be compacted without change in its moisture content?

Ans: (a) 74% (b) 17.5 kN/m^3

3.18 - A loose, uncompacted sand fill 1.83m in depth has a relative density of 40%. Laboratory tests indicated that the minimum and maximum void ratios of the sand are 0.46 and 0.90, respectively. The specific gravity of solids of the sand is 2.65.

a- What is the dry unit weight of the sand?

b- If the sand is compacted to a relative density of 75%, what is the decrease in thickness of the 1.83m fill?

Ans: (a) 15.1 kN/m^3 (b) 16.1cm

Chapter 4: Plasticity of Fine-Grained Soils

4.1 Introduction

The physical and mechanical behavior of fine-grained soils is linked to four distinct states: solid, semisolid, plastic and liquid in; order of increasing water content. If a soil initially is in a liquid state, it locates at point A in Figure (4.1). As the soil dries, its water content reduces and consequently, its volume.

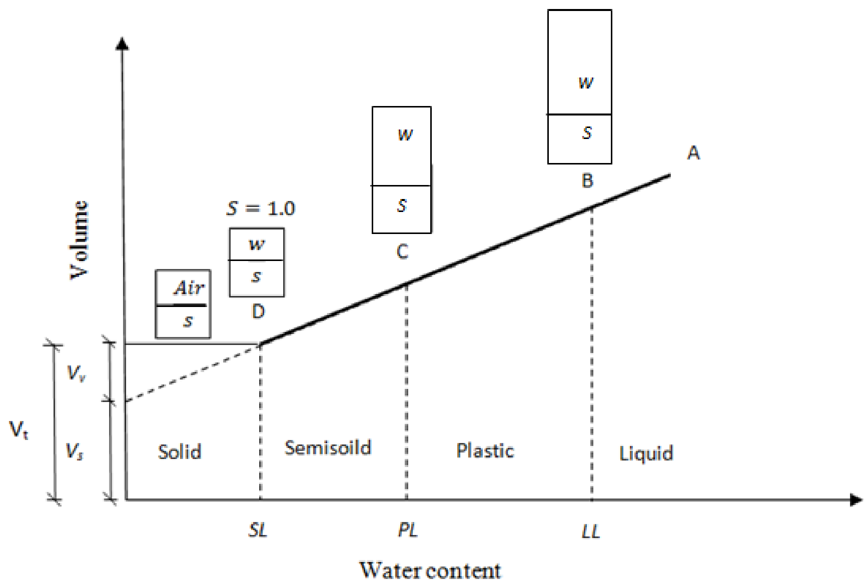


Figure (4.1) Change in soil states as a function of water content

At point B, the soil becomes so stiff that it can no longer be flow as a liquid. The boundary water content at point B is called the liquid limit (LL). As the soil continues to dry, there is a range of water content at which the soil can be molded into any desired shape without rupture. The soil at this state is said to exhibit plastic behaviour-the ability to deform continuously without rupture. But if drying is continued beyond the range of water content for plastic behavior, the soil becomes a semisolid. The soil cannot be molded now without visible cracks appearing. The water content at which the soil changes from a plastic to a semisolid is known as the plastic limit (PL). The rang of water contents over which the soil deforms plastically is known as the plasticity index, (PI);

$$PI = LL - PL \tag{4.1}$$

As the soil continues to dry, it comes to a final state called the solid state. At this state, no further volune change occurs since nearly all the water in the soil has been removed. The water content at which the soil changes from a semisolid to a solid state is called the shrinkage limit (SL). The shrinkage limit is useful for the determination of the swelling and shrinkage potential of soils. The liquid and plastic limits are called the Atterberg limits named after their originator, Swedish soil scientist, Atterberg (1911).

Since engineers are interested in the strength and deformation of materials, we can associate specific strength characteristic to each of the soil states. At one extreme, the liquid state, the soil has the lowest strength and the largest deformation. At the other extreme, the solid state, the soil has the largest strength and the lowest deformation. A measure of soil strength using the Atterberg limits is known as the liquidity index (LI) and is expressed as:

$$LI = \frac{w - PL}{PI} \tag{4.2}$$

The liquidity index is the ratio of the difference in water content between the natural or in-situ water content of a soil and its plastic limit to its plasticity index. Table below shows a description of soil strength based on the values of (LI).

Values of LI	Description of soil strength
LI < 0	Semisolid state – high strength , brittle , (sudden) fracture is expected
0 < LI < 1	Plastic state – intermediate strength, soil deforms like a plastic material
LI > 1	Liquid state – low strength , soil deforms like a viscous fluid

4.2 Liquid Limit (*LL*)

The liquid limit is determined from an apparatus Figure (4.2) that consists of a semispherical brass cup that is repeatedly dropped onto a hard rubber base from a height of 10 mm by a cam-operated mechanism. The apparatus was developed by A. Casagrande (1932) and the procedure for the test is called the Casagrande cup method.

A dry powder of the soil is mixed with distilled water into a paste and placed in the cup to a thickness of about 12.5 mm. The soil surface is smoothed and a groove is cut into the soil using a standard grooving tool. The crank operating the cam is turned at a rate of 2 revolutions per second and the number of blows required to close the groove over a length of 12.5 mm is counted and recorded. A specimen of soil within the closed portion is extracted for detemination of the water content. The liquid limit is defined as the water content at which the groove cut into the soil will close over a distance of 12.5 mm following 25 blowes. This is difficult to achieve in a single test.

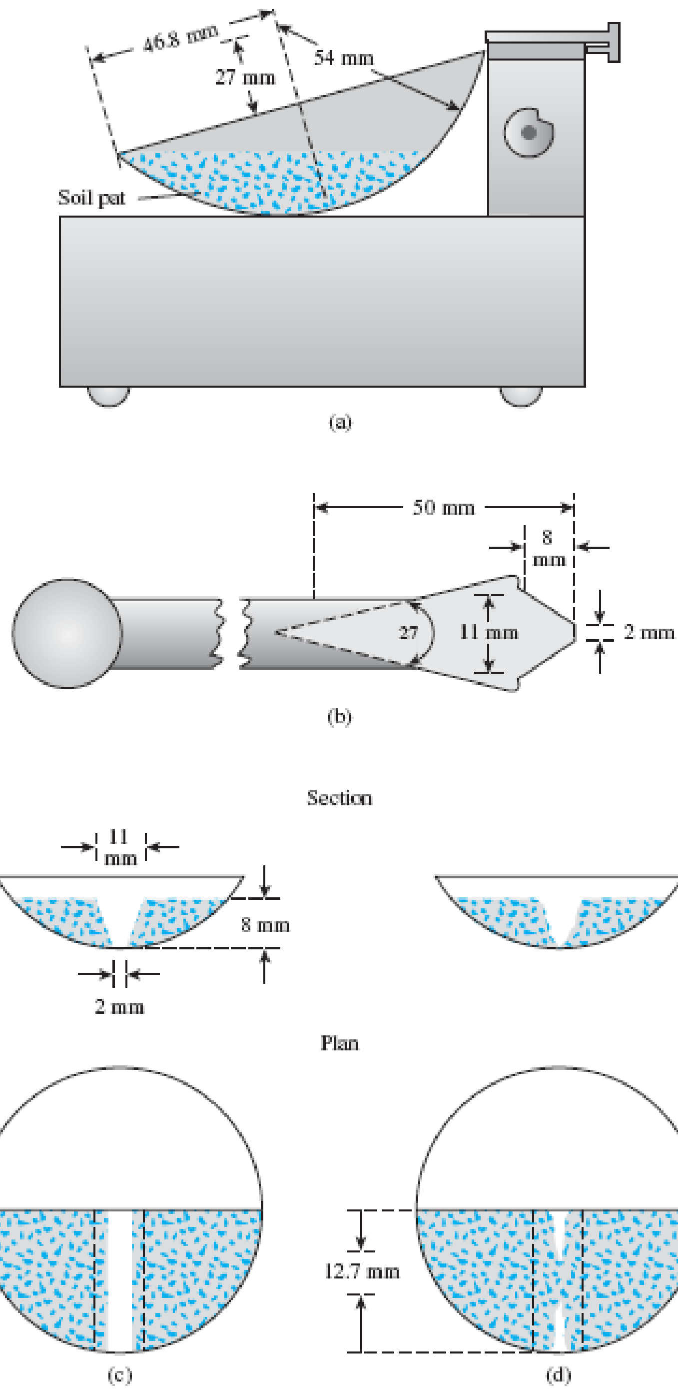


Figure (4.2) Liquid limit test: (a) Liquid limit device; (b) grooving tool; (c) soil pat befor test; (d) soil after test

Four or more tests at different water contents are usually required for terminal blows (number of blows to close the groove over a distance of 12.5 mm) ranging from 10 to 40. The results are presented in a plot of water content (ordinate, arithmetic scale) versus terminal blows (abscissa, logarithm scale) as shown in Figure (4.3).

The best-fit straight line to the data points, usually called the flow line, is drown. The liquid limit is read from the graph as the water content on the liquid state line corresponding to 25 blows.

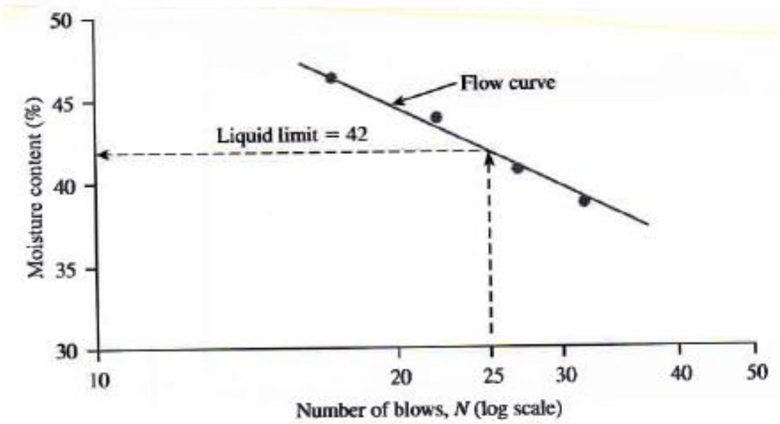


Figure (4.3) Flow curve for liquid limit determination of a clayey silt

4.3 Plastic Limit (*PL*)

The plastic limit is determined by rolling a small clay sample [Figure (4.4)] into threads and finding the water content at which threads approximately 3 mm in diameter will just start to crumble. Two or more determinations are made and the average water content is reported as the plastic limit.

Figure (4.4) Rolling of soil mass on ground glass plate to determine plastic limit

4.4 Shrinkage Limit (SL)

The shrinkage limit is determined as follows. A mass of wet soil, m_1, is placed in a porcelain dish 44.5 mm in a diameter and 12.5 mm high and then oven-dried. The volume of oven-dried soil is determined by using mercury to occupy the vacant spaces caused by shrinkage. The mass of the mercury is determined and the volume decrease casued by shrinkage can be calculated from the known density of mercury. The shrinkage limit is calculated from

$$SL = \left(\frac{m_1 - m_2}{m_2} - \frac{V_1 - V_2}{m_2}\frac{\gamma_w}{g}\right) \times 100 \qquad (4.3)$$

Where m_1 is the mass of the wet soil, m_2 is the mass of the oven-dried soil, V_1 is the volume of wet soil, V_2 is the volume of the oven-dried soil, and g is the acceleration due to gravity (9.8 m/s^2). The range of water content from the plastic to the shrinkage limits is called the shrinkage index (SI).

$$SI = PL - SL \qquad (4.4)$$

4.5 Plasticity Chart

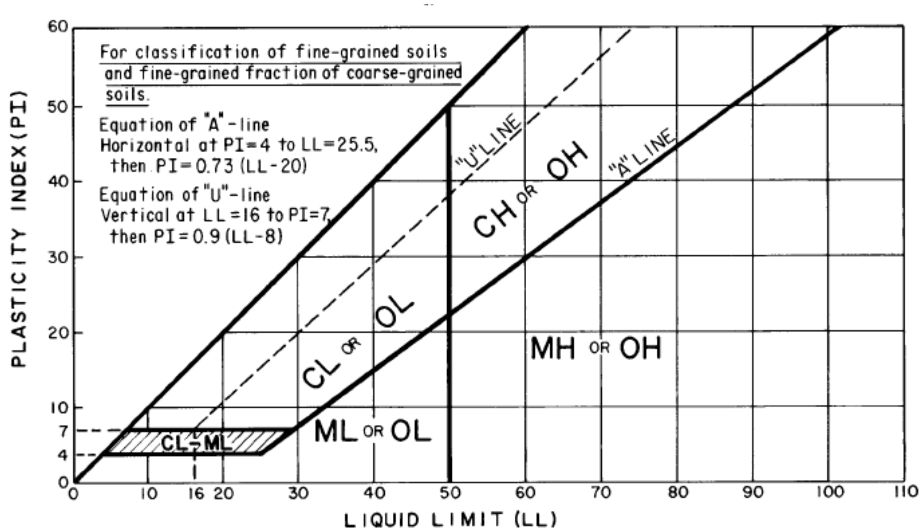
Experimaental results from soils tested from different parts of the world were plotted on a graph of plasticity index (ordinate) versus liquid limit (abscissa). It was found that clay, silt, and organic soils lie in distinct regions of the graph. A line defined by the equation:

$$PI = 0.73(LL - 20)\%$$

called the "A-line", delineates the boundaries between clays (above the line) and silts and organic soils (below the line). A second line, the U-line expresses as:

$$PI = 0.9(LL - 8)$$

defines the upper limit of the correlation between plasticity index and liquid limit. It the results of your soil tests fall above the U-line, you should be suspicious of your results and repeat your tests.



Plasticity chart for fine-grained soils

Example 4.1

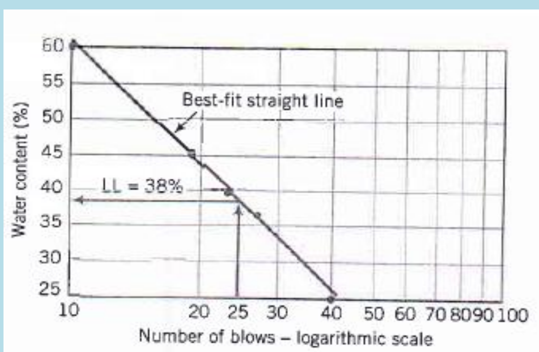
A liquid limit test conducted on a soil sample in the cup device gave the following results:

Number of blows	10	19	23	27	40
Water content (%)	60	45.2	39.8	36.5	25.2

Two determinations for the plastic limit gave water contents of 20.3 % and 20.8 %. Determine (a) the liquide limit and plastic limit, (b) the plasticity index, (c) the liquidity index if the natural water content is 27.4%, and (d) the void ratio at the liquid limit, if G_s = 2.7. if the soil were to be loaded to failure, would you expect a brittle failure?

Solution:

(a) Plot the data



Plot of the flow curve by the Casagrande cup method

The water content on the liquid state line corresponding to a terminal blow of 25 gives the liquid limit,

$$LL = 38\ \%$$

The plastic limit is

$$PL = \frac{20.3 + 20.8}{2} = 20.6\ \%$$

(b) Calculate *PI*

$$PI = LL - PL = 38 - 20.6 = 17.4\%$$

(c) Calculate *LI*

$$LI = \frac{(w - PL)}{PI} = \frac{27.4 - 20.6}{17.4} = 0.39$$

(d) Calculate the void ratio

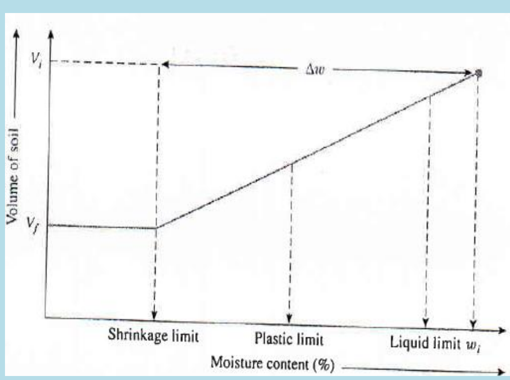
Assume the soil is saturated at the liquid limit. For a saturated soil, $e = wG_s$. Thus,

$$e_{LL} = LLG_s = 0.38 \times 2.7 = 1.03$$

Brittle failure is not expected as the soil is in a plastic state ($0 < LI < 1$).

Example 4.2

A saturated soil has the following characteristics: initial volume ($V_i = V_1$) = 19.65 cm^3. Final volume ($V_f = V_2$) = 13.5 cm^3, mass of wet soil (m_1) = 36g, and mass of dry soil (m_2) = 25g. Determine the shrinkage limit.



Solution:

$$SL = \left(\frac{m_1 - m_2}{m_2} - \frac{V_1 - V_2}{m_2}\rho_w\right) \times 100$$

$$= (\frac{36-25}{25} - \frac{19.65-13.5}{25} \times 1) \times 100$$

= 19.4 %

Example 4.3

A sample of saturated clay has a volume of 97 cm^3 and a mass of (0.202 kg). When completely dried out, the volume reduces to (87 cm^3) and its mass (0.167kg). Find:

a- Initial water content
b- Specific gravity
c- Shrinkage limit

Solution:

a-

$$w = \frac{M_w}{M_s} = \frac{202-167}{167} = 0.21 = 21\%$$

b-

At fully saturated state;

$$Se = w\,G_s \quad \Rightarrow \quad 1 \times e = G_s \times 0.21 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also we have:

$$\rho = \frac{M}{V} = \frac{202}{97} = 2.08\ gm/cm^3$$

$$\rho = \left(\frac{G_s+e}{1+e}\right)\rho_w \Rightarrow 2.08 = \frac{G_s+e}{1+e}\left(1\,\frac{gm}{cm^3}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Solve (1) and (2) yields;

G_s = 2.69 , e = 0.565

c-

Method (1):

$$SL = \left(\frac{m_1 - m_2}{m_2} - \frac{V_1 - V_2}{m_2}\rho_w\right) \times 100 = \left(\frac{202-167}{167} - \frac{97-87}{167}(1)\right) \times 100 = 15\ \%$$

Method (2)

At dry state,

$$\rho_d = \frac{M_s}{V} = \frac{167}{87} = 1.92\ gm/cm^3$$

$$\rho_d = \frac{G_s}{1+e}\rho_w$$

$$\Rightarrow 1.92 = \frac{2.69}{1+e}(1) \quad \Rightarrow \quad e_{dry} = 0.4$$

$$e_{SL} = e_{dry} = 0.4$$

$$Se = G_s w \qquad (w=SL)$$

$$1 \times 0.4 = 2.69 \times SL \quad \Rightarrow \quad SL = 0.15 = 15\%$$

Example 4.4 (H.W)

A dry sample of soil has the following properties:

LL= 52%, PL = 30%, G_s = 2.7, e = 0.53. Find:

Shrinkage limit, dry density and dry unit weight.

Ans:

SL = 19.6%

ρ_d = 1.764 gm/cm^3

γ_d = 17.3 kN/m^3

Problems

4.1 Following are the results from the liquid and plastic limit tests for a soil.
Liquid limit test:

Number of blows, *N*	Moisture content (%)
16	36.5
20	34.1
28	27.0

Plastic limit test: PL= 12.2%

a- Draw the flow curve and obtain the liquid limit.
b- What is the plasticity index of the soil?
Ans: (b) *PI* = 16.3 %

4.2 Determine the liquidity index of the soil described in Problem 4.1, if $w_{\text{in situ}}$ = 31%.
Ans: *LI* = 1.15

4.3 Following are the results from the liquid and plastic limit tests for a soil.
Liquid limit test:

Number of blows, *N*	Moisture content (%)
15	42
20	40.8
28	39.1

Plastic limit test: PL= 12.2%

a- Draw the flow curve and obtain the liquid limit.
b- What is the plasticity index of the soil?
Ans: (b) *PI* = 21 %

4.4 Refer to Problem 4.3. Determine the liquidity index of the soil when the *in situ* moisture content is 26%.

Ans: *LI* = 0.35

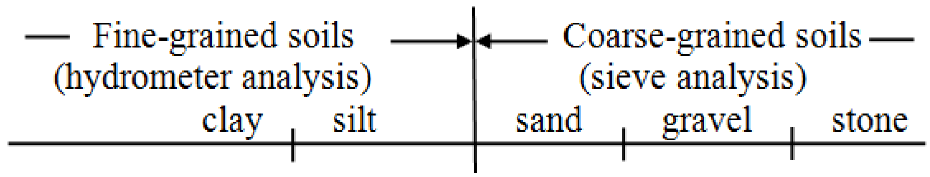
4.5 A saturated soil has the following characteristics: initial volume (V_i) = 24.6 cm^3, final volume (V_f) = 15.9 cm^3, mass of wet soil (M_l) = 44 g, and mass of dry soil (M_2) = 30.1 g. Determine the shrinkage limit.

Ans: *SL* =17.3%

Chapter 5: Classification of Soil

5.1 Mechanical Analysis (Particle Size Analysis) of Soils

Mechanical analysis is the determination of the size range of particles present in a soil, expressed as a percentage of the total dry weight. Two methods generally are used to find the particle-size distribution of soil: (1) sieve analysis-for coarse-grained soils, and (2) hydrometer analysis-for fine-grained soils.



US-standard	(Diameter)	0.002	0.075	4.75	75mm
(ASTM D422)	(Sieve No.)		#200	#4	3″
BS-Standard (BS 1377)	(Diameter)	0.002	0.06	2.0	60mm

<u>Sieve analysis</u>

Sieve analysis consists of shaking the soil sample through a set of sieves that have progressively smaller openings. U.S. standard sieve numbers and the sizes of openings are given in Table (5.1).

Table (5.1) U.S. standard Sieve sizes

Sieve no.	Opening (mm)	Sieve no.	Opening (mm)
4	4.75	35	0.500
5	4.00	40	0.425
6	3.35	50	0.355
7	2.80	60	0.250
8	2.36	70	0.212
10	2.00	80	0.180
12	1.70	100	0.150
14	1.40	120	0.125
16	1.18	140	0.106
18	1.00	170	0.090
20	0.850	200	0.075
25	0.710	270	0.053
30	0.600		

To conduct a sieve analysis, one must first oven dry the soil and then shaken a known weight of soil through a stack of sieves (Figure 5.1) with openings of decreasing size from top to bottom (a pan is placed below the stack). The smallest-size sieve that should be used is the US NO. 200 sieve. The soil retained on each sieve is weighted and the percentage of soil retained on each sieve is calculated. The results are plotted on a graph of percent of particles finer than a given sieve as the ordinate versus the logarithm of the particle sizes.

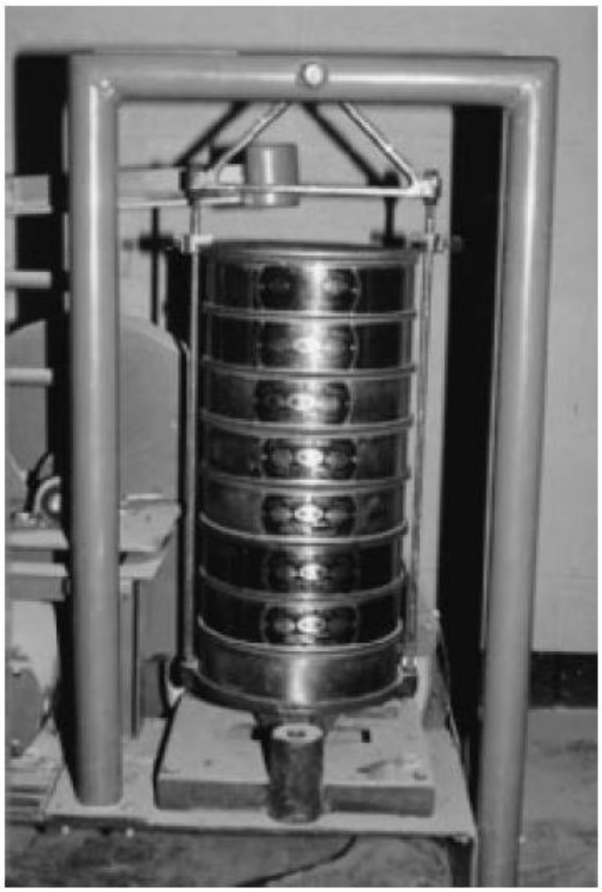


Figure (5.1) A set of sieves for test in the laboratory

<u>Hydrometer Analysis</u>

Hydrometer analysis is based on the principle of sedimentation of soil grains in water. When a soil specimen is dispersed in water, the particles settle at different velocities, depending on their shape, size, weight, and the viscosity of the water. For simplicity, it is assumed that all the soil particles are spheres and that the velocity of soil particles can be expressed by *Stokes' law*, according to which

$$v = \frac{\rho_s - \rho_w}{18\eta} D^2 \tag{5.1}$$

where v = velosity

ρ_s = density of soil particles

ρ_w = density of water

η = viscosity of water

D = diameter of soil particles

Thus, from Eq. (5.1),

$$D = \sqrt{\frac{18\eta v}{\rho_s - \rho_w}} = \sqrt{\frac{18\eta}{\rho_s - \rho_w}}\sqrt{\frac{L}{t}} \tag{5.2}$$

where $v = \frac{Distance}{Time} = \frac{L}{t}$

Note that

$$\rho_s = G_s \rho_w \tag{5.3}$$

Thus, combining Eqs. (5.2) and (5.3) gives

$$D = \sqrt{\frac{18\eta}{(G_s - 1)\rho_w}}\sqrt{\frac{L}{t}} \tag{5.4}$$

If the units of η are (g.sec)/cm^2, ρ_w is in g/cm^3, L is in cm, t is in min, and D is in mm, then

$$\frac{D(mm)}{10} = \sqrt{\frac{18\eta[\frac{\text{g.sec}}{cm^2}]}{(G_s - 1)\rho_w(\frac{\text{g}}{cm^3})}}\sqrt{\frac{L(cm)}{t(\text{min})\times 60}}$$

or

$$D = \sqrt{\frac{30\eta}{(G_s - 1)\rho_w}}\sqrt{\frac{L}{t}}$$

Assume ρ_w to be approximately equal to 1 g/cm^3, so that

$$D(mm) = K\sqrt{\frac{L(cm)}{t(\text{min})}} \tag{5.5}$$

where

$$K = \sqrt{\frac{30\eta}{(G_s - 1)}} \tag{5.6}$$

Note that the value of K is a function of G_s and η, which are dependent on the temperature of the test. Table (5.2) gives the variation of K with the test temperature and the specific gravity of soil solids.

