University of Basrah

College of Engineering

Department of Mechanical Engineering

Advanced Vibration Lecture-2

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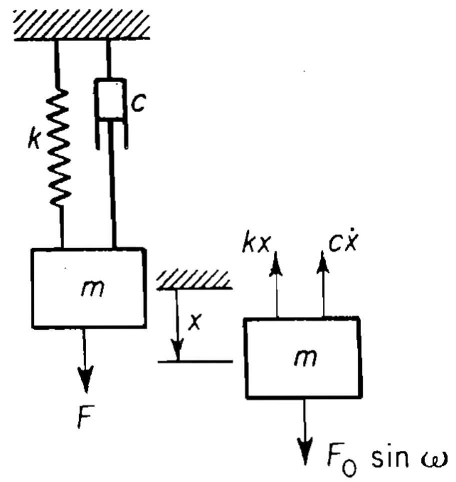
Forced Vibration of Single Degree of Freedom Systems with Damping

Harmonically excited vibration

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$



The solution consists of two parts, the homogenous solution (due to initial conditions) and the particular solution or steady-state solution.

The steady state solution is given by:

$$x = X_0 \sin(\omega t - \phi)$$

$$-m\omega^2 X_0 \sin(\omega t - \phi) + c\omega X_0 \cos(\omega t - \phi) + kX_0 \sin(\omega t - \phi) = F_0 \sin \omega t$$

$$(k - m\omega^2) X_0 \sin(\omega t - \phi) + c\omega X_0 \cos(\omega t - \phi) = F_0 \sin \omega t$$

$$(k - m\omega^2) X_0 \sin(\omega t - \phi) + c\omega X_0 \cos(\omega t - \phi) = F_0 \sin[(\omega t - \phi) + \phi]$$

$$(k - m\omega^2) X_0 \sin(\omega t - \phi) + c\omega X_0 \cos(\omega t - \phi) = F_0 \left[\sin(\omega t - \phi)\cos\phi + \cos(\omega t - \phi)\sin\phi\right]$$

By comparison:

$$(k - m\omega^2) X_0 = F_0 \cos\phi \quad and$$

$$c\omega X_0 = F_0 \sin\phi$$

Squaring both equations and add together, then taking root, we obtain:

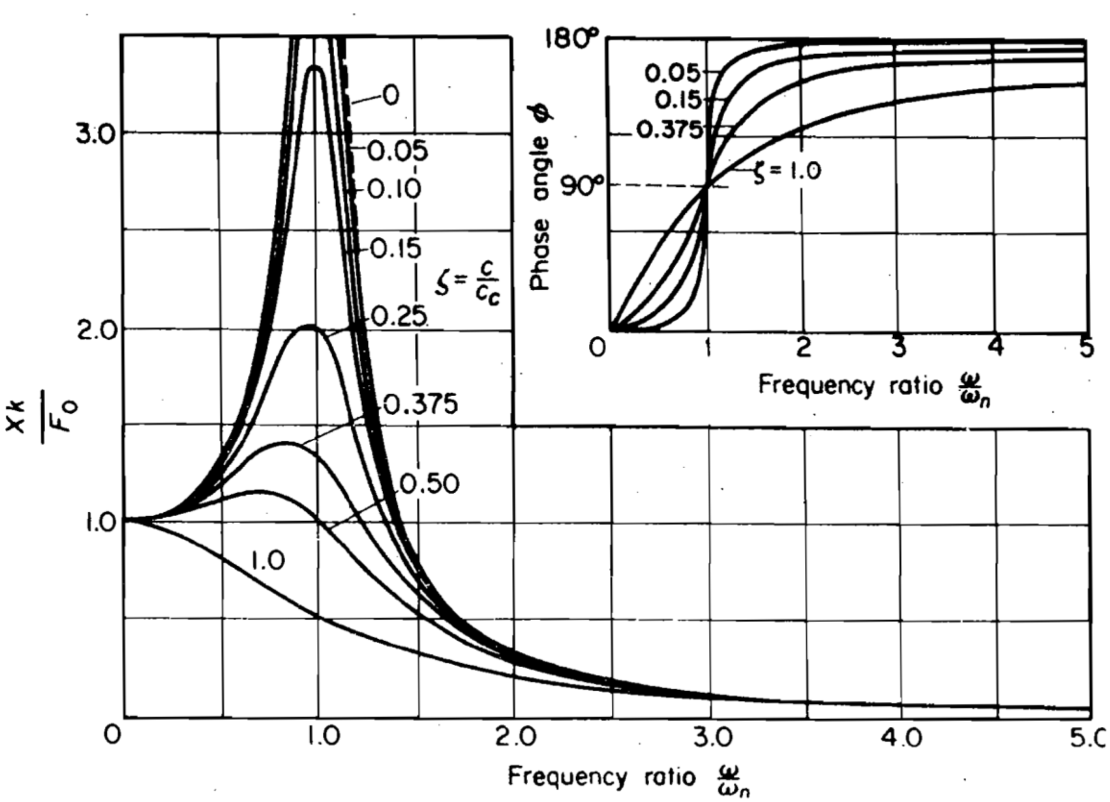
$$X_0 = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

