Principles of Flow

(G306-Second semester 2021-2022)

Lecture -4

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Introduction

Ground-water flow analyses, which in turn are based on some straightforward physical principles that govern subsurface flow. An empirical relationship called Darcy's law and conservation of mass form the basis for many useful hand calculations and computer simulations that can be made to analyze groundwater flow.

Estimating Average Hydraulic Conductivities

For flow analyses, it is often necessary to estimate average values of hydraulic conductivity based on a set of measured values. One common situation is that of approximately parallel layers of materials with differing hydraulic conductivities, as illustrated in Figure 3.13. The x axis is parallel to the layers and the z axis is normal to the layers.

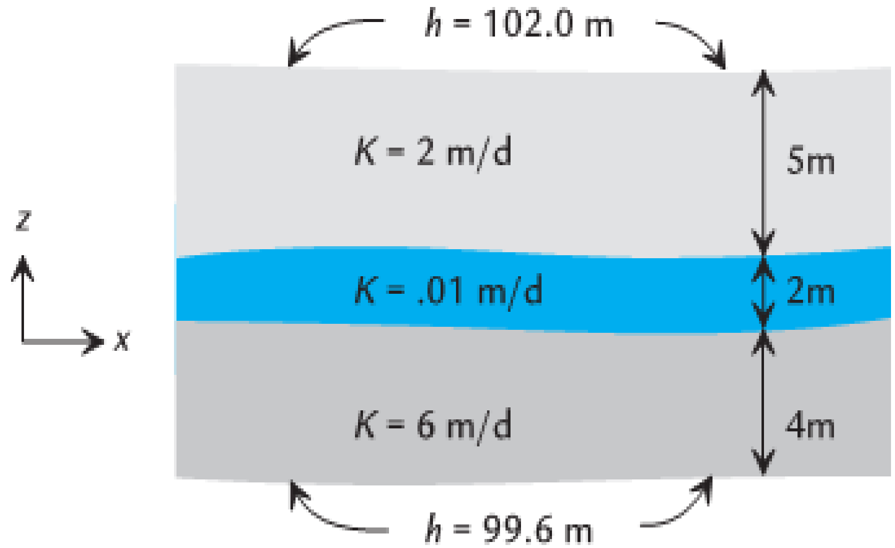
The layered system can be represented as one homogeneous anisotropic layer with one value of K_x and one value of K_z to represent the overall horizontal and vertical resistance to flow. The average values of K_x and K_z in the equivalent homogeneous system are calculated so that, under the same hydraulic gradients, the discharges are the same as in the heterogeneous, layered system.

Consider the flow parallel to the layers in the x direction. Assuming that the hydraulic gradient in the x direction is the same in all the layers, the discharge through a slice of the system that is one unit thick in the y direction is

$$Q_x = \sum -K_{xi} \frac{\partial h}{\partial x} d_i \qquad (3.17)$$

Where Kxi is the x-direction conductivity in the i the layer and di is the thickness of the its layer. This equation results from applying Darcy's law to each layer and then summing the discharges in all the layers.

Figure 3.13 Vertical section through three layers. Layers are parallel to the *x* axis and normal to the *z* axis.



The discharge through the entire system represented as a single layer with an equivalent overall horizontal hydraulic conductivity Kxe and identical total thickness is also calculated from Darcy's law:

$$Q_x = -K_{xe} \frac{\partial h}{\partial x} \sum d_i \tag{3.18}$$

Setting Eqs. 3.17 and 3.18 equal and then solving for Kxe gives

$$\boxed{K_{xe} = \frac{\sum K_{xi} d_i}{\sum d_i}} \tag{3.19}$$

The **equivalent average horizontal hydraulic conductivity** is the thickness-weighted arithmetic average of the horizontal conductivities of the layers.