Table (5.2) values of K for use in equation for computing diameter of particle in hydrometer analysis

Temperature, °C	Specific Gravity of Soil Particles								
	2.45	2.50	2.55	2.60	2.65	2.70	2.75	2.80	2.85
16	0.01510	0.01505	0.01481	0.01457	0.01435	0.01414	0.01394	0.01374	0.01356
17	0.01511	0.01486	0.01462	0.01439	0.01417	0.01396	0.01376	0.01356	0.01338
18	0.01492	0.01467	0.01443	0.01421	0.01399	0.01378	0.01359	0.01339	0.01321
19	0.01474	0.01449	0.01425	0.01403	0.01382	0.01361	0.01342	0.1323	0.01305
20	0.01456	0.01431	0.01408	0.01386	0.01365	0.01344	0.01325	0.01307	0.01289
21	0.01438	0.01414	0.01391	0.01369	0.01348	0.01328	0.01309	0.01291	0.01273
22	0.01421	0.01397	0.01374	0.01353	0.01332	0.01312	0.01294	0.01276	0.01258
23	0.01404	0.01381	0.01358	0.01337	0.01317	0.01297	0.01279	0.01261	0.01243
24	0.01388	0.01365	0.01342	0.01321	0.01301	0.01282	0.01264	0.01246	0.01229
25	0.01372	0.01349	0.01327	0.01306	0.01286	0.01267	0.01249	0.01232	0.01215
26	0.01357	0.01334	0.01312	0.01291	0.01272	0.01253	0.01235	0.01218	0.01201
27	0.01342	0.01319	0.01297	0.01277	0.01258	0.01239	0.01221	0.01204	0.01188
28	0.01327	0.01304	0.01283	0.01264	0.01244	0.01255	0.01208	0.01191	0.01175
29	0.01312	0.01290	0.01269	0.01249	0.01230	0.01212	0.01195	0.01178	0.01162
30	0.01298	0.01276	0.01256	0.01236	0.01217	0.01199	0.01182	0.01165	0.01149

The percent finer, N, can be estimated from:

$$N = \frac{G}{G - G_l}\frac{\gamma_w}{W_s}(R - G_l) \times 100\% \tag{5.7}$$

where w_s = weight of solid sample

R = hydrometer reading in sedimentation jar

G_l = specific gravity of the liquid (water) in which soil particles are suspended

5.2 Particle-Size Distribution Curve

A particle-size distribution curve can be used to determine the following parameters for a given soil (Figure 5.2):

1. *Effective size* (D_{10}): This parameter is the diameter in the particle-size distribution curve corresponding to 10% finer. The effective size of a granular soil is a good measure to estimate the hydraulic conductivity and drainage through soil.

2. *Uniformity coefficient* (C_u): This parameter is defined as

$$C_u = \frac{D_{60}}{D_{10}} \tag{5.8}$$

where D_{60} = diameter corresponding to 60% finer.

3. Coefficient of gradation (Cc): This parameter is defined as

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} \tag{5.9}$$

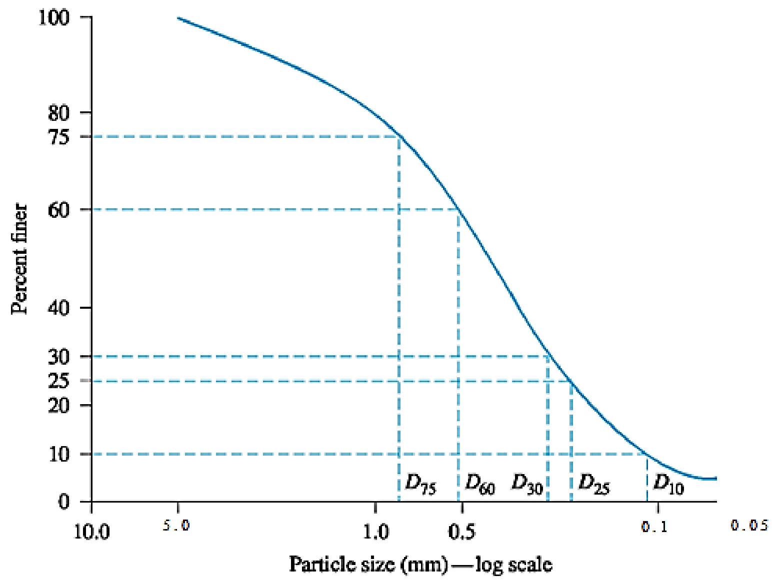


Figure (5.2) Definition of D_{60}, D_{30}, and D_{10}

The particle-size distribution curve shows not only the range of particle sizes present in a soil, but also the type of distribution of various-size particles. Such types of distributions are demonstrated in Figure (5.3). Curve I represents a type of soil in which most of the soil grains are the same size. This is called *poorly graded soil*. Curve II represents a soil in which the particle sizes are distributed over a wide range, termed *well graded*. A well-graded soil has a uniformity coefficient greater than about 4 for gravels and 6 for sands, and a coefficient of gradation between 1 and 3 (for gravels and sands). A soil might have a combination of two or more uniformly graded fractions. Curve III represents such a soil. This type of soil is termed *gap graded*.

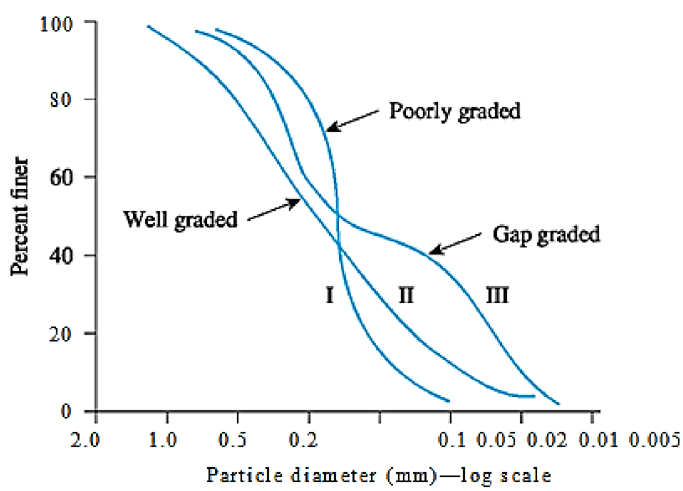


Figure (5.3) Different types of particle-size distribution curves

Example 5.1

Following are the results of a sieve analysis. Make the necessary calculations and draw a particle-size distribution curve.

U.S. sieve no.	Mass of soil retained on each sieve (g)
4	0
10	40
20	60
40	89
60	140
80	122
100	210
200	56
Pan	12

Solution

The following table can now be prepared.

U.S. sieve (1)	Opening (mm) (2)	Mass retained on each sieve (g) (3)	Cumulative mass retained above each sieve (g) (4)	Percent finer[a] (5)
4	4.75	0	0	100
10	2.00	40	0 + 40 = 40	94.5
20	0.850	60	40 + 60 = 100	86.3
40	0.425	89	100 + 89 = 189	74.1
60	0.250	140	189 + 140 = 329	54.9
80	0.180	122	329 + 122 = 451	38.1
100	0.150	210	451 + 210 = 661	9.3
200	0.075	56	661 + 56 = 717	1.7
Pan	–	12	717 + 12 = 729 = ΣM	0

[a] $$\frac{\sum M - \text{col. } 4}{\sum M} \times 100 = \frac{729 - \text{col. } 4}{729} \times 100$$

The particle-size distribution curve is shown in Figure 5.4

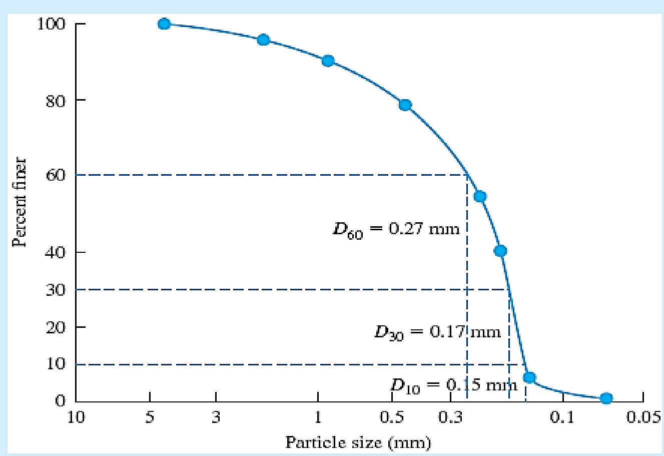


Figure (5.4) Particle-size distribution curve

Example 5.2

For the particle-size distribution curve shown in Figure (5.4) determine

a- D_{10}, D_{30}, and D_{60}
b- Uniformity coefficient, C_u
c- Coefficient of gradation, C_c

Solution

Part a

From Figure (5.4)

$D_{10} = 0.15$ mm

$D_{30} = 0.17$ mm

$D_{60} = 0.27$ mm

Part b

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.27}{0.15} = 1.8$$

Part c

$$C_c = \frac{D_{30}^2}{D_{60} \times D_{10}} = \frac{(0.17)^2}{(0.27)(0.15)} = 0.71$$

Example 5.3

For the particle-size distribution curve shown in Figure (5.4), determine the percentages of gravel, sand, silt, and clay size particles persent. Use Unified Soil Classification system.

Solution

From Figure (5.4), we can prepare the following table.

Size (mm)	percent finer	
75	100	
		100 – 100 = 0% gravel
4.75	100	
		100 – 1.7 = 98.3% sand
0.075	1.7	
		1.7 – 0 = 1.7% silt and clay
-	0	

Example 5.4

An air dry soil sample weighing 2000g is brought to the soils laboratory for sieve analysis. The laboratory data are as follows:

U.S. sieve size	Size Opening (mm)	Weight Retained (g)
3/4 in.	19.0	0
3/8 in.	9.5	158
No. 4	4.75	308
No.10	2.00	608
No. 40	0.425	652
No. 100	0.150	224
No. 200	0.075	42
Pan	……..	8

Plot the grain-size distribution curve for this soil sample.

Solution

(1) Sieve Number	(2) Sieve Opening (mm)	(3) Mass Retained (g)	(4) Cumulative Mass retained (g)	(5) Percentage Passing
3/4 in.	19.0	0	0	100
3/8 in.	9.5	158	158	92.1
No. 4	4.75	308	466	76.7
No.10	2.00	608	1074	46.3
No. 40	0.425	652	1726	13.7
No. 100	0.150	224	1950	2.5
No. 200	0.075	42	1992	0.4
Pan	……..	8	$\Sigma M = 2000$	

$$col.(5) = \frac{\Sigma M - col.(4)}{\Sigma M} \times 100$$

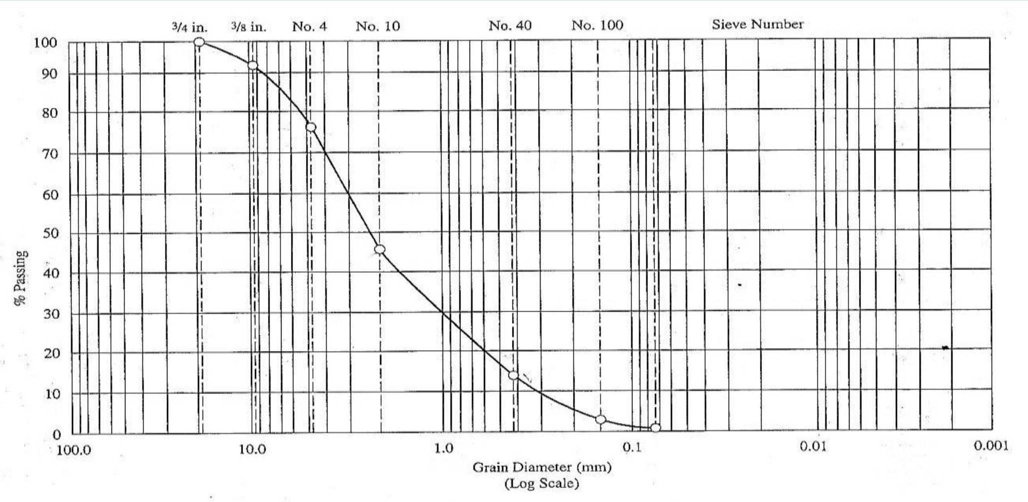


Figure (5.5) Grain-size Distribution curve for Example (5.4)

5.3 Unified Soil Classification System (ASTM D-2487)

The original form of this system was proposed by Casagrande in 1942 for use in the airfield construction works undertaken by the Army Corps of Engineers during World War II. In cooperation with the U.S. Bureau of Reclamation, this system was revised in 1952. At present, it is used widely by engineers (ASTM Test Designation D-2487). The Unified classification system is presented in Figures (5.6 through 5.8).

This system classifies soils into two broad categories:

1. Coarse-grained soils that are gravelly and sandy in nature with less than 50% passing through the No. 200 sieve. The group symbols start with a prefix of G or S. G stands for gravel or gravelly soil, and S for sand or sandy soil.
2. Fine-grained soils are with 50% or more passing through the No. 200 sieve. The group symbols start with prefixes of M, which stands for inorganic silt, C for inorganic clay, or O for organic silts and clays. The symbol Pt is used for peat, muck, and other highly organic soils.

Other symbols used for the classification are:

- W—well graded
- P—poorly graded
- L—low plasticity (liquid limit less than 50)
- H—high plasticity (liquid limit more than 50)

For proper classification according to this system, some or all of the following information must be known:

1. Percent of gravel—that is, the fraction passing the 75-mm sieve and retained on the No. 4 sieve (4.75-mm opening)
2. Percent of sand—that is, the fraction passing the No. 4 sieve (4.75-mm opening) and retained on the No. 200 sieve (0.075-mm opening)
3. Percent of silt and clay—that is, the fraction finer than the No. 200 sieve (0.075-mm opening)
4. Uniformity coefficient (C_u) and the coefficient of gradation (C_c)
5. Liquid limit and plasticity index of the portion of soil passing the No. 40 sieve

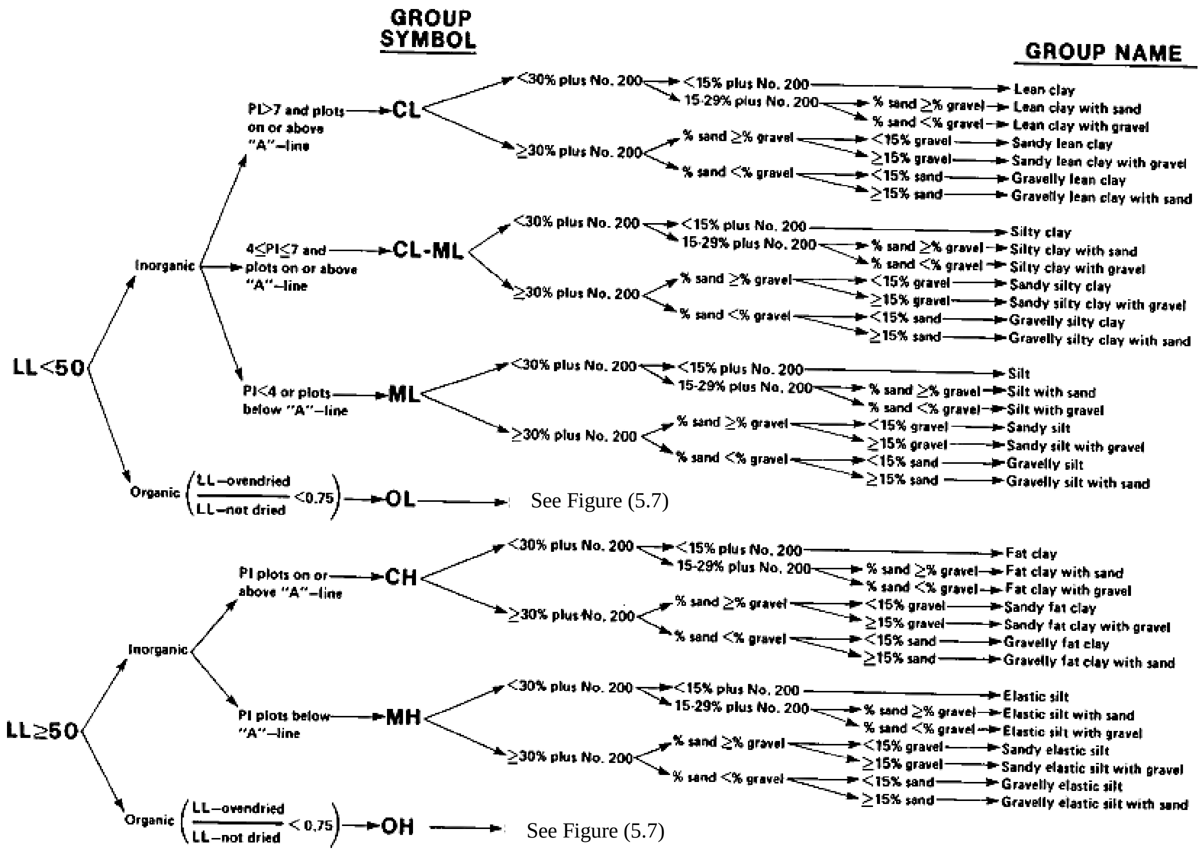


Figure (5.6) flow chart for classifying fine-grained soil (50% or more passes No. 200 sieve)

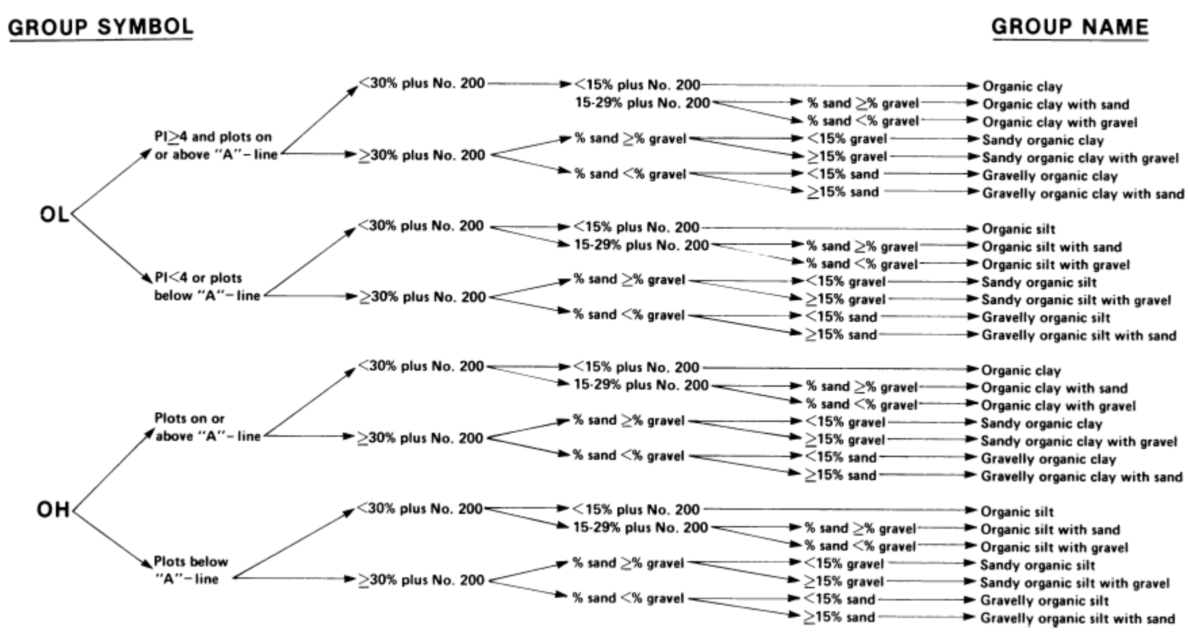


Figure (5.7) flow chart for classifying organic fine-grained soil (50% or more passes No. 200 seive)

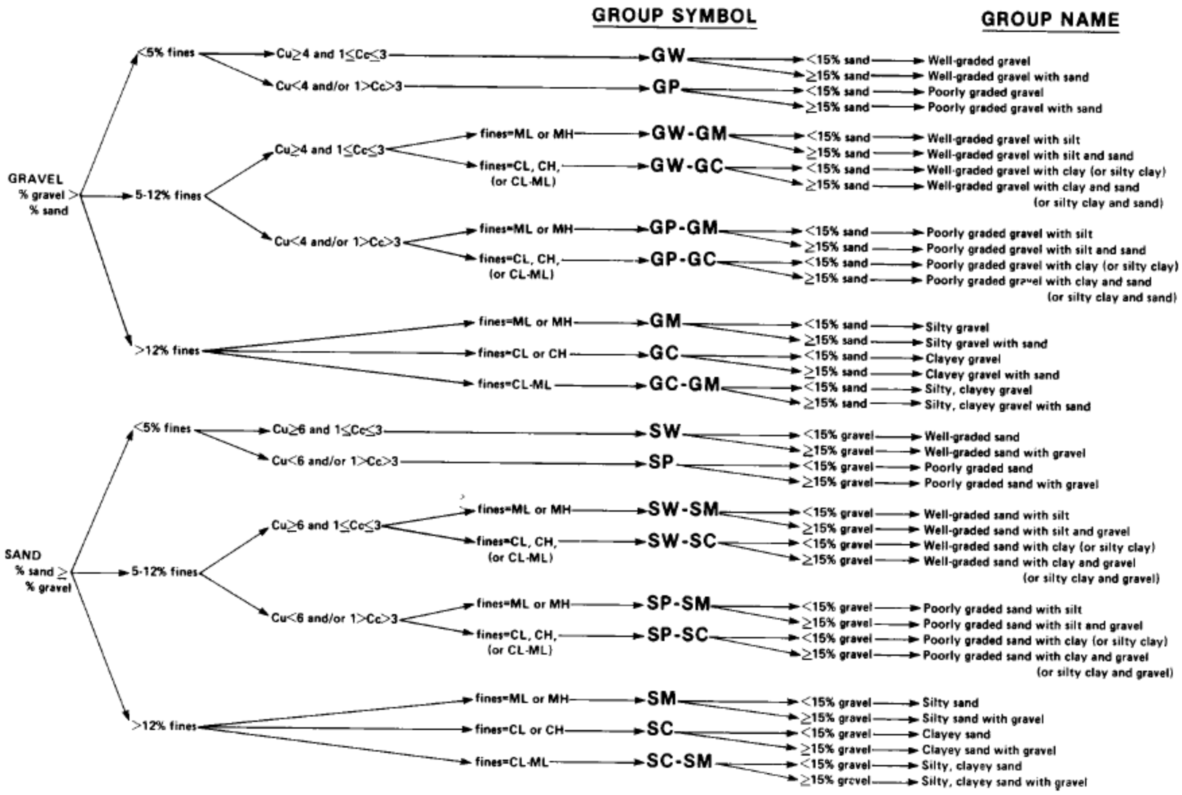


Figure (5.8) flow chart for classifying coarse -grained soils (More than 50% retained on No. 200 seive)

Example 5.5

A sample of soil was tested in the laboratory with the following results:

1- Liquid limit = 30.0 %
2- Plastic limit = 12.0 %
3- Sieve analysis data:

U.S. sieve size	Percentage Passing
3/8 in.	100
No. 4	76.5
No. 10	60.0
No. 40	39.7
No. 200	15.2

Classify the soil by the USCS.

Solution

Gravel; $P_{3/4} - P_4 = 100 - 76.5 = 23.5\%$

or $(R_4 - R_{3/4})$

Sand; $P_4 - P_{200} = 76.5 - 15.2 = 61.3\%$

or $(R_{200} - R_4)$

Fines; $P_{200} = 15.2\%$

Fines < 50% ⇒ Go to (**coarse-grained soils**) chart

Sand > Gravel ⇒ Go to (**sand**) block

Fines > 12% ⇒ consider plasticity chart

Plasticity chart

L.L = 30.0

P.I = 30.0 – 12 =18

} ⇒ Point above A-line ⇒ CL

Group symbol ≡ **SC**

Gravel ≥ 15% ⇒ (**clayey sand with gravel**)

⇒ The soil is **: clayey sand with gravel (SC)**

Example 5.6

A sample of soil was tested in the laboratory with the following results:

1- Liquid limit = NP (nonplastic)
2- Plastic limit = NP (nonplastic)
3- Sieve analysis data:

U.S. sieve size	Percentage Passing
1 in.	100
3/4 in.	85
1/2 in.	70
3/8in.	60
No. 4	48
No. 10	30
No. 40	16
No.100	10
No. 200	2

Classify the soil by the USCS.

Solution

Gravel: $P_{3''} - P_4 = 100 - 48 = 52$ %
Sand: $P_4 - P_{200} = 48 - 2 = 46$ %
Fines: $P_{200} = 2$ %
Fines < 50% ⇒ Go to (**coarse-grained soils**) chart

Gravel > Sand ⇒ Go to (**Gravel**) block

Fines < 5% ⇒ C_u and C_c need to be calculated.

