Working Stress Design

Dr. Majed Ashoor

Working Stress Method

Historically the Design method of (Working stress, Allowable Stress, or Service loads) is the first method used in design of Steel and Reinforced Concrete Structures.

The Method is Using (Working or Service Loads) & (Working or Allowable Stress) for design.

Working or Service Loads mean: The ordinary daily loads that expected to be applied on specific structures (without any factors of increasing the loads or factor of safety)

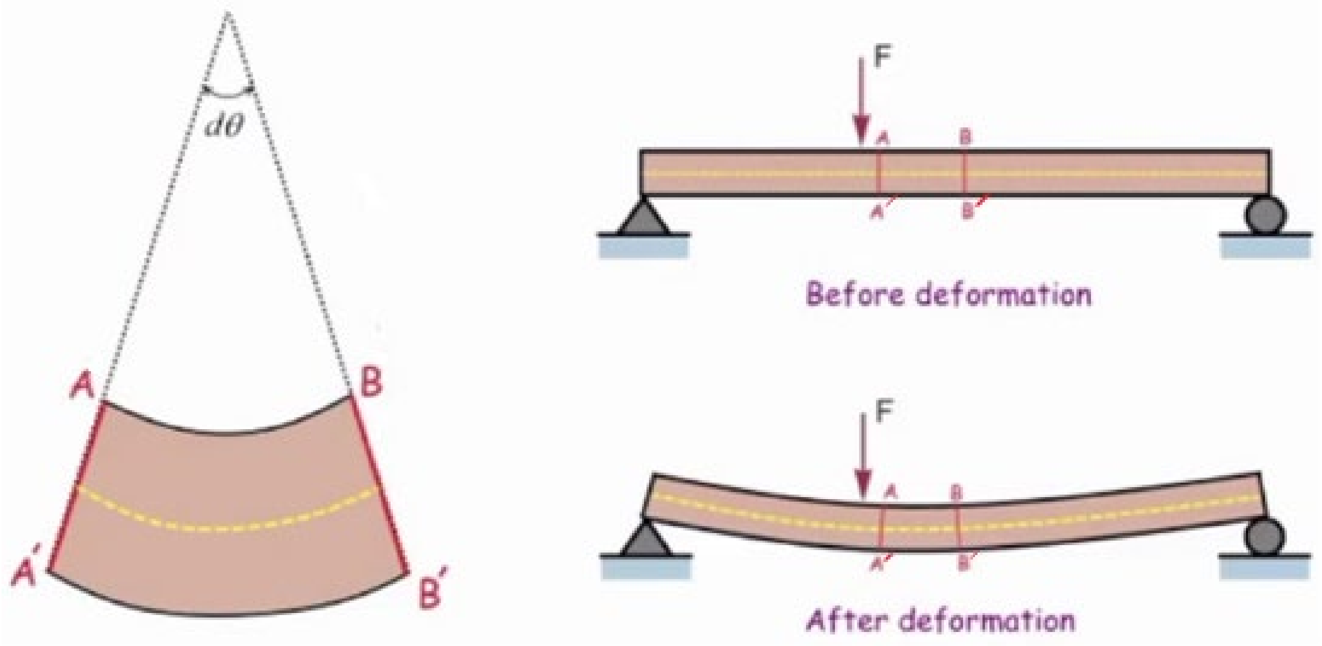
Working or Allowable Stress means: The level of stresses that would be Accepted by the “Designer” or more accurately by the “Code” in the specific member of the structure, that give some Factor of Safety against the failure or against un acceptable deformation or deflection.

Although the recent codes are using another methods for design (The Ultimate Strength Method), the method of Working Stress is still important for the following reasons:

1. The method still being used in Steel Structures, Prestress Concrete and some other applications.
2. It is necessary to calculate the serviceability of the concrete structures (Cracks & Deflection).
3. It is used in concrete tanks that used to reserve water or other fluids, specifically to investigate cracks.
4. Calculate deflection at service loads.

The Flexural Members

As we studied in the Strength of Materials when a flexural member is subjected to Bending Moment it will deform or deflect in a curvature form. Some of the fibers of the material (at the top or Bottom) will be exerted compression stresses, and the other fibers (at the other side) will be exerted tension stresses. The plane or the line between these two zones is called “The Neutral Axis” which will not be deformed at all and will not have tension neither compression stresses.



The Flexural Formulae

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

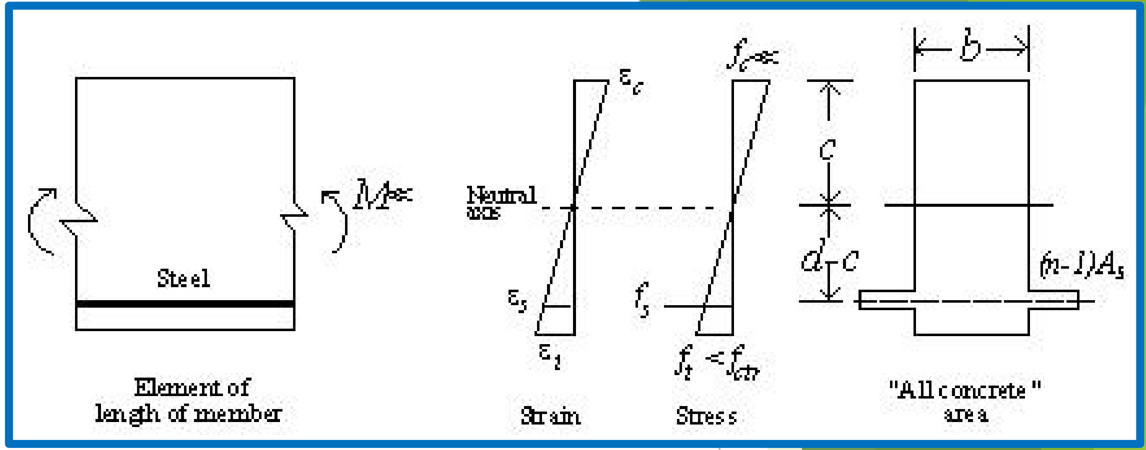
$$\frac{M}{I} = \frac{\sigma}{y}$$

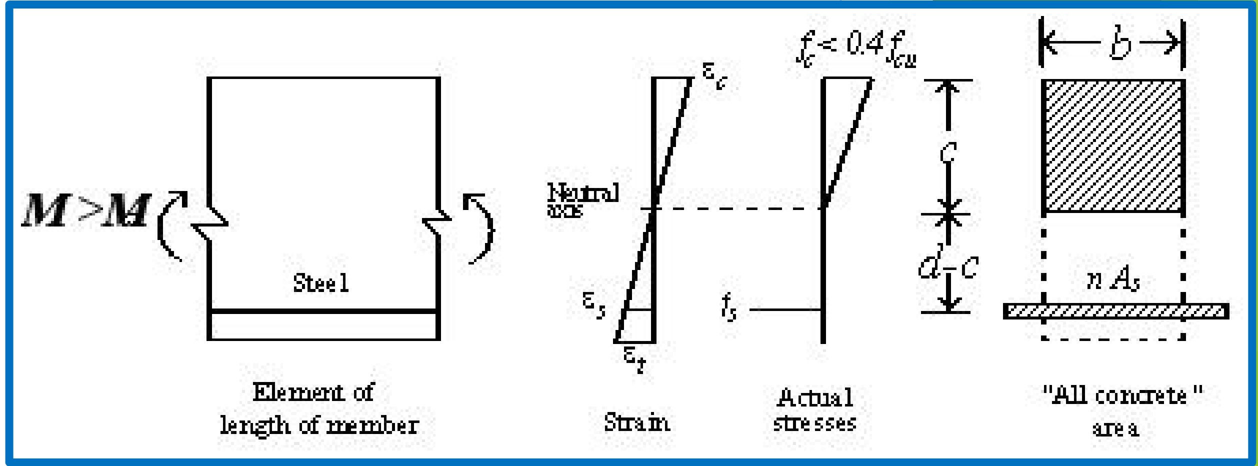
$$\sigma = \frac{M}{I} y$$

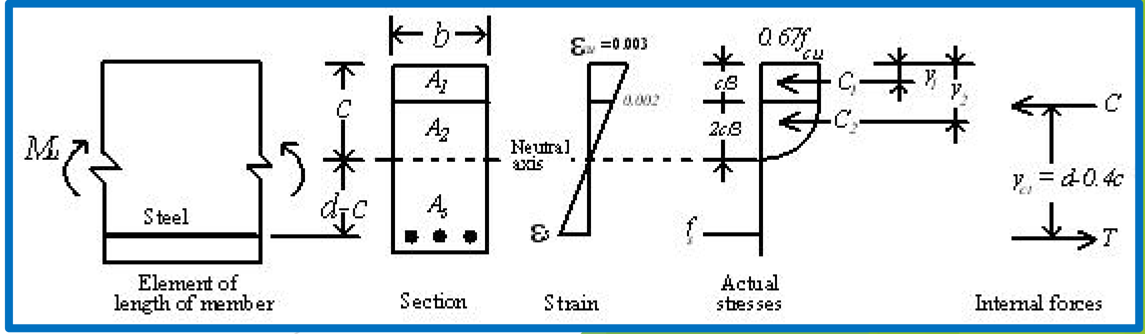
$$\frac{I}{y_{max}} = Z$$

$$\sigma_{max} = \frac{M}{Z}$$

$$n = \frac{E_s}{E_c}$$







The flexural formulae is working directly for the homogenous sections (Steel, Wood Members), However for Composite sections like (Reinforced Concrete), the section need to be transformed to an (Equivalent Section)

The flexural formulae needs also the material to be in the range of (Linear Stress-Strain Relationship) and does not work for the range of nonlinearity.

$f_c \leq 0.45\, f_c'$ $\quad (Allowable\ concrete\ compression\ stress)$

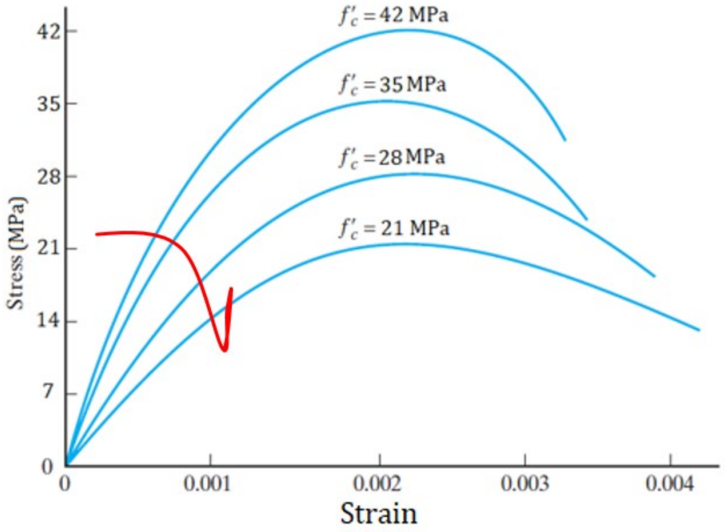
$f_s \leq 0.5\, f_y$ $\quad$ (Allowable steel stress)

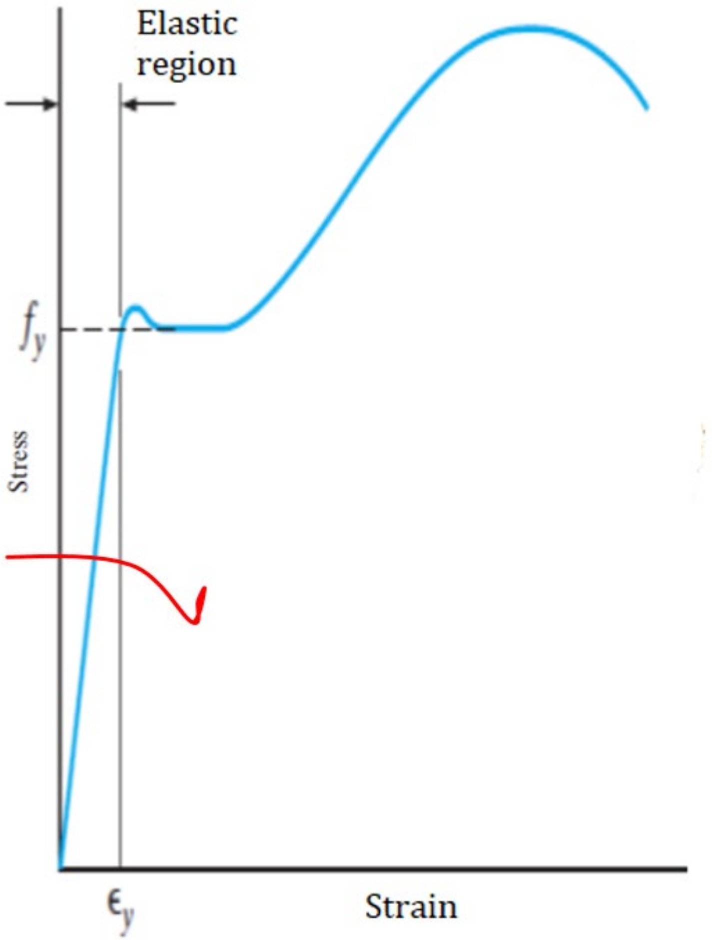
$E_s = 200{,}000\ MPa$

$E_c = 4730\sqrt{f_c'}\ MPa$ $\quad$ (Code)

$f_{cr} = 0.625\sqrt{f_c'}\ MPa$ $\quad$ (Code)

$Modulus\ of\ Rupture$

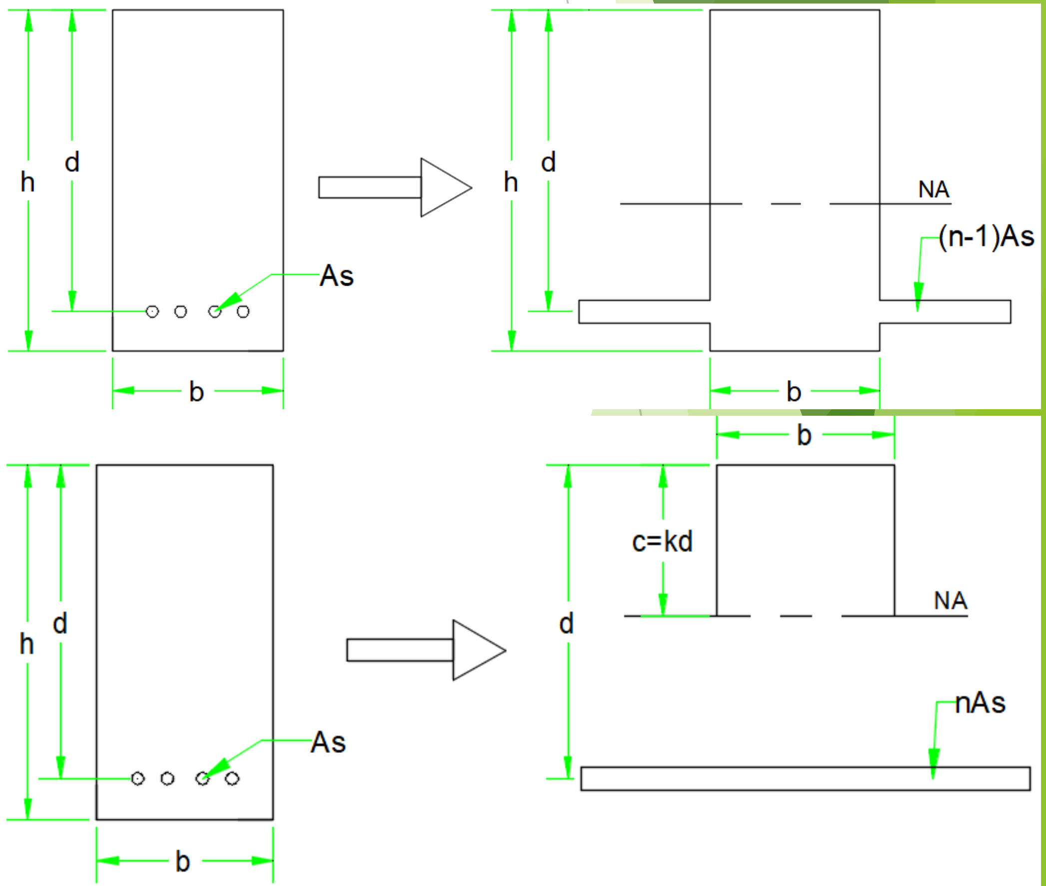




Stress-Strain Curves for Concrete and Steel

General Solution Steps:

- Transform the original section (two materials) into an equivalent section (one material)
- Calculate the location of the Neutral Axis (N.A)
- Calculate second moment of area (the moment of Inertia) of the equivalent section (about N.A)
- Apply the Flexural Formulae to calculate the stresses everywhere in the section
- Check the resulting stresses with the Code limitations



Useful Equations for Cracked Sections

$$\rho = \frac{As}{bd}$$

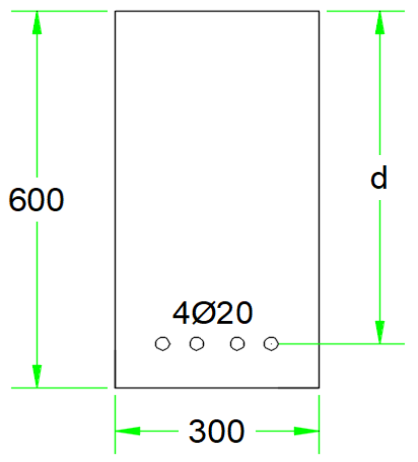
$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$c = k\, d$$

$$n = \frac{E_s}{E_c}$$

Example 1: Determine the crack moment for the section shown below, and the stress state.

