Ground water movement

G306-Seconed semester 2021-2022)

Lecture -3

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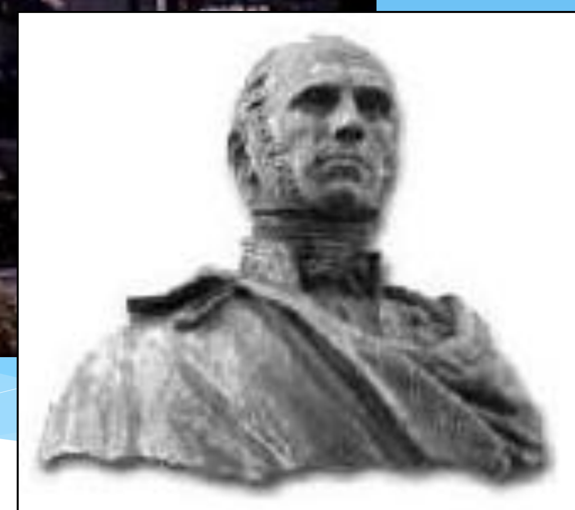
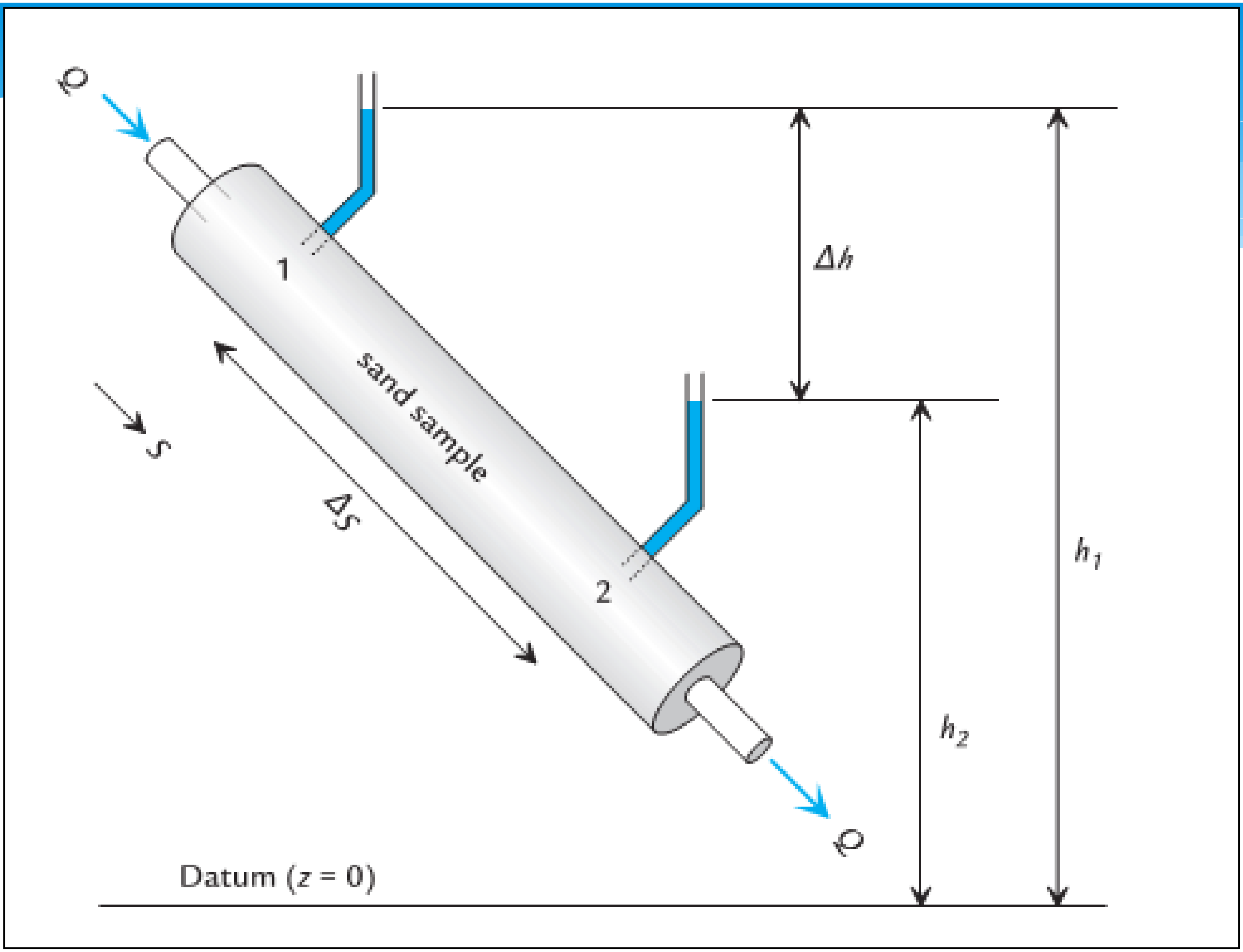


Place Darcy, Dijon, France.