From the particle size distribution curve:

$$C_u = \frac{D_{60}}{D_{10}} = \frac{9.5}{0.15} = 63.3$$

$$C_c = \frac{D_{30}^2}{D_{60}.D_{10}} = \frac{(2.0)^2}{(9.5)(0.15)} = 2.8$$

$C_u \geq 4$ and $1 \leq C_c \leq 3$ ⇒ Group symbol ≡ **GW**

Sand ≥ 15 ⇒ (Well-graded gravel with sand)

⇒ The soil is : **well-graded gravel with sand (GW)**

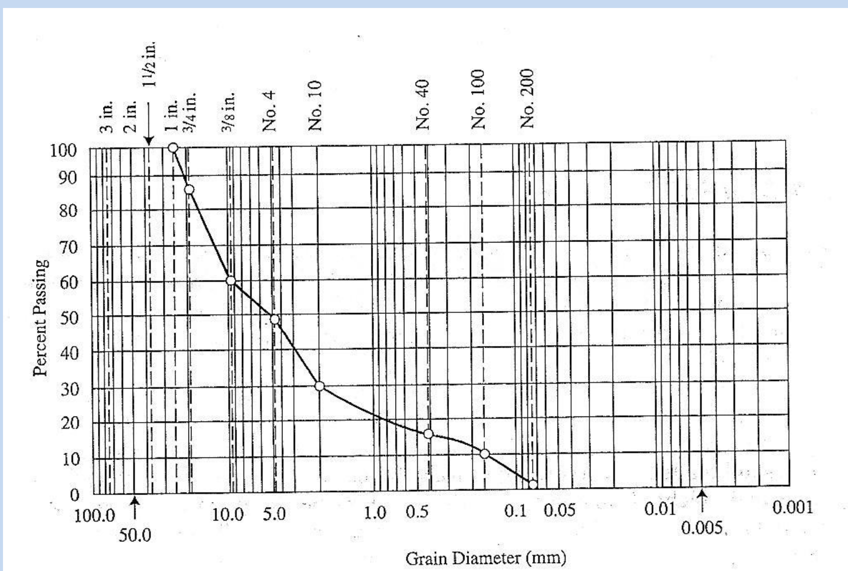


Figure (5.9) Grain-size Distribution curve for Example (5.6)

Example 5.7

A sample of inorganic soil was tested in the laboratory with the following results:

4- Liquid limit = 42.3 %
5- Plastic limit = 15.8 %
6- Sieve analysis data:

U.S. sieve size	Percentage Passing
No. 4	100
No. 10	93.2
No. 40	81.0
No. 200	60.2

Classify the soil sample by the USCS.

Solution

Gravel: $P_{3''} - P_4 = 100 - 100 = 0$

Sand: $P_4 - P_{200} = 100 - 60.2 = 39.8$ %

Fines: $P_{200} = 60.2$ %

Fines > 50% ⇒ Go to (**Fine-grained soils**) chart

L.L = 42.3% < 50

Inorganic soil

Plasticity chart

L.L = 42.3 %

P.I = 42.3 – 15.8 = 26.5% > 7.0 } Point above A-line

Group symbol ≡ **CL**

Plus No.200 (coarce-grained soil) = (0 + 39.8) = 39.8% > 30%

% sand > % gravel

% gravel < 15%

⇒ **Sandy lean clay (CL)**

Example 5.8

Figure (5.10) gives the grain-size distribution of two soils. The liquid and plastic limits of minus No. 40 sieve fraction of the soil are as follows:

	Soil A	Soil B
Liquid limit	30	26
Plastic limit	22	20

Determine the group symbols and group names according to the Unified soil Classification system.

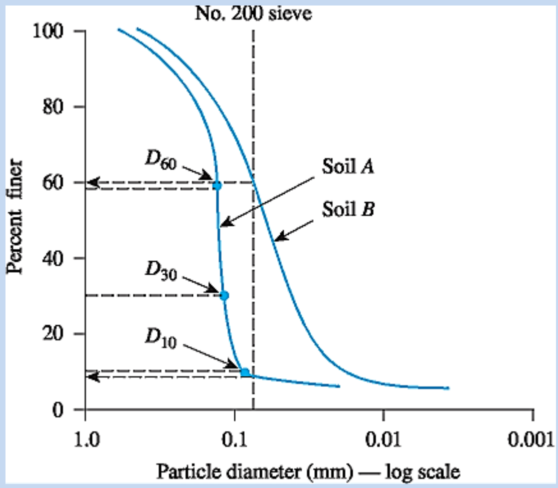


Figure (5.10) Particle –size distribution of two soils

Solution

Soil A

P_{200} = 8%

P_4 = 100%

$P_{3''}$ = 100%

Gravel = $P_{3''} - P_4$ = 0

Sand = $P_4 - P_{200}$ = 100 – 8 = 92%

Fines = P_{200} = 8%

Fines < 50% ⇒ Goto (**Coarse-grained soils**) chart

Gravel < Sand ⇒ Goto (**Sand**) block

5 < Fine < 12% ⇒ C_u and C_c need to be calculated

From the Figure (5.10);
D_{10} = 0.085 mm, D_{30} = 0.12 mm, D_{60} = 0.135 mm. Thus ;

$$C_u = \frac{D_{60}}{D_{10}} = \frac{0.135}{0.085} = 1.59 < 6$$

$$C_c = \frac{D_{30}^2}{D_{60}.D_{10}} = \frac{(0.12)^2}{(0.135)(0.085)} = 1.25 > 1$$

⇒ **Poorly graded sand**

Plasticity chart

LL = 30
PI = 30 – 22= 8

⇒ Point above A-line (CL)

Group symbol ≡ SP-SC

Gravel < 15% ⇒ **poorly graded sand with clay**
⇒ The soil is: poorly graded sand with clay (SP-SC)

<u>Soil B</u>

P_{200} = 61%

P_4 = 100%

$P_{3''}$ = 100%

Gravel = $P_{3''}$ – P_4 = 0

Sand = P_4 – P_{200} = 100 – 61 = 39%

Fines = P_{200} = 61%

Fines > 50% ⇒ Goto (**Fine-grained soils**) chart

Plasticity chart

LL = 26
PI = 26 – 20= 6

Group symbol ≡ CL-ML

Plus No.200 = 100– 61 = 39% > 30%

Sand > Gravel

Gravel < 15%

⇒ (**Sandy silty clay**)

The soil is : Sandy silty clay (CL-ML)

Example 5.9

For a given soil, the following are known:

- Percentage passing 3″ = 100
- Percentage passing No. 4 sieve = 70
- Percentage passing No. 200 sieve = 30
- Liquid limit = 33
- Plastic limit = 12

Classify the soil using the Unified Soil Classification system. Given the group symbol and the group name.

Solution

$P_{200} = 30\%$

$P_4 = 70\%$

$P_{3''} = 100\%$

Gravel = $P_{3''} - P_4 = 30\%$

Sand = $P_4 - P_{200} = 70 - 30 = 40\%$

Fines = $P_{200} = 30\%$

Fines < 50% ⇒ Goto (**Coarse-grained soils**) chart

Sand > Gravel ⇒ Goto (**Sand**) block

Fines = 30% ⇒ consider plasticity chart

Plasticity chart

$LL = 33$ } ⇒ above A-line (CL)

$PI = 33 - 12 = 21$

Group symbol ≡ SC

Gravel >15% ⇒ **Clayey sand with gravel**

⇒ The soil is: Clayey sand with gravel (SC)

Problems

5.1 For a soil, suppose that D_{10} = 0.08 mm, D_{30} = 0.22 mm, and D_{60} = 0.41 mm. Calculate the uniformity coefficient and the coefficient of gradation.

Ans: C_u = 5.13, C_c = 1.48

5.2 Repeat (Problem **5.1**) with the following: D_{10} = 0.24 mm, D_{30} = 0.82 mm, and D_{60} =1.81 mm.

Ans: C_u = 7.54, C_c = 1.55

5.3 Repeat (Problem **5.1**) with the following: D_{10} = 0.18 mm, D_{30} = 0.32 mm, and D_{60} = 0.78 mm.

Ans: C_u = 4.33, C_c = 0.73

5.4 The following are the results of a sieve analysis:

U.S. sieve No.	Mass of soil retained (g)
4	0
10	18.5
20	53.2
40	90.5
60	81.8
100	92.2
200	58.5
pan	26.5

a. Determine the percent finer than each sieve and plot a grain-size distribution curve.
b. Determine D_{10}, D_{30}, and D_{60} from the grain-size distribution curve.
c. Calculate the uniformity coefficient, C_u.
d. Calculate the coefficient of gradation, C_c.

Ans: (b) D_{10} = 0.12mm, D_{30} =0.21mm, and D_{60}=0.41mm (c) C_u=3.42 (d) C_c= 0.9

5.5 Repeat Problem 5.4 with the following:

U.S. sieve No.	Mass of soil retained (g)
4	0
10	44
20	56
40	82
60	51
80	106
100	92
200	85
pan	35

Ans: (b) D_{10} =0.089mm, D_{30} =0.18mm, D_{60}=0.29mm (c) C_u=3.26 (d) C_c= 1.26

5.6 Repeat Problem 5.4 with the following:

U.S. sieve No.	Mass of soil retained (g)
4	0
10	41.2
20	55.1
40	80.0
60	91.6
100	60.5
200	35.6
pan	21.5

Ans: (b) D_{10} = 0.13mm, D_{30} =0.26mm, D_{60}=0.51mm (c) C_u=3.92 (d) C_c= 1.02

5.7 Repeat Problem 5.4 with the following:

U.S. sieve No.	Mass of soil retained (g)
4	0
6	0
10	0
20	9.1
40	249.4
60	179.8
100	22.7
200	15.5
pan	23.5

Ans: (b) D_{10} = 0.23mm, D_{30} =0.33mm, D_{60}=0.48mm (c) C_u=2.09 (d) C_c= 0.99

5.8 The particle-size characteristics of a soil are given in this table. Draw the particle-size distribution curve.

Size (mm)	Percent
0.425	100
0.033	90
0.018	80
0.01	70
0.0062	60
0.0035	50
0.0018	40
0.001	35

Determine the percentages of gravel, sand, silt, and clay:According to the USDA system.

Ans: Gravel=0%, Sand = 6%, Silt = 52%, Clay = 42%

5.9 In a hydrometer test, the results are as follows: G_s = 2.60, temperature of water = 24 °C, and *R* = 43 at 60 min after the start of sedimentation (see Figure 2.24). What is the diameter, *D*, of the smallest-size particles that have settled beyond the zone of measurement at that time (that is, *t* = 60 min)?
Ans; D = 0.0052 mm

5.10 Repeat Problem 5.9 with the following values: G_s = 2.70, temperature = 23 °C, t = 120 min, and *R* = 25.
Ans: D = 0.0041 mm

5.11 Classify the following soil using the U.S. Department of Agriculture textural classification chart.

	Particle-size distribution (%)		
Soil	Sand	Silt	Clay
A	20	20	60
B	55	5	40
C	45	35	20
D	50	15	35
E	70	15	15

Ans:

Classification of soil				
A	B	C	D	E
Clay	Sandy clay	Loam	Sandy clay loam	Sandy loam

5.12 Classify the following soils using the Unified soil classification system. Give group symbols and group names.

Soil No.	Sieve analysis (percent finer) No.4	No. 200	Liquid limit	Plasticity limit	Comments
1	94	3	—	NP	$C_u = 4.48$ and $C_c = 1.22$
2	100	77	63	25	
3	100	86	55	28	
4	100	45	36	22	
5	92	48	30	8	
6	60	40	26	4	
7	99	76	60	32	

Ans:

Soil	Symbol	Group name
1	SP	Poorly graded sand
2	MH	Elastic silt with sand
3	CH	Fat clay
4	SC	Clayey sand
5	SC	Clayey sand
6	GM-GC	Silty clayey gravel with sand
7	CH	Fat clay with sand

Chapter 6: Soil Compaction

In the construction of highway embankments, earth dams, and many other engineering structures, loose soils must be compacted to increase their unit weights. Compaction increases the strength characteristics of soils, which increase the bearing capacity of foundations constructed over them. Compaction also decreases the amount of undesirable settlement of structures and increases the stability of slopes of embankments.

6.1 Compaction — General Principles

Compaction, in general, is the densification of soil by removal of air, which requires mechanical energy. The degree of compaction of a soil is measured in terms of its dry unit weight. When water is added to the soil during compaction, acts as a softening agent on the soil particles. The soil particles slip over each other and move into a densely packed position. The dry unit weight after compaction first increases as the moisture content increases (See Figure 6.1). When the moisture content is gradually increased and the same compactive effort is used for compaction, the weight of the soil solids in a unit volume gradually increases.

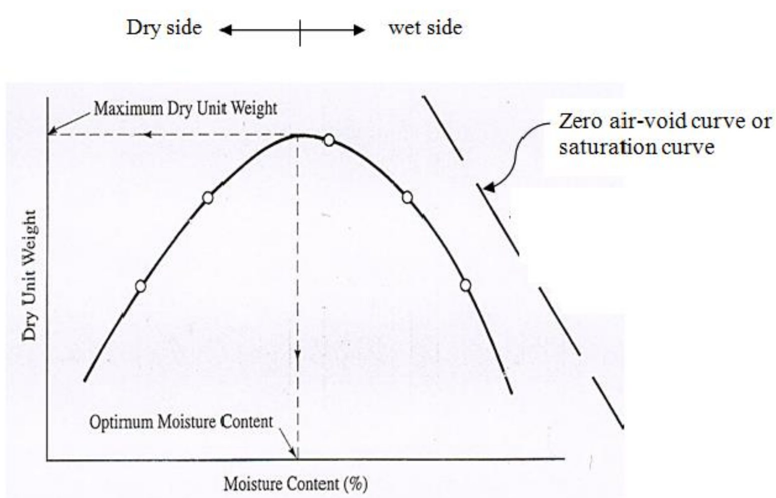


Figure 6.1 Principles of compaction

Beyond a certain moisture content, Figure (6.1), any increase in the moisture content tends to reduce the dry unit weight. This phenomenon occurs because the water takes up the spaces that would have been occupied by the solid particles. The moisture content at which the maximum dry unit weight is attained is generally referred to as the *optimum moisture content*.

The laboratory test generally used to obtain the maximum dry unit weight of compaction and the optimum moisture content is called the *Proctor compaction test* (Proctor, 1933).

6.2 Standard Proctor Test

In the Proctor test, the soil is compacted in a mold that has a volume of 944 cm^3 (Figure 6.2). The diameter of the mold is 101.6 mm (4 in.). The soil is mixed with varying amounts of water and then compacted in three equal layers by a hammer (Figure 6.2b) that delivers 25 blows to each layer. The hammer has a mass of 2.5 kg (6.5 lb) and has a drop of 305 mm (12 in.). Figure 6.2c is a photograph of the laboratory equipment required for conducting a standard Proctor test.

For each test, the moist unit weight of compaction, γ, can be calculated as

$$\gamma = \frac{W}{V_{(m)}} \tag{6.1}$$

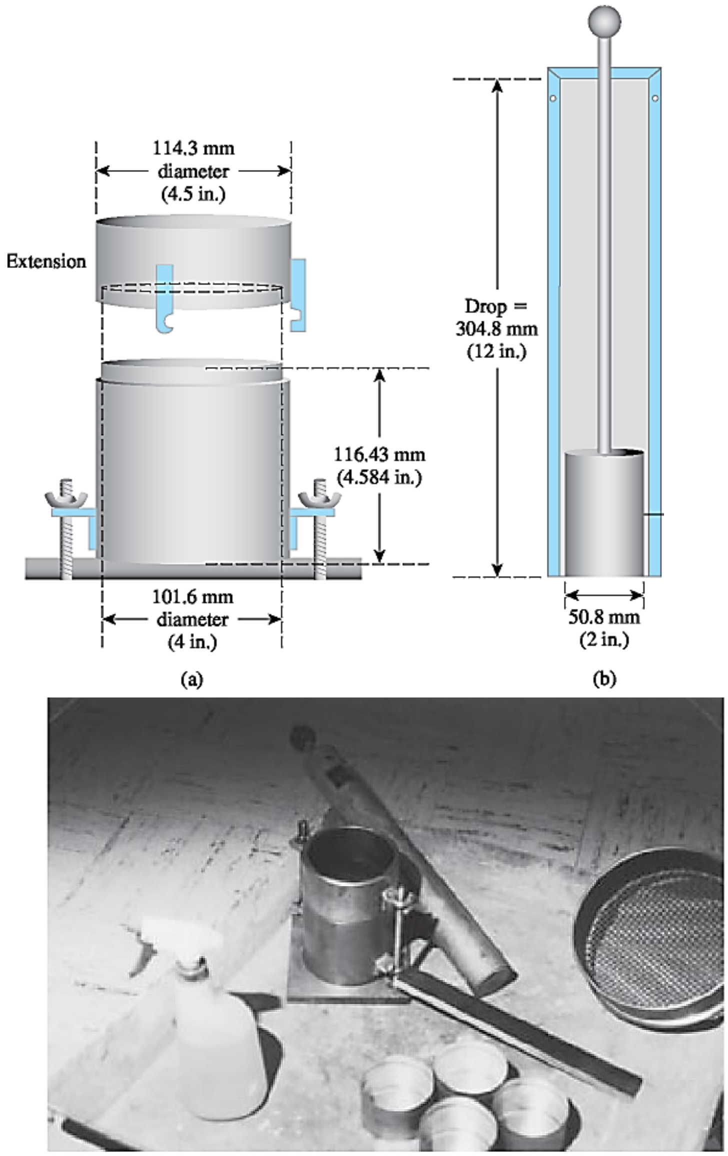
where W = weight of the compacted soil in the mold
V(m) = volume of the mold (944 cm^3)

For each test, the moisture content of the compacted soil is determined in the laboratory. With the known moisture content, the dry unit weight can be calculated as

$$\gamma_d = \frac{\gamma}{1+\frac{w(\%)}{100}} \tag{6.2}$$

where w (%) = percentage of moisture content.

The values of γ_d determined from Eq. (6.2) can be plotted against the corresponding moisture contents to obtain the maximum dry unit weight and the optimum moisture content for the soil. Figure (6.3) shows such a plot for a silty-clay soil.



(c)

Figure 6.2 Standard Proctor test equipment: (a) mold; (b) hammer; (c) photograph of laboratory equipment used for test

The procedure for the standard Proctor test is elaborated in ASTM Test Designation D-698 (ASTM, 2007) and AASHTO Test Designation T-99 (AASHTO, 1982).

For given *moisture content w and degree of saturation S*, the dry unit weight of compaction can be calculated as follows. From Chapter 3 [Eq. (3.16)], for any soil,

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

Where G_s *= specific gravity of soil solids*
γ_w *= unit weight of water*
e = void ratio

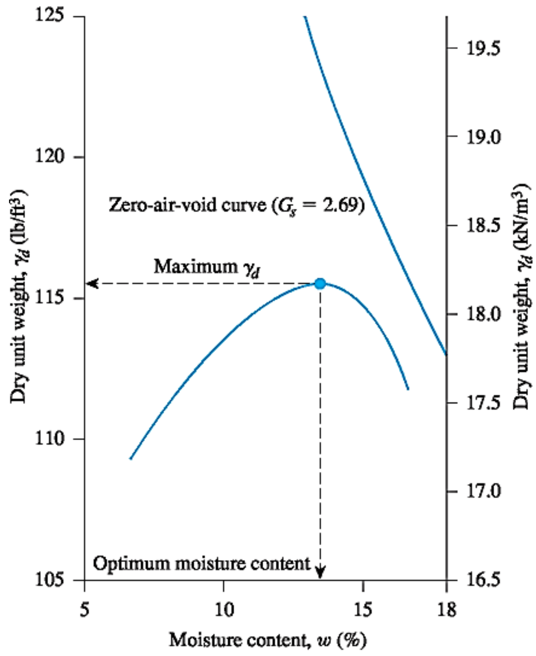


Figure 6.3 Standard Proctor compaction test results for a silty clay

and, from Eq. (3.18),

$$Se = G_s w$$

or

$$e = \frac{G_s w}{S}$$

Thus,

$$\gamma_d = \frac{G_s \gamma_w}{1 + \frac{G_s w}{S}} \qquad (6.3)$$

For given moisture content, the theoretical maximum dry unit weight is obtained when no air is in the void spaces—that is, when the degree of saturation equals 100%. Hence, the maximum dry unit weight at a given moisture content with zero air voids can be obtained by substituting *S* = 1 into Eq. (6.3), or

$$\gamma_{zav} = \frac{G_s \gamma_w}{1 + wG_s} = \frac{\gamma_w}{w + \frac{1}{G_s}} \qquad (6.4)$$

Where γ_{zav} *=zero-air-void unit weight.*

To obtain the variation of γ_{zav} with moisture content, use the following procedure:

1. Determine the specific gravity of soil solids.
2. Know the unit weight of water (γ_w).
3. Assume several values of w, such as 5%, 10%, 15%, and so on.
4. Use Eq. (6.4) to calculate γ_{zav} for various values of w.

Figure (6.3) also shows the variation of γ_{zav} with moisture content and its relative location with respect to the compaction curve. Under no circumstances should any part of the compaction curve lie to the right of the zero-air-void curve.

6.3 Factors Affecting Compaction

The preceding section showed that moisture content has a strong influence on the degree of compaction achieved by a given soil. Besides moisture content, other important factors that affect compaction are soil type and compaction effort (energy per unit volume).

Effect of Soil Type

The soil type—that is, grain-size distribution, shape of the soil grains, specific gravity of soil solids, and amount and type of clay minerals present—has a great influence on the maximum dry unit weight and optimum moisture content. Figure (6.4) shows typical compaction curves obtained from four soils. The laboratory tests were conducted in accordance with ASTM Test Designation D-698.

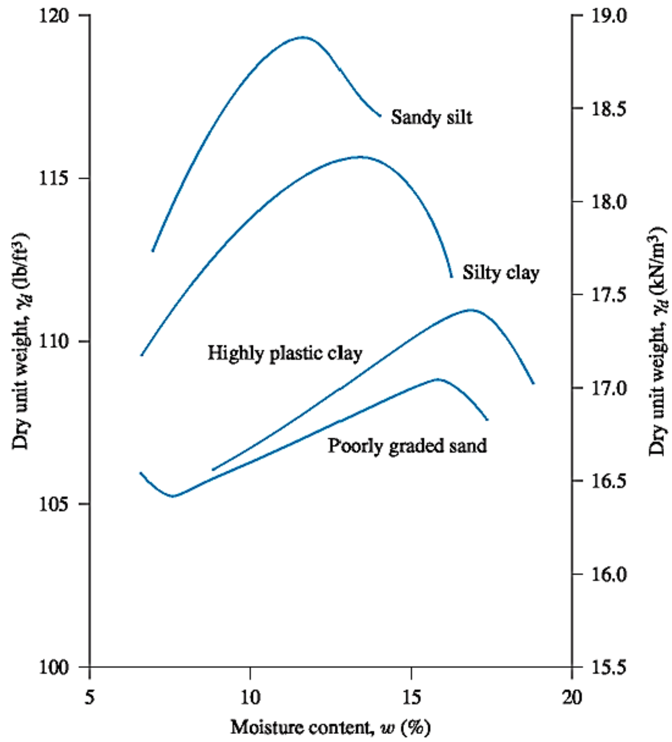


Figure 6.4 Typical compaction curves for four soils (ASTM D-698)

Effect of Compaction Effort

The compaction energy per unit volume used for the standard Proctor test described in Section 6.2 can be given as

$$\boldsymbol{E} = \frac{\begin{pmatrix}\text{Number of blows}\\ \text{per layer}\end{pmatrix}\times\begin{pmatrix}\text{Number of}\\ \text{layers}\end{pmatrix}\times\begin{pmatrix}\text{weight of}\\ \text{hammer}\end{pmatrix}\times\begin{pmatrix}\text{Height of drop}\\ \text{of hammer}\end{pmatrix}}{\text{Volume of mold}} \tag{6.5}$$

or

$$E = \frac{(25)(3)(\frac{2.5\times 9.81}{1000}kN)(0.305m)}{944\times 10^{-6}m^3} = 594\ kN.\frac{m}{m^3} \cong 600\ kN.m/m^3$$

If the compaction effort per unit volume of soil is changed, the moisture–unit weight curve also changes. This fact can be demonstrated with the aid of Figure (6.5), which shows four compaction curves for sandy clay. The standard Proctor mold and hammer were used to obtain these compaction curves. The number of layers of soil used for compaction was three for all cases. However, the number of hammer blows per each layer varied from 20 to 50, which varied the energy per unit volume.

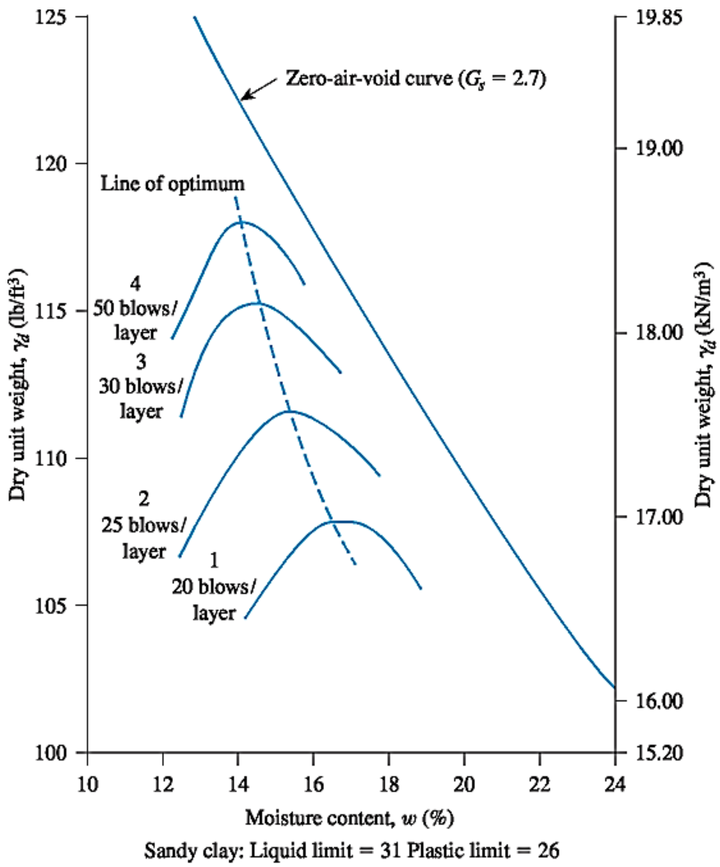


Figure 6.5 Effect of compaction energy on the compaction of a sandy clay

From the preceding observation and Figure (6.5), we can see that

1. As the compaction effort is increased, the maximum dry unit weight of compaction is also increased.
2. As the compaction effort is increased, the optimum moisture content is decreased to some extent.