Es=200000 MPa	f'c=28 MPa	fy=414 MPa	As=4ϕ20mm
b=300 mm	h=600 mm	Cover=50 mm	



$$A\emptyset 20 = \frac{\pi\, D^2}{4} = \frac{\pi}{4}(20)^2 = 314\, mm^2$$

$$A_s = 4 \times 314 = 1256\, mm^2$$

$$E_c = 4730\sqrt{f_c'} = 4730\sqrt{28} = 25028.8\, MPa$$

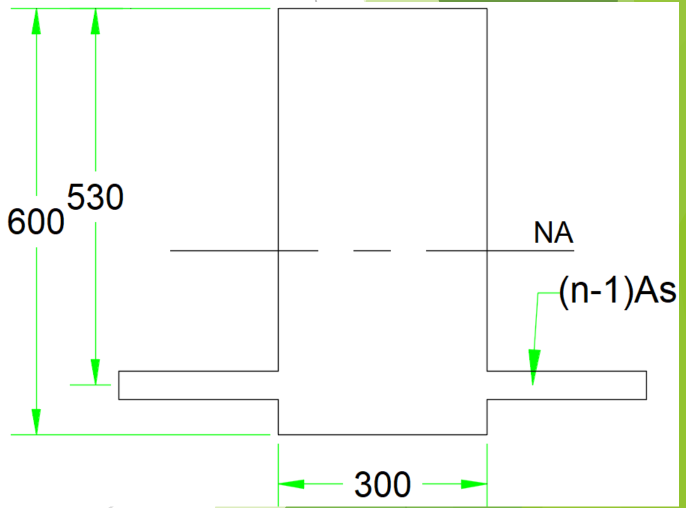
$$f_{c(allowable)} = 0.45 f_c' = 0.45 \times 28 = 12.6\, MPa$$

$$f_{s(allowable)} = 0.5 f_y = 0.5 \times 414 = 207\, MPa$$

$$n = \frac{E_s}{E_c} = \frac{200000}{25028.8} = 7.99 \cong 8$$

$$d = h - Cover - 10 - \frac{\emptyset}{2}$$

$$d = 600 - 50 - 10 - \frac{20}{2} = 530\, mm$$



$$\bar{y} = \frac{b\,h\,\frac{h}{2} + (n-1)A_s\,d}{b\,h + (n-1)A_s}$$ (from top fiber)

$$\bar{y} = \frac{\frac{300 \times 600 \times 600}{2} + (8-1) \times 1256 \times 530}{300 \times 6000 + (8-1) \times 1256} = 310.7\, mm$$

$$I_{gr} = \frac{b\,h^3}{12} + bh\left(\bar{y} - \frac{h}{2}\right)^2 + (n-1)A_s\,(d-\bar{y})^2$$

$$I_{gr} = \frac{300 \times 600^3}{12} + 300 \times 600\left(310.7 - \frac{600}{2}\right)^2 + (8-1) \times 1256 \times (530 - 310.7)^2$$

$$I_{gr} = 5.843 \times 10^9\ mm^4$$

$$y_{top} = \bar{y} = 310.7\ mm$$

$$y_{bot} = h - \bar{y} = 600 - 310.7 = 289.3\ mm$$

$$y_{steel} = y_{bot} - cover - 10 - \frac{\emptyset}{2} = 289.3 - 70 = 219.3\ mm$$

$$f_{cr} = 0.625\sqrt{f_c'} = 0.625 \times \sqrt{28} = 3.31\ MPa$$

$$f_b = f_{cr} = \frac{M_{cr}\,y_b}{I_{gr}}$$

$$3.31 = \frac{M_{cr} \times 289.3}{5.843 \times 10^9}$$

$$M_{cr} = 66.852 \times 10^6\ N.mm$$

$$M_{cr} = 66.852\ kN.m$$

$$f_c = \frac{M_{cr}\, y_t}{I_{gr}}$$

$$f_c = \frac{66.852 \times 10^6 \times\ 310.7}{5.843 \times 10^9} = 3.55 \ll 12.6\ MPa$$

$$f_s = n f_c =\ n \frac{M\ (h - \bar{y} - cover - 10 - \emptyset/2)}{I_{gr}}$$

$$f_s = 8 \times \frac{66.852 \times 10^6 \times 219.3}{5.843 \times 10^9} = 20.07 \ll 207\ MPa$$

Example 2: Determine the stress state for the section in the previous example if it is subjected to service moment M=100 kN.m

From the previous example: Mcr=66.85 kN.m

$$M = 100\ kN.m > 66.85\ (the\ section\ will\ crack)$$

$$\rho = \frac{As}{bd} = \frac{1256}{300 \times 530} = 0.0079$$

$$\rho n = 0.0079 \times 8 = 0.0632$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

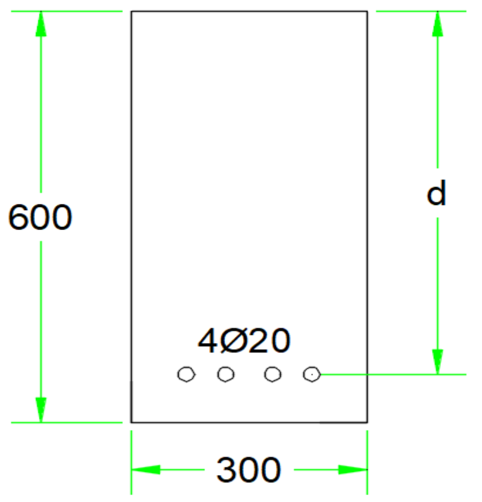
$$k = \sqrt{(0.0632)^2 + 2 \times 0.0632} - 0.0632 = 0.297$$

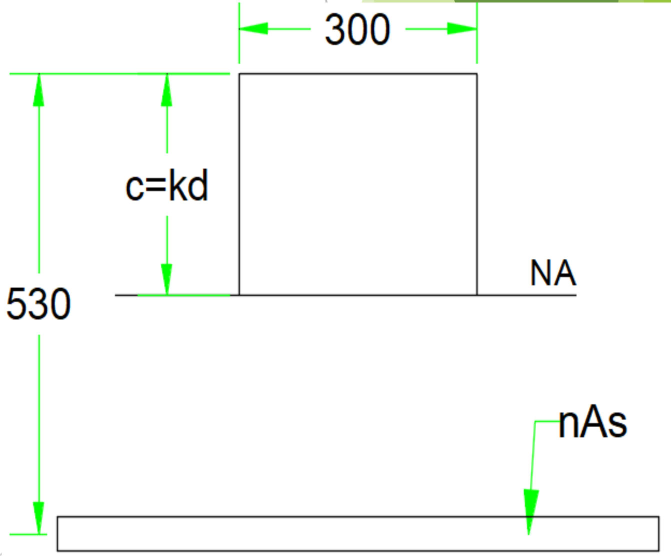
$$c = k\ d = 0.2975 \times 530 = 157.68$$

$$I_{cr} = \frac{b\ c^3}{3} + n\ A_s\ (d - c)^2$$

$$I_{cr} = \frac{300 \times (157.68)^3}{3} + 8 \times\ 1256 \times\ (530 - 157.68)^2$$

$$I_{cr} = \ 392039506 + 1392875689 = 1784915195 = 1.785 \times 10^9$$





$$f_{c(top)} = \frac{M\,y_t}{I_{cr}} = \frac{M\,c}{I_{cr}}$$

$$f_c = \frac{100 \times 10^6 \times \ 157.68}{1.785 \times 10^9} = 8.833 < 12.6\ MPa$$

$$f_s = \ n f_c = n\frac{M\,(d - c)}{I_{cr}}$$

$$f_s = \ 8 \times \frac{100 \times 10^6 \times (530 - 157.68)}{1.785 \times 10^9} = 166.866\ MPa < 207$$

Thank you…

Ultimate Strength Design Method

(Singly Reinforced Beams)

By: Dr. Majed Ashoor

Ultimate Strength Design Philosophy

design strength ≥ required strength

$$\phi S_n \geq U$$

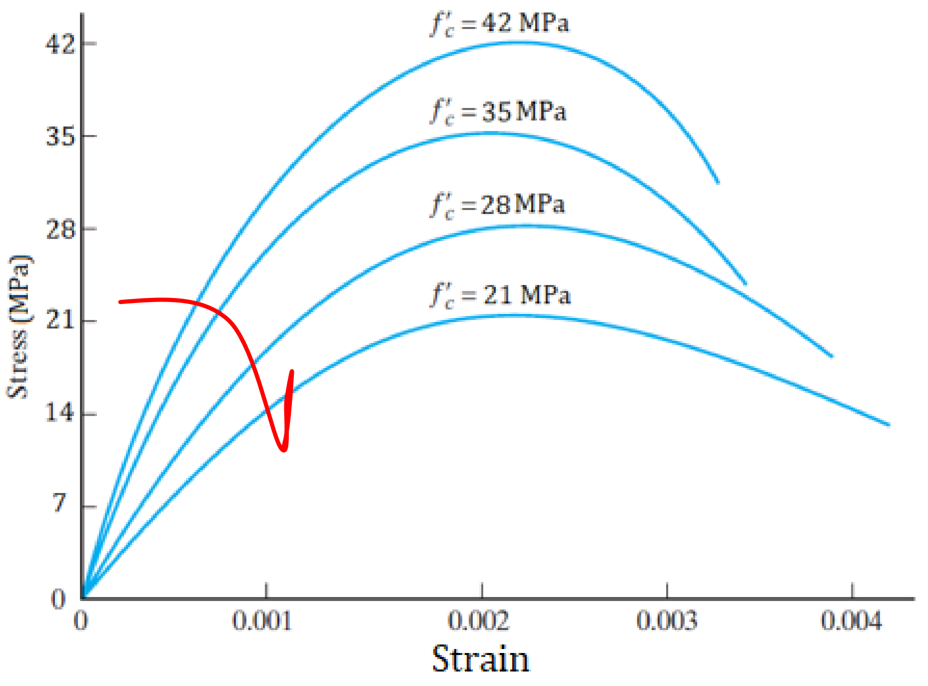
$$\phi M_n \geq M_u$$
$$\phi V_n \geq V_u$$
$$\phi T_n \geq T_u$$
$$\phi P_n \geq P_u$$

Table: 21.2.1 Strength Reduction Factor Ø

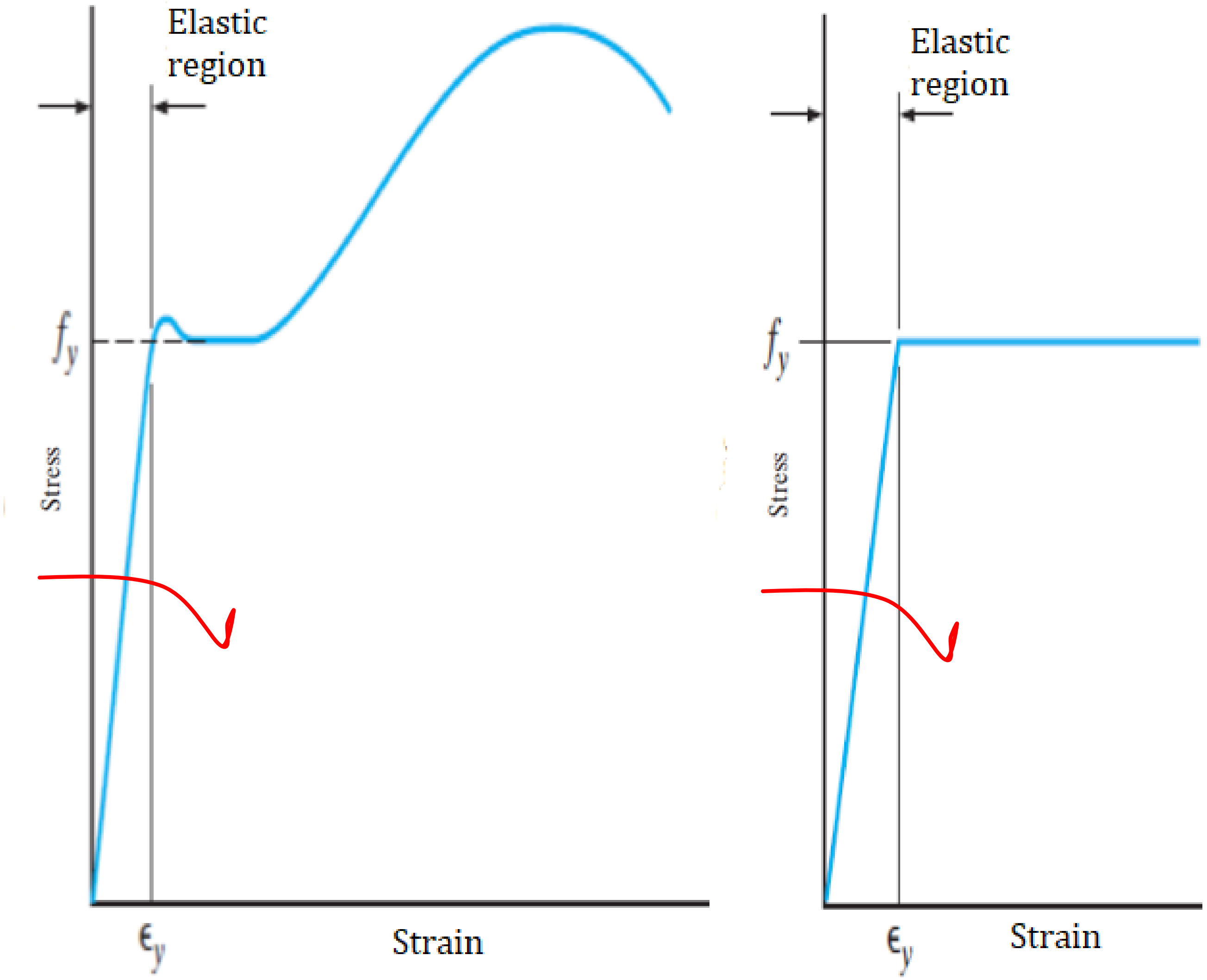
Action or Structural Element	Ø
Moment (Tension Controlled)	0.90
Shear	0.75
Torsion	0.75
Bearing	0.65
Plain Concrete Elements	0.60

