University of Basrah

College of Engineering

Department of Mechanical Engineering

Advanced Vibration Lecture-1

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1. **Syllabus**
2. **Fundamentals of Mechanical Vibration**
3. **Oscillatory and Harmonic Motion**

   Oscillatory motion: frequency, amplitude and phase

   Displacement, velocity and acceleration

   Periodic functions

   Fourier series and transform

4. **Free Vibration of Single Degree of Freedom Systems with Damping**

   What id Degree of Freedom?

Mechanical Vibration

Mechanical vibration is the motion of a machine or its part back and forth from its position of rest. The most classical example is that of a body with mass *M* to which a spring with a stiffness *k* is attached. Until a force is applied to the mass *M* and causes it to move, there is no vibration.

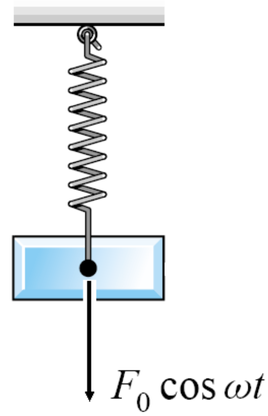


Figure 1.1 Mass-spring System

Mechanical vibration is the term used to describe the movement produced in mechanical parts due to the effect of external or internal forces on that part. Each part can be considered composed of one or more spring-mass-damper system subjected to an exciting force. The amplitude of vibration is a function of system parameters and severity of the exciting force.

Oscillatory Motion

For the oscillatory motion shown in Figure beside, the motion of the mass from its neutral position, to the top limit of travel, back through its neutral position, to the bottom limit of travel and the return to its neutral position, represents one cycle of motion. This one cycle of motion contains all the information necessary to measure the vibration of this system. Continued motion of the mass will simply repeat the same cycle. This motion is called periodic and harmonic, and the relationship between the displacement of the mass and time is expressed in the form of a sinusoidal equation:

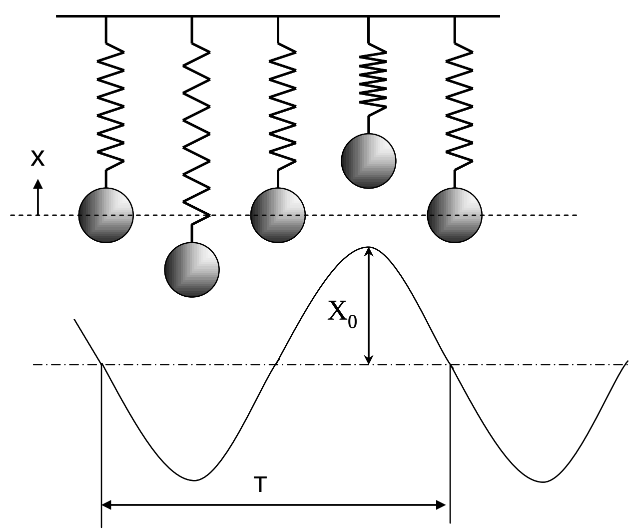


Figure 1.2 Oscillatory

$$x = X_0 \sin \omega t$$

x = displacement at any given instant t;
X_0 = maximum displacement or peak amplitude;

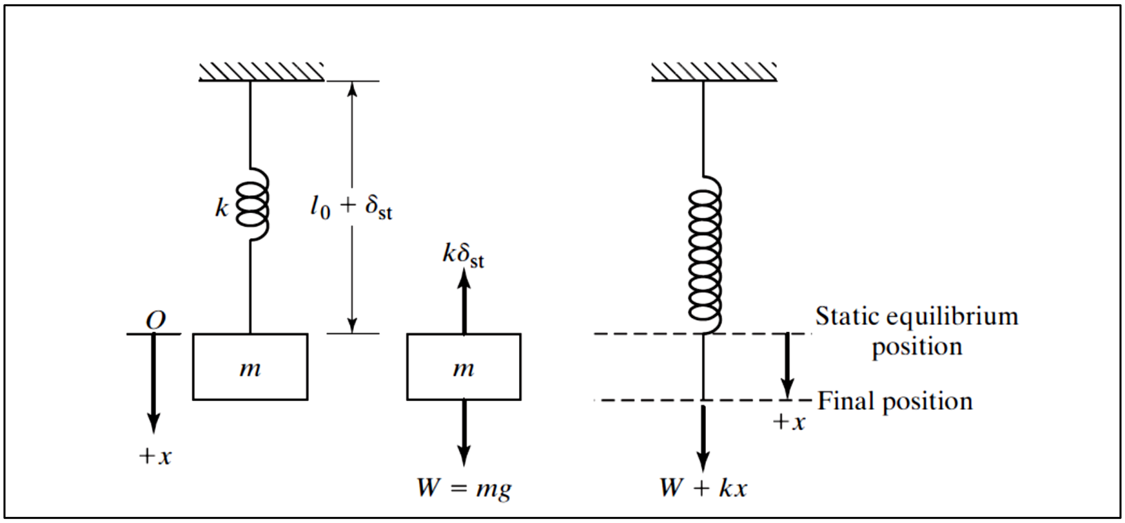
ω = 2 π f, the radian frequency and measured in rad/s

f = frequency (cycles/s – hertz – Hz); t = time (seconds)

In the above Figure, T is the periodic time (period) is seconds, i.e., the time required for complete one period. The frequency of the signal is simply the reciprocal of the periodic time, i.e. $f = \frac{1}{T}$.