6.4 Modified Proctor Test

With the development of heavy rollers and their use in field compaction, the standard Proctor test was modified to better represent field conditions. This revised version sometimes is referred to as the *modified Proctor test* (ASTM Test Designation D-1557 and AASHTO Test Designation T-180).

Table (6.1) Summarizes differences between standard and modified proctor tests

Specifications	Standard	Modified
Mold volume (cm^3)	944	944
No. of soil layers	3	5
Mass of rammer (*kg*)	2.5	4.5
Drop distance (*mm*)	305	457
No. of blows/ layer	25	25

Figure (6.6) shows a comparison between the hammers used in standard and modified Proctor tests. The compaction energy for this type of compaction test can be calculated as 2700 $kN\text{-}m/m^3$.

Because it increases the compactive effort, the modified Proctor test results in an increase in the maximum dry unit weight of the soil. The increase in the maximum dry unit weight is accompanied by a decrease in the optimum moisture content.

Figure (6.6) Comparison between standard Proctor hammer (left) and modified Proctor hammer (right)

Example 6.1

The results of a standard compaction test are:

w(%)	6.2	8.1	9.8	11.5	12.3	13.2
γ (kN/m³)	16.9	18.7	19.5	20.5	20.4	20.1

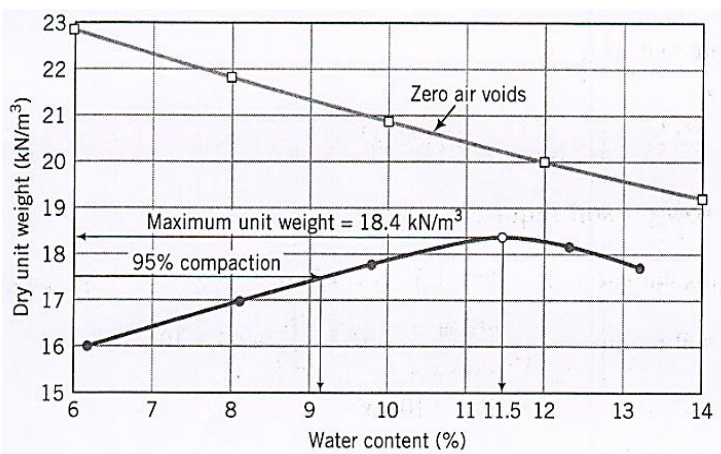
if G_s = 2.7, find

a. The maximum dry unit weight and optimum water content.
b. S at $(\gamma_d)_{max}$.
c. γ_d and w at 95% compaction.
d. Plot the zero-void line (or curve).

Solution;

a-

w %	γ (kN/m³)	$\gamma_d = \frac{\gamma}{1+w} (\frac{kN}{m^3})$
6.2	16.9	15.9
8.1	18.7	17.3
9.8	19.5	17.8
11.5	20.5	18.4
12.3	20.4	18.2
13.2	20.1	17.8



From the (w-γ_d) graph, we get:

$(\gamma_d)_{max}$ = 18.4 kN/m³ & $w_{opt.}$ =11.5%

b-

$$\gamma_d = \frac{G_s \gamma_w}{1 + \frac{G_s w}{S}}$$

or

$$S = \frac{w G_s (\gamma_d)_{max}/\gamma_w}{G_s - (\gamma_d)_{max}/\gamma_w} = \frac{0.115 \times 2.7 \times (\frac{18.4}{9.8})}{2.7 - (\frac{18.4}{9.8})} = 0.71 = 71\%$$

c-

at 95% compaction:

γ_d = 18.4× 0.95 = 17.5 kN/m^3

w = 9.2 % (From the graph)

d-

zero air-void line:

w (%)	6	8	10	12	14
$\gamma_d = \frac{G_s\gamma_w}{1+\frac{G_s w}{S=1}}$	22.8	21.8	20.8	20.0	19.2

6.5 Field Compaction

Most of the compaction in the field is done with rollers. The four most common types of rollers are;

1. Smooth-Wheel (or smooth-drum) rollers (Figure 6.7):
   - Suitable for base coarse and for smooth finished grade (sandy and clayey soils)
   - Not suitable for producing high unit weights when used on thicker layers.
2. Pneumatic rubber-tired rollers (Figure 6.8):
   - Provide very high contact pressure.
   - Used for sandy and clayey soils.
   - Compaction is achieved by a combination of pressure and kneading.
3. Sheepsfoot rollers (Figure 6.9);
   - Most effective in compacting fine-grained soils by kneading action.
4. Vibratory rollers (Figure 6.10);
   - Most efficient in compacting granular soils.

Figure (6.7) Smooth-wheel roller

Figure (6.8) Pneumatic rubber-tired roller

Figure (6.9) Sheepsfoot roller

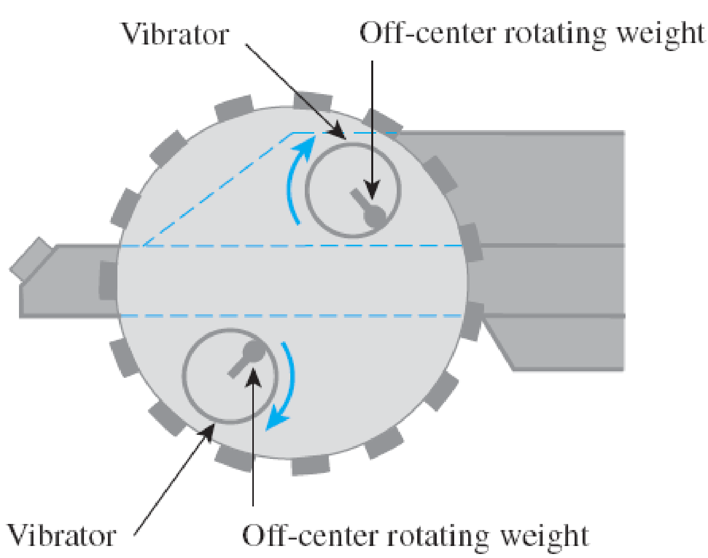


Figure (6.10) Principles of vibratory rollers

Handheld vibrating plates can be used for effective compaction of granular soils over a limited area. Vibrating plates are also gang-mounted on machines. These plates can be used in less restricted areas.

6.6 Specifications for Field Compaction

In most specifications for earthwork, the contractor is instructed to achieve a compacted field dry unit weight of 90 to 95% of the maximum dry unit weight determined in the laboratory by either the standard or modified Proctor test. This is a specification for relative compaction, which can be expressed as

$$R(\%) = \frac{\gamma_{d(field)}}{\gamma_{d(max-lab)}} \times 100 \tag{6.6}$$

where R = relative compaction.

For the compaction of granular soils, specifications sometimes are written in terms of the required relative density D_r or the required relative compaction. Relative density should not be confused with relative compaction. From Chapter 3, we can write

$$D_r = \left[\frac{\gamma_{d(field)} - \gamma_{d(\min)}}{\gamma_{d(\max)} - \gamma_{d(\min)}}\right]\left[\frac{\gamma_{d(\max)}}{\gamma_{d(field)}}\right] \tag{6.7}$$

Comparing Eqs. (6.6) and (6.7), we see that

$$R = \frac{R_0}{1 - D_r(1 - R_0)} \tag{6.8}$$

where

$$R_0 = \frac{\gamma_{d(\min)}}{\gamma_{d(\max)}} \tag{6.9}$$

6.7 Determination of Field Unit Weight

A geotechnical engineer needs to check that field compaction meets specifications. The standard procedures available to check the amount of compaction achieved in the field include:

1. Core cutter (ASTM D-2937)
   - Useful for fine-grained soils.
2. Sand cone method (ASTM D-1556) (Figure 6.11)
   - Used for both fine-grained and coarse-grained soils.
3. Rubber balloon method (ASTM D-2167) (Figure 6.12)

- Same as above, but water is used instead of sand.

4. Nuclear method (ASTM D-2922) (Figure 6.13)
   - Nondistructive and fast approach to obtain the unit weight and water content of the soil.
   - More costly than the other apparatuses and of potential hazards to individuals handling radioactive materials.

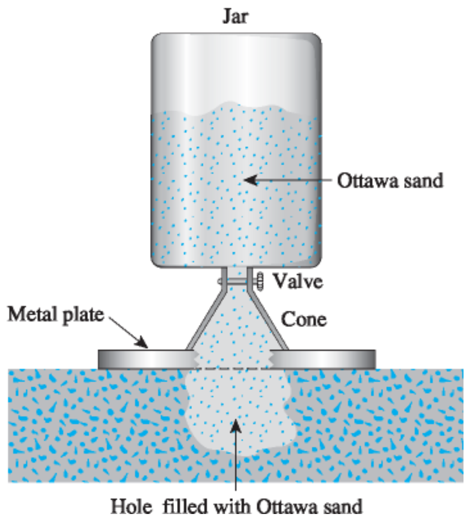


Figure (6.11) Field unit weight determined by sand cone method

Figure (6.12) Calibrated vessel used with rubber balloon

Figure (6.13) nuclear density meter

Problems

6.1 Given G_s = 2.75, calculate the zero-air-void unit weight for a soil in kN/m^3 at w = 5%, 8%, 10%, 12%, and 15%.

Ans:

w(%)	γ_{zav} (kN/m^3)
5	23.72
8	22.11
10	21.16
12	20.28
15	19.1

6.2 Repeat Problem **6.1** with G_s = 2.65.

Ans:

w(%)	γ_{zav} (kN/m^3)
5	22.95
8	21.45
10	20.55
12	19.72
15	18.6

6.3 Calculate the variation of dry density (kg/m^3) of a soil (G_s = 2.67) at w = 10% and 20% for degree of saturation (S) = 80%, 90%, and 100%.

Ans:

w(%)	ρ_d at S (%)(kg/m^3)		
	80	90	100
10	2002	2059	2107
20	1601	1676	1741

6.4 The results of a standard Proctor test are given in the following table. Determine the maximum dry density (kg/m^3) of compaction and the optimum moisture content.

Volume of Proctor mold (cm^3)	Mass of wet soil in the mold (kg)	Moisture content (%)
943.3	1.68	9.9
943.3	1.71	10.6
943.3	1.77	12.1
943.3	1.83	13.8
943.3	1.86	15.1
943.3	1.88	17.4
943.3	1.87	19.4
943.3	1.85	21.2

Ans: $\rho_{d(\max)}$ = 1718 kg/m^3 , $w_{opt.}$ =15.8 %

6.5 A field unit weight determination test for the soil described in Problem 6.4 yielded the following data: moisture content = 10.5% and moist density = 1705 kg/m^3. Determine the relative compaction.

Ans: R = 89.81 %

6.6 The *in situ* moisture content of a soil is 18% and the moist unit weight is 16.5 kN/m^3. The specific gravity of soil solids is 2.75. This soil is to be excavated and transported to a construction site for use in a compacted fill. If the specifications call for the soil to be compacted to a minimum dry unit weight of 16.3 kN/m^3 at the same moisture content of 18%, how many cubic meters of soil from the excavation site are needed to produce 10,000 m^3 of compacted fill? How many 20-ton truckloads are needed to transport the excavated soil?

Ans: N= 981

6.7 A proposed embankment fill requires 8000 m^3 of compacted soil. The void ratio of the compacted fill is specified as 0.7. Four borrow pits are available as described in the following table, which lists the respective void ratios of the soil and the cost per cubic meter for moving the soil to the proposed construction site. Make the necessary calculations to select the pit from which the soil should be bought to minimize the cost. Assume G_s to be the same at all pits.

Borrow pit	Void ratio	Cost ($/m^3)
A	0.82	8
B	1.1	5
C	0.90	9
D	0.78	12

Ans: Pit B

6.8 The maximum and minimum dry unit weights of a sand were determined in the laboratory to be 16.3 kN/m^3 and 14.6 kN/m^3, respectively. What would be the relative compaction in the field if the relative density is 78%?

Ans: R = 97.4%

6.9 The maximum and minimum dry densities of a sand were determined in the laboratory to be 1682 kg/m^3 and 1510 kg/m^3, respectively. In the field, if the relative density of compaction of the same sand is 70%, what are its relative compaction (%) and dry density (kg/m^3)?

Ans: R = 96.7%

6.10 The relative compaction of a sand in the field is 90%. The maximum and minimum dry unit weights of the sand are 17.0 kN/m^3 and 14.65 kN/m^3, respectively. For the field condition, determine

a. Dry unit weight

b. Relative density of compaction

c. Moist unit weight at a moisture content of 12%

Ans: (a) $\gamma_{d(field)}$ =15.3 kN/m^3 (b) D_r = 31.08% (c) $\gamma_{(field)}$ =17.14 kN/m^3

Chapter 7: Permeability

Soils are permeable due to the existence of interconnected voids through which water can flow from points of high energy to points of low energy. The study of the flow of water through permeable soil media is important in soil mechanics. It is necessary for estimating the quantity of underground seepage under various hydraulic conditions, for investigating problems involving the pumping of water for underground construction, and for making stability analyses of earth dams and earth-retaining structures that are subject to seepage forces.

7.1 Bernoulli's Equation

From fluid mechanics, we know that, according to Bernoulli's equation, the total head at a point in water under motion can be given by the sum of the pressure, velocity, and elevation heads, or

$$h = \underbrace{\frac{u}{\gamma_w}}_{\text{Pressure head}} + \underbrace{\frac{v^2}{2g}}_{\text{Velocity head}} + \underbrace{Z}_{\text{Elevation head}} \tag{7.1}$$

where h = total head
u = pressure
v = velocity
g = acceleration due to gravity
γ_w = unit weight of water

Note that the elevation head, Z, is the vertical distance of a given point above or below a datum plane. The pressure head is the water pressure, u, at that point divided by the unit weight of water, γ_w.

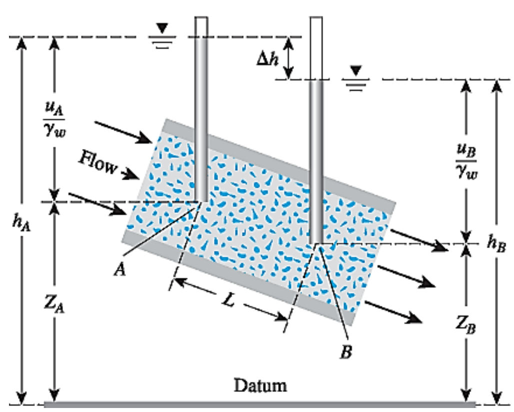


Figure 7.1 Pressure, elevation, and total heads for flow of water through soil

If Bernoulli's equation is applied to the flow of water through a porous soil medium, the term containing the velocity head can be neglected because the seepage velocity is small, and the total head at any point can be adequately represented by

$$h = \frac{u}{\gamma_w} + Z \qquad (7.2)$$

Figure (7.1) shows the relationship among pressure, elevation, and total heads for the flow of water through soil. Open standpipes called *piezometers* are installed at points *A* and *B*. The levels to which water rises in the piezometer tubes situated at points A and B are known as the *piezometric levels* of points *A* and *B*, respectively. The pressure head at a point is the height of the vertical column of water in the piezometer installed at that point.
The loss of head between two points, *A* and *B*, can be given by

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right) \qquad (7.3)$$

The head loss, Δh, can be expressed in a nondimensional form as

$$i = \frac{\Delta h}{L} \qquad (7.4)$$

where *i* = hydraulic gradient
L = distance between points A and B—that is, the length of flow over which the loss of head occurred

7.2 Darcy's Law

Darcy (1856) proposed that average flow velocity through soils is proportional to the gradient of the total head

$$v = ki \qquad (7.5)$$

Where *v* = discharge velocity, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angle to the direction of flow
k = hydraulic conductivity (otherwise known as the coefficient of permeability)

In Eq. (7.5), *v* is the discharge velocity of water based on the gross cross-sectional area of the soil. However, the actual velocity of water (that is, the seepage velocity) through the void spaces is greater than *v*. A relationship between the discharge velocity and the seepage velocity can be derived by referring to Figure (7.2), which shows a soil of length *L* with a gross cross-sectional area *A*. If the quantity of water flowing through the soil in unit time is q, then

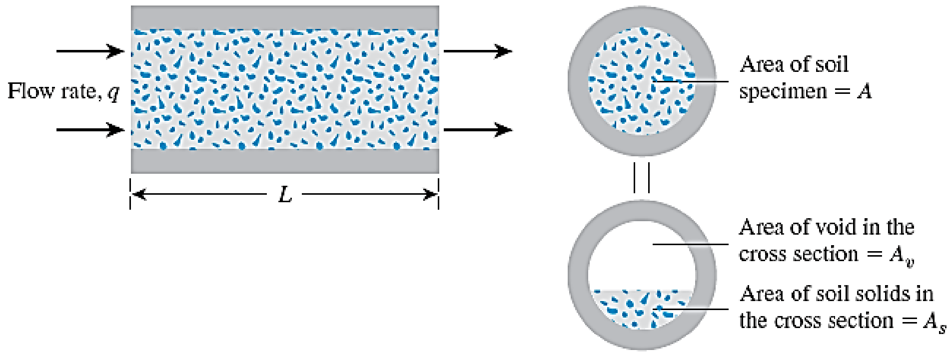


Figure 7.2 Derivation of Eq. (7.7)

$$q = vA = A_v v_s \tag{7.6}$$

Where v_s = seepage velocity
A_v = area of void in the cross section of the specimen

Thus;

$$v_s = \frac{A}{A_v} v = \frac{LA}{LA_v} v = \frac{V}{V_v} v = \frac{v}{n} = v\left(\frac{1+e}{e}\right) \tag{7.7}$$

where e = void ratio
n = porosity

7.3 Hydraulic Conductivity

Hydraulic conductivity is generally expressed in cm/sec. It depends on several factors:

1. Soil type (fine-grained or coarse-grained).
2. Grain size distribution.
3. Void ratio.
4. Pore size distribution.
5. Fluid viscosity.
6. Degree of saturation.
7. Roughness of mineral particles.

7.4 Laboratory Determination of Hydraulic Conductivity

Two standard laboratory tests are used to determine the hydraulic conductivity of soil— the constant-head test and the falling-head test.

Constant-Head Test

A typical arrangement of the constant-head permeability test is shown in Figure 7.3. In this type of laboratory setup, the water supply at the inlet is adjusted in such a way that the difference of head between the inlet and the outlet remains constant during the test period. After a constant flow rate is established, water is collected in a graduated flask for a known duration.

The total volume of water collected may be expressed as

$$Q = Avt = A(ki)t \tag{7.8}$$

where Q = volume of water collected

A = area of cross section of the soil specimen

t = duration of water collection

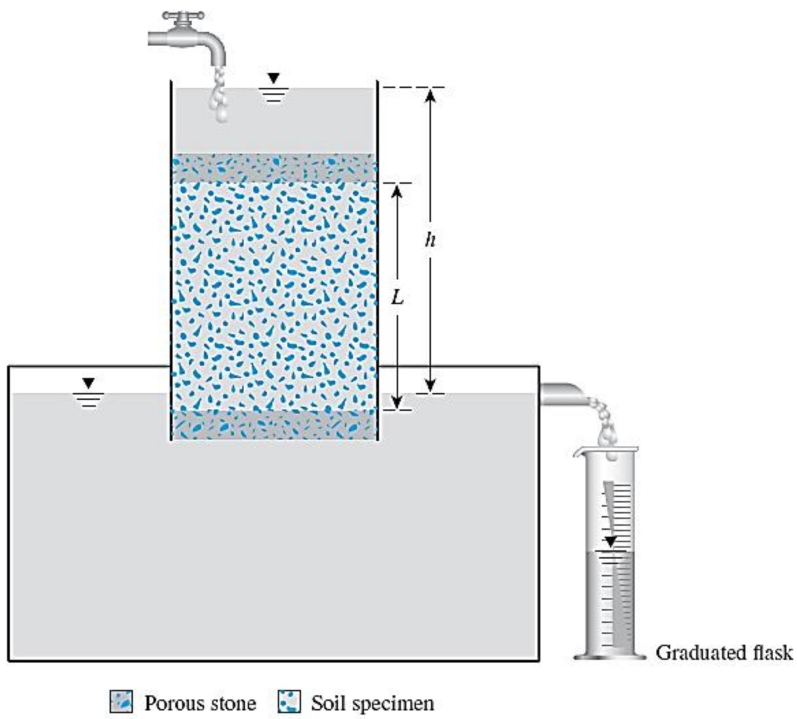


Figure (7.3) Constant-head permeability test

And because

$$i = \frac{h}{L} \tag{7.9}$$

where L = length of the specimen, Eq. (7.9) can be substituted into Eq. (7.8) to yield

$$Q = A\left(k\frac{h}{L}\right)t \qquad (7.10)$$

or

$$k = \frac{QL}{Aht} \qquad (7.11)$$

This test is usually used to determine k for coarse-grained soils.

Falling-Head Test

A typical arrangement of the falling-head permeability test is shown in Figure 7.4. Water from a standpipe flows through the soil. The initial head difference h_1 at time $t = 0$ is recorded, and water is allowed to flow through the soil specimen such that the final head difference at time $t = t_2$ is h_2.

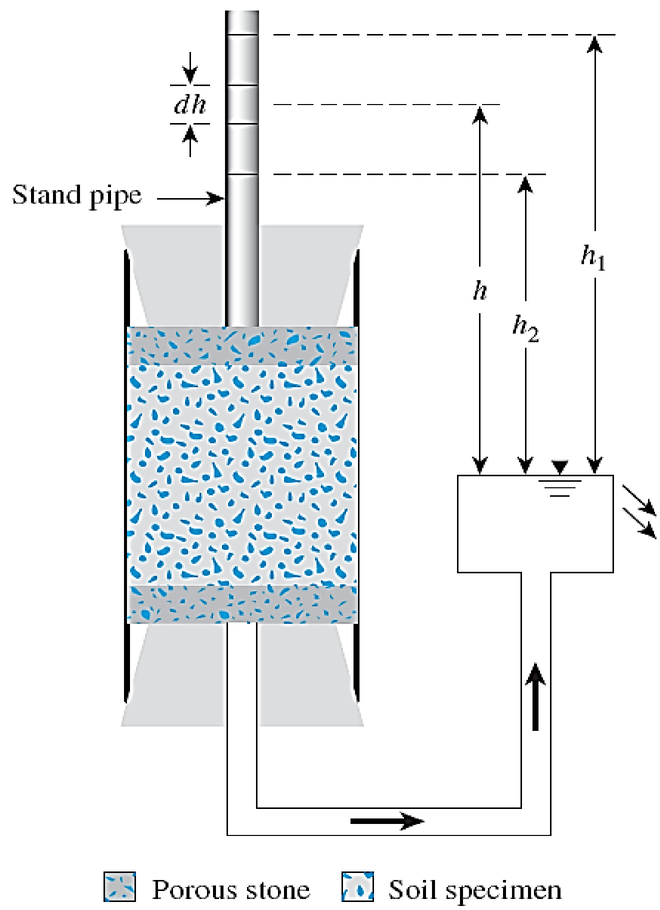


Figure (7.4) Falling-head permeability

The rate of flow of the water through the specimen at any time t can be given by

$$q = k\frac{h}{L}A = -a\frac{dh}{dt} \tag{7.12}$$

where q = flow rate
a = cross-sectional area of the standpipe
A = cross-sectional area of the soil specimen

Rearrangement of Eq. (7.12) gives

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right) \tag{7.13}$$

$$\int_0^t dt = \frac{aL}{Ak}\int_{h_1}^{h_2}\left(-\frac{dh}{h}\right)$$

or

$$k = \frac{aL}{At}\ln\left(\frac{h_1}{h_2}\right) = 2.303\frac{aL}{At}\log_{10}\frac{h_1}{h_2} \tag{7.14}$$

This test is usually used to determine k for fine-grained soils.

Example 7.1

Refer to the constant-head permeability test arrangement shown in Figure (7.3). A test gives these values:

- L= 30 cm
- A= area of the specimen = 177 cm^2
- Constant-head difference, h= 50 cm
- Water collected in a period of 5 min = 350 cm^3

Calculate the hydraulic conductivity in cm/sec.

Solution:

From eq. (7.11)

$$k = \frac{QL}{Aht}$$

Given Q= 350 cm^3, L=30 cm, A= 177 cm^2, h= 50 cm, and t= 5 min, we have;

$$k\frac{(350)(30)}{(177)(50)(5)(60)} = 3.95 \times 10^3\ cm/sec$$

Example 7.2

For a falling-head permeability test, the following values are given:

- Length of specimen = 20 cm
- Area of soil specimen = 10 cm^2
- Area of standpipe = 0.4 cm^2
- Head difference at time $t_{=0}$ = 50 cm
- Head difference at time $t_{=180\ sec}$ = 30 cm

Determine the hydraulic conductivity of the soil in cm/sec.