Table 5.3.1—Load combinations

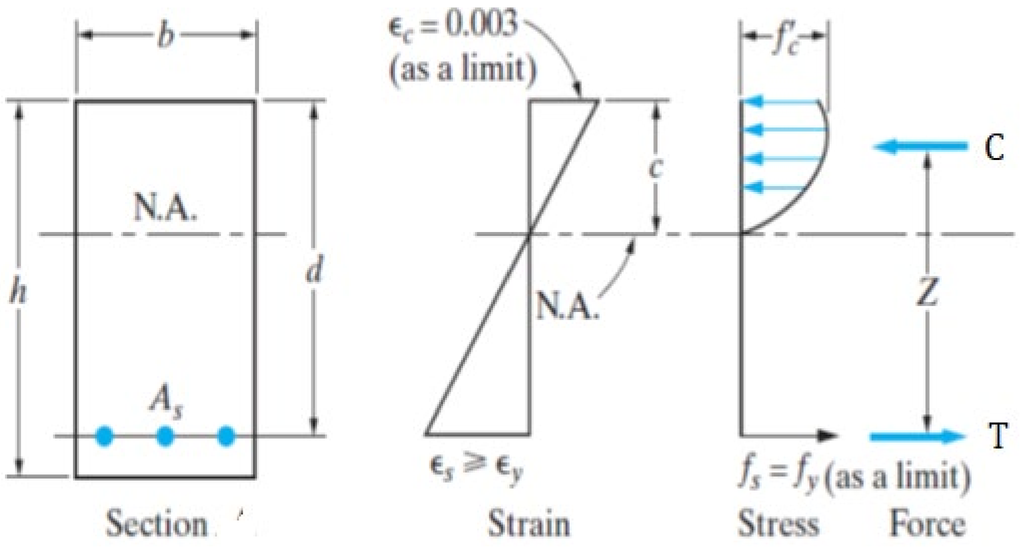
Load combination	Equation	Primary load
$U = 1.4D$	(5.3.1a)	D
$U = 1.2D + 1.6L$		L
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1b)	L
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$	(5.3.1c)	L_r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1d)	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	(5.3.1e)	E
$U = 0.9D + 1.0W$	(5.3.1f)	W
$U = 0.9D + 1.0E$	(5.3.1g)	E

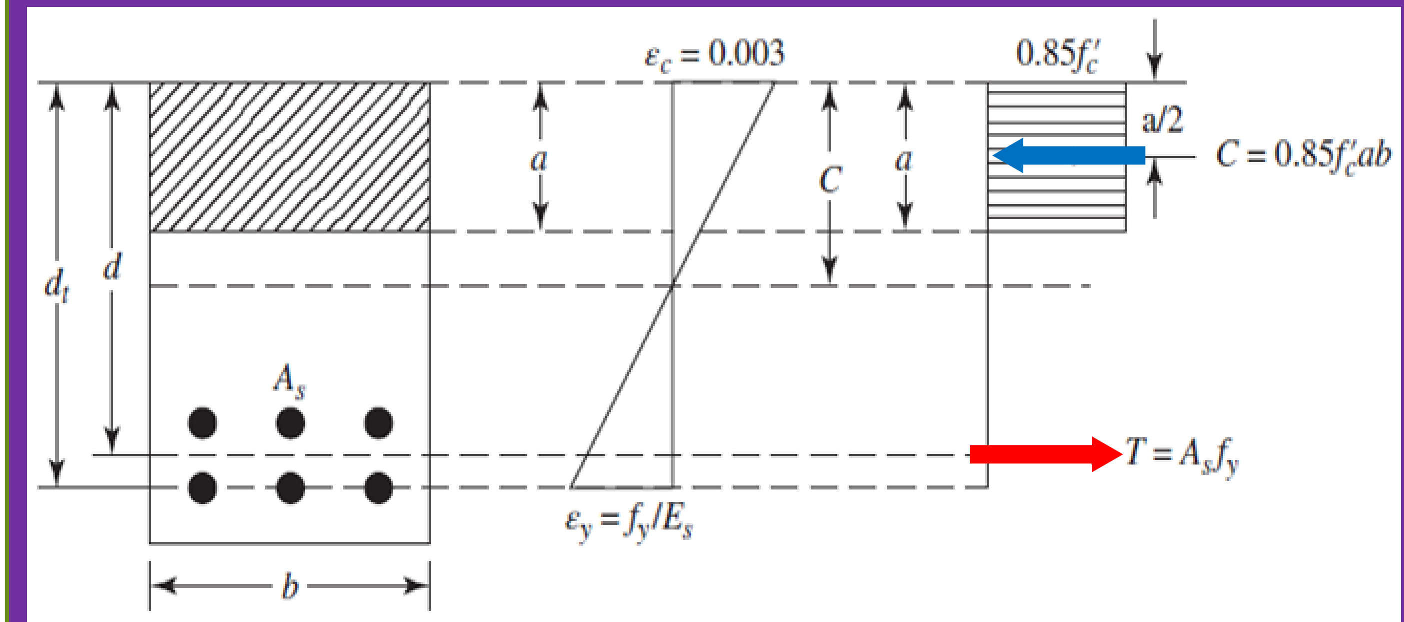


Typical Stress-Strain Curves for Concrete



Actual and Idealized Stress-Strain Curves for Steel





Idealized Stress and Strain Distribution of a.Rec.Section in Bending

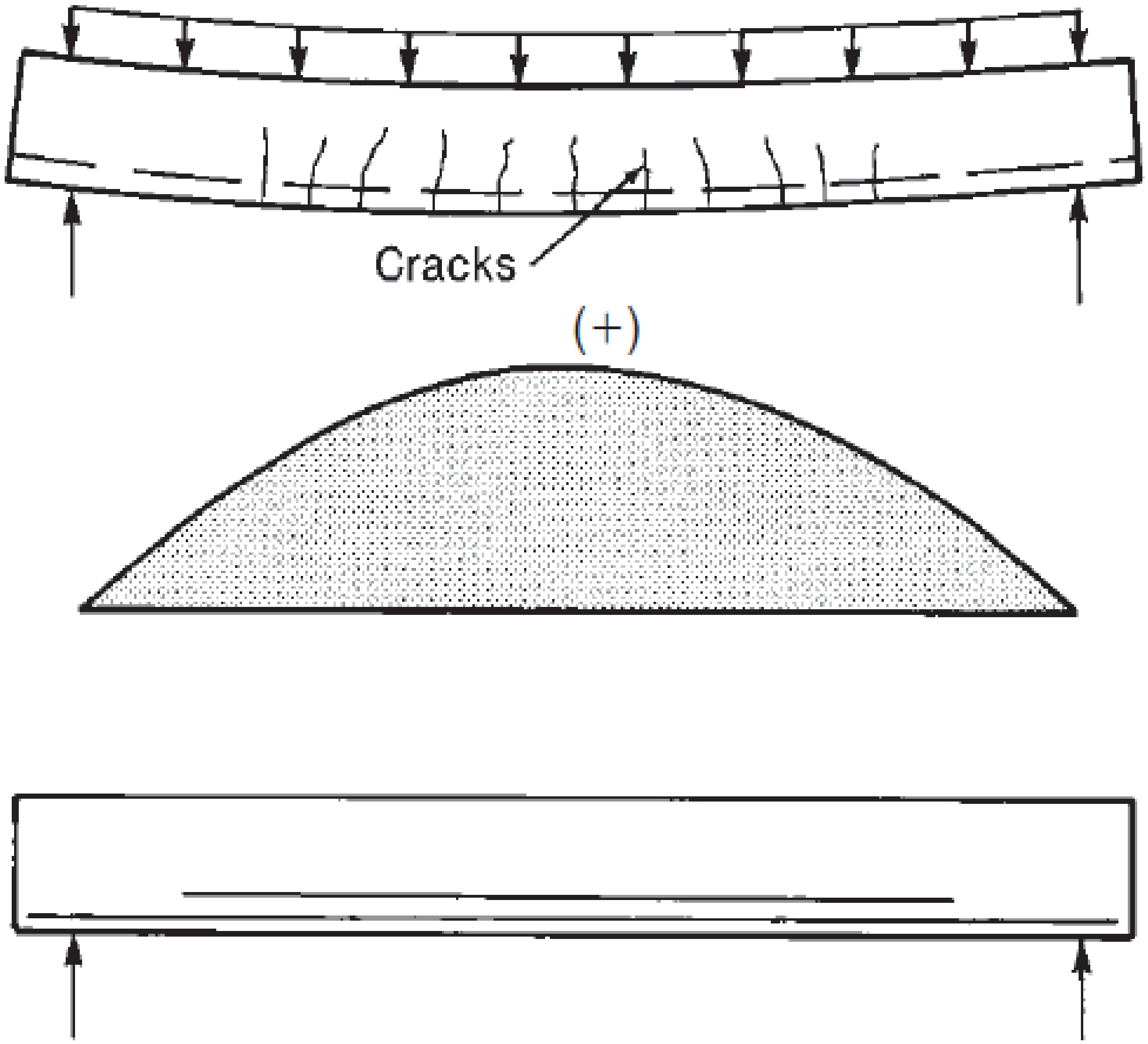
$$A_s\, f_y = 0.85\, f'_c\, a\, b \qquad (1)$$

$$M_n = A_s\, f_y \left(d - \frac{a}{2}\right) \qquad (OR) \qquad M_n = 0.85\, f'_c\, a\, b \left(d - \frac{a}{2}\right)$$

$$\varepsilon_t = 0.003 \left(\frac{d_t - c}{c}\right) \qquad (3)$$

$a = \beta_1 c$ *by definition:*

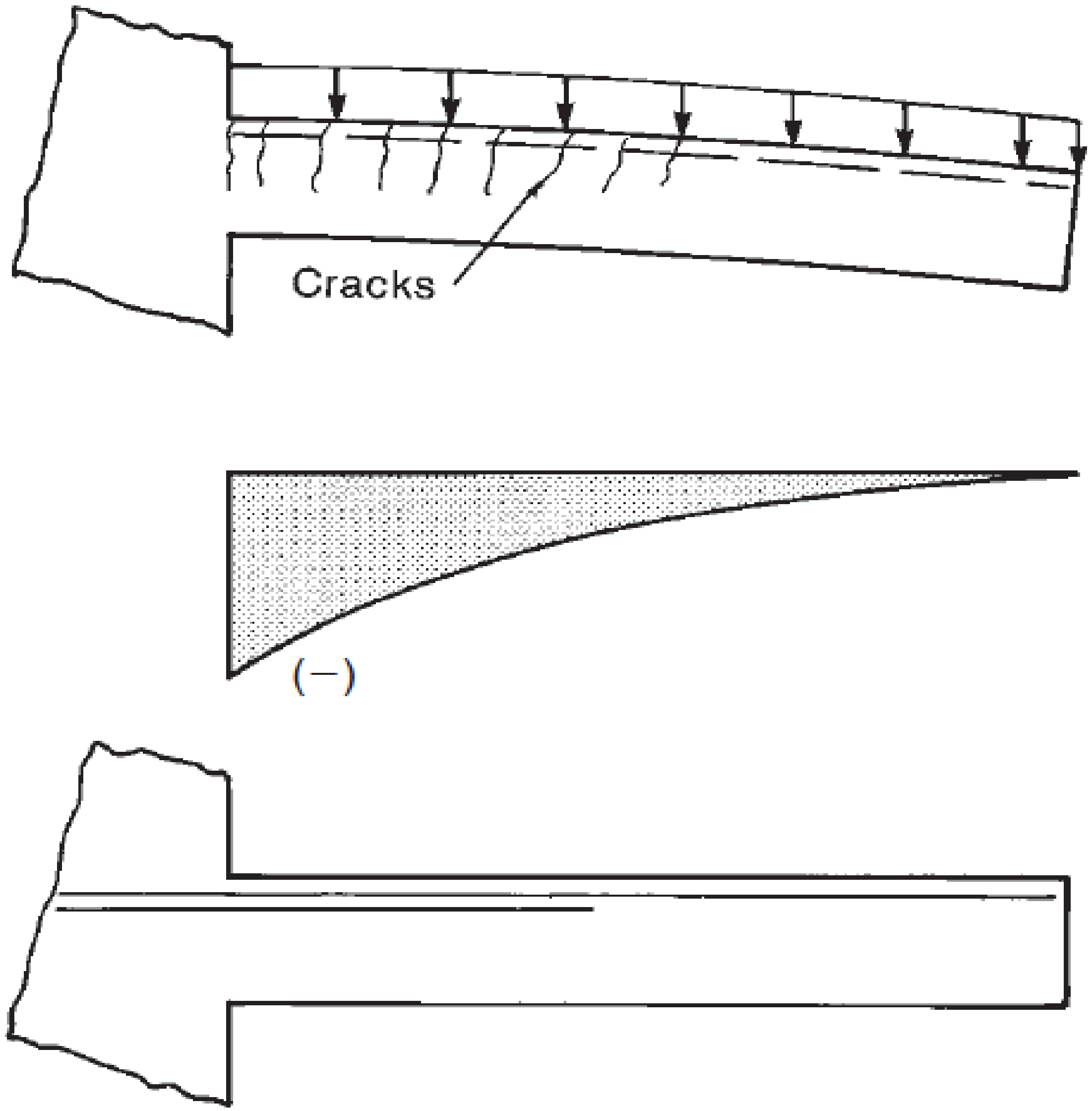
$$\beta_1 = \begin{cases} 0.85 & for\;\; f'_c \leq 28.0\; MPa \\ 0.85 - \dfrac{0.05\,(f'_c - 28)}{7} \nless 0.65 & for\;\; f'_c > 28.0\; MPa \end{cases}$$



(a) Deflected shape.

(b) Moment diagram.

(c) Reinforcement location.



(a) Deflected shape.

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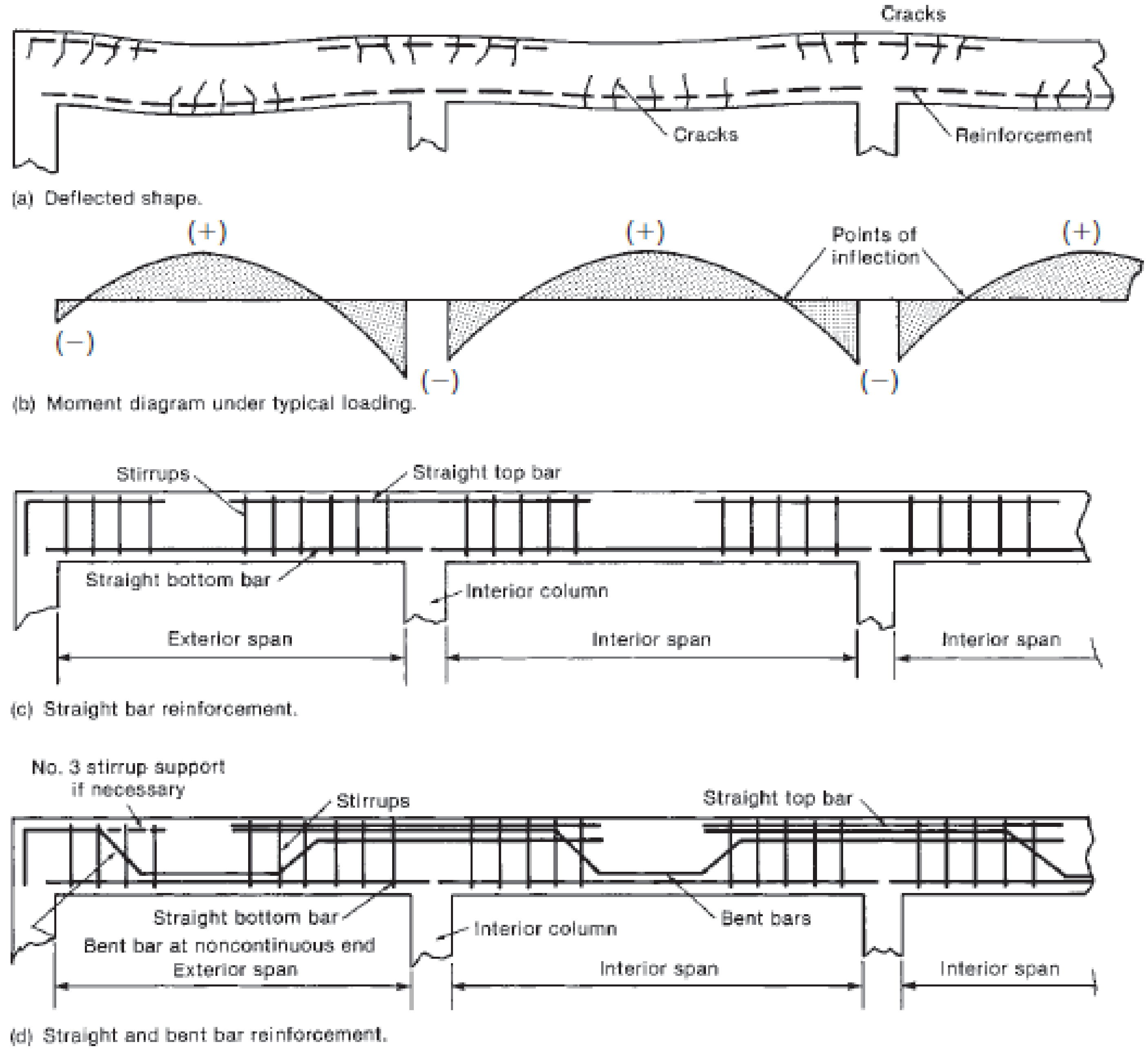

Cracks
Cracks
Reinforcement
(a) Deflected shape.
(+)
(+)
Points of
inflection
(+)
(−)
(−)
(−)
(b) Moment diagram under typical loading.
Stirrups
Straight top bar
Straight bottom bar
Interior column
Exterior span
Interior span
Interior span
(c) Straight bar reinforcement.
No. 3 stirrup support
if necessary
Stirrups
Straight top bar
Straight bottom bar
Bent bar at noncontinuous end
Interior column
Bent bars
Exterior span
Interior span
Interior span
(d) Straight and bent bar reinforcement.