The experimental verification of Darcy's law can be performed with flowing at a rate (Q)through a cylinder of cross-sectional area (A) packed with sand and having manometer (which is small open tubes) as shown in figure.



Schematic illustrating steady flow through a sand sample. The manometers measure heads h1 and h2 at locations 1 and 2 within the column.

The total energy heads, or fluid potentials above a datum plane may be expressed by the Bernoulli equation:-

$$\frac{p_1}{\gamma} + \frac{v_1}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2}{2g} + z_2 + \Delta h \quad \ldots \ldots 1$$

P=pressure , $\gamma = specific\ weight\ of\ water$

V=velocity of flow , g=acceleration of gravity Z= elevation, Δh=head loss

Because the velocities in porous media are usually low, velocity heads may be neglected ($\frac{v_1}{2g} = 0)$ *without appreciable error*

So the head loss becomes:-

$$\Delta h = \left(\frac{P1}{\gamma} + Z1\right) - \left(\frac{P2}{\gamma} + Z2\right) \quad \ldots\ldots\ldots 2$$

Therefore, the resulting head loss is defined as the potential loss within the cylinder

Now, Darcy measurements show that the proportionalities:

$$Q \propto \Delta h, \qquad Q \propto \frac{1}{\Delta s} \qquad (3.1)$$

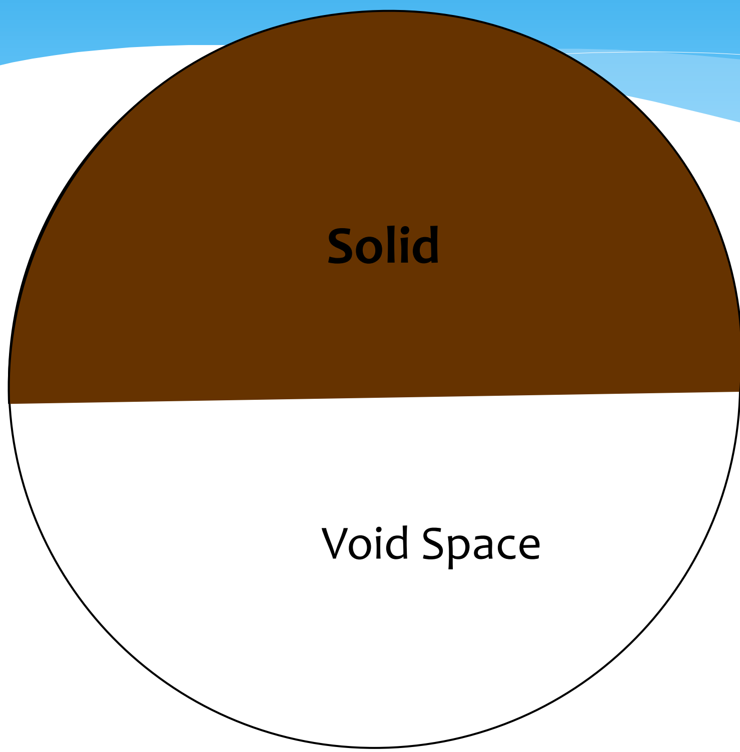
Introducing a proportionality constant k leads to the equation:

$Q=KA\ \Delta h \backslash \Delta S$ ……. 3

* Expressed in general terms where $\Delta s=\Delta l$
* **$Q=KA\ dh \backslash dl$ ……………..4**
* Or simply
* **$V=Q/A=K\ dh \backslash dl$ …………………5**
* Where:
* $dh \backslash dl$ = hydraulic gradient
* V=darcy velocity or specific discharge (the volumetric flow rate per surface area of sample)