Solution:

From Eq. (7.14)**,**

$$k = 2.303 \frac{aL}{At} log_{10} \frac{h_1}{h_2}$$

We are given a= 0.4 cm^2, L= 20 cm, A= 10 cm^2, t= 180 sec, h_1= 50 cm and h_2= 30 cm

$$k = 2.303 \frac{(0.4)(20)}{(10)(180)} log_{10}\left(\frac{50}{30}\right) = 2.27 \times 10^{-3}\ cm/sec$$

Example 7.3

A permeable soil layer is underlain by an impervious layer, as shown in figure (7.5a). With $k = 5.3 \times 10^{-5}$ m/sec for the permeable layer, calculate the rate of seepage through it in $m^3/hr/m$ width if H= 3m and $\alpha = 8$.

Solution: From figure (7.5 b)

$$i = \frac{head\ loss}{length} = \frac{L.tan\alpha}{\left(\frac{L}{cos\alpha}\right)} = sin\alpha$$

$q = kiA = (k)(\sin\alpha)(3\cos\alpha)(1)$

$k = 5.3 \times 10^{-5}$ m/sec

$q = (5.3 \times 10^{-5})(sin8)(3cos8)(3600) = 0.0789\ m^3/hr/m$

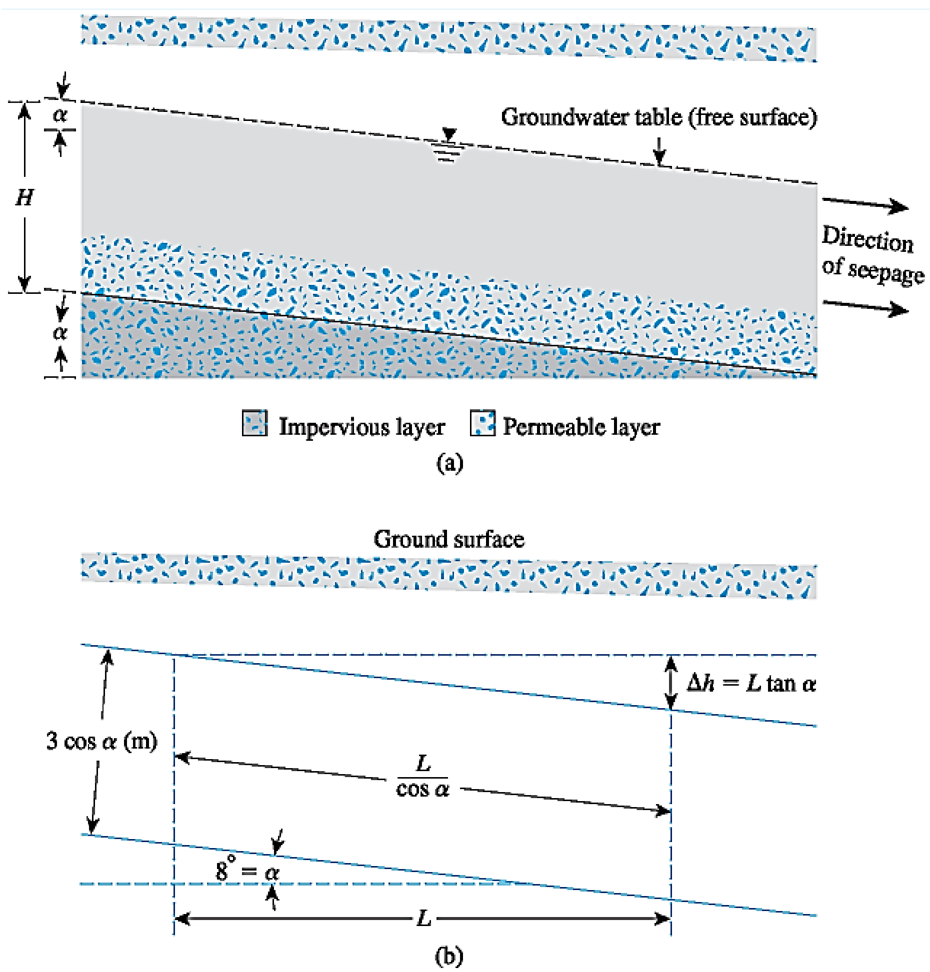


Figure (7.5)

Example 7.4

Find the flow rate in m^3/sec/m length (at right angle to the cross section shown) through the permeable soil layer shown in Figure (7.6) given H = 8 m, H_1= 3m, h = 4m, L = 50m, $\alpha = 8$, and k = 0.08 cm/sec.

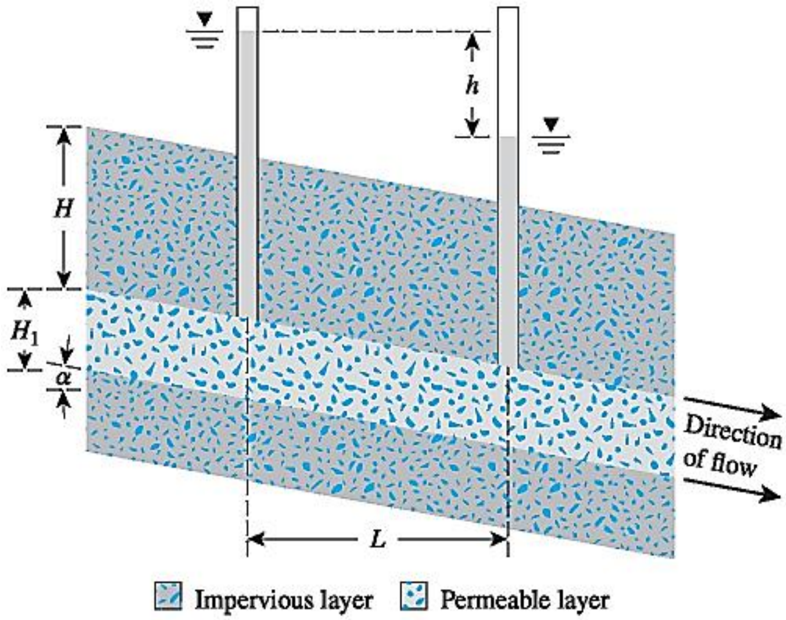


Figure 7.6 Flow through permeable layer

Solution:

Hydraulic gradient $(i) = \frac{h}{\left(\frac{L}{cos\alpha}\right)}$

From Eqs. (7.9) and (7.10)

$q = kiA = (k)(\frac{hcos\alpha}{L})(H_1 cos\alpha \times 1)$

$q = (0.08 \times 10^{-2}$ m/sec$)(\frac{4cos8}{50})(3cos8\times1) = 0.19 \times 10^{-3}$ m^3/sec/m

Example 7.5

A soil sample 10 cm in diameter is placed in a tube 1m long. A constant supply of water is allowed to flow into one end of the soil at *A* and the outflow at *B* is collected by beaker. The average amount of water collected is 1 cm^3 for every 10 seconds. Determine the:

a. Hydraulic gradient
b. Flow rate
c. Average velocity
d. Seepage velocity, if $e = 0.6$
e. Hydraulic conductivity

Solution :

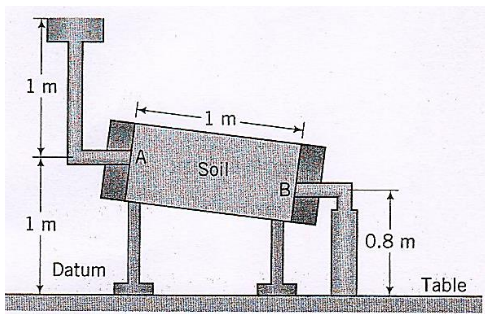


Figure (7.7)

(a) $h_A = Z_A + \frac{U_A}{\gamma_w} = 1 + 1 = 2\text{m}$

$h_B = Z_B + \frac{U_B}{\gamma_w} = 0.8 + 0 = 0.8\text{m}$

$\Delta h = 2 - 0.8 = 1.2\text{m}$

$L = 1.0$ m

$i = \frac{\Delta h}{L} = \frac{1.2}{1.0} = 1.2$

(HW) Repeat the solution by choosing the datum passing through point B.

(b) $q = \frac{Q}{t} = \frac{1}{10} = 0.1\,\frac{cm^3}{sec}$

(c) $q = vA$

$A = \frac{\pi}{4}(10)^2 = 78.5\text{cm}^2$

Thus; $v = \frac{q}{A} = \frac{0.1}{78.5} = 0.0013\,\frac{cm}{sec}$

(d) $v_s = \frac{v}{n}$

$n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.38$

Thus; $v_s = \frac{0.0013}{0.38} = 0.0034\,\frac{cm}{sec}$

(e) $k = \frac{v}{i} = \frac{0.0013}{1.2} = 10.8 \times 10^{-4}\,\frac{cm}{sec}$

HW

7.6. A sample of sand, 5 cm in diameter and 15 cm long, was prepared at a porosity of 60% in a constant-head apparatus. The total head was kept constant at 30 cm and the amount of water collected in 5 seconds was 40 cm^3. The test temperature was 20^0C. Calculate the hydraulic conductivity and the seepage velocity.

Ans; k = 0.2 cm/sec , v_s = 0.67 cm/sec

7.7. The data from a falling-head test on a silty clay are:

Cross-sectional area of soil = 80 cm^2

Length of soil = 10 cm

Initial head = 90 cm, Final head = 84 cm

Duration of test = 15 minutes, Diameter of tube = 6 mm

Determine k.

Ans; $k = 2.7\times10^{-6}$ cm/sec

7.5 Equivalent Hydraulic Conductivity in Stratified Soil

In a stratified soil deposit where the hydraulic conductivity for flow in a given direction changes from layer to layer, an equivalent hydraulic conductivity can be computed to simplify calculations.

a. Flow in the Horizontal Direction

Figure (7.8) shows n layers of soil with flow in the *horizontal direction*. Let us consider a cross section of unit length passing through the n layers and perpendicular to the direction of flow. The total flow through the cross section in unit time can be written as

$$q = v.1.H$$
$$= v_1.1.H_1 + v_2.1.H_2 + v_3.1.H_3 + \cdots + v_n.1.H_n \qquad (7.15)$$

Where v = average discharge velocity

$v_1, v_2, v_3, \ldots, v_n$ = discharge velocities of flow in layers denoted by the subscripts

If $\boldsymbol{k_{H1}, k_{H2}, k_{H3}, \ldots, k_{Hn}}$ are the hydraulic conductivities of the individual layers in the horizontal direction and $k_{H(eq)}$ is the equivalent hydraulic conductivity in the horizontal direction, then, from *Darcy's law*,

$$v = k_{H(eq)} i_{eq};\ \ v_1 = k_{H1} i_1;\ v_2 = k_{H2} i_2;\ v_3 = k_{H3} i_3;\ \ldots\ v_n = k_{Hn} i_n$$

Substituting the preceding relations for velocities into Eq. (7.15) and noting that $i_{eq} = i_1 = i_2 = i_3 = \ldots = i_n$ results in

$$k_{H(eq)} = \frac{1}{H}\left(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots .. + k_{H_n}H_n\right) \quad (7.16)$$

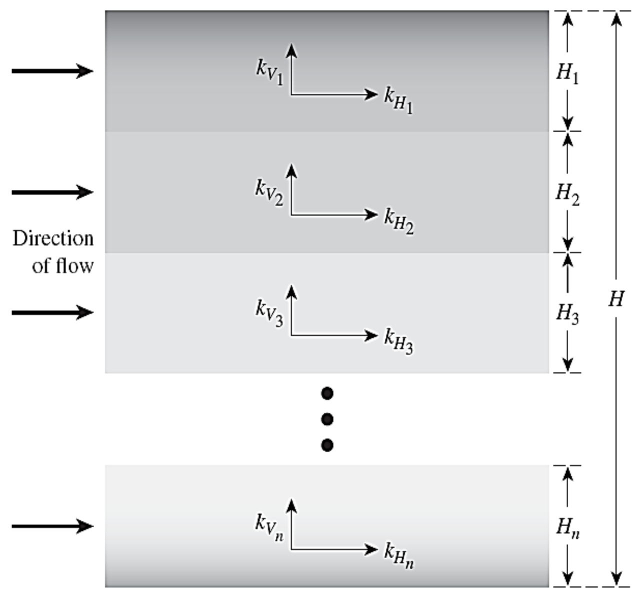


Figure (7.8) Equivalent hydraulic conductivity determination—horizontal flow in stratified soil

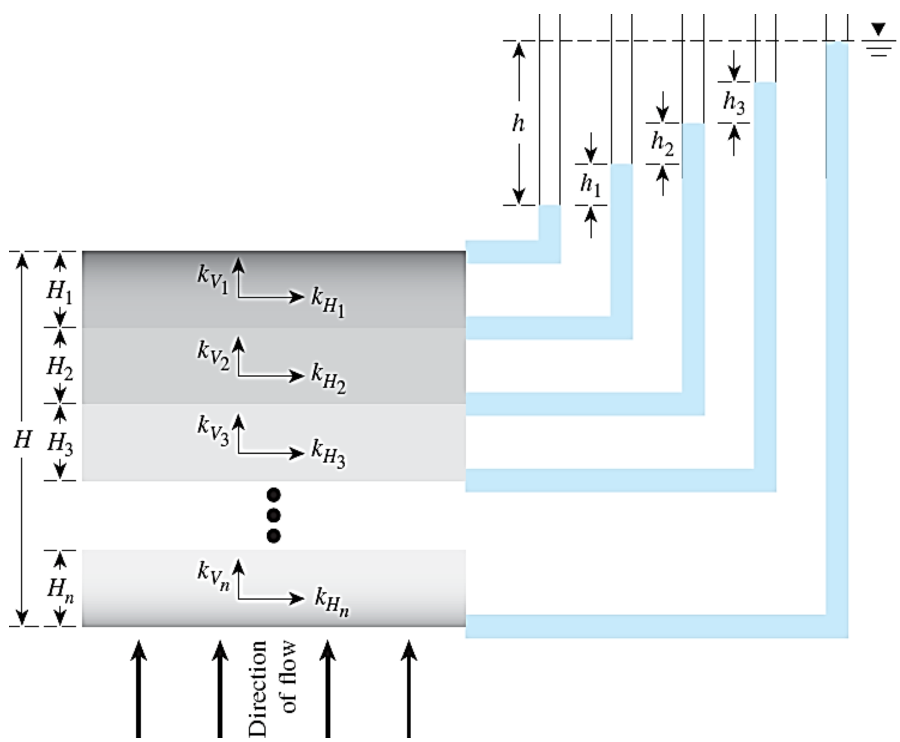


Figure 7.9 Equivalent hydraulic conductivity determination—vertical flow in stratified soil

b. Flow in the Vertical Direction

Figure 7.9 shows n layers of soil with flow in the vertical direction. In this case, the velocity of flow through all the layers is the same. However, the total head loss, *h*, is equal to the sum of the head losses in all layers. Thus,

$$v = v_1 = v_1 = v_1 = \cdots = v_n \tag{7.17}$$

and

$$h = h_1 + h_2 + h_3 + \cdots + h_n \tag{7.18}$$

Using Darcy's law, we can rewrite Eq. (7.17) as

$$k_{v(eq)}\left(\frac{h}{H}\right) = k_{v_1} i_1 = k_{v_2} i_1 = k_{v_3} i_3 = \cdots = k_{v_n} i_n \tag{7.19}$$

where $k_{v_1}, k_{v_2}, k_{v_3}, \ldots, k_{v_n}$ are the hydraulic conductivities of the individual layers in the vertical direction and $k_{v(eq)}$ is the equivalent hydraulic conductivity.

Again, from Eq. (7.18),

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n \tag{7.20}$$

Solving Eqs. (7.19) and (7.20) gives

$$k_{v(eq)} = \frac{H}{\left(\frac{H_1}{k_{v_1}}\right) + \left(\frac{H_2}{k_{v_2}}\right) + \left(\frac{H_3}{k_{v_3}}\right) + \cdots + \left(\frac{H_n}{k_{v_n}}\right)} \tag{7.21}$$

Example 7.8 (H.W)

A layered soil is shown in Figure (7.10). Given:

$H_1 = 2$m $k_1 = 10^{-4}$ cm/sec, $H_2 = 3$m $k_2 = 3.2 \times 10^{-2}$ cm/sec

$H_3 = 4$m $k_3 = 4.1 \times 10^{-5}$ cm/sec

Estimate the ratio of equivalent hydraulic conductivity, $\frac{k_{H(eq)}}{k_{v(eq)}}$

Ans: 139.96

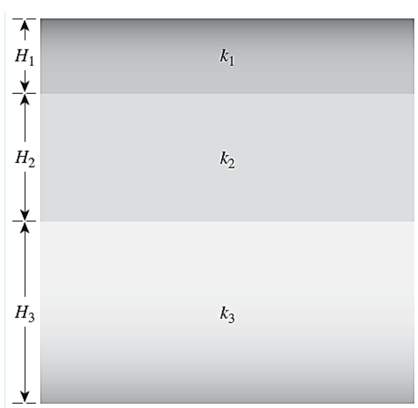


Figure (7.10) A layered soil profile

Example 7.9 (HW)

Figure (7.11) shows three layers of soil in a tube that is 100 mm × 100 mm in cross section. Water is supplied to maintain a constant-head difference of 300 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as follows:

Soil	k(cm/sec)
A	10^{-2}
B	3×10^{-3}
C	4.9×10^{-4}

Find the rate of water supply in cm^3/hr

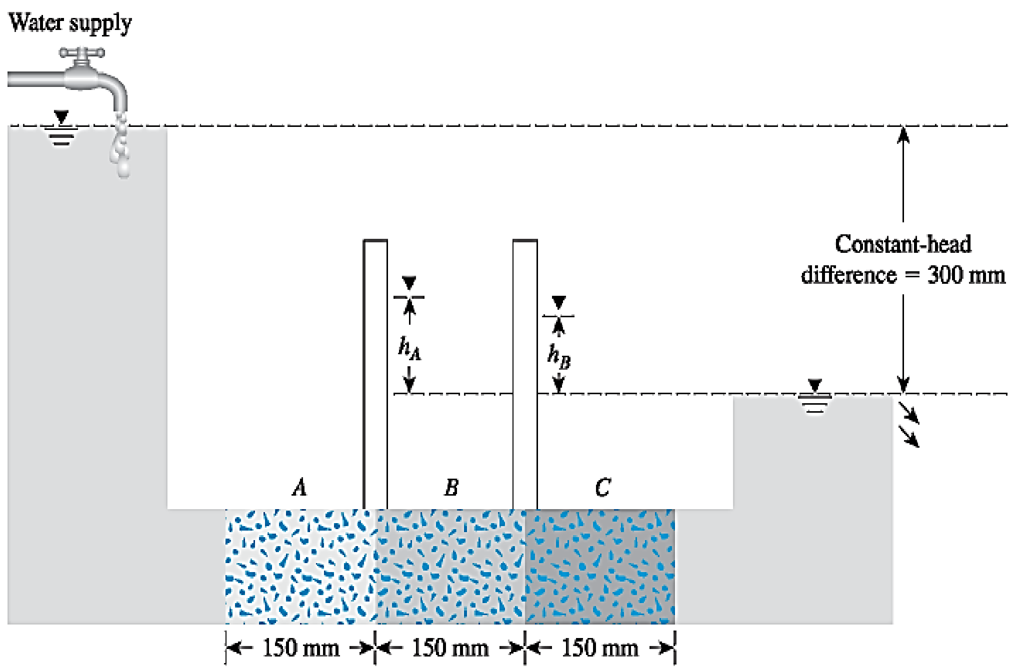


Figure (7.11) Three layers of soil in a tube 100 mm × 100 mm in cross section

Ans: q = 291.24 cm^3/hr

7.6 Permeability Test in the Field by Pumping from Wells

In the field, the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells. Figure (7.12) shows a case where the top permeable layer, whose hydraulic conductivity has to be determined, is unconfined and underlain by an impermeable layer. During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well. Continuous observations of the water level in the test well and in the observation wells are made after the start of pumping, until a steady state is reached. The steady state is established when the water level in the test and observation wells becomes constant. The expression for the rate of flow of groundwater into the well, which is equal to the rate of discharge from pumping, can be written as

$$q = k\left(\frac{dh}{dr}\right)2\pi rh \qquad (7.22)$$

or

$$\int_{r_2}^{r_1}\frac{dr}{r} = \left(\frac{2\pi k}{q}\right)\int_{h_2}^{h_1} h\,dh$$

Thus,

$$k = \frac{2.303\,q\,log_{10}(\frac{r_1}{r_2})}{\pi(h_1^2-h_2^2)} \qquad (7.23)$$

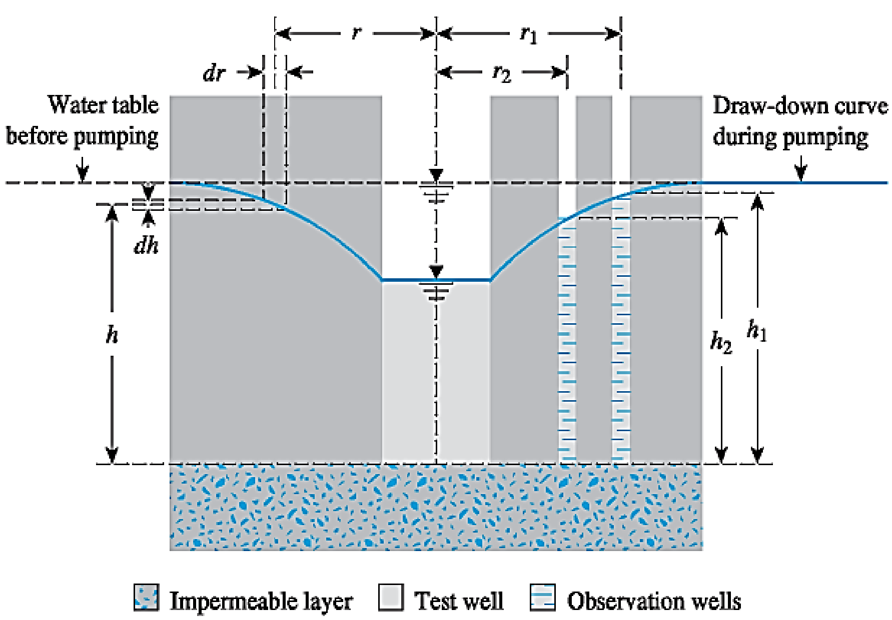


Figure (7.12) Pumping test from a well in an unconfined permeable layer underlain by an impermeable stratum.

7.7 Flow through an Aquifer

The average hydraulic conductivity for a confined aquifer can also be determined by conducting a pumping test from a well with a perforated casing that penetrates the full depth of the aquifer and by observing the piezometric level in a number of observation wells at various radial distances (Figure 7.13). Pumping is continued at a uniform rate q until a steady state is reached.

Because water can enter the test well only from the aquifer of thickness H, the steady state of discharge is

$$q = k\left(\frac{dh}{dr}\right)2\pi rH \qquad (7.24)$$

or

$$\int_{r_2}^{r_1}\frac{dr}{r} = \int_{h_2}^{h_1}\left(\frac{2\pi kH}{q}\right)dh$$

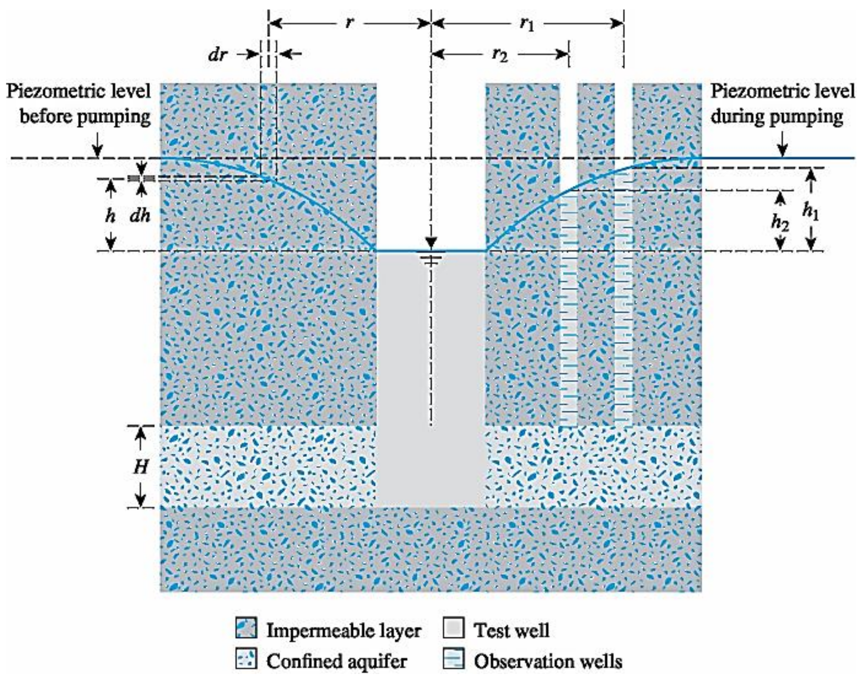


Figure (7.13) Pumping test from a well penetrating the full depth in a confined aquifer

This gives the hydraulic conductivity in the direction of flow as

$$k = \frac{q\log_{10}\left(\frac{r_1}{r_2}\right)}{2.727H(h_1 - h_2)} \qquad (7.25)$$

Problems

7.1 Refer to Figure (7.3). For a constant-head permeability test in sand, the following are given:
* *L*= 300 mm
* *A*= 175 cm^2
* *h*= 500 mm
* Water collected in 3 min = 620 cm^3
* Void ratio of sand = 0.58

Determine

a. Hydraulic conductivity, *k* (cm/sec)
b. Seepage velocity

Ans: (a) k =0.012 cm/sec (b) v_s = 0.054 cm/sec

7.2 For a falling-head permeability test, the following are given: length of specimen = 380 mm; area of specimen= 6.5 cm^2; k = 0.175 cm/min. What should be the area of the standpipe for the head to drop from 650 cm to 300 cm in 8 min?