Design of Single Reinforced Rectangular Beam Sections

(Design of SRRS)

By: Dr. Majed Ashoor

Before getting into design it is good to introduce the following new three terms or definitions:

$$\rho = \frac{A_s}{bd}$$

$$m = \frac{f_y}{0.85\, f'_c}$$

$$R_n = \frac{M_n}{bd^2}$$

$$A_s\, f_y = 0.85\, f'_c\, a\, b \qquad (1)$$

$$M_n = A_s\, f_y \left(d - \frac{a}{2}\right) \qquad (2)$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) \qquad (3)$$

$$a = \rho\, m\, d \qquad (1')$$

$$R_n = \frac{M_n}{bd^2} = \rho\, f_y \left(1 - \frac{1}{2}\rho m\right) \qquad (2')$$

$$\rho = \left(\frac{0.003}{0.003 + \varepsilon_s}\right)\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \qquad (3')$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2m\, R_n}{f_y}}\right) \qquad (4')$$

As an example of usefulness of the above (dash equations) we can calculate ρb and ρmax for (fy=420MPa) by substituting εs=0.002 and εs=0.005 in eq.(3') respectively:

$$\rho_b = \frac{3}{5}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \quad for\ fy = 420\ MPa\ because\ \varepsilon y = 0.002$$

$$\rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \quad for\ all\ Cases$$

Or in design problems we can start estimating the As by assuming εs=0.007 to stay in Tension Control Zone so:

$$\rho_{start} = \frac{3}{10}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

Design problems will mainly depend on eq.(2')

$$\boldsymbol{R_n = \frac{M_n}{bd^2} = \rho\, f_y\left(1 - \frac{1}{2}\rho m\right)} \quad (2')$$

Table 9.3.1.1—Minimum depth of nonprestressed beams

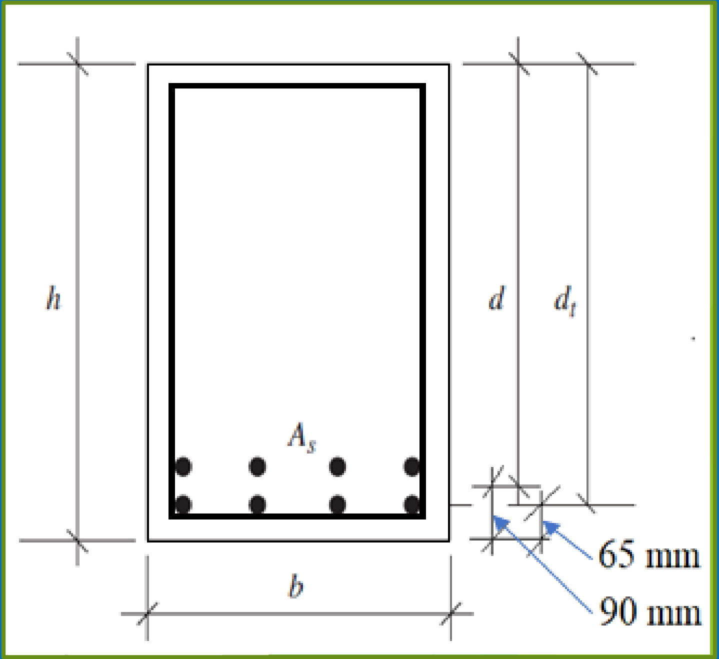
Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

$$h = [1.5\ to\ 2] \times b$$

Minimum concrete cover for beams=40mm

ACI 25.2.1 the minimum clear bar spacing:

$$S_{min} \geq \begin{cases} 25mm \\ d_b \\ \left(\frac{4}{3}\right) d_{agg} \end{cases}$$



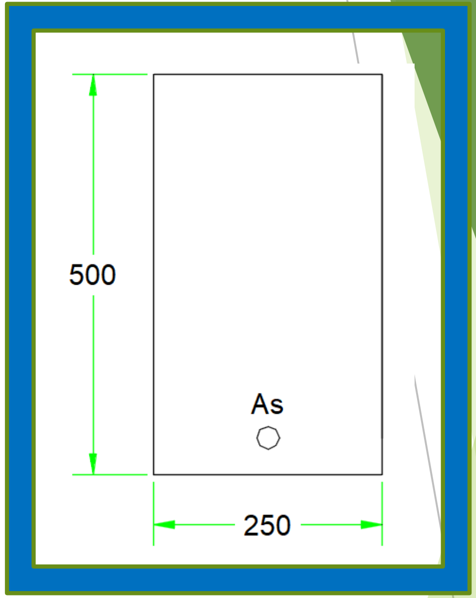
Ex1: Find the necessary reinforcement for a given section that has a width of 250mm and total depth of 500mm, if it is subjected to a factored moment of 180kN.m. Given f'c=21Mpa, fy=375MPa.

Solution:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{375}{200000} = 0.001875$$

$$m = \frac{f_y}{0.85\, f_c'} = \frac{375}{0.85 \times 21} = 21.0$$

$$M_n = \frac{180}{0.9} = 200\ kN.m$$



Assume one layer of reinforcement

$$d = h - 65 = 500 - 65 = 435mm$$

$$R_n = \frac{M_n}{bd^2}$$

$$R_n = \frac{200 \times 10^6}{250 \times 435^2} = 4.227$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2m\,R_n}{f_y}}\right)$$

$$\rho = \frac{1}{21}\left(1 - \sqrt{1 - \frac{2 \times 21 \times 4.227}{375}}\right) = 0.013$$

$$\rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

$$\rho_{max} = \frac{3 \times 0.85}{8 \times 21.0}(1) = 0.0151 > 0.013\ (ok\ Tension\ control)$$

$$A_s = \rho b d = 0.013 \times 250 \times 435 = 1413.75\ mm^2$$

Try Ø25→Ab=490 mm^2

$$n = \frac{As}{Ab} = \frac{1413.75}{490} = 2.88$$

Use 3 Ø25

$$As_{provided} = 3 \times 490 = 1470\ mm^2$$

$$\rho_{provided} = \frac{1470}{250 \times 435} = 0.0135 < 0.0151\ (ok\ T.C)$$

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{375} = 0.00373 < 0.0135\ (ok)$$

Check for bar spacing:

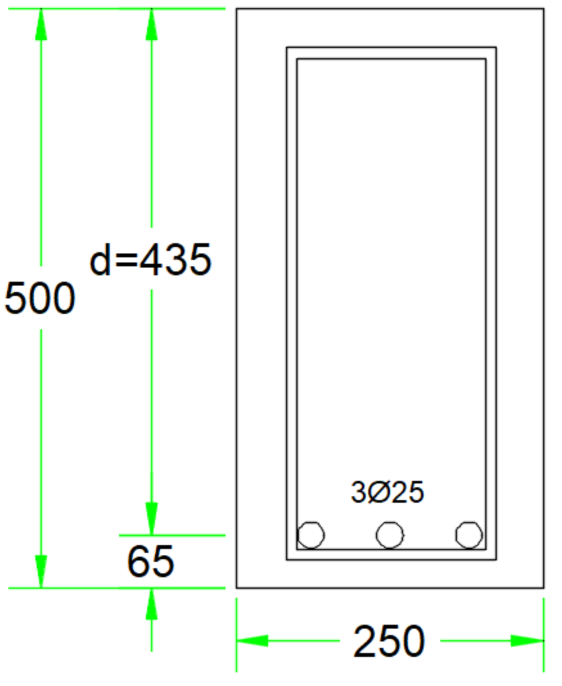
$$S_{min} = \begin{cases} 25mm \\ d_b = 25 \\ \left(\frac{4}{3}\right) d_{agg} = \frac{4}{3} \times 20 = 26.66 \end{cases}$$

$$s = \frac{250 - 100 - 3 \times 25}{2} = 37.5 > 26.66\ (ok)$$

$$M_n = \rho\ f_y\ bd^2 \left(1 - \frac{1}{2}\rho m\right)$$

$$M_n = 0.0135 \times 375 \times 250 \times 435^2 \left(1 - \frac{1}{2} \times 0.0135 \times 21.0\right) = 205.54kN.m$$

$$M_u = \emptyset\ M_n = 0.9 \times 205.54 = 184.98\ kN.m > 180\ (OK)$$



Ex2: Design a rectangular section to resist a factored moment of 300kN.m. Given f'c=28MPa, and fy=420MPa.

Solution:

$$M_n = \frac{M_u}{\emptyset} = \frac{300}{0.9} = 333.33\ kN.m$$

$$m = \frac{f_y}{0.85\ f_c'} = \frac{420}{0.85 \times 28} = 17.647$$

$$R_n = \frac{M_n}{bd^2} = \rho\ f_y\left(1 - \frac{1}{2}\rho m\right)$$

Start with ρ depending on εs=0.007 and assuming one layer of reinforcement:

$$\rho_{start} = \frac{3}{10}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

$$\rho_{start} = \left(\frac{3}{10}\right)\frac{0.85}{17.647}(1) = 0.0144$$

$$R_n = \rho\ f_y\left(1 - \frac{1}{2}\rho m\right)$$

$$R_n = 0.0144 \times 420 \left(1 - \frac{1}{2} \times 0.0144 \times 17.647\right) = 5.277$$

$$R_n = \frac{M_n}{bd^2}$$

$$5.277 = \frac{333.33 \times 10^6}{bd^2}$$

$$bd^2 = 63166571.92\, mm^3$$

Assume b to get d:

b	d	d/b
250	502.66	2.01
300	458.86	1.529
350	424.82	1.213

$$A_s = \rho bd = 0.0144 \times 300 \times 460 = 1987.2\, mm^2$$

Try Ø22→Ab=380

$$n = \frac{1987.2}{380} = 5.23$$

Use 6Ø22 in two layers:

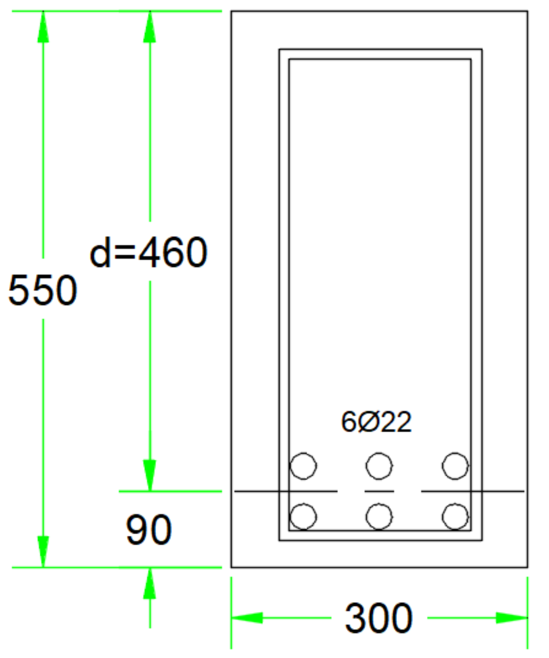
$$As_{provided} = 6 \times 380 = 2280\ mm^2$$

$$\rho_{provided} = \frac{2280}{300 \times 460} = 0.0165$$

$$\rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

$$\rho_{max} = \frac{3}{8} \times \frac{0.85}{17.647} \times \left(\frac{489}{460}\right) = 0.0192 > 0.0165\ (ok\ T.C)$$

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{420} = 0.00333 < 0.0165\ (ok)$$



$$h = d + 90 = 460 + 90 = 550\ mm$$

$$s = \frac{300 - 100 - 3 \times 22}{2} = 67 > 26.66\ (ok)$$

$$M_n = 0.0165 \times 420 \times 300 \times 460^2\left(1 - \frac{1}{2} \times 0.0165 \times 17.647\right) = 375.87\ kN.m$$

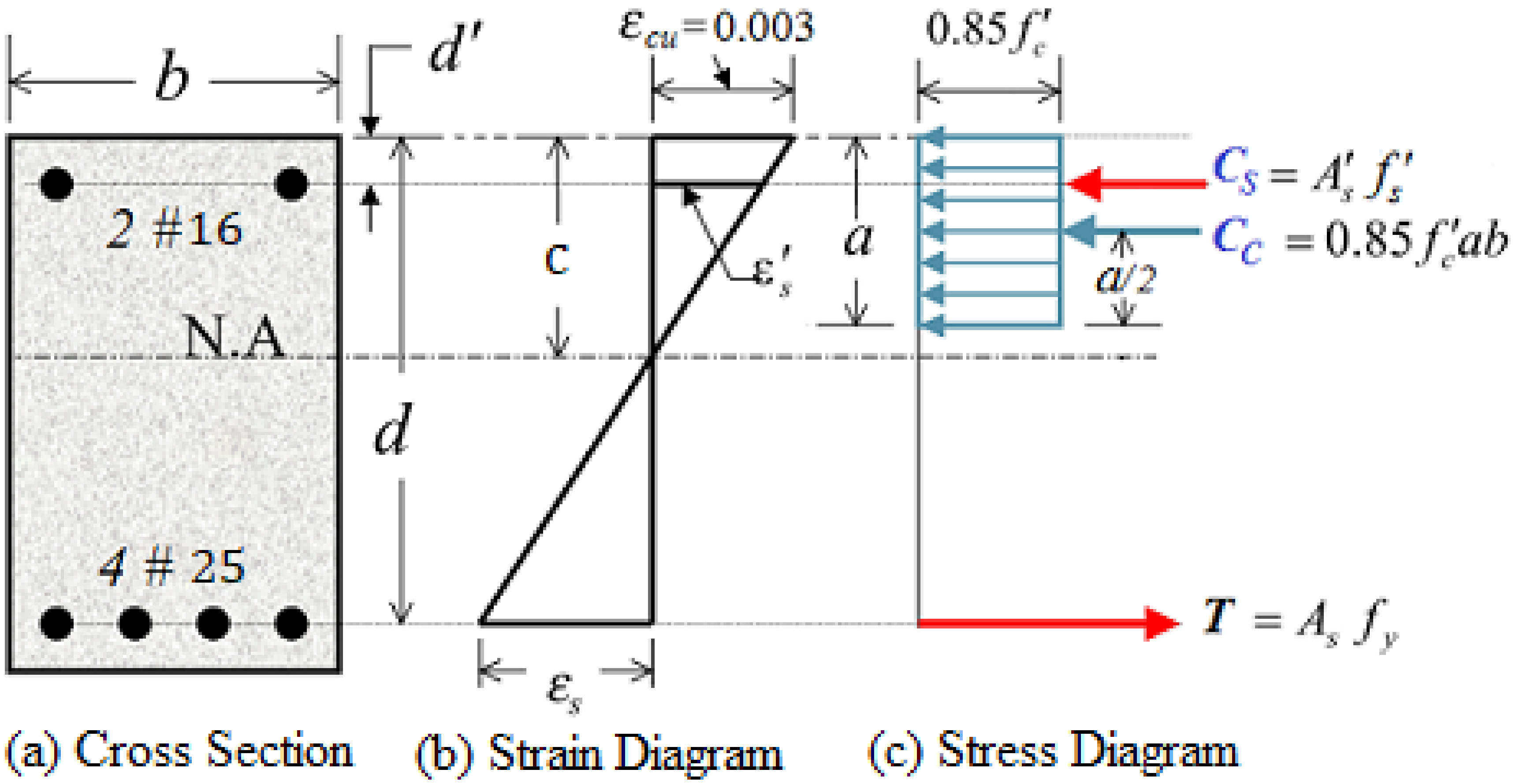
$$M_u = \emptyset\ M_n = 0.9 \times 375.87 = 338.283\ kN.m > 300\ (OK)$$