Dividing both equations:

$$\phi = \tan^{-1} \frac{c\omega}{k - m\omega^2}$$

$$X = \frac{F_0}{k\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$\phi = \tan^{-1} \frac{2\zeta\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$



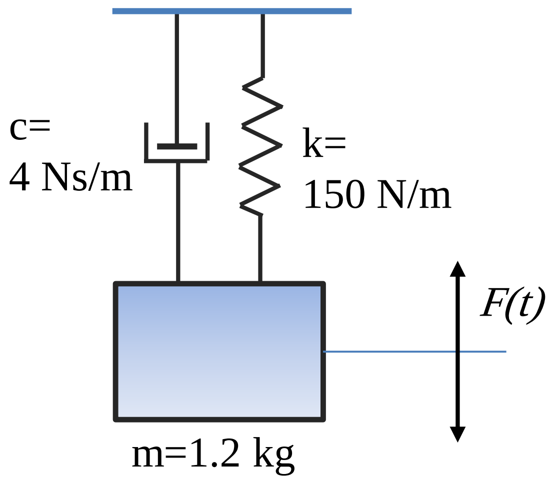
Example:

For the system shown:

(1) Find the response when *F(t)* = 10 *sin* (5t)

(2) Find the response at resonance

(3) The value of *c* for which the amplitude is reduced to 50% of its value at the resonance.



Solution:

(1)

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{150}{1.2}} = 11.18 \text{ rad/s}$$

$$\zeta = \frac{4}{2\times1.2\times11.18} = 0.1491$$

$$X = \frac{F_0}{k\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} = \frac{10}{150\sqrt{\left(1-\left(\frac{5}{11.18}\right)^2\right) + \left(2\times0.1491\times\frac{5}{11.18}\right)^2}}$$

$$= 0.08333m$$

$$\phi = \tan^{-1}\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2} = \tan^{-1}\frac{2\times0.1491\times\frac{5}{11.18}}{1-\left(\frac{5}{11.18}\right)^2} = 9.46^\circ$$

$x(t) = 0.0833 \sin(5t - 9.46°)$

(2) Repeat above when $\omega = \omega_n = 11.18$

$$X = \frac{F_0}{k \times 2\zeta} = \frac{10}{150 \times 2 \times 0.1491}$$

$X = 0.2236$ m

$\phi = 90°$

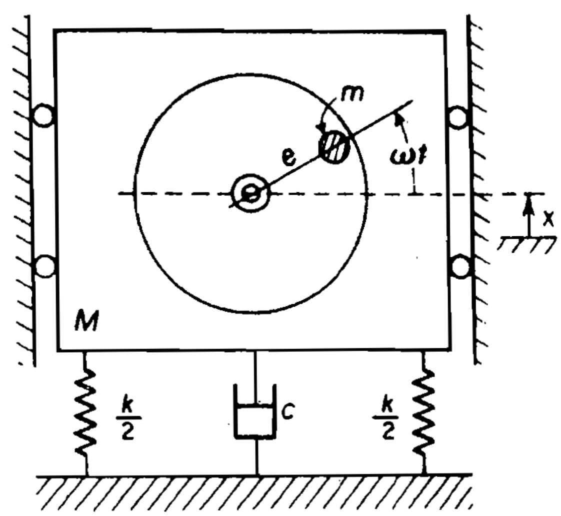
(3)

$$X = 0.5 \times 0.2236 = \frac{10}{150 \times 2 \times \zeta}$$

$$\zeta = 0.29815$$

$$c = \zeta \times 2 \times 1.2 \times 11.18 = 8\ Ns/m$$

Rotating Unbalance



When we have rotating unbalance:

$$X_0 = \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}$$

$$\phi = \tan^{-1} \frac{c\omega}{k - m\omega^2}$$

$$X = \frac{me\omega^2 \frac{1}{M}}{\frac{k}{M}\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