Ans: 0.31 cm^2

7.3 A sand layer of the cross-sectional area shown in Figure (7.14) has been determined to exist for a 800-m length of the levee. The hydraulic conductivity of the sand layer is 2.8 m/day. Determine the quantity of water which flows into the ditch in m^3/min.

Ans: 0.206 m^3/min

7.4 A permeable soil layer is underlain by an impervious layer, as shown in Figure (7.15). With $k = 5.2 \times 10^{-4}$ cm/sec for the permeable layer, calculate the rate of seepage through it in m^3/hr/m length if H = 3.8 m and α = 8.

Ans: 0.7739×10^{-6} m^3/hr/m

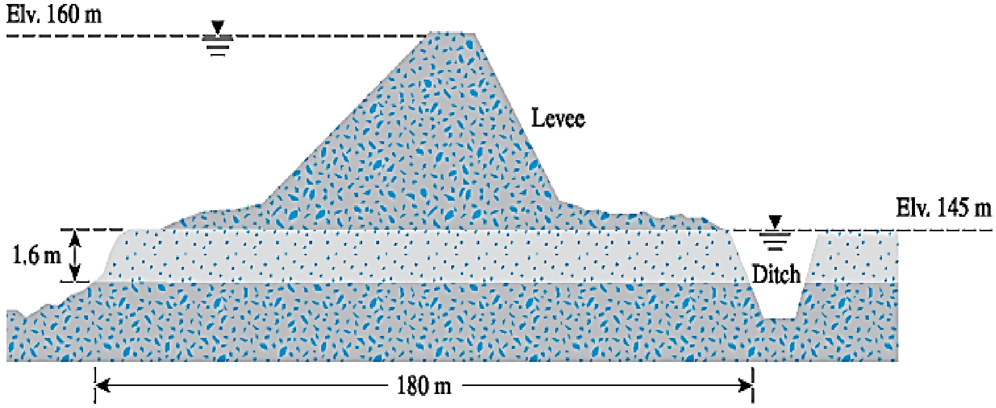


Figure (7.14)

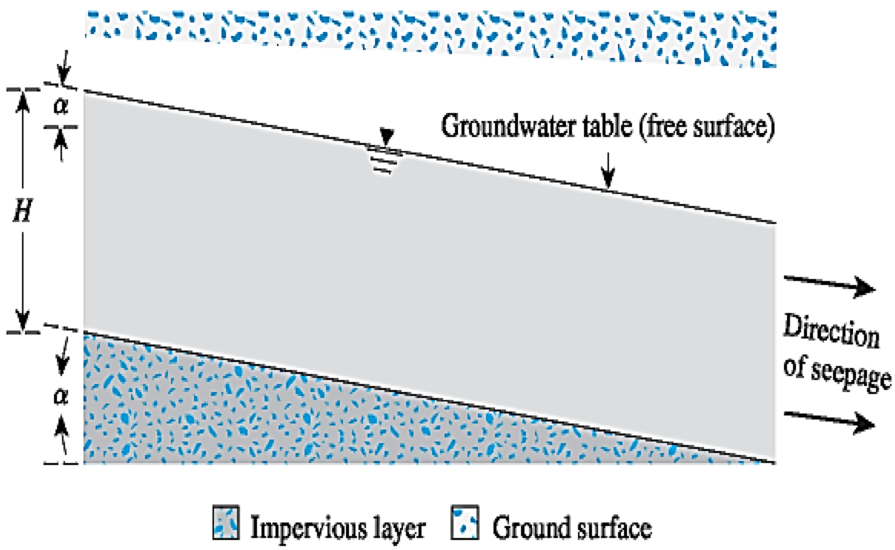


Figure (7.15)

7.5 Refer to Figure (7.16). Find the flow rate in m^3/sec/m length (at right angle to the cross section shown) through the permeable soil layer. Given: H = 5 m, H_1 =2.8 m, h = 3.1 m, L = 60 m, α = 5, k = 0.05 cm/sec.

Ans: 7.18×10^{-5} m^3/sec/m

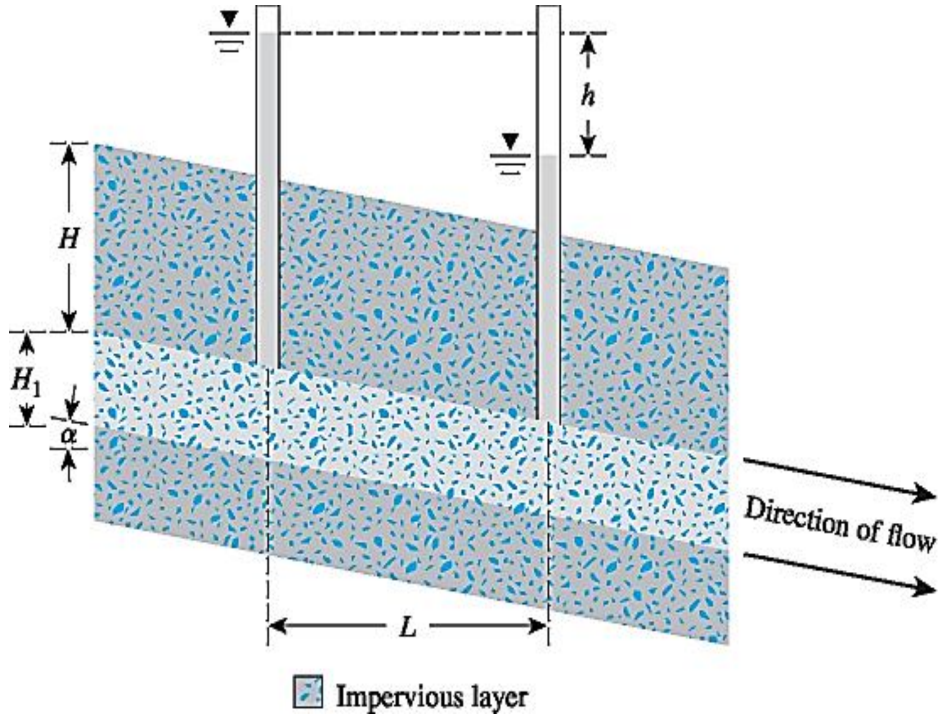


Figure (7.16)

7.6 A layered soil is shown in Figure (7.17). Given that

*H_1= 1.5m * $k_1 = 10^{-5}$ cm/sec

*H_2= 2.5m * $k_2 = 3.0\times10^{-3}$ cm/sec

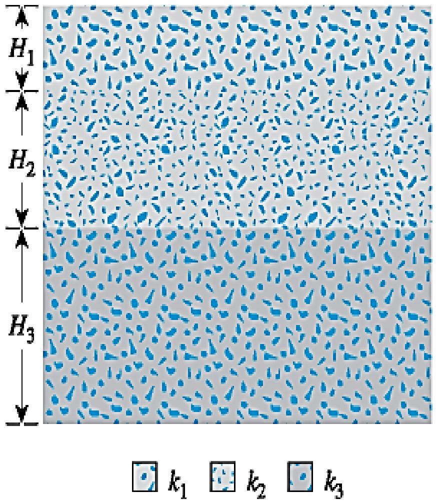
*H_3= 3.0m * $k_2 = 3.5\times10^{-5}$ cm/sec

Estimate the ratio of equivalent hydraulic conductivity, $\frac{k_{H(eq)}}{k_{v(eq)}}$

Ans: 0.3684

7.7 A layered soil is shown in Figure (7.18). Estimate the ratio of equivalent hydraulic conductivity, $\frac{k_{H(eq)}}{k_{v(eq)}}$

Ans: 3.53



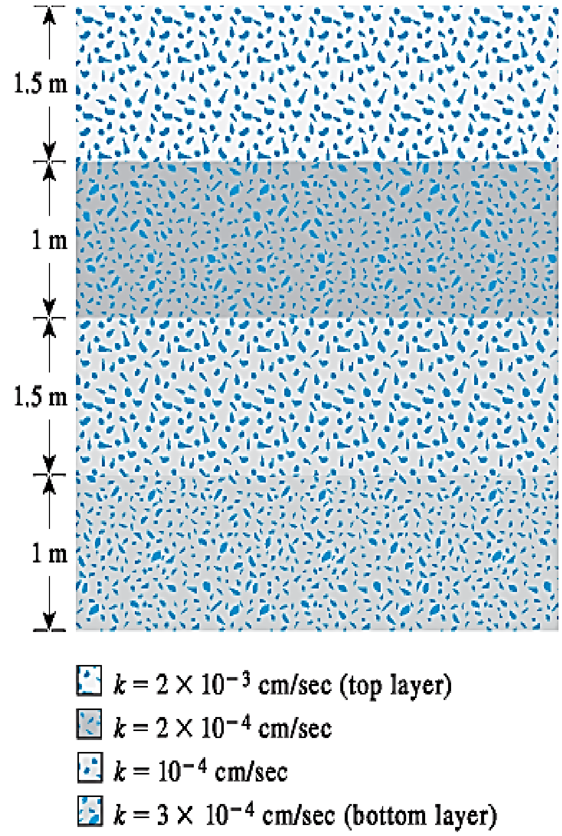


Figure (7.17) Figure (7.18)

Chapter 8: Seepage

In the preceding chapter, we considered some simple cases for which direct application of Darcy's law was required to calculate the flow of water through soil. In many instances, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow. In such cases, the groundwater flow is generally calculated by the use of graphs referred to as flow nets. The concept of the flow net is based on Laplace's equation of continuity, which governs the steady flow condition for a given point in the soil mass. In the following sections of this chapter, the derivation of Laplace's equation of continuity will be presented along with its application to seepage problems.

8.1 Laplace's Equation of Continuity

To derive the Laplace differential equation of continuity, let us consider a single row of sheet piles that have been driven into a permeable soil layer, as shown in Figure (8.1a). The row of sheet piles is assumed to be impervious. The steady state flow of water from the upstream to the downstream side through the permeable layer is a two-dimensional flow. For flow at a point A, we consider an elemental soil block. The block has dimensions *dx*, *dy*, and *dz* (length *dy* is perpendicular to the plane of the paper); it is shown in an enlarged scale in Figure (8.1b). Let v_x and v_z be the components of the discharge velocity in the horizontal and vertical directions, respectively. The rate of flow of water into the elemental block in the horizontal direction is equal to $v_x\, dz\, dy$, and in the vertical direction it is $v_z\, dx\, dy$. The rates of outflow from the block in the horizontal and vertical directions are, respectively,

$$\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dzdy$$

and

$$\left(v_z + \frac{\partial v_z}{\partial z} dz\right) dxdy$$

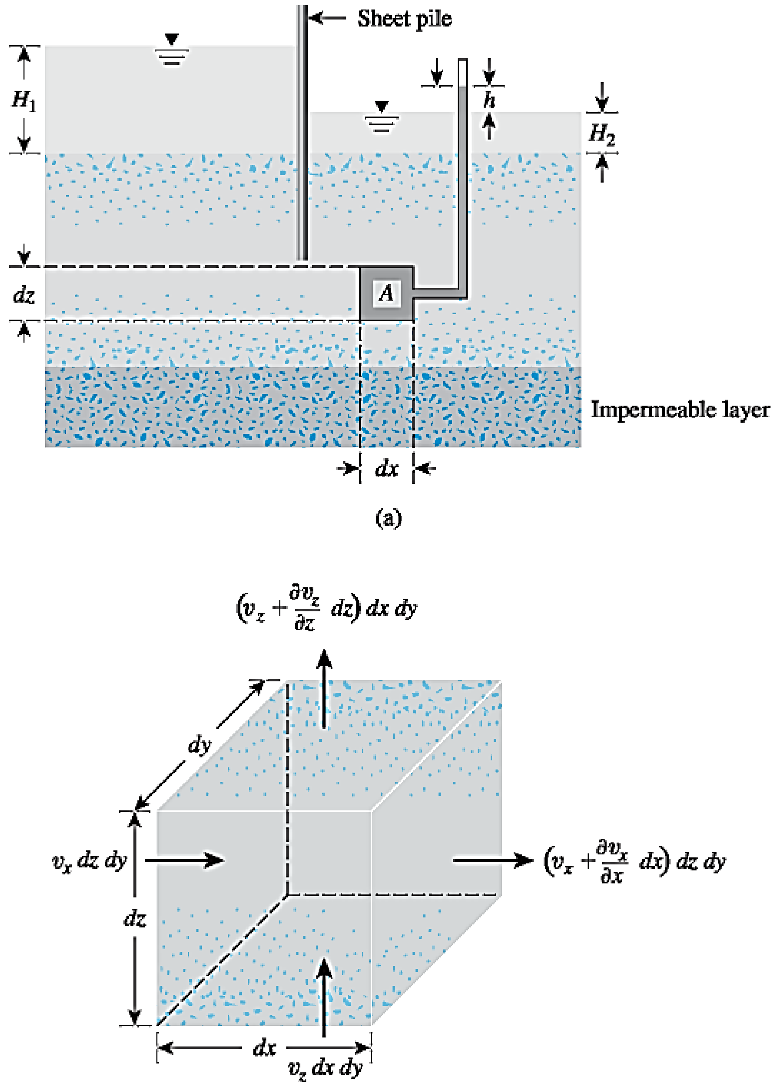


Figure 8.1 (a) Single-row sheet piles driven into permeable layer; (b) flow at A

Assuming that water is incompressible and that no volume change in the soil mass occurs, we know that the total rate of inflow should equal the total rate of outflow. Thus,

$$\left[\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dzdy + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dxdy\right] - [v_x dzdy + v_z dxdy] = 0$$

or

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0 \qquad (8.1)$$

With Darcy's law, the discharge velocities can be expressed as

$$v_x = k_x i_x = k_x \frac{\partial h}{\partial x} \qquad (8.2)$$

and

$$v_z = k_z i_z = k_z \frac{\partial h}{\partial z} \qquad (8.3)$$

where k_x and k_z are the hydraulic conductivities in the horizontal and vertical directions, respectively.

From Eqs. (8.1), (8.2), and (8.3), we can write

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \qquad (8.4)$$

If the soil is isotropic with respect to the hydraulic conductivity—that is, $k_x = k_z$ —the preceding continuity equation for two-dimensional flow simplifies to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad (8.5)$$

8.2 Flow Nets

The continuity equation [Eq. (8.5)] in an isotropic medium represents two orthogonal families of curves—that is, the flow lines and the equipotential lines. *A flow line* is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium. An *equipotential line* is a line along which the potential head at all points is equal. Thus, if piezometers are placed at different points along an equipotential line, the water level will rise to the same elevation in all of them. Figure (8.2a) demonstrates the

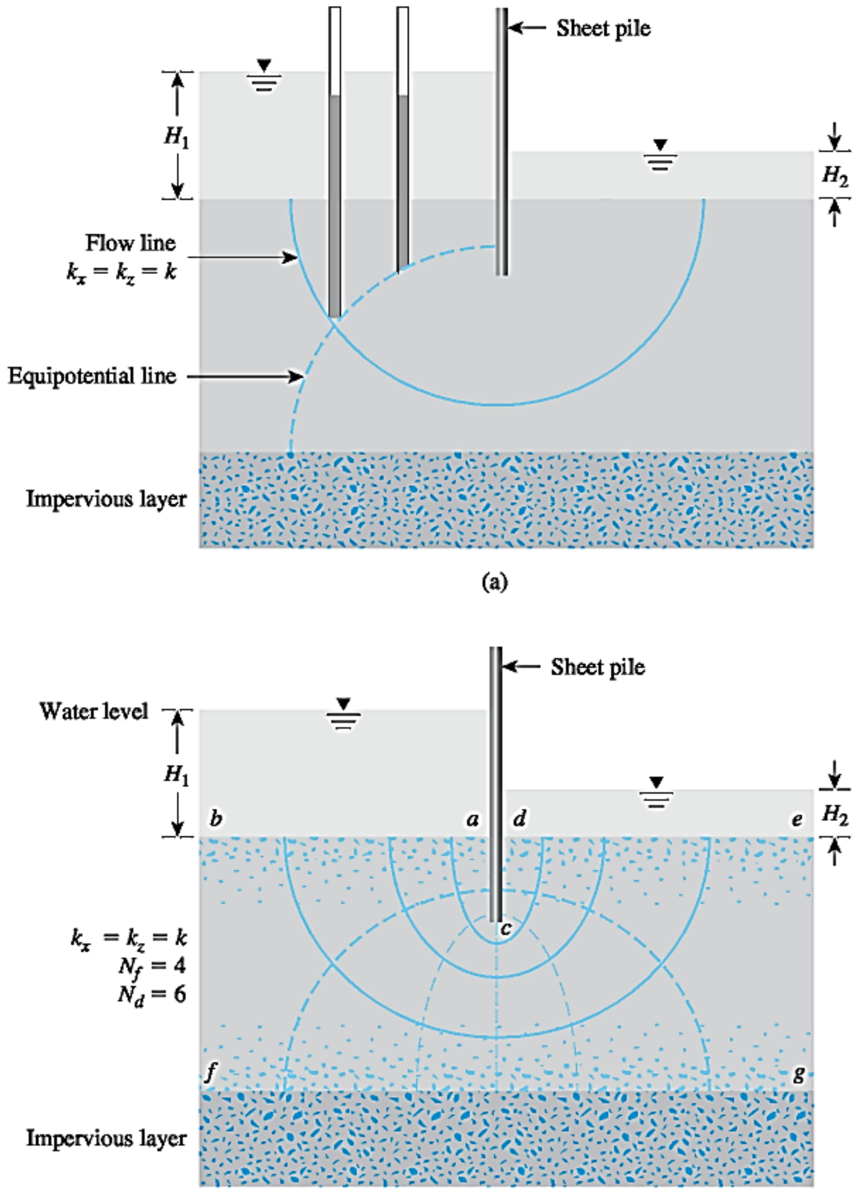


Figure 8.2 (a) Definition of flow lines and equipotential lines; (b) completed flow net

definition of flow and equipotential lines for flow in the permeable soil layer around the row of sheet piles shown in Figure (8.1) (for $k_x = k_z = k$).

A combination of a number of flow lines and equipotential lines is called a *flow net*. As mentioned in the introduction, flow nets are constructed for the calculation of groundwater flow and the evaluation of heads in the media. To complete the graphic construction of a flow net, one must draw the flow and equipotential lines in such a way that

1. The equipotential lines intersect the flow lines at right angles.
2. The flow elements formed are approximate squares.

Figure (8.2b) shows an example of a completed flow net. One more example of flow net in isotropic permeable layer are given in Figure (8.3). In these figures, N_f is the number of flow channels in the flow net, and N_d is the number of potential drops (defined later in this chapter).

Drawing a flow net takes several trials. While constructing the flow net, keep the boundary conditions in mind. For the flow net shown in Figure (8.2b), the following four boundary conditions apply:

Condition 1: The upstream and downstream surfaces of the permeable layer (lines ***ab*** and ***de***) are equipotential lines.

Condition 2: Because ***ab*** and ***de*** are equipotential lines, all the flow lines intersect them at right angles.

Condition 3: The boundary of the impervious layer—that is, line ***fg***—is a flow line, and so is the surface of the impervious sheet pile, line ***acd***.

Condition 4: The equipotential lines intersect ***acd*** and ***fg*** at right angles.

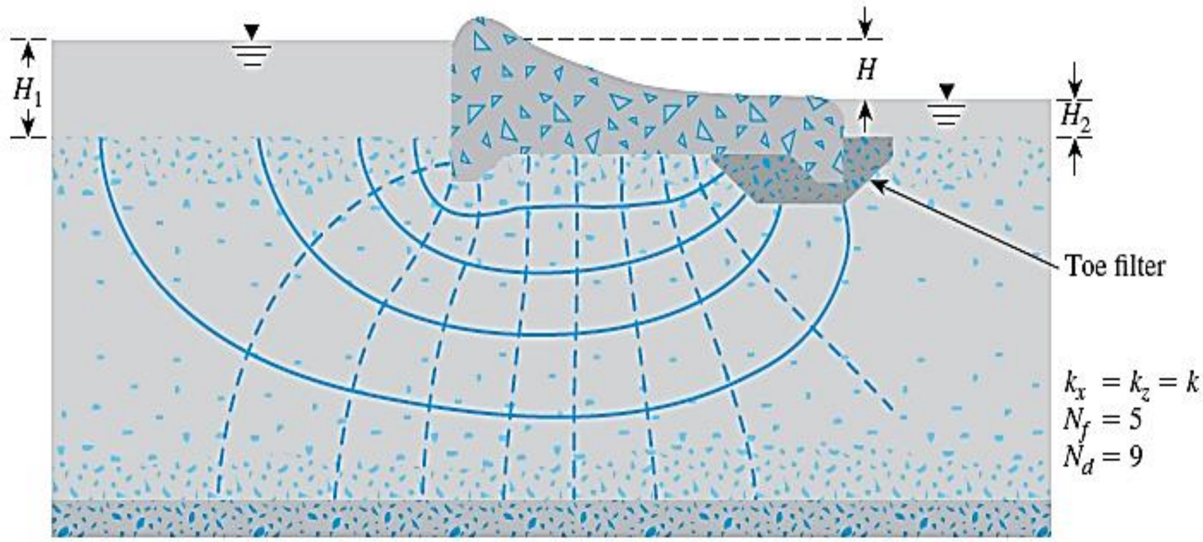


Figure 8.3 Flow net under a dam with toe filter

8.3 Seepage Calculation from a Flow Net

In any flow net, the strip between any two adjacent flow lines is called a *flow channel*. Figure (8.4) shows a flow channel with the equipotential lines forming square elements. Let h_1, h_2, h_3, h_4,.. ., h_n be the piezometric levels corresponding to the equipotential lines. The rate of seepage through the flow channel per unit length (perpendicular to the vertical section through the permeable layer) can be calculated as follows. Because there is no flow across the flow lines,

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \cdots = \Delta q \tag{8.6}$$

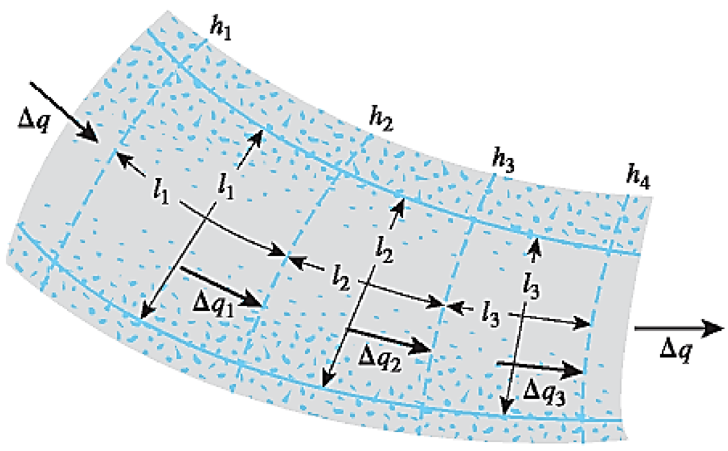


Figure 8.4 Seepage through a flow channel with square elements

From Darcy's law, the flow rate is equal to *kiA*. Thus, Eq. (8.6) can be written as

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right) l_1 = k\left(\frac{h_2 - h_3}{l_2}\right) l_2 = k\left(\frac{h_3 - h_4}{l_3}\right) l_3 = \cdots \tag{8.7}$$

Eq. (8.7) shows that if the flow elements are drawn as approximate squares, the drop in the piezometric level between any two adjacent equipotential lines is the same. This is called the *potential drop*. Thus,

$$\Delta h = h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \cdots = \frac{H}{N_d} \tag{8.8}$$

and

$$\Delta q = k\frac{H}{N_d} \tag{8.9}$$

where H = head difference between the upstream and downstream sides
N_d = number of potential drops

In Figure (8.2b), for any flow channel, $H = H_1 - H_2$ and $N_d = 6$.

If the number of flow channels in a flow net is equal to N_f, the total rate of flow through all the channels per unit length can be given by

$$q = k\frac{HN_f}{N_d} \qquad (8.10)$$

8.4 Hydraulic Gradient

The hydraulic gradient over each square in flow net can be calculated by dividing the head loss, Δh, by the length, l, of the cell, that is:

$$i = \frac{\Delta h}{l} \qquad (8.11)$$

Where, $$\Delta h = \frac{H}{N_d}$$

Since l is not constant, the hydraulic gradient is not constant. The maximum hydraulic gradient occurs where l is a minimum, that is:

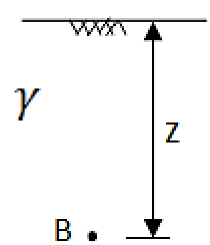
$$i_{max} = \frac{\Delta h}{l_{min}} \qquad (8.12)$$

Where l_{min} is the minimum length of the cells within the flow domain. Usually, l_{min} occurs at exit points or around corners.

8.5 Seepage Stress and Seepage Force

The effective stress at a given point in a saturated soil is given by:

$$\acute{\sigma}_z = \sigma_z - u_z = \gamma.z - \gamma_w.z = (\gamma - \gamma_w) = \acute{\gamma}.z = \gamma_{sub}.z$$



If water is seeping, the effective stress at any point in a soil mass will differ from that in the static case. It will increase or decrease, depending on the direction of seepage. The seepage force per unit volume is equal to: $\gamma_w.i$. This force produces a stress with a soil mass at depth z equal to: $\gamma_w.i.z$. Thus, the effective stress will be:

$$\acute{\sigma}_z = \acute{\gamma}.z \pm \gamma_w.i.z \qquad (8.13)$$

\+ for downward seepage.
− for upward seepage.