Thank you…

Design of
Double Reinforced
Rectangular Beam Sections

(Design of DRRS)

By: Dr. Majed Ashoor



$$A_s f_y = 0.85\, f_c' \,\beta_1\, c\, b + A_s' \, f_s'$$

$$\rho_{max} = \left(\frac{3}{8}\right)\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

$$M_{n(max)} = \rho_{max}\, f_y b d^2 \left(1 - \frac{1}{2}\rho_{max} m\right)$$

$$M_{nmax} \geq \frac{M_u}{\emptyset}$$

Yes → SRRS

No → DRRS

$$M_n = M_{n1} + M_{n2}$$

ρ_{start}

$$\rho_{start} = \left(\frac{3}{10}\right)\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

→ $As_1 = \rho_{start}\, b\, d$

$$M_{n1} = \rho_{start}\, f_y b d^2 \left(1 - \frac{1}{2}\rho_{start} m\right)$$

$$M_{n2} = \frac{M_u}{\emptyset} - M_{n1}$$

$$M_{n2} = A_{s2} f_y (d - d')$$

$$A'_s = As_2 \times \frac{f_y}{f'_s}$$

$$M_n = M_{n1} + M_{n2}$$

$$a = \rho\, m\, d$$

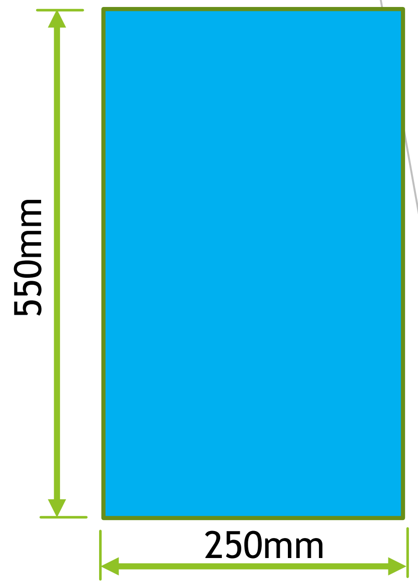
$$a_{start} = \rho_{start}\, m\, d$$

$$c_{start} = \frac{a_{start}}{\beta_1}$$

$$f'_s = 0.003\, E_s \left(\frac{c - d'}{c}\right)$$

Ex1: A beam section is limited to a width b=250mm and a total depth of h=550mm and has to resist a factored moment of 307 kN.m. Calculate the required reinforcement, Given: f'c=21 MPa, fy=350 MPa

Solution:



$$m = \frac{f_y}{0.85\, f_c'} = \frac{350}{0.85 \times 21} = 19.607$$

$$\rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

$$\rho_{max} = \frac{3}{8} \times \frac{0.85}{19.607}(1) = 0.0162$$

$$M_n = \rho\, f_y\, bd^2\left(1 - \frac{1}{2}\rho m\right)$$

$$d = h - 65 = 550 - 65 = 485mm$$

$$M_n = 0.0162 \times 350 \times 250 \times 485^2\left(1 - \frac{1}{2}0.0162 \times 19.607\right) = 280.293kN.m$$

$$\emptyset M_n = 0.9 \times 280.293 = 252.26 < 307 \quad (Not\ OK)$$

The section should be designed as a DOUBLY REINFORCED SECTION

Assume two layers of tension steel:

$$d = h - 90 = 550 - 90 = 460\ mm$$

$$d_t = h - 65 = 550 - 65 = 485\ mm$$

$$d' = 65\ mm$$

$$\rho_{start} = \frac{3}{10}\frac{\beta 1}{m}\left(\frac{dt}{d}\right)$$

$$\rho_{start} = \frac{3}{10} \times \frac{0.85}{19.607}\left(\frac{485}{460}\right) = 0.0137$$

$$As_1 = \rho_{start}\ b\ d = 0.0137 \times 250 \times 460 = 1575.5\ mm^2$$

$$M_{n1} = \rho\ f_y b d^2\ \left(1 - \frac{1}{2}\rho m\right)$$

$$M_{n1} = 0.0137 \times 350 \times 250 \times 460 \times 460\left(1 - \frac{1}{2}0.0137 \times 19.607\right) = 219.786 kN.m$$

$$M_{n2} = \frac{M_u}{\emptyset} - M_{n1}$$

$$M_{n2} = \frac{307}{0.9} - 219.786 = 121.325\ kN.m$$

$$M_{n2} = A_{s2} f_y (d - d')$$

$$121.325 \times 10^6 = A_{s2} \times 350(460 - 65)$$

$$A_{s2} = 877.577\ mm^2$$

$$As = As_1 + As_2 = 1575.5 + 877.577 = 2453.07\ mm^2$$

$$a = \rho m d = 0.0137 \times 19.607 \times 460 = 123.563 mm$$

$$c = \frac{a}{\beta 1} = \frac{123.563}{0.85} = 145.368 mm$$

$$f_s' = \left[0.003\left(\frac{c - d'}{c}\right)\right] E_s$$

$$f_s' = \left[0.003\left(\frac{145.368 - 65}{145.368}\right)\right] 200000 = 331.716 < f_y$$

$$A_s' = As_2 \times \frac{f_y}{f_s'}$$

$$A_s' = 877.577 \times \frac{350}{331.716} = 926\ mm^2$$

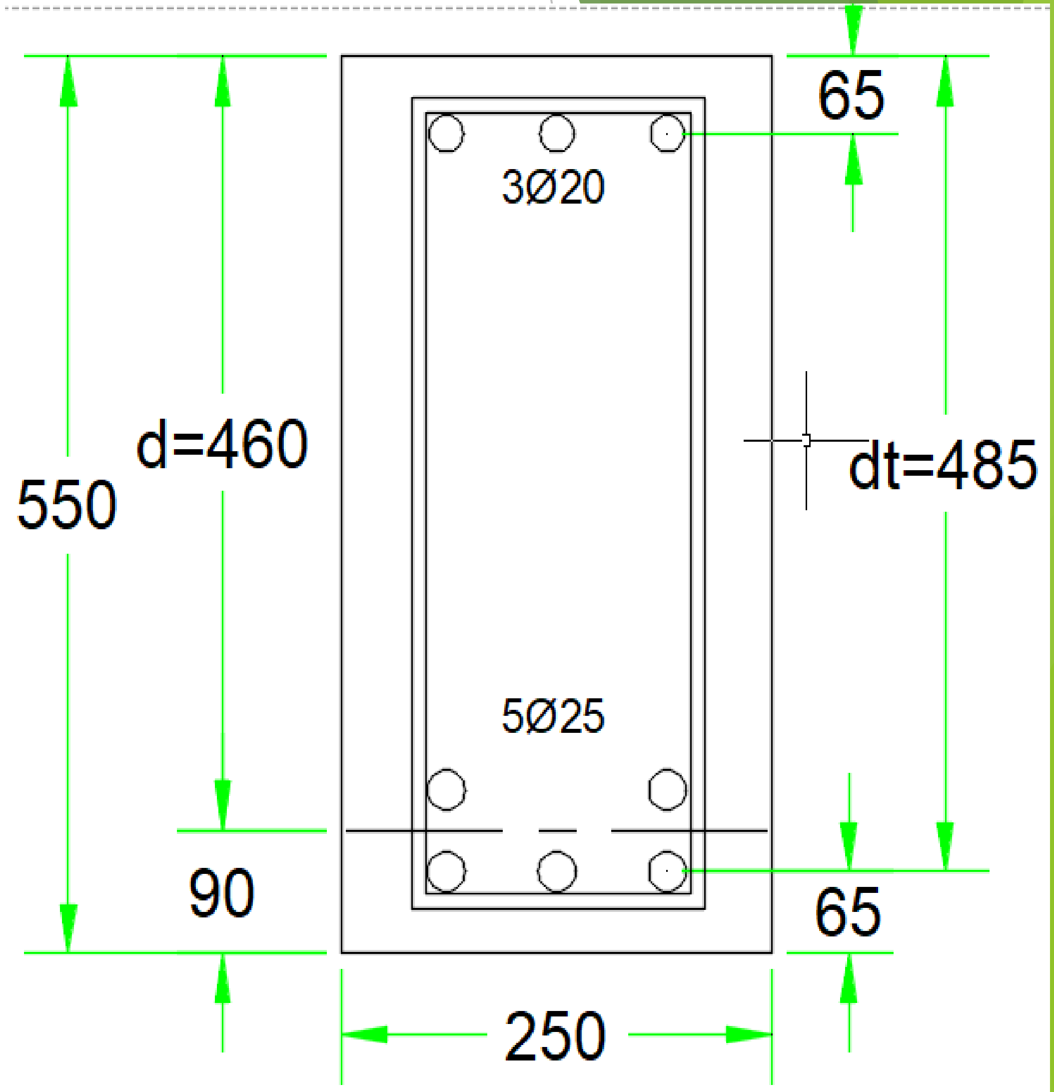
Use: 3Ø20 for compression:

$$A_s' = 3 \times 314 = 942\,mm^2 > 926\,mm^2$$

Use: 5Ø25 for Tension:

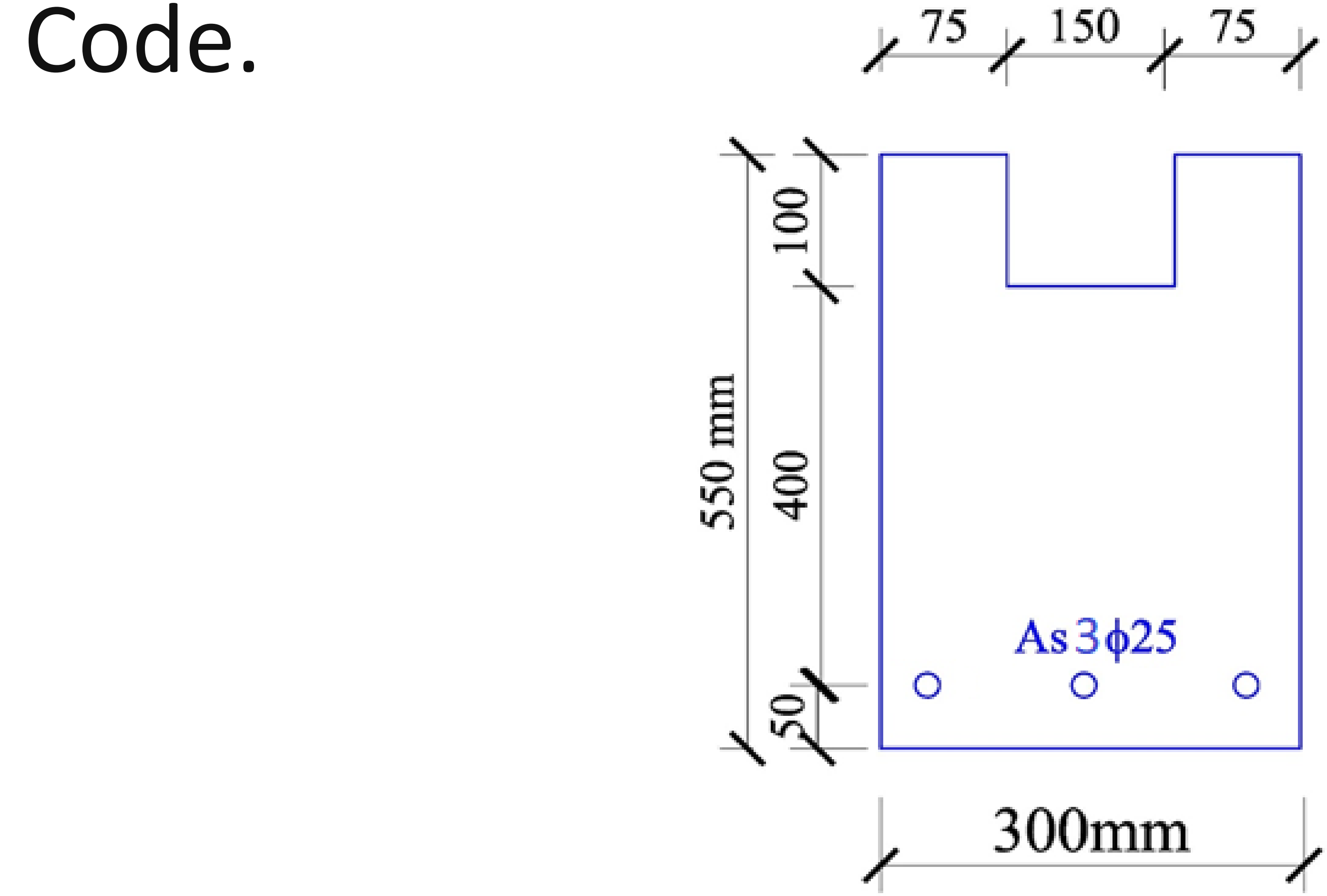
$$A_s = 5 \times 490.8 = 2454\,mm^2 \approx 2453.07$$

$$s = \frac{150 - 3 \times 25}{2} = 37.5 > 26.66\,(ok)$$



Thank you…

EX7. Determine the design moment strength of section shown below , Given $f_c' = 28\,MPa$ and $f_y = 420\,MPa$ and check the specification of the section according to ACI Code.



Solution:

$$\rho = \frac{A_s}{effective\ area\ (Ae)}$$

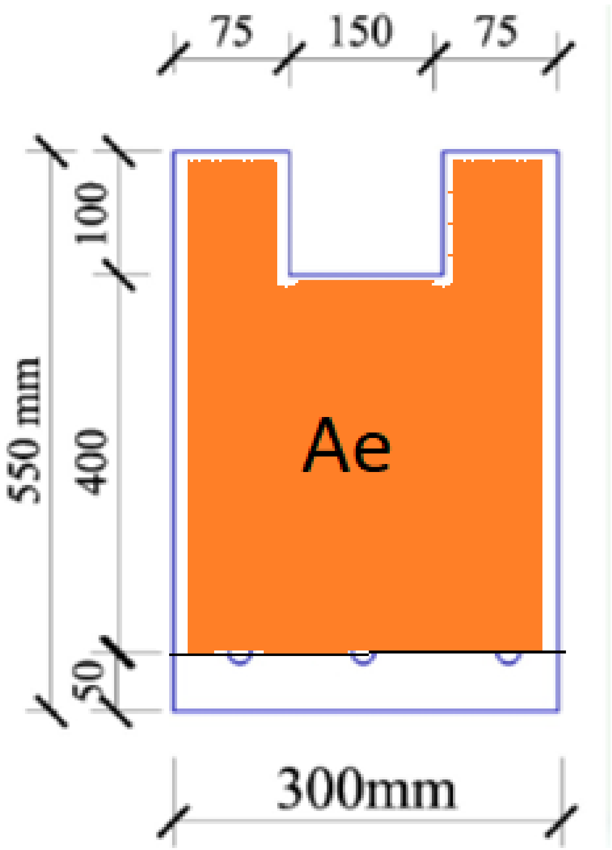
$$A_s = 3 \times \pi \ \times \frac{25^2}{4} = 1470\, mm^2$$