Undamped System

Newton Second Law Method (Force Balance)



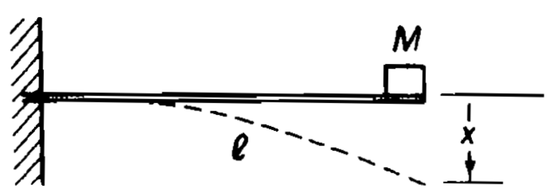
The equation of motion: $m\ddot{x} = W - k(x + \delta_{st})$

Since $W = k\delta_{st}$

then $m\ddot{x} + kx = 0$

hence, $\omega_n = \sqrt{\frac{k}{m}}$

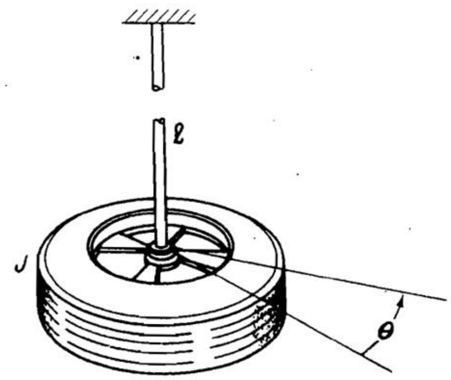
Cantilever Beam



$x = \frac{Pl^3}{3EI} = \frac{P}{k}$, hence $k = \frac{3EI}{l^3}$

$M\ddot{x}+kx=0$ leads to: $\omega_n=\sqrt{\frac{k}{M}}=\sqrt{\frac{3EI}{Ml^3}}$

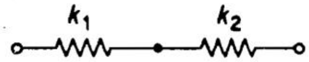
Torsional Vibration



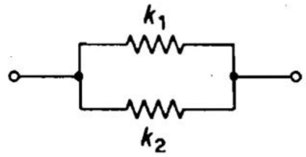
$J\ddot{\theta}=-K\theta$ Where $K=\frac{GI_p}{l}$

Where J is the rotational moment of inertia of the mass and K is the rotational stiffness of the shaft, Ip is the polar moment of inertia of the shaft cross section area.

Hence: $\omega_n=\sqrt{\frac{K}{J}}=\sqrt{\frac{GI_p}{J\,l}}$



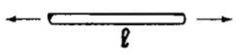
$$k = \frac{1}{1/k_1 + 1/k_2}$$



$$k = k_1 + k_2$$

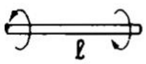
$$k = \frac{EI}{l}$$

I = moment of inertia of cross-sectional area
l = total length



$$k = \frac{EA}{l}$$

A = cross-sectional area

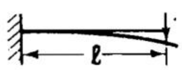


$$k = \frac{GJ}{l}$$

J = torsion constant of cross section

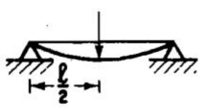
$$k = \frac{Gd^4}{64nR^3}$$

n = number of turns

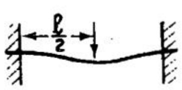


$$k = \frac{3EI}{l^3}$$

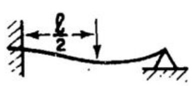
k at position of load



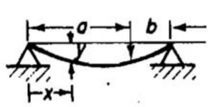
$$k = \frac{48EI}{l^3}$$



$$k = \frac{192EI}{l^3}$$

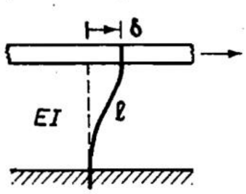


$$k = \frac{768EI}{7l^3}$$

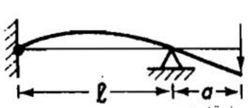


$$k = \frac{3EIl}{a^2b^2}$$

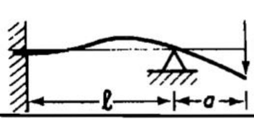
$$y_x = \frac{Pbx}{6EIl}(l^2 - x^2 - b^2)$$