Velocity through Porous Medium

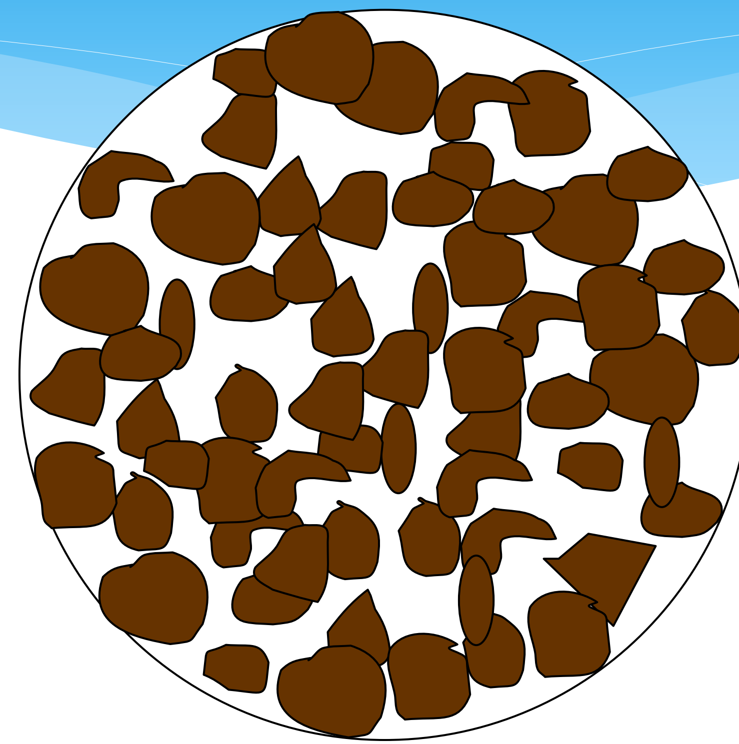
Pipe



"Porosity" = 0.5

$$v = \frac{Q}{A_x}$$

Porous Medium



Porosity = 0.5

$$v = \frac{q}{n_e}$$

Laminar and Turbulent Flow

Darcy's law holds when groundwater velocities are small enough that flow is laminar and not turbulent. Turbulent flow is characterized by chaotic eddies, like in the atmosphere or a flowing stream. Figure 3.8 illustrates these types of flow. A measure of whether a flow tends toward laminar or turbulent behavior is the Reynolds number Re, a dimensionless parameter used in fluid mechanics:

$$R_e = \frac{\rho v d}{\mu} \quad (3.10)$$

Where ρ is the fluid density, v is its velocity, μ is the dynamic viscosity of the fluid, and d is a characteristic length such as mean pore diameter or mean grain size.