8.6 Static Liquefaction, Heaving, Boiling and Piping

If the effective stress becomes zero, the soil loses its intergranular frictional strength and behaves like a viscous fluid. The soil state at which the effective stress is zero is called static liquefaction (boiling, quicksand, piping or heaving).

$$\acute{\sigma}_z = \acute{\gamma}.z - \gamma_w.i.z = 0 \qquad (8.14)$$

High localized hydraulic gradient statically liquefies the soil, which progresses to the water surface in the form of a pipe, and water then rushes beneath the structure through the pipe, leading to instability and failure.

8.7 Critical Hydraulic Gradient

The hydraulic gradient that brings a soil mass to static liquefaction is called the critical hydraulic gradient:

$$\acute{\sigma}_z = \acute{\gamma}.z - \gamma_w.i.z = 0$$

$$\Rightarrow i = i_{cr} = \frac{\overline{\gamma}}{\gamma_w} = \left(\frac{G_s-1}{1+e}\right)\frac{\gamma_w}{\gamma_w} = \frac{G_s-1}{1+e} \qquad (8.15)$$

Since G_s is constant, the critical hydraulic gradient is a function of the void ratio. In designing structures that are subjected to steady state seepage, it is absolutely essential to ensure that the critical hydraulic gradient cannot develop. The factor of safety against boiling is defined as:

$$(F.S)_{boiling} = \frac{i_{cr}}{i_{exit}} \qquad (8.16)$$

8.8 Uplift Pressure under Hydraulic Structures

The pore water pressure at any point j is calculated as follows:

$$u_j = (h_p)_j \gamma_w \qquad (8.17)$$

where $(h_p)_j$ is the pressure head at point j given by:

$$(h_p)_j = H - (N_d)_j.\Delta h - z_j \qquad (8.18)$$

Example 8.1

A flow net for flow around a single row of sheet piles in a permeable soil layer is shown in Figure (8.5). Given that $k_x = k_z = k = 5 \times 10^{-3}$ cm/sec, determine

a. How high (above the ground surface) the water will rise if piezometers are placed at points *a* and *b*.
b. The total rate of seepage through the permeable layer per unit length
c. The approximate average hydraulic gradient at *c*.

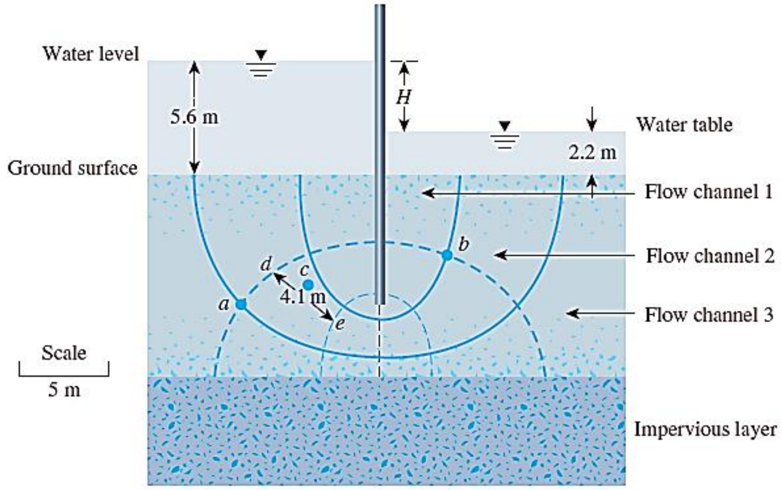


Figure 8.5 Flow net for seepage around a single row of sheet piles

Solution: Part a

From Figure (8.5), we have N_d =6, H_1=5.6m, and H_2=2.2 m. So the head loss of each potential drop is

$H = H_1 - H_2 = 5.6 - 2.2 = 3.4m$

$$\Delta h = \frac{H}{N_d} = \frac{3.4}{6} = 0.567m$$

At point a, we have gone through one potential drop. So the water in the piezometer will rise to an elevation of

(5.6- 0.567) = **5.033 m above the ground surface**

At point b, we have five potential drops. So the water in the piezometer will rise to an elevation of

(5.6- 5 x 0.567) = **2.765 m above the ground surface**

Part b

From Eq. (8.10),

$$q = k\frac{HN_f}{N_d} = (5 \times 10^{-5})(3.4)\frac{2.38}{6} = 6.74 \times 10^{-5}\ m^3/sec/m$$

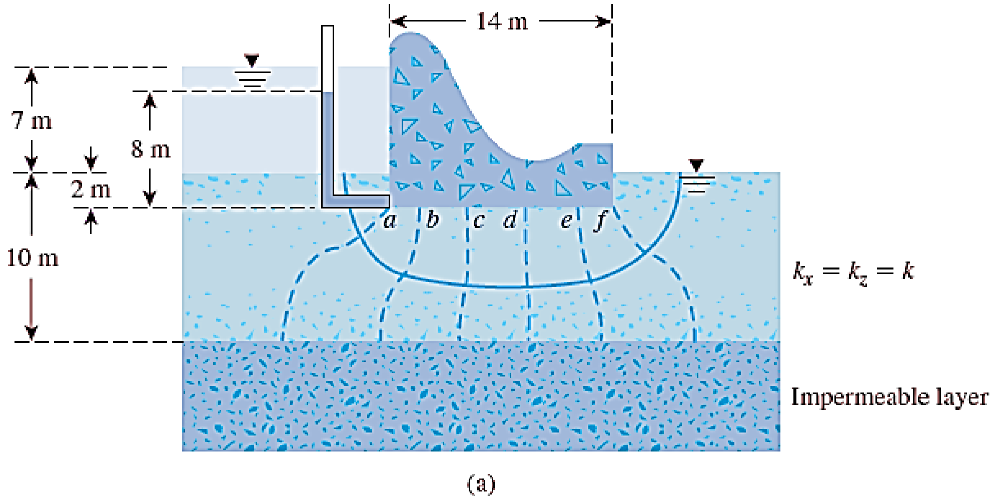
Part c

The average hydraulic gradient at c can be given as

$$i = \frac{head\ loss}{average\ length\ of\ flow\ between\ d\ and\ e} = \frac{\Delta h}{\Delta l} = \frac{0.567m}{4.1m} = 0.138$$

(Note: The average length of flow has been scaled)

Example 8.2
Draw the uplift pressure distribution under the hydraulic structure shown.



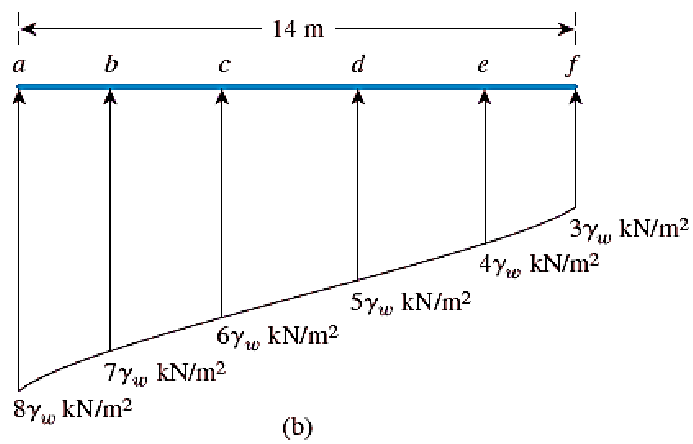


Figure 8.6 (a) A weir; (b) uplift force under a hydraulic structure

H = 7-0 = 7m

$N_d = 7$, $\Delta h = \frac{H}{N_d} = \frac{7}{7} = 1.0$ m

point	z_j	$(N_d)_j$	$(h_p)_j = H - (N_d)_j.\Delta h - z_j$	$U_j = (h_p)_j.\gamma_w$
a	-2	1	8	$8\gamma_w$
b	-2	2	7	$7\gamma_w$
c	-2	3	6	$6\gamma_w$
d	-2	4	5	$5\gamma_w$
e	-2	5	4	$4\gamma_w$
f	-2	6	3	$3\gamma_w$

Example 8.3

A dam, shown in Fig.(8.7), retains 10 m of water. A sheet pile wall (cutoff curtain) on the up steam side, which is used to reduce seepage under the dam, penetrates 7 m into a 20.3 m thick silty sand stratum. Below the silty sand is a thick deposit of clay. The average hydraulic conductivity of the silty sand is 2.0×10^{-4} cm/sec. Assume that the silty sand is homogeneous and isotropic.

a. Draw the flownet under the dam
b. Calculate the flow, *q*.
c. Calculate and draw the porewater pressure distribution at the base of the dam.
d. Determine the uplift force.
e. Determine and draw the porewater pressure distribution on the upstream and downstream faces of the sheet pile wall.

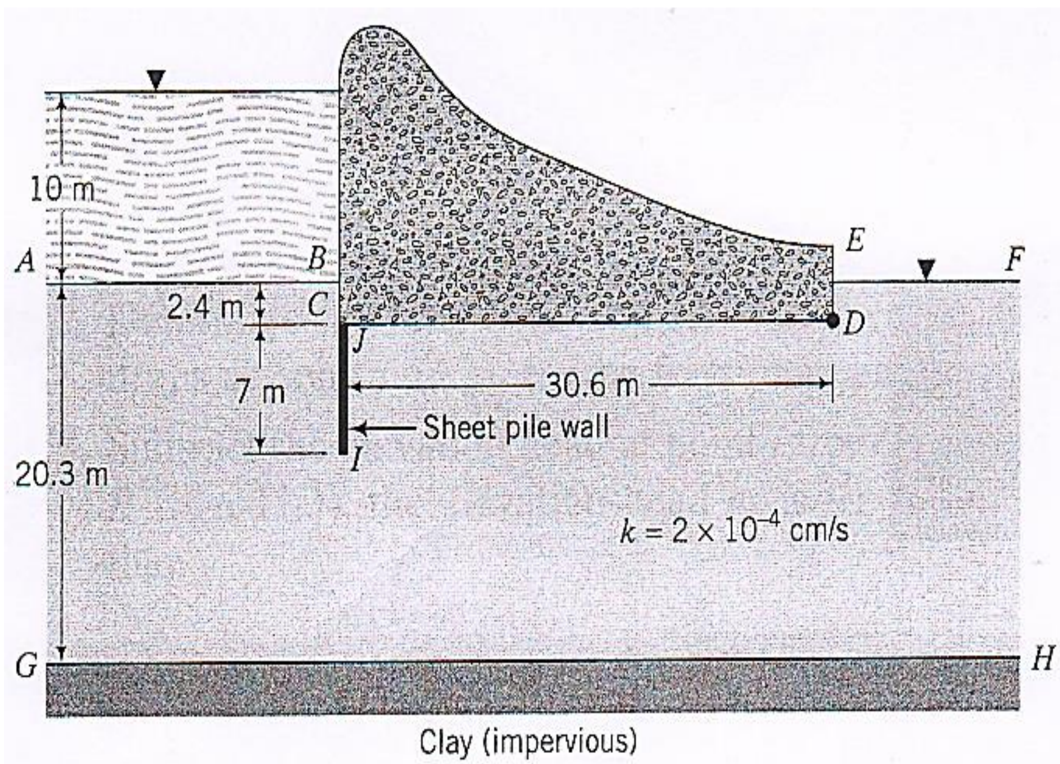


Figure 8.7

f. Determine the resultant lateral force on the sheet pile wall due to the powerwater.
g. Determine the maximum hydraulic gradient.
h. Will piping occur if the void ratio of the silty sand is 0.8.
i. What is the effect of reducing the depth of penetration of the sheet pile wall?

Solution

Step 1: Draw the dam to scale (see Fig. 8.8).

Step 2: Identify the impermeable and permeable boundaries.

With reference to Fig.(8.7), *AB* and *EF* are permeable boundaries and are therefore equipotential lines. *BCIJDE* and *GH* are impermeable boundaries and are therefore flow lines.

Step 3: Sketch the flownet.

Draw about three flow lines and then draw a suitable number of equipotential lines. Remember that flow lines are orthogonal to equipotential lines and the area between two consecutive flow lines and two consecutive equipotential lines is approximately a square. Use a circle template to assist you in estimating the square. Adjust/add/subtract flow lines and equipotential lines until you are satisfied that the flownet consists essentially of curvilinear squares. See sketch of flownet in Fig. (8.8).

Step 4: Calculate the flow.

Select the downstream end, *EF*, as the datum.

H= 10 m

N_d = 14, N_f = 4

$$q = k\frac{HN_f}{N_d} = 2\times10^{-4}\times(10\times10^2)\times\frac{4}{14} = 0.057\ cm^3/sec$$

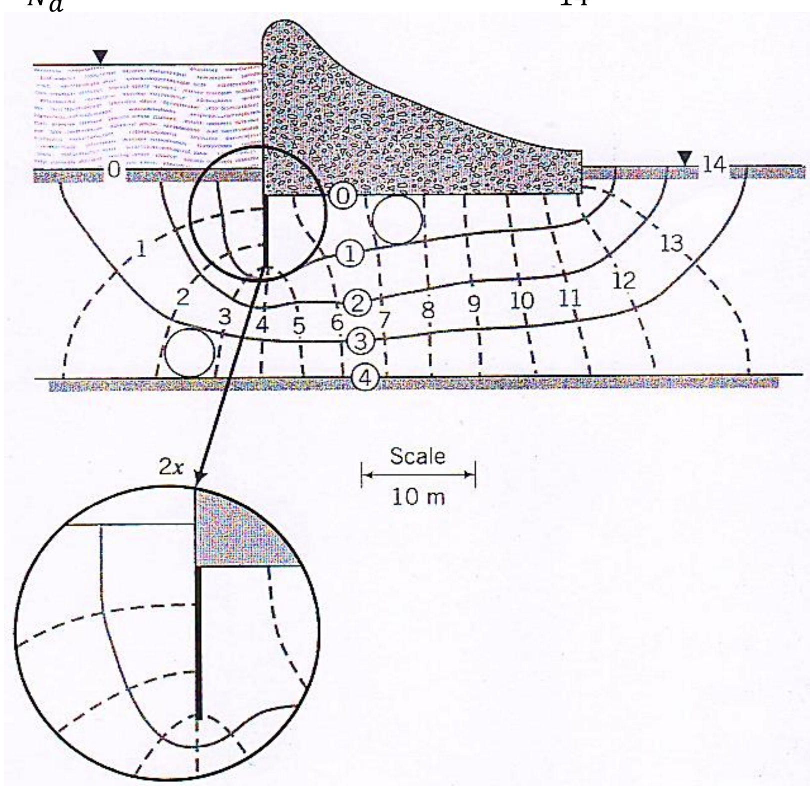


Figure (8.8)

Step 5: determine the porewater pressure under the base of the dam.

Divide the base into a convenient number of equal intervals. Let us use 10 intervals; that is

$$\Delta x = \frac{30.6}{10} = 3.06m$$

Determine the porewater pressure at each nodal point. Use a table for convenience or, better yet, use a spreadsheet.

$$\Delta h = \frac{H}{N_d} = \frac{10}{14} = 0.714m$$

The calculation in the table below was done using a spreadsheet program.

Flow under a dam: Δh = 0.714 m

Parameters	Under base of dam										
x (m)	0	3.06	6.12	9.18	12.24	15.3	18.36	21.42	24.48	27.54	30.6
N_d (m)	5.60	5.80	6.20	6.90	7.40	8.00	8.80	9.40	10.30	11.10	12.50
$N_d \Delta h$ (m)	4.00	4.14	4.43	4.93	5.28	5.71	6.28	6.71	7.35	7.93	8.93
z (m)	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40	−2.40
h_p (m) = $H - N_d \Delta h - z$	8.40	8.26	7.97	7.47	7.12	6.69	6.12	5.69	5.05	4.47	3.48
u (kPa) = $h_p \gamma_w$	82.3	80.9	78.1	73.2	69.7	65.5	59.9	55.7	49.4	43.9	34.1

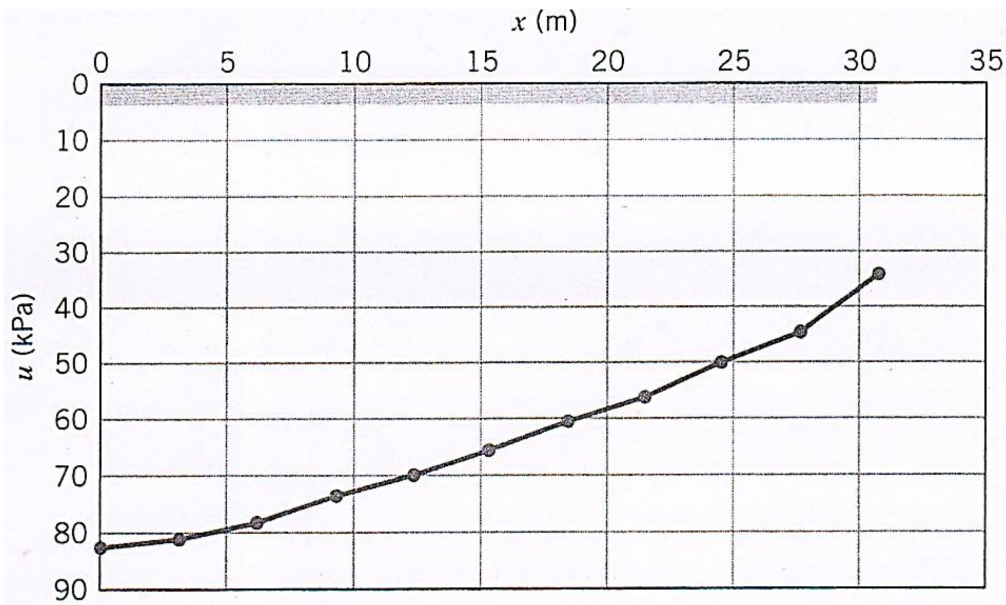


Figure (8.9)

Plot the porewater pressure distribution. See Fig. (8.9)

Step 6: Calculate the uplift force and its location.

Using Simpson's rule, we find

$$A = \frac{\Delta x}{3}\left[y_0 + y_n + 2\sum_{i=2,4,..} y_i + 4\sum_{i=1,3,..} y_i\right]$$

$$\boldsymbol{P_w = \frac{3.06}{3}[82.3 + 34.1 + 2(78.1 + 69.7 + 59.9 + 49.4) + 4(80.9 + 73.2 + 65.5 + 55.7 + 43.9)]}$$

= 1946.4 kN/m

Step 7: Determine the pore water pressure distribution on the sheet pile wall. Divide the front face of the wall into six intervals of 7/6=1.17 m and the back face into one interval. Six intervals were chosen because it is convenient for the scaling using the scale that was used to draw the flownet. The greater the intervals, the greater the accuracy. Only one interval is used for the back face of the wall because there are no equipotential lines that meet there. Use a spreadsheet to compute the porewater pressure distribution and the hydrostatic forces. The distributions of porewater pressure at the front and back of the wall are shown in Figs.(8.10). Use Simpson's rule to calculate the hydrostatic force on the front face of the wall. The porewater pressure distribution at the back face is a trapezoid and the area is readily calculated.

Parameters	Front of wall							Back of wall	
x (m)	0	1.17	2.33	3.50	4.67	5.83	7.00	7.00	0.00
N_d (m)	0.70	1.00	1.30	1.60	1.90	2.40	3.00	5.00	5.60
$N_d \Delta h$ (m)	0.50	0.71	0.93	1.14	1.36	1.71	2.14	3.57	4.00
z (m)	−2.40	−3.57	−4.73	−5.90	−7.07	−8.23	−9.40	−9.40	−2.40
h_p (m) = $H - N_d \Delta h - z$	11.90	12.85	13.81	14.76	15.71	16.52	17.26	15.83	8.40
u (kPa) = $h_p \gamma_w$	116.6	126.0	135.3	144.6	154.0	161.9	169.1	155.1	82.3
	Front	Back	Difference						
P_w (kN/m)	1011.7	830.9	180.8						

Step 8: determine the maximum hydraulic gradient.

The smallest value of l occurs at the exit. By measurement, l_{min} = 2 m

$$i_{max} = \frac{\Delta h}{l_{min}} = \frac{0.714}{2} = 0.36$$

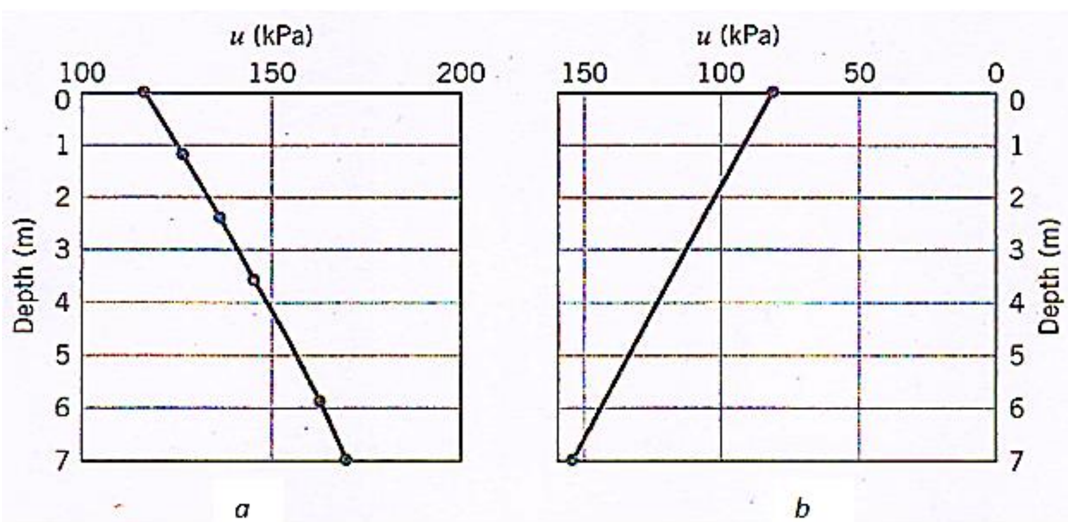


Figure 8.10 porewater pressure distributions (a) at front of wall and (b) at back of wall

Step 9: Determine if piping would occure.

Equation (8.15): $i_{cr} = \frac{G_S - 1}{1+e} = \frac{2.7-1}{1+0.8} = 0.89$

Since $\boldsymbol{i_{max}} < \boldsymbol{i_{cr}}$, piping will not occur.

Factor of safety against piping: $\frac{0.89}{0.36} = 2.5$

Step 10: State the effect of reducing the depth of penetration of the sheet pile wall. If the depth is reduced, the value of $\boldsymbol{\Delta h}$ increases and $\boldsymbol{i_{max}}$ is likely to increase.

Example 8.4

A bridge pier is to be constructed in a river bed by constructing a cofferdam as shown in Fig.(8.11 a). A cofferdam is temporary enclosure consisting of long, slender elements of steel, concrete, or timber members to support the sides of enclosure. After constructing the cofferdam, the water within it will be pumped out. Determine (a) the flow net rate using $k = 1\times10^{-4}$ cm/sec and (b) the factor of safety against piping. The void ratio of the sand is 0.59.

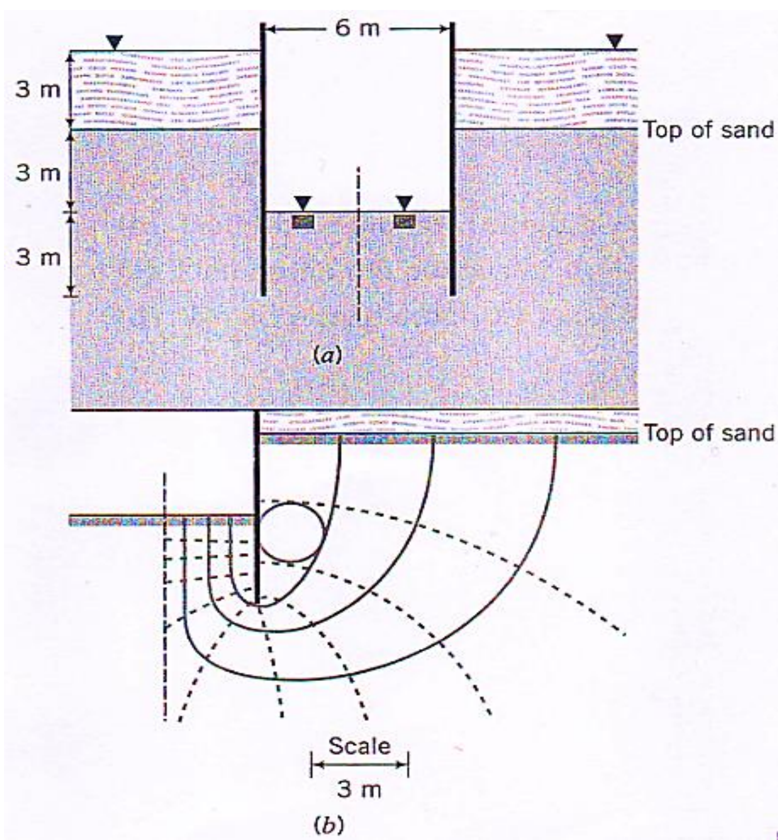


Figure (8.11)

Solution:

Step 1: draw the cofferdam to scale and sketch the flownet. [see Fig.8.11 b]

Step 2: Determine the flow.

$$H = 6\text{m}; \quad N_f = 4, \quad N_d = 10$$

$$q = 2k\frac{HN_f}{N_d} = 2 \times 1 \times 10^{-4} \times 10^{-2} \times 6 \times \frac{4}{10} = 4.8 \times 10^{-6}\ cm^3/sec$$

(Note: the factor 2 is needed because you have to consider both halves of the structure; the factor 10^{-2} is used to convert cm/sec to m/sec).

