University of Basrah
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Mathematics and Statistics\ first stage

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Differentiation Rules:

Let y, f and g are differentiable functions of x, then :-

1. ***If $y = k$, where k is constant, $y' = 0$***
2. ***If $y = k * f(x)$, then $y' = k * f'(x)$.***
3. ***If $f(x) = x^n$, (n positive integer or negative) then for every real value of x,***
   $f'(x) = n\, x^{n-1}$.
4. ***If $y = f(x) \mp g(x)$, then $y' = f'(x) \mp g'(x)$.***
5. ***If $y = f(x) * g(x)$, then $y' = f(x) * g'(x) + g(x) * f'(x)$.***
6. ***If $y = \frac{f(x)}{g(x)}$, where $(x) \neq 0$ for every x,***
   then $y' = \frac{g(x) * f'(x) - f(x) * g'(x)}{(g(x))^2}$.
7. ***If $y = (f(x))^n$, (Any real number n) then $y' = n\,(f(x))^{n-1}.f'(x)$***

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Derivative of higher orders:-

Frist order derivative of $y = f(x) \rightarrow y'$, $\frac{dy}{dx}, f'(x)$.

Second order derivative of $y = f(x) \rightarrow y''$, $\frac{d^2y}{dx^2}$, $f''(x)$.

Third order derivative of $y = f(x) \rightarrow y'''$, $\frac{d^3y}{dx^3}$, $f'''(x)$.

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The n_{th} order derivative of $y = f(x)$ $\rightarrow y^{(n)}, \frac{d^n y}{dx^n}, f^{(n)}(x)$

Examples:- Find derivative of following functions:

1. $y = 4x^3 + 5x$. $\Rightarrow y' = 12x^2 + 5.$

2. $y = (x^2 + 1)(3x + 5)$ $\Rightarrow y' = (x^2 + 1) * (3) + (3x + 5) * 2x$
$= 3x^2 + 3 + 6x^2 + 10x = 9x^2 + 10x + 3.$

3. $y = (x^2 + x + 1)^3$
$\Rightarrow y' = 3(x^2 + x + 1)^2 * (2x + 1)$

4. $y = \frac{x^2}{2x-1}$
$\Rightarrow y' = \frac{(2x-1)*(2x)-x^2*2}{(2x-1)^2}$
$= \frac{4x^2-2x-2x^2}{(2x-1)^2} = \frac{2x^2-2x}{(2x-1)^2} = \frac{2x(x-1)}{(2x-1)^2}$

5. $y = \sqrt{x+3} \rightarrow y = (x+3)^{\frac{1}{2}}$
$\Rightarrow y' = \frac{1}{2}(x+3)^{\frac{-1}{2}} * 1 = \frac{1}{2\sqrt{x+3}}$

6. $y = \frac{1}{(x^2+3)^2} \rightarrow y = (x^2+3)^{-2}$
$\Rightarrow y' = -2(x^2+3)^{-3} * (2x)$
$= \frac{-4x}{(x^2+3)^3}.$

7. *Find y', y'', y''' of $y = 6\sqrt[3]{x^2}$*
$y = 6(x)^{\frac{2}{3}} \rightarrow y = 6(x)^{\frac{2}{3}}$
$\Rightarrow y' = 6 * \frac{2}{3}(x)^{\frac{-1}{3}} = \frac{4}{\sqrt[3]{x}}$
$\Rightarrow y'' = 4 * \left(\frac{-1}{3}\right) * (x)^{\frac{-4}{3}} = \frac{-4}{3\sqrt[3]{x^4}}$

$\Rightarrow y''' = \frac{-4}{3} * \left(\frac{-4}{3}\right) * (x)^{\frac{-7}{3}} = \frac{16}{9\sqrt[3]{x^7}}$.

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8. *let* $f(x) = x^2(x-4)^3$, *find* $f'(x)$?

$\Rightarrow f'(x) = x^2 * 3(x-4)^2 + (x-4)^3 * (2x)$

$= x(x-4)^2(3x + 2(x-4))$

$= x(x-4)^2(3x + 2x - 8).$

$= x(x-4)^2(5x-8)$

9. $f(x) = x^3 * \sqrt[3]{(7x^2 + x - 1)^2} = x^3\,(7x^2 + x - 1)^{2/3}$

$f'(x) = x^3 * \frac{2}{3}(7x^2 + x - 1)^{\frac{-1}{3}} * (14x + 1) + (7x^2 + x - 1)^{2/3} * (3x^2).$

Trigonomtric Functions

$sinx$, $cosx$, $tanx$, $cotx$, $secx$, *and* $cscx$.