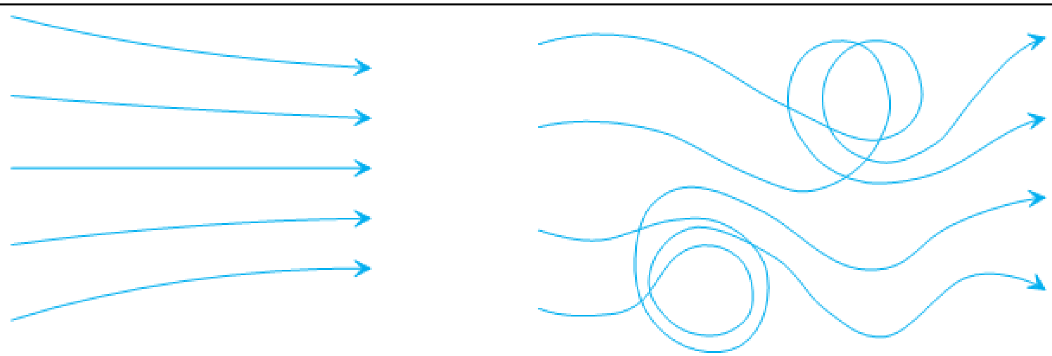


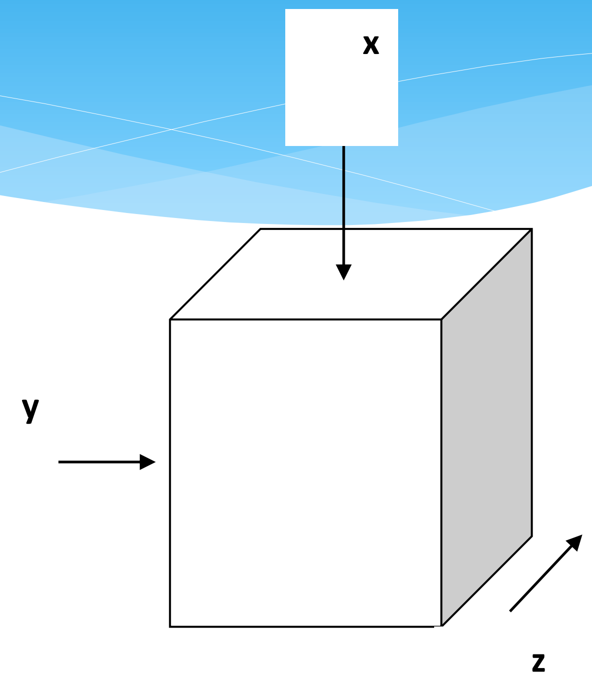
Figure 3.8 Flow paths in a laminar flow (left) and in a turbulent flow (right).

concludes from experimental data that if Re is less than some value between 1 and 10, flow in granular media is laminar, and Darcy's law applies. Turbulent flows develop in media with large pores and high groundwater velocities. Such conditions occur in extremely coarse granular materials like in karst limestone and dolomite rock, in large fractures in crystalline rock, and in volcanics with flow tubes and other large pores.

General flow equation

In general form Darcy's law be written (Δl=Δs) :

$$v = k\frac{\partial h}{\partial s}\ldots\ldots\ldots\ldots\ldots 1$$



S=is the distance along the average direction of flow

In anisotropic porous medium where permeable which vary with flow direction.

The velocity components in rectangular coordinate system may be given by:-

$$v_x = k_x\frac{\partial h}{\partial x} \quad v_y = k_y\frac{\partial h}{\partial y} \quad v_z = k_z\frac{\partial h}{\partial z} \ldots\ldots\ldots 2$$

Where:
$k_{x,}$ $k_{y,}$ k_z = the coefficients of permeability in x,y,z directions respectively

in homogeneous aquifer where the permeability is the same in all direction so:-

$$v_x = k\frac{\partial h}{\partial x} \quad v_y = k\frac{\partial h}{\partial y} \quad v_z = k\frac{\partial h}{\partial z} \qquad \text{.........3}$$

In hydrodynamic the velocity potential ϕ is (a scale or function) of space and time such that it's a negative derivative with respect to any direction is the fluid velocity in that direction

$$\Phi = -kh \quad \text{.....4}$$

So equation 3 will be

$$v_x = -\frac{\partial \Phi}{\partial x} \quad v_y = -\frac{\partial \Phi}{\partial y} \quad v_z = -\frac{\partial \Phi}{\partial z} \quad \text{........5}$$

This means that a velocity potential exists for ground water flow.

In steady state flow

All ground water flow must satisfy the equation of continuity, which in general form:

$$\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] = \frac{\partial \rho}{\partial t} \quad \text{..................6}$$

Where : ρ=fluid density, t= time

In steady state flow there is no conditions with respect to time and regarding water as an incompressible fluids so ρ=constant

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \quad \ldots\ldots\ldots\ldots 7$$

By substituted eq.5 in eq.7

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \ldots\ldots\ldots\ldots 8$$ **(laplace equation)**

Replacing ϕ=-kh leads to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad \ldots\ldots\ldots\ldots 9$$ or $\nabla^2 h = 0$

(general partial differential equation for steady state flow in homogeneous isotropic media)

Unsteady state flow

In order to derive the corresponding for unsteady state flow it will be necessary to consider the storage coefficient (S) for confined aquifer but for unconfined aquifer we defined as the specific yield

$$\partial \rho = \frac{\rho S}{b\gamma} \partial p \quad \ldots \ldots .11$$

By inserting this equation in the continuity equation .6 we got

$$-\left[\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z}\right] = \frac{\rho S}{b\gamma} \cdot \frac{\partial p}{\partial t} \ldots \ldots .12$$

This equation may be expanded and values of velocity components from eq. 3 inserted to give:

$$k\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\right)=\frac{\rho S}{b\gamma}\cdot\frac{\partial p}{\partial t} \quad \ldots\ldots\ldots 13$$

When ρ is assumed constant. Rewriting and substituting p=γh then

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=\frac{S}{kb}\cdot\frac{\partial p}{\partial t} \quad \ldots\ldots\ldots 14$$

laplace equation for partial differential equation for unsteady flow of water in compressible confined of uniform thickness b , or in grad uniform

$$\nabla^2 h=\frac{S}{T}\frac{\partial h}{\partial t} \quad \ldots\ldots\ldots 15$$