$$A_e = \ b\, d - 150\ \times 100$$

$$A_e = \ 300\ \times\ 500 - 150\ \times 100$$

$$A_e = \ 135000\, mm^2$$

$$\rho = \frac{1470}{135000} = 0.0109$$

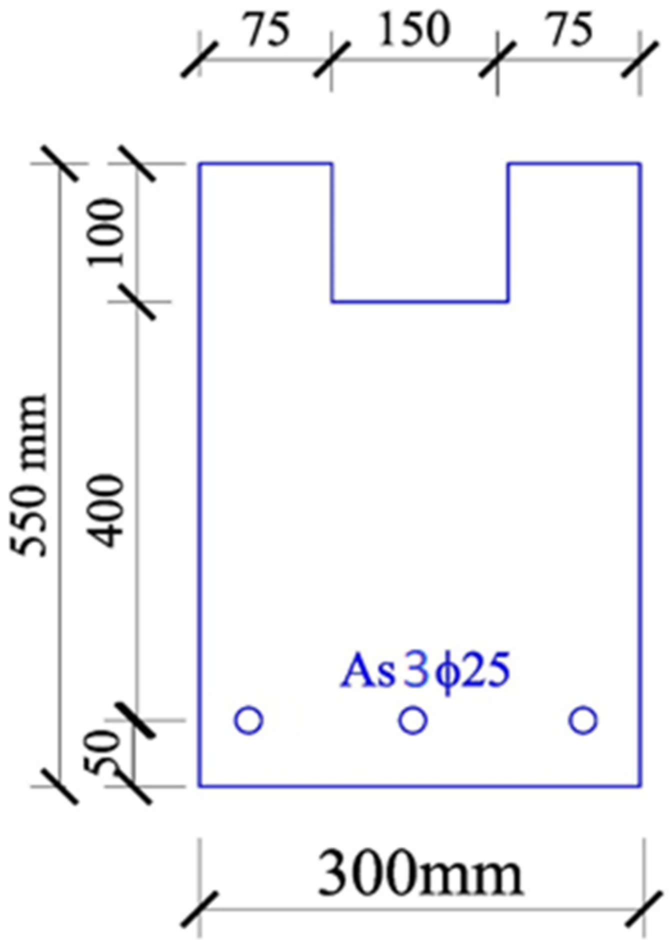


Check ACI requirements

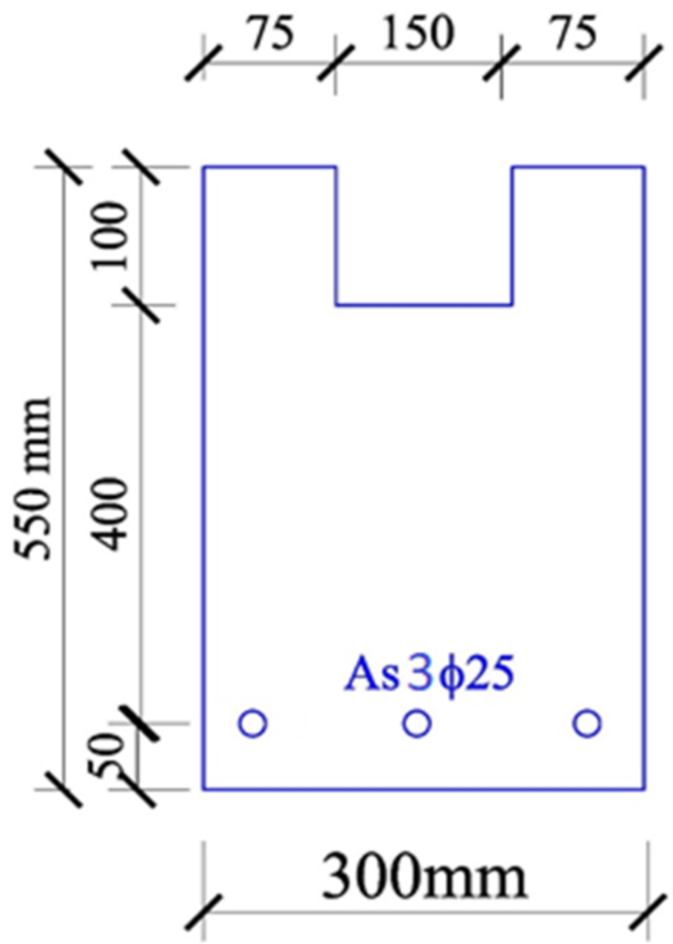
Check e_{min}

$$\rho_{min} = \begin{cases} \left(\frac{0.25\sqrt{f_c'}}{fy}\right) & \text{for } f_c' > 30 \text{ MPa} \\ \left(\frac{1.4}{fy}\right) & \text{for } f_c' \leq 30 \text{ MPa} \end{cases}$$

$\therefore\ e_{min} = \frac{1.4}{fy} = 0.003333 < e\ (0.0109)$ OK



Check ACI requirements



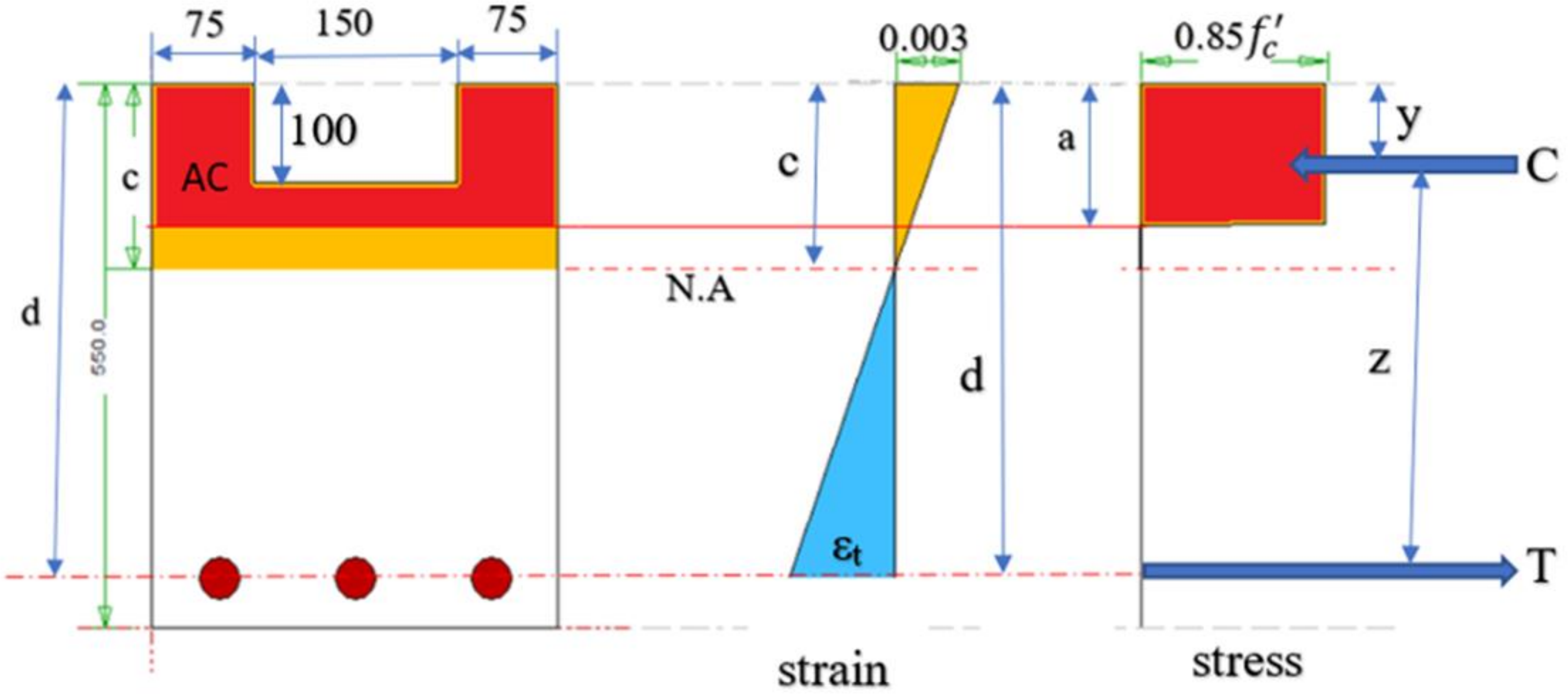
check the ductility

The check of ductility should be w.r.t

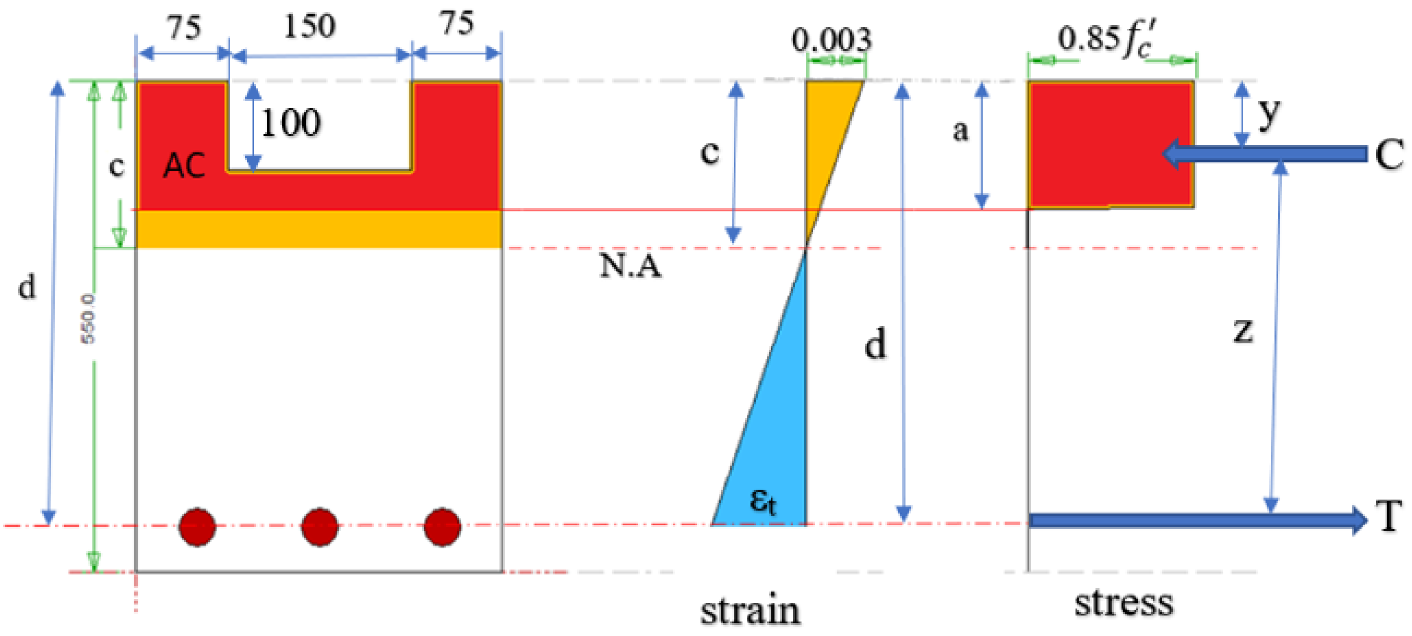
ε_t (strain at steel reinforcement ≥ 0.005)

note: the equations of ρ_{max} and ρ_b for rectangular section.

Section Analysis



Section Analysis



$$C = T$$

$$0.85 f_c' AC = A_s f_y$$

$$AC = ab - 150 \times 100$$

$$0.85 f_c' \,(ab - 15000) = A_s f_y$$

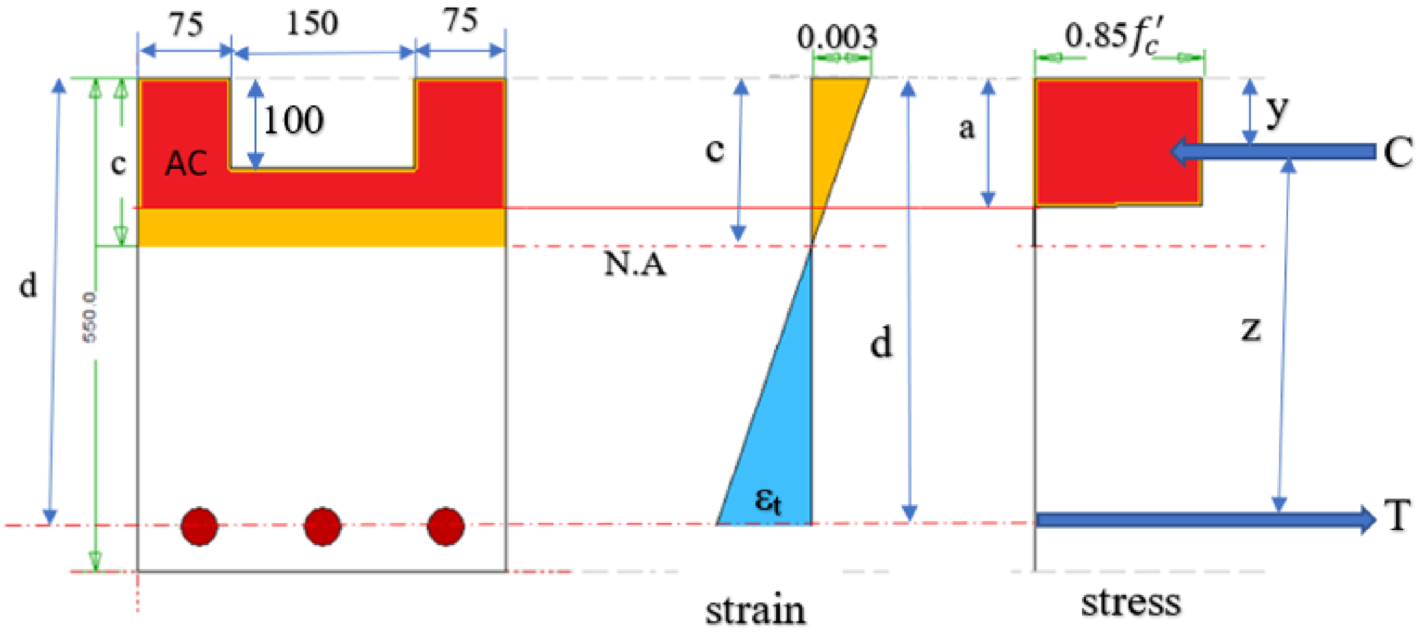
$$a = \frac{\left(\frac{A_s f_y}{0.85 f_c'} - 15000\right)}{b} = \frac{\left(\frac{1470\times420}{0.85\times28} + 15000\right)}{300}$$

$$= 136.47\ mm$$

$$c = \frac{a}{\beta_1} = \frac{136.471}{0.85} = 160.55\ mm$$

Section Analysis

By finding c, we can check the ductility of section with respect to ε_t of steel reinforcement



$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right)$$

$$= 0.003\left(\frac{500 - 160.55}{160.55}\right)$$

$$= 0.00634 > 0.005 \quad OK$$

The section is Tension Controlled

$$and \;\; \emptyset = 0.9$$

Section Analysis

$$\therefore\ AC = 136.47 \times 300 - 150 \times 100 = 25941\ mm^2$$

$$\therefore C = 0.85 f_c' AC = 0.85 \times 28 \times 25941 = 617400\ N$$

$$\therefore T = A_s f_y = 1470 \times 420 = 617400\ N$$

$$M_n = (T\ or\ C) \times z\ (\ distance\ between\ T\ and\ C)$$

$$To\ find\ the\ distance\ between\ T\ and\ C\ (z), we\ need\ to\ find\ y$$

$$y = \frac{300 * 136.47 * \left(\frac{136.47}{2}\right) - 150 * 100 * (\frac{100}{2})}{300 * 136.47 - 150 * 100}$$