$$k = \frac{12EI}{l^3}$$



$$k = \frac{3EI}{(l + a)a^2}$$



$$k = \frac{24EI}{a^2(3l + 8a)}$$

Energy Method

In a conservative system, the total energy remains constant, hence

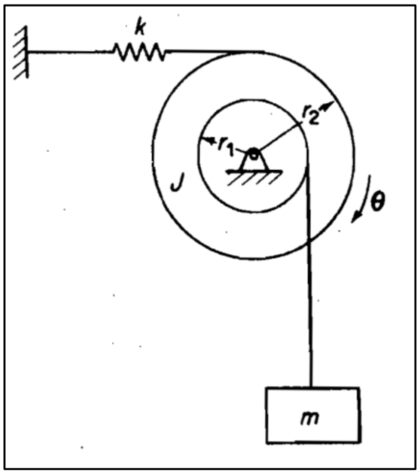
$$\frac{d}{dt}\left(T+U\right)=0$$

Where T is the kinetic energy, U is the potential energy. For example, in the mass-spring system:

$T=\frac{1}{2}m\dot{x}^2 \; and \; U=\frac{1}{2}kx^2$, hence

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx\right)=m\not{\dot{x}}\ddot{x}+kx\not{\dot{x}}=0 \Rightarrow m\ddot{x}+kx=0$$

<u>Example:</u>



$$T=\frac{1}{2}m(r_1\dot{\theta})^2+\frac{1}{2}J\dot{\theta}^2 \; and:$$

$$U=\frac{1}{2}k(r_2\theta)^2$$

The equation of motion is:

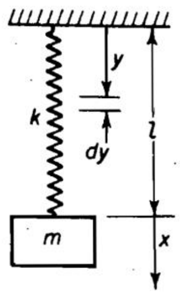
$$(mr_1^2+J)\ddot{\theta}+kr_2^2\theta=0$$

Hence

$$\omega_n = \sqrt{\frac{kr_2^{\ 2}}{mr_1^{\ 2} + J}}$$

Rayleigh Principal

Effective spring mass

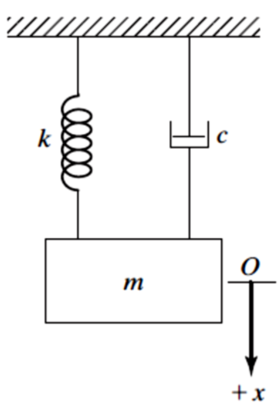


Mass per unit length is $\frac{m_s}{l}$, the kinetic energy of the spring is:

$T_s = \frac{1}{2}\int_0^l \left(\dot{x}\,\frac{y}{l}\right)^2 \left(\frac{m_s}{l}dy\right) = \frac{1}{2}\,\frac{m_s}{3}\,\dot{x}^{\ 2}$, hence

$$\omega_n = \sqrt{\frac{k}{m + \frac{m_s}{3}}}$$

Viscously Damped Systems



$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

$$\zeta = \frac{C}{C_c} = \frac{C}{2m\omega_n}$$

The roots of the characteristic equation are: $s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

<u>Overdamped</u> (ζ > 1):

$$x = Ae^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right)t} + Be^{\left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right)t}$$

<u>Critically damped</u> (ζ = 1):

$$x = \left(A + Bt\right)e^{-\zeta\omega_n t}$$

<u>Underdamped</u> (ζ < 1):

$$x = Ae^{\left(-\zeta\omega_n + \omega_n i\sqrt{1-\zeta^2}\right)t} + Be^{\left(-\zeta\omega_n - \omega_n i\sqrt{1-\zeta^2}\right)t}$$

This can be written as:

$$x = e^{-\zeta\omega_n t}\left[C\sin\omega_d t + D\cos\omega_d t\right]$$

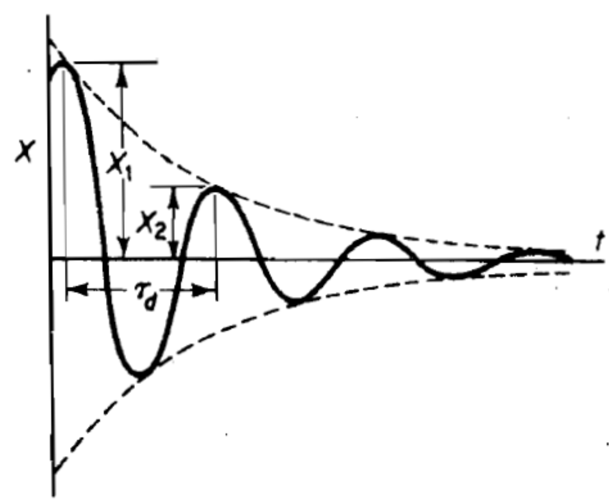
$$or$$

$$x = Xe^{-\zeta\omega_n t}\sin\left(\omega_d t + \phi\right)$$

Where: $\omega_d = \omega_n\sqrt{1-\zeta^2}$

Logarithmic Decrement

$$\delta = \ln\frac{x_1}{x_2} = \frac{1}{n}\ln\frac{x_1}{x_{n+1}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$



Solution of Equation of Motion of Free SDoF system