Perpendicular to the layers, the vertical hydraulic gradient will vary from layer to layer, but the specific discharge q_z must not vary from layer to layer. If q_z were not the same in each layer, discharge would have to disappear or materialize at the layer boundaries. The specific discharge q_z in each of the i layers is given by

$$q_z = -K_{zi} \frac{\Delta h_i}{d_i} \tag{3.20}$$

where K_{zi} is the z-direction conductivity in the ith layer and Δh_i is the head drop across the ith layer. Rearranging this equation for the head drop across a layer gives

$$\Delta h_i = -\frac{q_z d_i}{K_{zi}} \tag{3.21}$$

The proper equivalent conductivity K_{ze} must have the same specific discharge over an equivalent single layer with the same total thickness and total head drop.

$$\boxed{q_z = -\frac{K_{ze} \sum \Delta h_i}{\sum d_i}} \tag{3.22}$$

Combining the two previous equations and solving for K_{ze} gives

$$\boxed{K_{ze} = \frac{\sum d_i}{\sum (d_i / K_{zi})}} \tag{3.23}$$

Equations 3.19 and 3.23 provide general equations for estimating the equivalent homogeneous, anisotropic hydraulic conductivities of a single layer that represents a series of layers for simplified flow calculations.

Example :- Consider the three layers illustrated in Figure 3.13. Each layer is considered to and the be isotropic, with K=Kx =Kz. The head at the top of the uppermost layer is 102.0m head at the bottom of the lower most layer is 99.6 m.

Calculate the equivalent Kxe and Kze for this layered system.

Calculate the vertical specific discharge qz, the head at the interface between the upper and middle layers, and the head at the interface between the middle and lower layers.

Using Eq. 3.19, the equivalent horizontal conductivity is

$$K_{xe} = \frac{(2\ \mathrm{m/d})(5\ \mathrm{m}) + (0.01\ \mathrm{m/d})(2\ \mathrm{m}) + (6\ \mathrm{m/d})(4\ \mathrm{m})}{11\ \mathrm{m}}$$
$$= 3.1\ \mathrm{m/d}$$

and using Eq. 3.23, the equivalent vertical conductivity is

$$K_{ze} = 11\ \mathrm{m} \Bigg/ \left(\frac{5\ \mathrm{m}}{2\ \mathrm{m/d}} + \frac{2\ \mathrm{m}}{0.01\ \mathrm{m/d}} + \frac{4\ \mathrm{m}}{6\ \mathrm{m/d}}\right)$$
$$= 0.054\ \mathrm{m/d}$$

The vertical specific discharge is calculated from Darcy's law as

$$
\begin{aligned}
q_z &= -K_{ze}\frac{\Delta h}{\Delta z} \\
&= -0.054\ \mathrm{m/d}\left(\frac{102.0 - 99.6\ \mathrm{m}}{11\ \mathrm{m}}\right) \\
&= -0.012\ \mathrm{m/d}
\end{aligned}
$$

That q_z is negative indicates flow in the negative z direction. Using this specific discharge and Darcy's law, the head drop across the upper layer can be calculated, resulting in

$$
\begin{aligned}
\Delta h &= -q_z\frac{1}{K}\Delta z \\
&= -(-0.012\ \mathrm{m/d})\left(\frac{1}{2\ \mathrm{m/d}}\right)(5\ \mathrm{m}) \\
&= 0.030\ \mathrm{m}
\end{aligned}
$$

Transmissivity

Often it is only practical to measure the hydraulic conductivity as an integrated parameter over the thickness of a given layer. This parameter is called the transmissivity of the layer. If the hydraulic conductivity tangential to the layer Kt can be assumed constant over the thickness b of a layer, the transmissivity T of the layer is simply:

$$T = K_t b \qquad (3.25)$$

If a layer is composed of m strata of thickness b_i and hydraulic conductivity $(K_t)_i$, the total transmissivity of the layer is the sum of the transmissivities of each stratum:

$$\begin{aligned} T &= \sum_{i=1}^{m} T_i \\ &= \sum_{i=1}^{m} (K_t)_i b_i \qquad (3.26) \end{aligned}$$

The dimensions of transmissivity are $[L^2/T]$. Transmissivity is a measure of how easily a layer transmits water.