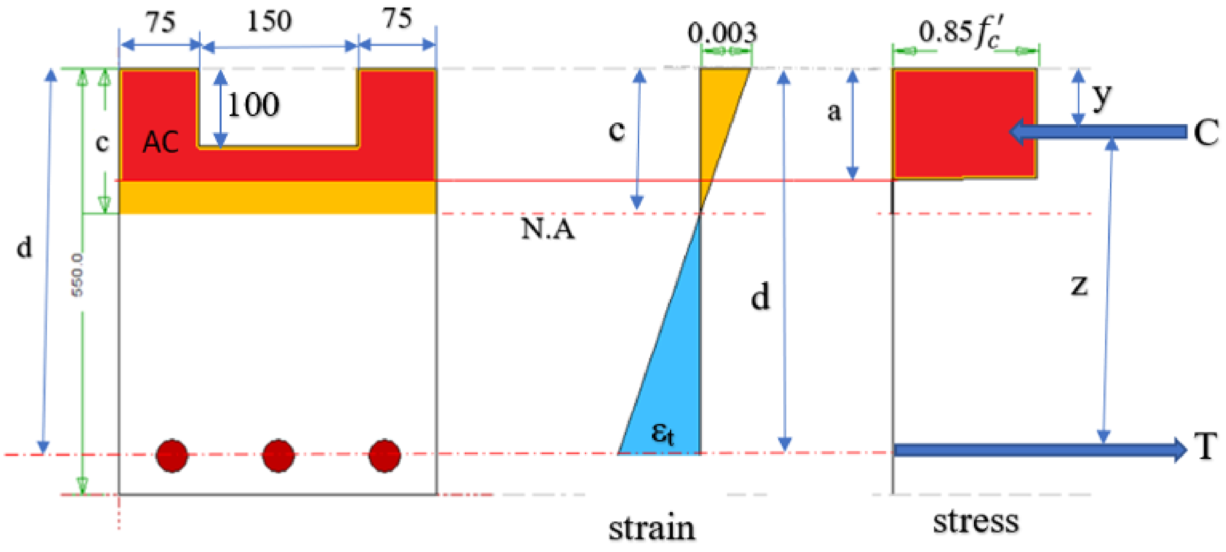
$$= 78.78\ mm$$

$$\therefore z = d - y = 500 - 78.78 = 421.22\ mm$$

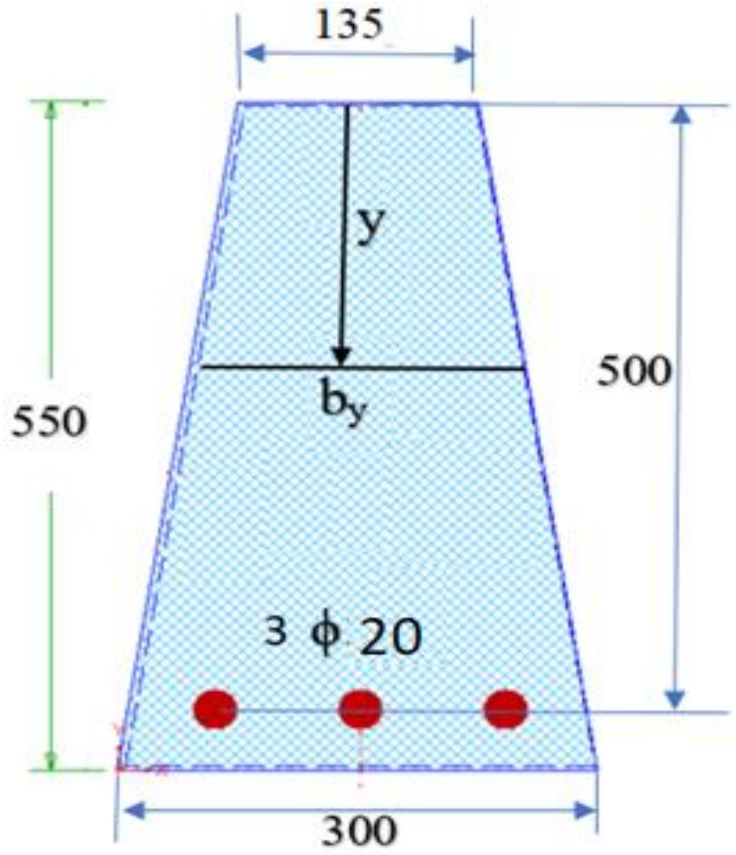
$$\therefore\ M_n = T \times z = 617400 \times 421.22$$

$$= 260061228\ N.mm \approx 260\ kN.m$$

$$\therefore Design\ moment\ \emptyset\ M_n = 0.9 \times 260 = 234\ kN.m$$



Example (8) : Determine the design moment strength of section shown below , Given $f_c' = 28\ MPa$ and $f_y = 420\ MPa$ and check the specification of the section according to ACI Code.



Solution:

$$A_S = 3\pi \frac{20^2}{4} = 942\ mm^2$$

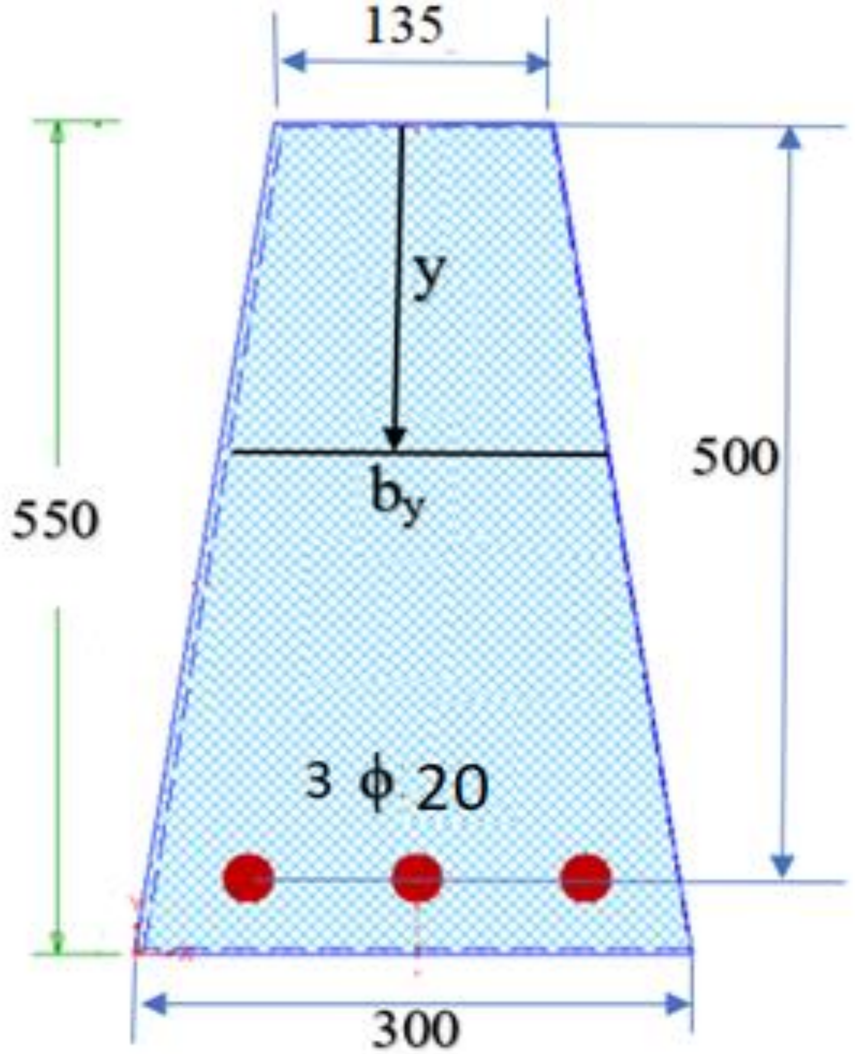
Find width of beam w.r.t. y

$$b_y = 135 + c1\ y + c2$$

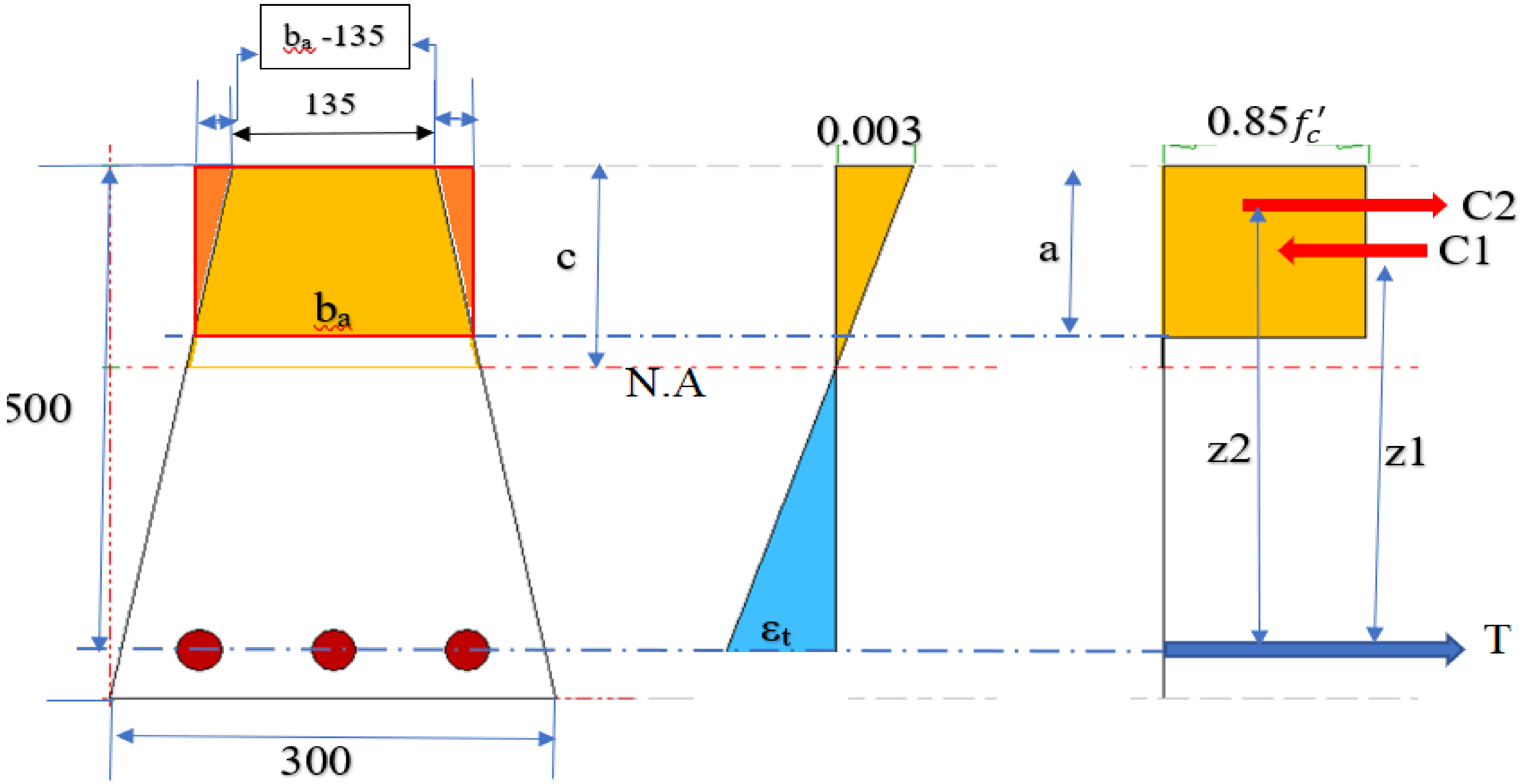
$$at\ y = 0\ , b_y = 135 \ \ \rightarrow c2 = 0$$

$$at\ y = 550, b_y = 300 \ \rightarrow c1 = 0.3$$

$$\therefore\ b_y = 135 + 0.3\ y$$



Section Analysis



Section Analysis

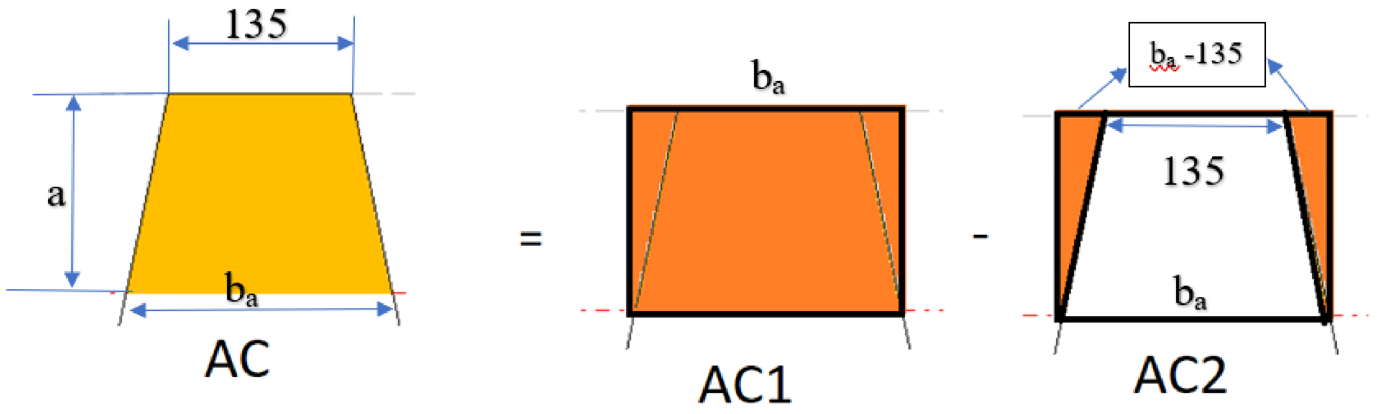
$C = T$

$0.85 f_c' AC = A_s f_y$

$AC = AC1 - AC2$

$AC1 = b_a \, a$

$AC2 = (b_a - 135)/a/2$



$$\therefore \; AC = b_a \, a - (b_a - 135)\frac{a}{2} = 0.5 b_a \, a + 67.5 \, a$$

$but \quad b_a = 135 + 0.3 \, a$

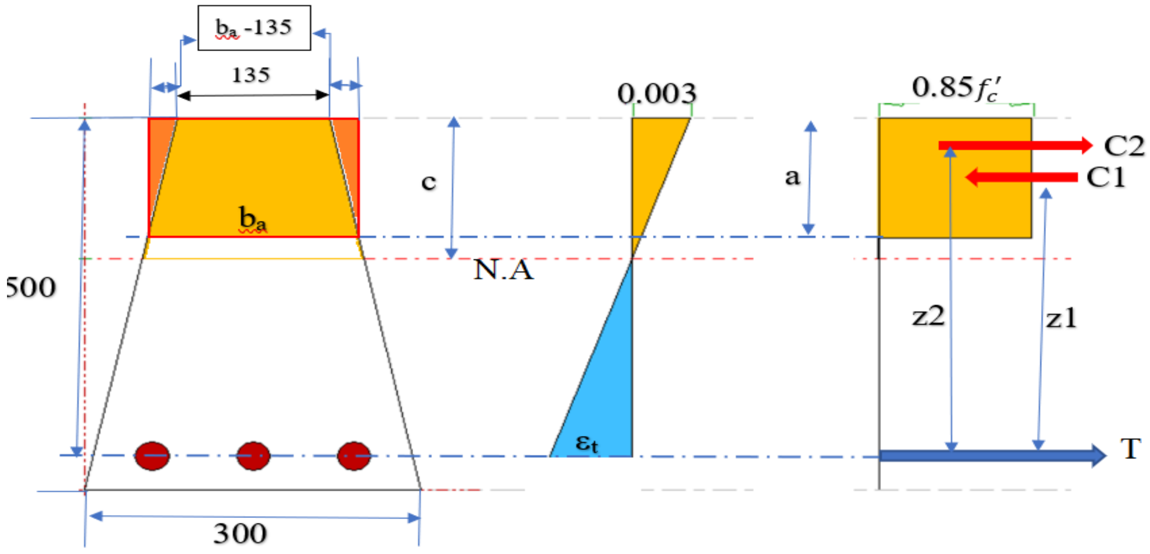
$$\therefore \; AC = 0.5(135 + 0.3 \, a) \, a + 67.5 \, a = 135 \, a + 0.15 a^2$$

$$\therefore \; 0.85 f_c' \times (135a + 0.3a^2) = A_s f_y$$

$$3.57 \, a^2 + 3213 \, a - 395640 = 0$$

$$a = 109.75 \, mm$$

$$c = \frac{a}{\beta_1} = \frac{109.75}{0.85} = 129.12 \, mm$$



Section Analysis

$M_n = (T \; or \; C) \times z \; (\; distance \; between \; T \; and \; C)$

$M_n = C1 \times z1 - C2 \; \times z2$

$C1 = \; 0.85 f_c' AC1 = 0.85 f_c' \; \; b_a \; a$

$b_a = \; 135 + 0.3 \; a = 135 + 0.3 \times 109.75 = 167.93 \; mm$

$\therefore C1 = \; 0.85 \times 28 \; \times 167.93 \; \times 109.75 = 438642 \; N$

