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Mathematics first stage

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The Natural Logarithm Function

Rules of arithmetic logarithms :- for any $x > 0$ *and* $y > 0$ ***and any exponent n:***

1. $ln(x * y) = ln\, x + ln\, y$
2. $ln\left(\frac{x}{y}\right) = ln\, x - ln\, y$
3. $ln(x^n) = n\, ln\, x$
4. $ln\left(\frac{1}{x}\right) = -\, ln\, x$
5. $ln\left(\sqrt[n]{x}\right) = \frac{1}{n} ln\, x$

Derivative of Natural Logarithm Function

If u ***is a differentiable function of*** x ***, and*** $f = ln(u)$ ***then the derivative of*** f ***is :-***

$$\frac{df}{dx} = \frac{d}{dx}(ln(u)) = \frac{u'}{u}$$

Examples:- Find derivative of the following functions:-

1. $y = ln\,(5x)$

$\Rightarrow y' = \frac{5}{5x} = \frac{1}{x}$

2. $y = ln\,(5x^2 + 8)$

$\Rightarrow y' = \frac{10x}{5x^2+8}$

3. $y = ln(tan^3 x)$

$\Rightarrow y' = \frac{3tan^2 x * sec^2 x}{tan^3 x} = \frac{3\ sec^2 x}{tan\ x}$.

4. $y = ln(sin(2x))$

$\Rightarrow y' = \frac{2\ cos\ 2x}{sin\ 2x} = 2\ cot\ 2x$

5. $y = ln(ln\ 2x)$

$\Rightarrow y' = \frac{\frac{2}{2x}}{ln\ 2x} = \frac{1}{x * ln\ 2x}$

6. $y = sin^{-1}(ln\ 2x)$

$\Rightarrow y' = \frac{1}{\sqrt{1-(ln\ 2x)^2}} * \frac{2}{2x}$

7. $y = ln(cos^{-1}(x))$

$\Rightarrow y' = \frac{\frac{-1}{\sqrt{1-x^2}}}{cos^{-1} x} = \frac{-1}{\sqrt{1-x^2}} * \frac{1}{cos^{-1}(x)}$.

The Exponential Function (e^x)

Exponential function :defined as the inverse of the natural logarithm function .

Law of exponential function (e^x) for all real numbers x and y :

1. $e^{x+y} = e^x * e^y$
2. $e^{x-y} = \frac{e^x}{e^y}$
3. $e^{-x} = \frac{1}{e^x}$
4. $(e^x)^y = e^{x*y}$.

Derivative of exponential function:-

If $u(x)$ is a differentiable function of x , and let $f(x) = e^{u(x)}$, then the derivative of f is :-

$$\frac{df}{dx} = \frac{d}{dx}(e^u) = e^u * \frac{du}{dx}$$

Examples: Find y' of the following functions:

1. $y = e^{3x}$ ⇒ $y' = e^{3x} * 3 = 3e^{3x}$.

2. $y = e^{5x^2+1}$ ⇒ $y' = e^{5x^2+1} * 10x = 10x * e^{5x^2+1}$.

3. $y = cos\,(e^{2x})$ ⇒ $y' = -sin\,(e^{2x}) * e^{2x} * 2 = -2e^{2x} * sin\,(e^{2x})$.

4. $y = e^{tan(2x)}$ ⇒ $y' = e^{tan\,2x} * sec^2(2x) * 2 = 2 * sec^2(2x) * e^{tan\,2x}$.

5. $y = e^{\sqrt{x}}$ ⇒ $y' = e^{\sqrt{x}} * \left(\frac{1}{2\sqrt{x}}\right)$

6. $y = e^{\left(sin\,x+\sqrt[3]{x}\right)}$ ⇒ $y' = e^{(sin\,x+\sqrt[3]{x})} * (cos\,x + \frac{1}{3\sqrt[3]{x^2}})$.

7. $y = tan^{-1}(e^{3x})$ ⇒ $y' = \frac{1}{1+(e^{3x})^2} * e^{3x} * 3$.

8. $y = e^{sin^{-1}x}$ ⇒ $y' = e^{sin^{-1}x} * \frac{1}{\sqrt{1-x^2}}$.

The General Exponential Function

If a is a positive number, then $a^x = e^{\,lna^x} = e^{xlna}$ ***for example*** $2^{\sqrt{3}} = e^{\sqrt{3}ln2}$.

Derivative of General Exponential Function:

If $a > 0$ and u is a differentiable function of x , then a^u is a differentiable function of x and:

$$\frac{d}{dx}a^u = a^u * ln\,a * \frac{du}{dx}$$

Examples: Find y' of the following functions:

1. $y = 3^{2x-5} \longrightarrow y' = 3^{2x-5} * ln\ 3 * 2$

2. $y = 5^{sin\ x} \longrightarrow y' = 5^{sin\ x} * ln\ 5 * cos\ x$

3. $y = 5^{\sqrt{x}} \longrightarrow y' = 5^{\sqrt{x}} * ln\ 5 * \frac{1}{2\sqrt{x}}$

4. $y = x^2 * sin\left(2^{x^2}\right)$

$y' = x^2 * cos\left(2^{x^2}\right) * \left(2^{x^2} * ln2 * 2x\right) + sin\left(2^{x^2}\right) * 2x\,.$

5. $y = 2^{sinh\ x} \longrightarrow y' = 2^{sinh\ x} * ln\ 2 * cosh\ x\,.$

6. $y = x^x = e^{lnx^x} = e^{xln\ x}$

$y' = e^{xln\ x} * (x * \frac{1}{x} + ln\ x * 1)$

$= e^{xln\ x}\,(1 + ln\ x)$

$= x^x(1 + ln\ x)\,.$

7. $y = tan^{-1}\left(2^{3x^2}\right)$

$y' = \frac{1}{1 + (2^{3x^2})^2} * 2^{3x^2} * ln\ 2 * (6x).$