Important relations:

1. $cos(x \pm y) = cos\,x\,cos\,y \mp sin\,x\,sin\,y$
2. $sin(x \pm y) = sin\,x\,cos\,y \pm cos\,x\,sin\,y$
3. $sin^2x + cos^2x = 1$
4. $sec^2x - tan^2x = 1$
5. $csc^2x - cot^2x = 1$
6. $cos^2x = \frac{1}{2}(1 + cos\,2x)$
7. $sin^2x = \frac{1}{2}(1 - cos\,2x)$
8. $lim_{a\to 0}\ \frac{sin\,a}{a} = 1$
9. $lim_{a\to 0}\ \frac{cos\,a - 1}{a} = 0$
10. $tanx = \frac{sin\,x}{cos\,x}$
11. $cot\,x = \frac{cos\,x}{sin\,x}$
12. $sec\,x = \frac{1}{cos\,x}$
13. $csc\,x = \frac{1}{sinx}$

Derivatives of Trigonomtric Functions

Let $u = f(x)$ be a differentiable function of x, then :-

1. $\frac{d}{dx}\sin u = \cos u \frac{du}{dx}$
2. $\frac{d}{dx}\cos u = -\sin u * \frac{du}{dx}$
3. $\frac{d}{dx}\tan u = \sec^2 u * \frac{du}{dx}$
4. $\frac{d}{dx}\cot u = -\csc^2 u * \frac{du}{dx}$
5. $\frac{d}{dx}\sec u = \sec u * \tan u * \frac{du}{dx}$
6. $\frac{d}{dx}\csc u = -\csc u * \cot u * \frac{du}{dx}$

Examples:- Find derivative of the following functions:-

1. $y = \tan^4 x$

➡ $y' = 4\tan^3 x * \sec^2 x$.

2. $y = \sec^4(x^2+1)$

➡ $y' = 4\sec^3(x^2+1) * \sec(x^2+1) * \tan(x^2+1) * (2x)$

$= 8x\sec^4(x^2+1) * \tan(x^2+1)$.

3. $y = \cos(3x-2)$

➡ $y' = -\sin(3x-2) * (3) = -3\sin(3x-2)$.

4. Find y' and y'' if $y = \frac{\sin x}{1+\cos x}$?

➡ $y' = \frac{(1+\cos x)*\cos x - \sin x * (-\sin x)}{(1+\cos x)^2}$

$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$

➡ $y'' = \frac{(1+\cos x)*(0)-(-\sin x)}{(1+\cos x)^2} = \frac{\sin x}{(1+\cos x)^2}$.

$5.f(x) = (7sin\,5x - cos\sqrt{x})^3$

$\Rightarrow f'(x) = 3(7sin\,5x - cos\sqrt{x})^2 * (35\,cos\,5x + \frac{1}{2\sqrt{x}}\,sin\sqrt{x}).$

$6.\ f(x) = sin^3(5x)$

$\Rightarrow f'(x) = 3(sin5x)^2 * cos5x * (5)$

$= 15\,sin^2 5x * cos5x.$

$7.\ y = cot\,x * csc\,x$

$\Rightarrow y' = cot\,x * (-csc\,x * cot\,x) + csc\,x * (-csc^2 x)$

$= -csc\,x * cot^2 x - csc^3 x.$

$8.\ y = \sqrt{x^3 - sin\,x} = (x^3 - sin\,x)^{\frac{1}{2}}$

$\Rightarrow y' = \frac{1}{2}(x^3 - sin\,x)^{\frac{-1}{2}} * (3x^2 - cos\,x)$

$= \frac{3x^2 - cos\,x}{2\sqrt{x^3 - sin\,x}}.$

$9.\ y = sin(\sqrt[3]{x})$

$\Rightarrow y' = cos\sqrt[3]{x} * (\frac{1}{3\sqrt[3]{x^2}})$

$10.\ f(x) = cos^3 7x = (cos\,7x)^3$

$\Rightarrow f'(x) = 3\,(cos\,7x)^2 * (-sin\,7x) * 7$

$= -21\,cos^2 7x * sin\,7x.$

$11.\ f(x) = cos\,3x - tan\,5x + sec\,4x$

$\Rightarrow f'(x) = -sin\,3x * (3) - sec^2 5x * (5) + sec\,4x * tan\,4x * 4$

$= -3\,sin\,3x - 5\,sec^2 5x + 4\,sec\,4x * tan\,4x.$