Which can be re-arranged as:

$$\frac{X}{e}\frac{M}{m} = \frac{\omega^2/\omega_n^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

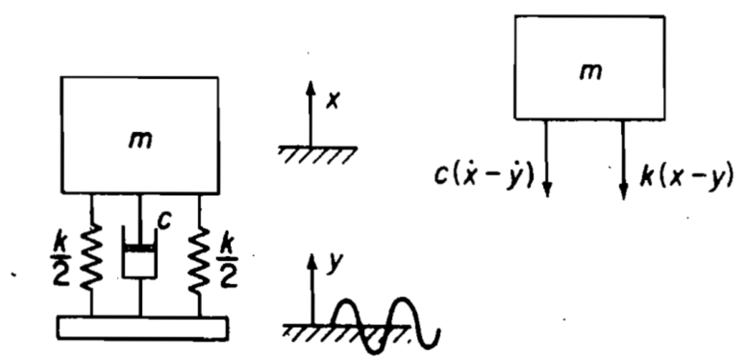
$$\phi = \tan^{-1}\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

Support Motion

$m\ddot{x} + c(\dot{x}-\dot{y}) + k(x-y) = 0$

Introducing $z = x - y$

$$m(\ddot{z}+\ddot{y}) + c\,\dot{z} + k\,z = 0$$
$$m\ddot{z} + +c\,\dot{z} + k\,z = -m\ddot{y}$$



Using the exponential form:

$y = Ye^{i\omega t}$

$z = Ze^{i(\omega t-\phi)} = \left(Ze^{-i\phi}\right)e^{i\omega t}$

$x = z + y = \left(Ze^{-i\phi} + Y\right)e^{i\omega t}$

$$-m\omega^2 Ze^{i(\omega t-\phi)} + ic\, Ze^{i(\omega t-\phi)} + k\, Ze^{i(\omega t-\phi)} = m\omega^2 Ye^{i\omega t}$$

Or:

$$\left(k - m\omega^2 + ic\omega\right)Ze^{-i\phi} = m\omega^2 Y$$

Hence:

$$Ze^{-i\phi} = \frac{m\omega^2 Y}{\left(k - m\omega^2 + ic\omega\right)}$$

Where $\phi = \tan^{-1}\frac{c\omega}{k - m\omega^2}$

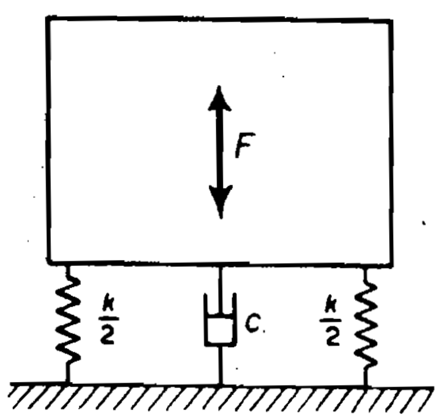
$$x = \left(Ze^{-i\phi} + Y\right)e^{i\omega t} = \left[\frac{m\omega^2 Y}{\left(k - m\omega^2 + ic\omega\right)} + Y\right]e^{i\omega t} = \frac{k + ic\omega}{\left(k - m\omega^2 + ic\omega\right)} Ye^{i\omega t}$$

$$\left|\frac{X}{Y}\right| = \frac{\sqrt{k^2 + \left(c\omega\right)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\psi = \tan^{-1}\frac{mc\omega^3}{k\left(k - m\omega^2\right) + \left(\omega c\right)^2}$$

Vibration Isolation and Measurement

Transmitted forced equals



$$F_T = \sqrt{\left(kX\right)^2 + \left(c\omega X\right)^2} = kX\sqrt{1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

But:

$$F_0 = k\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}\, X$$

Hence:

$$TR = \left| \frac{F_T}{F_0} \right| = \sqrt{\frac{1 + \left(2\zeta \frac{\omega}{\omega_n} \right)^2}{\left(1 - \left(\frac{\omega}{\omega_n} \right)^2 \right)^2 + \left(2\zeta \frac{\omega}{\omega_n} \right)^2}}$$

Solve

3.1 , 3.2, 3.3 , 3.4 , 3.5, 3.8 , 3.13, 3.15, 3.20