Step 3: Determine the maximum hydraulic gradient**.**

$l_{min} \approx 0.3$ m (this is an average value of the flow length at the exit of the sheet pile)

$$i_{max} = \frac{\Delta h}{l_{min}} = \frac{H}{N_d l_{min}} = \frac{6}{10\times 0.3} = 2$$

Step 4: Calculate the critical hydraulic gradient.

$$i_{cr} = \frac{G_S - 1}{1+e} = \frac{2.7-1}{1+0.59} = 1.07$$

Since $\boldsymbol{i_{max}} > \boldsymbol{i_{cr}}$, piping is likely to occur; the factor of safety is 1.07/2 ≈ 0.5

8.9 Flow Nets in Anisotropic Soil

In nature, most soils exhibit some degree of anisotropy. To account for soil anisotropy with respect to hydraulic conductivity, we must modify the flow net construction.

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

or

$$\frac{\partial^2 h}{\partial x^2} + \frac{k_z}{k_x}\frac{\partial^2 h}{\partial z^2} = 0$$

Let;

$$c = \sqrt{k_z/k_x} \qquad (8.19)$$

$$\acute{x} = cx \qquad (8.20)$$

$$\Rightarrow \quad \frac{\partial \acute{x}}{\partial x} = c$$

and

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial \acute{x}} . \frac{\partial \acute{x}}{\partial x} = c\frac{\partial h}{\partial \acute{x}}$$

$$\frac{\partial^2 h}{\partial x^2} = \frac{\partial}{\partial \acute{x}}\left(\frac{\partial h}{\partial x}\right) . \frac{\partial \acute{x}}{\partial x} = c^2 \frac{\partial^2 h}{\partial \acute{x}^2}$$

Substitute:

$$c^2 \frac{\partial^2 h}{\partial \acute{x}^2} + c^2 \frac{\partial^2 h}{\partial z^2} = 0 \qquad \Rightarrow \qquad \frac{\partial^2 h}{\partial \acute{x}^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

To find the equivalent hydraulic conductivity (k_e), we have;

$$v_x = -k_e \frac{\partial h}{\partial \acute{x}} = -k_x \frac{\partial h}{\partial x}$$

But, $$\frac{\partial h}{\partial x} = c\frac{\partial h}{\partial \acute{x}}$$

Thus: $$k_e = k_x . c \qquad \Rightarrow \quad k_e = \sqrt{k_x . k_z} \qquad (8.21)$$

and: $$q = k_e \frac{HN_f}{N_d}$$

To construct the flow net for anisotropic soil, use the following procedure:

Step 1: Adopt a vertical scale (that is, z axis) for drawing the cross section.

Step 2: Adopt a horizontal scale (that is, x axis) such that horizontal scale = $\sqrt{k_z/k_x} \times vertical\ scale.$

Step 3: With scales adopted as in Step 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.

Step 4: Draw the flow net for the permeable layer on the section obtained from Step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

Example 8.5

A dam section is shown in Figure (8.12a). The hydraulic conductivity of the permeable layer in the vertical and horizontal directions are 2×10^{-2} mm/s and 4×10^{-2} mm/s, respectively. Draw a flow net and calculate the seepage loss of the dam in m^3/day/m.

Solution

From the given data,

$k_z = 2\times10^{-2}$ mm/sec = 1.73 m/day

$k_x = 4\times10^{-2}$ mm/sec = 3.46 m/day

And H = 20m. For drawing the flow net,

$$c = \sqrt{k_z/k_x} = \sqrt{\frac{2\times10^{-2}}{4\times10^{-2}}} = \frac{1}{\sqrt{2}}$$

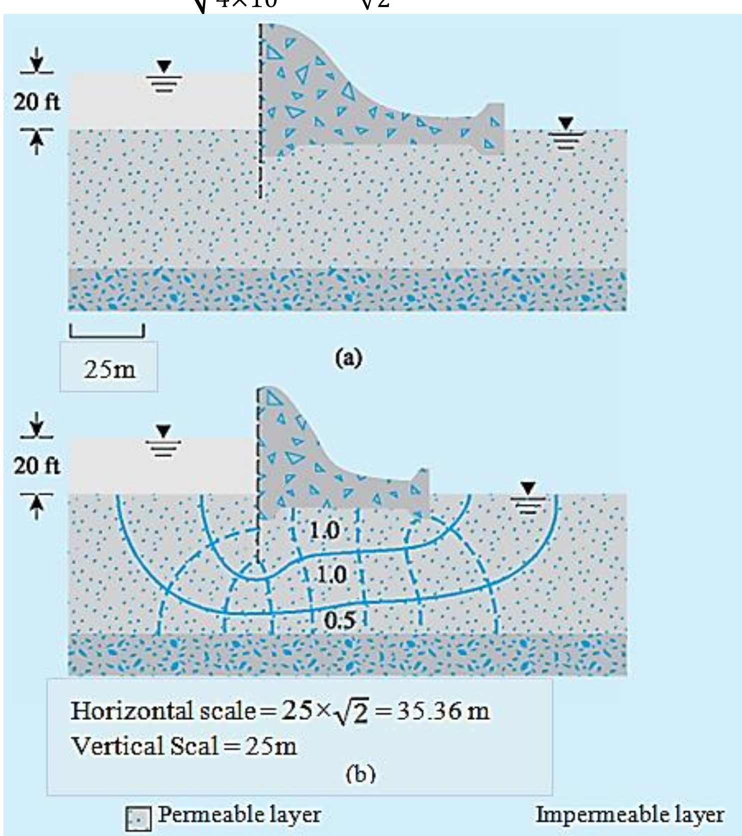


Figure (8.12)

On the basis of this, the dam section is replotted, and the flow net drawn as in Figure (8.12b). The rate of seepage is given by $q = \sqrt{k_x k_z}\frac{HN_f}{N_d}$ .From Figure (8.12b), N_d = 8 and N_f = 2.5 (the lowermost flow channel has a width-to-length ratio of 0.5). So,

$$q = \sqrt{(1.73)(3.46)}\left(\frac{20\times2.5}{8}\right) = 15.3 \quad m^3/day/m$$

8.10 Seepage through an Earth Dam

Flow through earth dams is an important design consideration. We need to ensure that the porewater pressure at the downstream end of the dam will not lead to instability and the exit hydraulic gradient does not lead to piping. The major exercise is to find the top flow line called the "phreatic surface". The pressure head on the phreatic surface is zero, Figure (8.13).

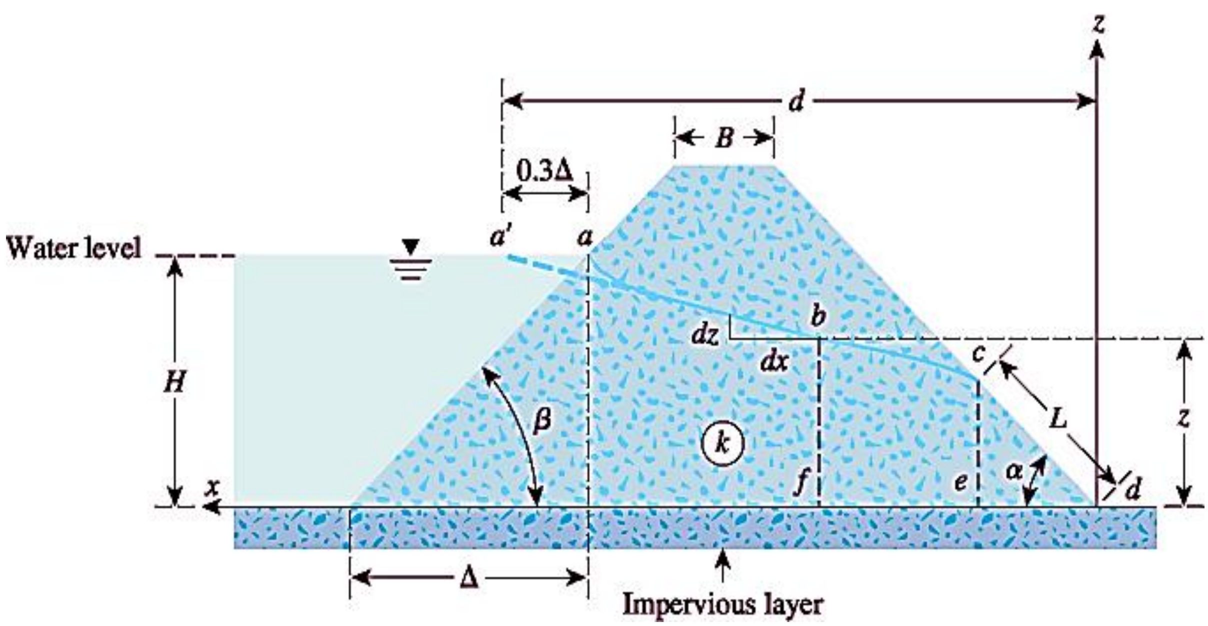


Figure (8.13) Flow through an earth dam constructed over an impervious base

Casagrande (1937) showed that the phreatic surface can be approximated by a parabola. The slop of the free surface can be assumed to be equal to the hydraulic gradient (at *c*):

$$i \approx \frac{dz}{dx} \tag{8.22}$$

Considering the triangle *cde*, we can give the rate of seepage per unit length of the dam (at right angle to the cross section shown in Figure (8.13) as

$$q = kiA$$

$$i = \frac{dz}{dx} = \tan\alpha$$

$$A = (\overline{ce})(1) = L\sin\alpha$$

So

$$q = k(\tan\alpha)(L\sin\alpha) = kL\tan\alpha\sin\alpha \tag{8.23}$$

Again, the rate of seepage (per unit length of the dam) through the section *bf* is

$$q = kiA = k\left(\frac{dz}{dx}\right)(z \times 1) = kz\frac{dz}{dx} \tag{8.24}$$

For continuous flow,

$$q_{Eq.(8.23)} = q_{Eq.(8.24)}$$

or

$$kz\frac{dz}{dx} = kL\tan\alpha\sin\alpha$$

or

$$\int_{z=L\,sin\alpha}^{z=H} kzdz = \int_{x=L\cos\alpha}^{x=d} (kL\tan\alpha\sin\alpha)dx$$

$$\frac{1}{2}(H^2 - L^2sin^2\alpha) = L\tan\alpha\sin\alpha(d - L\cos\alpha)$$

$$\frac{H^2}{2} - \frac{L^2sin^2\alpha}{2} = Ld\left(\frac{sin^2\alpha}{\cos\alpha}\right) - L^2sin^2\alpha$$

$$\frac{H^2\cos\alpha}{2sin^2\alpha} - \frac{L^2\cos\alpha}{2} = Ld - L^2\cos\alpha$$

or

$$L^2\cos\alpha - 2Ld + \frac{H^2\cos\alpha}{sin^2\alpha} = 0$$

So,

$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{cos^2\alpha} - \frac{H^2}{sin^2\alpha}} \tag{8.25}$$

Following is a step-by-step procedure to obtain the seepage rate q (per unit length of the dam):

Step 1: Obtain α.
Step 2: Calculate Δ (see Figure 8.13) and then 0.3Δ.
Step 3: Calculate d.
Step 4: With known values of α and d, calculate L from Eq. (8.25).
Step 5: With known value of L, calculate q from Eq. (8.23).

Example 8.6

Refer to the earth dam shown in Figure 8.13. Given that β = 45°, α = 30°, B = 10 m, H = 20 m, height of dam = 25 m, and $k = 2 \times 10^{-4}$ m/min, calculate the seepage rate, q, in m^3/day/m length.

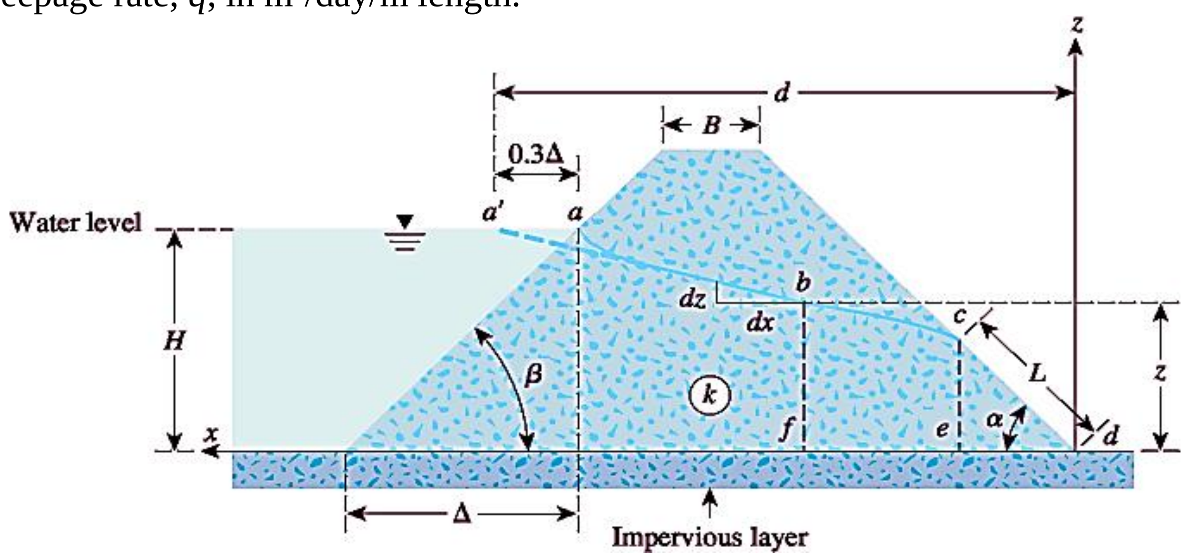


Figure (8.13)

Solution

We know that β = 45° and α= 30⁰. Thus,

$$\Delta = \frac{H}{\tan\beta} = \frac{20}{tan45^o} = 20m, \quad 0.3\Delta = (0.3)(20) = 6m$$

$$d = 0.3\Delta + \frac{(25-20)}{\tan\beta} + B + \frac{25}{tan\alpha}$$

$$d = 6 + \frac{(25-20)}{tan45^o} + 10 + \frac{25}{\tan 30^o} = 64.3\ m$$

From Eq. (8.25)

$$L = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{cos^2\alpha} - \frac{H^2}{sin^2\alpha}}$$

$$L = \frac{64.3}{\cos 30} - \sqrt{(\frac{64.3}{\cos 30})^2 - (\frac{20}{\sin 30})^2} = 11.7m$$

From Eq. (8.23)

$$q = kL\tan\alpha\sin\alpha = (2\times 10^{-4})(11.7)(\tan 30)(\sin 30)$$

$= 6.754\times 10^{-4}\ m^3/min/m = 0.973\ m^3/day/m$

8.11 Earth Dam with Drainage Blanket

Because the exit hydraulic gradient is often large, drainage blankets are used at the downstream end of dams to avoid piping. Seepage is controlled by the gradation of the coarse-grained soils used for the drainage blanket.

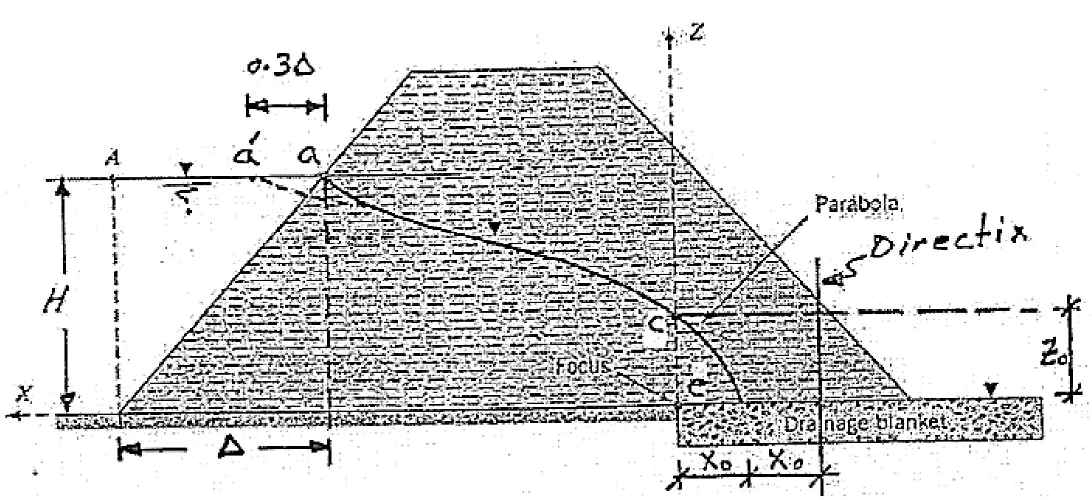


Figure (8.14) A horizontal drainage blanket at the toe of an earth dam

The flow through the dam shown in Figure (8.14) is; (through ce):

$$q = kiA = k\frac{dz}{dx}(z_o.1) \qquad (8.26)$$

From the geometry of the basic parabola: $z_o = 2x_o$ and the slope of the basic parabola $\{z^2 = 4x_o(x_o + x)\}$ $at\ c$ is;

$$\frac{dz}{dx} = \frac{2x_o}{z_o} = \frac{2x_o}{2x_o} = 1 \qquad (8.27)$$

Therefore, the flow through a dam with a horizontal drainage blanket is;

$$q = k.1.2x_o = 2x_ok \qquad (8.28)$$

Example 8.7

A homogeneous anisotropic embankment dam section is detailed in Figure (8.15), the coefficients of permeability in the x and z directions being 4.5×10^{-8} and 1.6×10^{-8} m/sec, respectively. Construct the flow net and determine the quantity of seepage through the dam. What is the pore water pressure at point P?

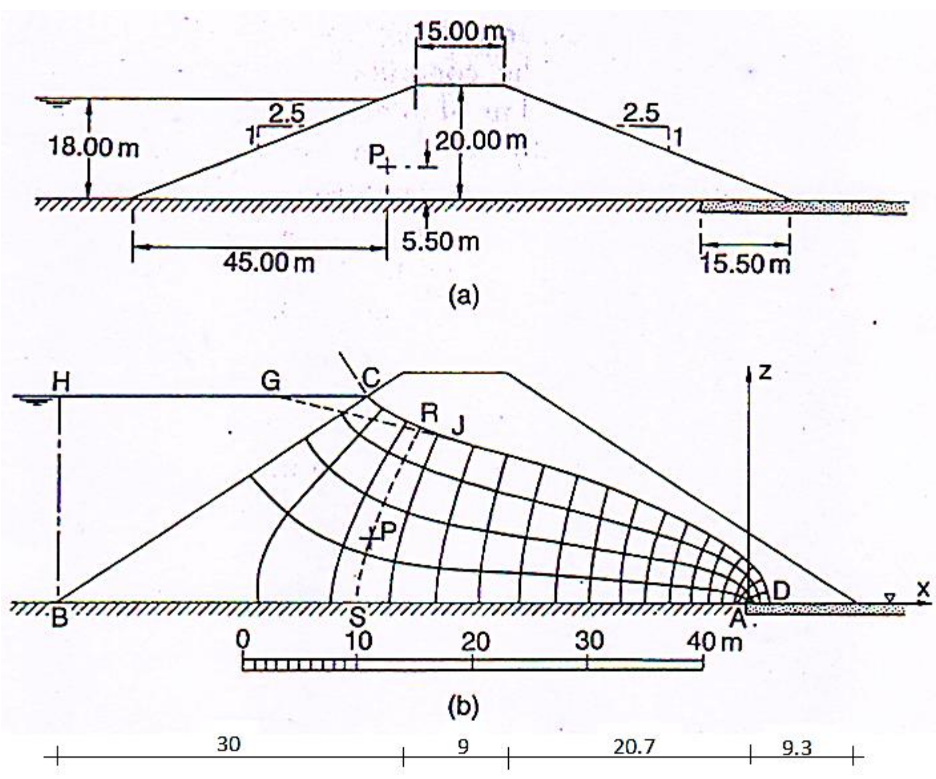


Figure (8.15)

Solution

The scale factor for transformation in the x-direction is,

$$c = \sqrt{\frac{k_z}{k_x}} = \sqrt{\frac{1.6}{4.5}} = 0.6$$

The equivalent isotropic permeability is;

$$k_e = \sqrt{k_x k_z} = \sqrt{(4.5 \times 1.6)} \times 10^{-8} = 2.7 \times 10^{-8}\ m/sec$$

The section is drawn to the transformed scale as in Figure (8.15 b)

$\Delta = 0.6 \times 45 = 27\ m$

$0.3\Delta = 0.3 \times 27 = 8.1\ m$

Thus, the coordinates of $\acute{a}$ are;

$x = 20.7 + 9 + (0.6\times2\times2.5) + 8.1 = 40.8\ m$

$z = 18.0\ m$

Substituting these coordinates in the equation of base parabola:

$$z^2 = 4x_o(x_o + x)$$

$$(18)^2 = 4x_o(x_o + 40.8) \Rightarrow x_o = 1.9m$$

<u>Phreatic surface</u>

Using the equation of basic parabola to draw the phreatic surface:

$\{z^2 = 4x_o(x_o + x)\} \Rightarrow z^2 = 7.6(1.9 + x)$

x	-1.9	0	5	10	20	30
z	0	3.8	7.24	9.51	12.9	15.57

The basic parabola is plotted in Figure (8.15 b). The upstream correction is made and flow net completed. In the flow net we have:

$N_f = 3.8$, $N_d = 18$

Hence;

$$q = k_e.H.\frac{N_f}{N_d} = 2.7 \times 10^{-8} \times 18 \times \frac{3.8}{18} = 1.0 \times 10^{-7}\ m^3/sec$$

The quantity of seepage can also be determined from Eq.(8.28)

$$q = 2x_o k = 2 \times 1.9 \times 2.7 \times 10^{-8} = 1.0 \times 10^{-7} m^3/sec$$

At point P,

$z_p = 5.5\ m$

$\Delta h = \frac{18}{18} = 1.0\ m$

$(N_d)_p = 2.4$

Thuse;

$$(h_p)_P = H - (N_d)_P.\Delta h - z_P$$

$$= 18 - 2.4 \times 1 - 5.5 = 10.1\ m$$

$$(u)_P = (h_p)_P \times \gamma_w$$

$$= 10.1 \times 9.8 = 99\ kN/m^2$$

Example 8.7 (H.W)

An embankment dam is shown in section in Figure (8.16), the coefficients of permeability in the horizontal and vertical directions being 7.5×10^{-6} and 2.7×10^{-6} m/sec, respectively. Construct the top flow line and determine the quantity of seepage through the dam.

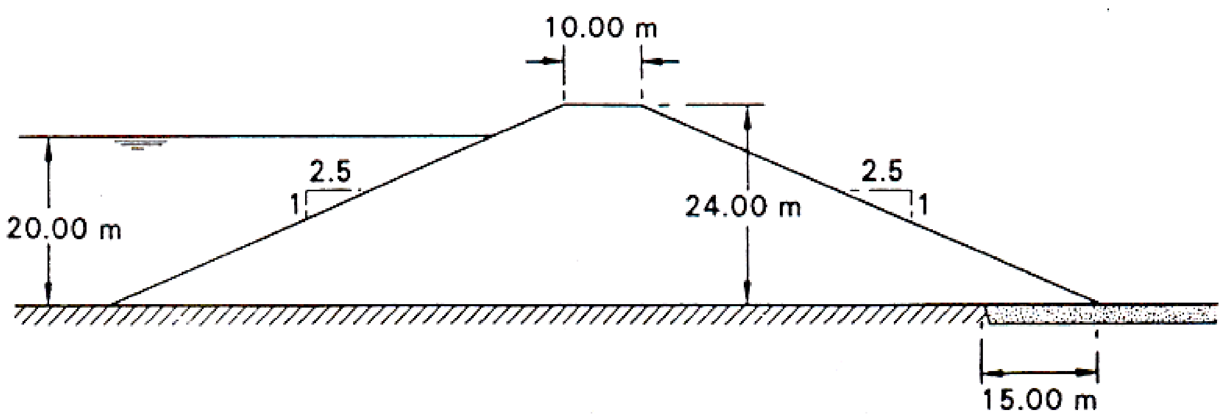


Figure (8.16)

Ans.;

$$\boldsymbol{x = \frac{z^2}{8} - 2.0}$$

$\boldsymbol{q = 1.8 \times 10^{-5}\ m^3/s/m}$

8.12 Control of Piping

Where high hydraulic gradients exist there is a possibility that the seeping water may cause internal erosion within the soil. Erosion can work its way back into the soil creating voids in the form of channels or "piping".

In order to increase the factor of safety against piping, two methods can be adopted:

1. The first procedure involves increasing the depth of the sheet pile at the top of the dam. This will increase the length of flow path resulting in drop in the pore pressure at the critical section.
2. The second procedure is to place a surcharge or filter on the top of the downstream side, the weight of which increases the downward force.

8.13 Filter Design

When seepage water flows from a soil with relatively fine grains into a coarser material, there is danger that the fine soil particles may wash away into the coarse material. Over a period of time, this process may clog the void spaces in the coarse material. Hence, the grain-size distribution of the coarse material should be properly manipulated to avoid this situation. For proper selection of the filter material, two conditions should be kept in mind:

1. The size of the voids in the filter material should be small enough to hold the larger particles of the protected material in place.
2. The filter material should have a high hydraulic conductivity to prevent building of large seepage forces and hydrostatic pressure in the filters.

Terzaghi and Peck (1948) provided the following criterion to satisfy condition (1).

$$\frac{D_{15}(F)}{D_{85}(S)} \leq 4 \ to \ 5$$

In order to satisfy condition (2), they suggested that;

$$\frac{D_{15}(F)}{D_{15}(S)} \geq 4 \ to \ 5$$