Al Basrah university
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Mathematics \ first stage

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16/1/2022

Derivatives of Inverse Trigonometric Function

Note:- $sin^{-1}x \neq (sin\ x)^{-1}$

Let $u = f(x)$

1. $\frac{d}{dx}\ sin^{-1}u = \frac{1}{\sqrt{1-u^2}} * \frac{du}{dx}$
2. $\frac{d}{dx} cos^{-1}u = \frac{-1}{\sqrt{1-u^2}} * \frac{du}{dx}$
3. $\frac{d}{dx} tan^{-1}u = \frac{1}{1+u^2} * \frac{du}{dx}$
4. $\frac{d}{dx} cot^{-1}u = \frac{-1}{1+u^2} * \frac{du}{dx}$
5. $\frac{d}{dx}\ sec^{-1}u = \frac{1}{\lvert u\rvert\sqrt{u^2-1}} * \frac{du}{dx}$
6. $\frac{d}{dx} csc^{-1}u = \frac{-1}{\lvert u\rvert\sqrt{u^2-1}} * \frac{du}{dx}$

Examples:- Find derivative of following:-

1. $y = sin^{-1}(x^2 + 3)$

$$\Rightarrow y' = \frac{1}{\sqrt{1-(x^2+3)^2}}(2x) = \frac{2x}{\sqrt{1-(x^2+3)^2}}$$

2. $y = \cos^{-1}(\sqrt{2x-1})$

$$\Rightarrow y' = \frac{-1}{\sqrt{1-(2x-1)}} * \left(\frac{1}{2}(2x-1)^{\frac{-1}{2}} * 2\right)$$

$$= \frac{-1}{\sqrt{2-2x}} * \frac{1}{\sqrt{2x-1}}$$

3. $y = \tan^{-1}(\cos\sqrt{x})$

$$\Rightarrow y' = \frac{1}{1+(\cos^2\sqrt{x})} * \left(\frac{-\sin\sqrt{x}}{2\sqrt{x}}\right) = \frac{-\sin\sqrt{x}}{2\sqrt{x}\,(1+\cos^2 x)}.$$

4. $y = (\sin^{-1} x)^4$

$$\Rightarrow y' = 4(\sin^{-1} x)^3 * \frac{1}{\sqrt{1-x^2}} = \frac{4(\sin^{-1} x)^3}{\sqrt{1-x^2}}$$

5. $y = 3x^2 + \cot^{-1} x$

$$\Rightarrow y' = 6x + \frac{-1}{1+x^2} = 6x - \frac{1}{1+x^2}$$

Hyperbolic Functions:

Some important relation :-

1. $\cosh^2(x) - \sinh^2(x) = 1$
2. $\operatorname{sech}^2(x) + \tanh^2(x) = 1$
3. $\coth^2(x) - \operatorname{csch}^2(x) = 1$

Derivatives of Hyperbolic Functions

Let $u = f(x)$

1. $\frac{d}{dx} sinh\,(u) = cosh\,(u) * \frac{du}{dx}$
2. $\frac{d}{dx} cosh\,(u) = sinh\,(u) * \frac{du}{dx}$
3. $\frac{d}{dx} tanh(u) = sech^2\,(u) * \frac{du}{dx}$
4. $\frac{d}{dx} coth(u) = -csch^2(u) * \frac{du}{dx}$
5. $\frac{d}{dx} sech(u) = -sech(u) * tanh(u) * \frac{du}{dx}$
6. $\frac{d}{dx} csch(u) = -csch(u) * coth(u) * \frac{du}{dx}$

Examples:- Find derivative of the following functions :-

1. $f(x) = tanh\,(5x^2 + 3)$

$f'(x) = sech^2(5x^2 + 3) * (10x)$

2. $f(x) = cosh\left(\sqrt{x}\right) + 5\,x^3$

$f'(x) = sinh\left(\sqrt{x}\right) * \left(\frac{1}{2\sqrt{x}}\right) + 15x^2$

3. $f(x) = coth(sin^{-1}\,x)$

$f'(x) = -csch^2\,(sin^{-1}\,x) * \frac{1}{\sqrt{1-x^2}}$

4. $f(x) = sech(3x^2 + 1) + coth(5x)$

$f'(x) = -sech\,(3x^2 + 1) * tanh\,(3x^2 + 1) * (6x) - csch^2(5x) * 5$

5. $f(x) = cosh(\sqrt{x^2 + 1})$

$f'(x) = sinh\left(\sqrt{x^2 + 1}\right) * \left(\frac{1}{2}\,(x^2 + 1)^{\frac{-1}{2}} * 2x\right)$

$= \frac{x * sinh(\sqrt{x^2+1})}{\sqrt{x^2+1}}$.