$C2 = \; 0.85 f_c' AC2 = 0.85 f_c' \; (b_a - 135) \; a/2$

$= 0.85 \times 28(167.93 - 135) \times 109.75/2 = 43007 \; N$

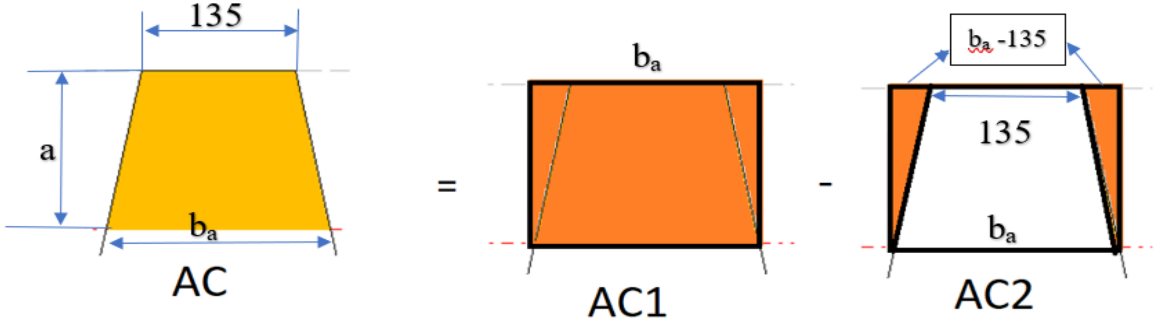
$$z1 = d - \frac{a}{2} = 500 - \frac{109.75}{2} = 445.13 \; mm$$

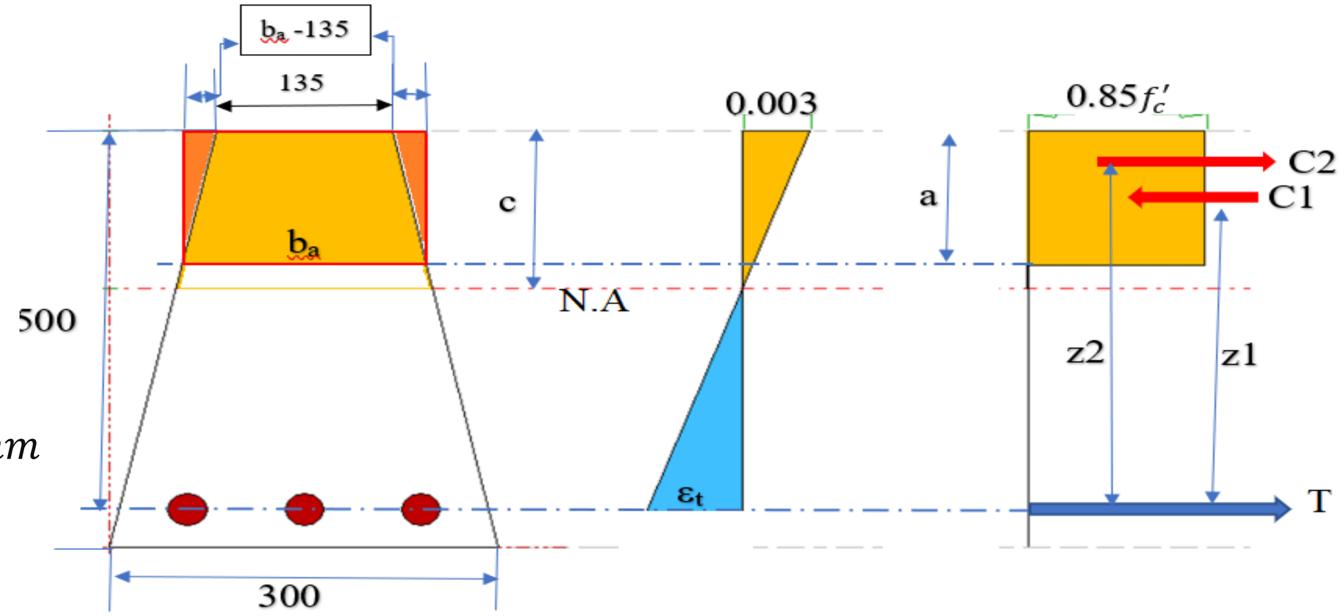
$$z2 = d - \frac{a}{3} = 500 - \frac{109.75}{3} = 463.42 \; mm$$

$M_n = C1 \times z1 - C2 \times \; z2$

$= 438642 \times 445.13 - 43007 \times 463.42 = 175322410 \; N.mm$

$M_n = 175 \; kN.m$





Check ACI requirements

$$\rho = \frac{A_s}{effective\ area\ (Ae)}$$

$$A_e = (\mathrm{b}_s + 135)\frac{d}{2}$$

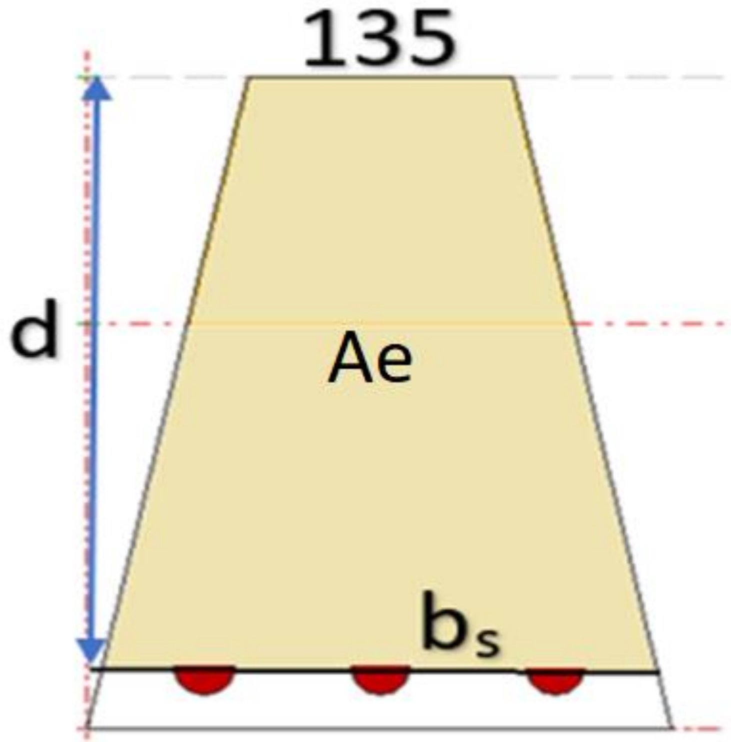
$$b_s = 135 + 0.3\,d = 135 + 0.3(500)$$

$$b_s = 285\,mm$$

$$\therefore A_e = (285 + 135)\frac{500}{2} = 105000\ \mathrm{mm}^2$$

$$\therefore \rho = \frac{942}{105000} = 0.00897$$

$e_{min} = \frac{1.4}{fy}$ = 0.003333 < e (0.00897) OK



Check ACI requirements

check the ductility of section with respect to ε_t of steel reinforcement

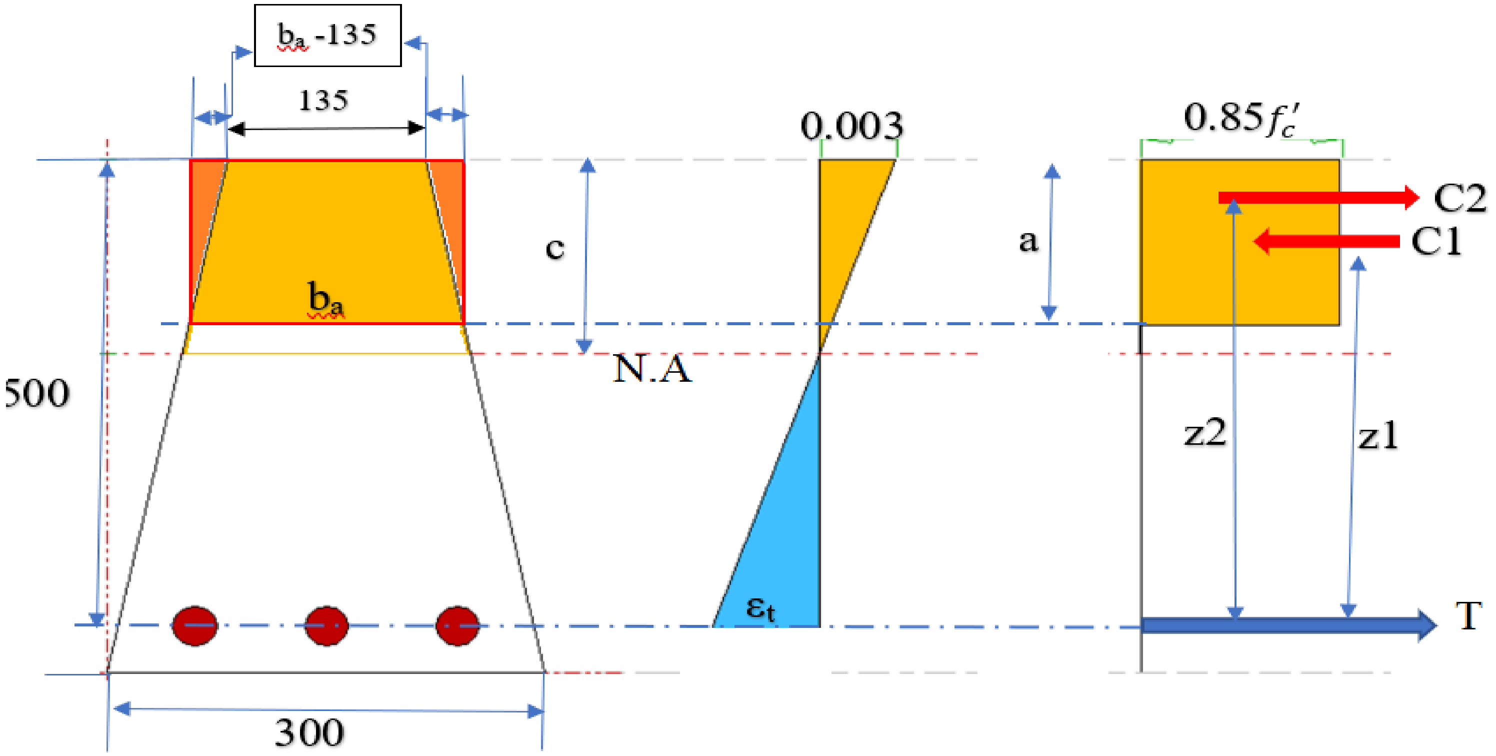
$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right)$$

$$= 0.003\left(\frac{500 - 129.12}{129.12}\right)$$

$$= 0.00863 > 0.005 \quad OK$$

The section is Tension Controlled

and $\emptyset = 0.9$



$\therefore$ *Design moment* $\emptyset\, M_n = 0.9 \times 175$
$= 157.5\ kN.m$

Examples for Analysis of Single Reinforced Rectangular Beam Sections

(Analysis of SRRS)

By: Dr. Majed Ashoor

The Minimum Reinforcement

ACI Code 9.6.1.2: As_{min} shall be the larger of (a) and (b)

$$a) \quad As_{min} = \frac{1.4}{f_y} b_w d$$

$$b) \quad As_{min} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d$$

OR

$$As_{min} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

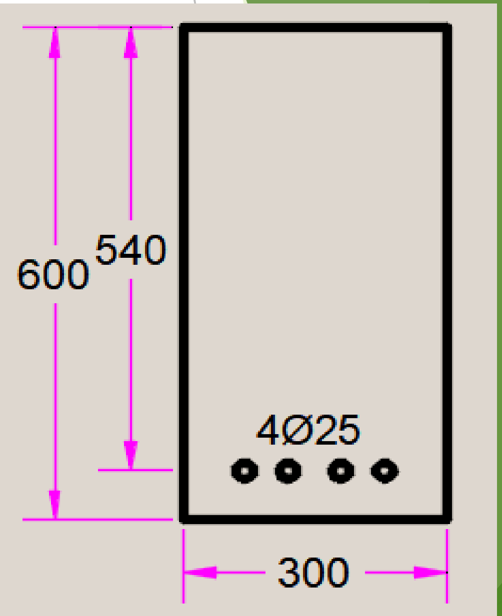
OR

$$As_{min} = \begin{cases} \dfrac{1.4}{f_y} b_w d & for\ \ f_c' \leq 31.0\ MPa \\ \dfrac{0.25\sqrt{f_c'}}{f_y} b_w d & for\ \ f_c' > 31.0\ MPa \end{cases}$$

Ex1: Determine the design moment strength and the position of the neutral axis of the rectangular section shown below, if the reinforcement used is 4Ø25, given f'c=28MPa, fy=420MPa.

Solution:

$$A\text{Ø}25 = \frac{\pi}{4}(25)^2 = 490\ mm^2 \quad \Rightarrow \quad As = 4 \times 490 = 1960\ mm^2$$



Check As Minimum:

$$As_{min} = \begin{cases} (a)\ \ \dfrac{1.4}{f_y} b_w d & for\ \ f_c' \le 31.0\ MPa \\ (b)\ \ \dfrac{0.25\sqrt{f_c'}}{f_y} b_w d & for\ \ f_c' > 31.0\ MPa \end{cases}$$

$$As_{min} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 300 \times 540 = 540\ mm^2 < 1960\ mm^2 \quad (OK)$$

Equilibrium Equation:

$$0.85\ f_c'\ a\ b = A_s\ f_y \Rightarrow a = \frac{A_s\ f_y}{0.85\ f_c'\ b} \Rightarrow a = \frac{1960\times420}{0.85\ \times28\times300} = 115.294\ mm$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \Rightarrow M_n = 1960 \times 420\left(540 - \frac{115.294}{2}\right) = 397.0729\ kN.m$$

$$c = \frac{a}{\beta_1} = \frac{115.294}{0.85} = 135.64\ mm$$

Check for Ductility:

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) \Rightarrow \varepsilon_t = 0.003\left(\frac{540}{135.64} - 1\right) = 0.00894\ > 0.005\quad (tension\ controlled\ ok)$$

So, the design moment strength is:

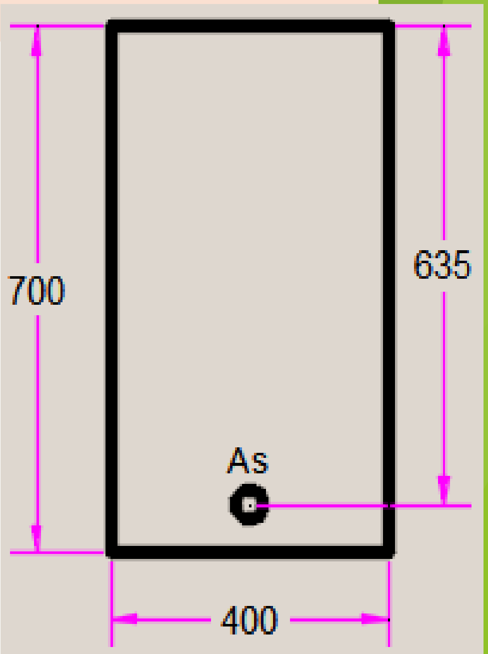
$$\emptyset M_n = 0.9 \times 397.0729 = 357.365\ kN.m$$

Ex2: For the section shown below with f'_c= 28 MPa and fy = 420 MPa, calculate

a. The balanced steel reinforcement

b. The maximum reinforcement area allowed by the ACI Code for a tension-controlled section.

c. The position of the neutral axis and the depth of the equivalent compressive stress block for the tension-controlled section in b.



Solution:

a)

$$\varepsilon_s = 0.003\left(\frac{d_t - c}{c}\right) \rightarrow 0.0021 = 0.003\left(\frac{635 - c}{c}\right)$$

$$\frac{0.0021}{0.003}c = 635 - c \rightarrow 1.7c=635 \rightarrow c=373.53mm$$

$$a = \beta_1\, c = 0.85 \times 373.53 = 317.5mm$$

$$0.85\, f_c'\, a\, b = A_s\, f_y$$

$$A_s = \frac{0.85\, f_c'\, a\, b}{f_y} = \frac{0.85 \times 28 \times 317.5 \times 400}{420} = 7196.66\, mm^2$$

b)

$$\varepsilon_s = 0.003\left(\frac{d_t - c}{c}\right) \qquad 0.005 = 0.003\left(\frac{635 - c}{c}\right) \qquad \frac{0.005}{0.003}c = 635 - c$$

2.666c=635 c=238.18 mm $a = \beta_1\, c = 0.85 \times 238.18 = 202.45\, mm$

$$0.85\, f_c'\, a\, b = A_s\, f_y$$

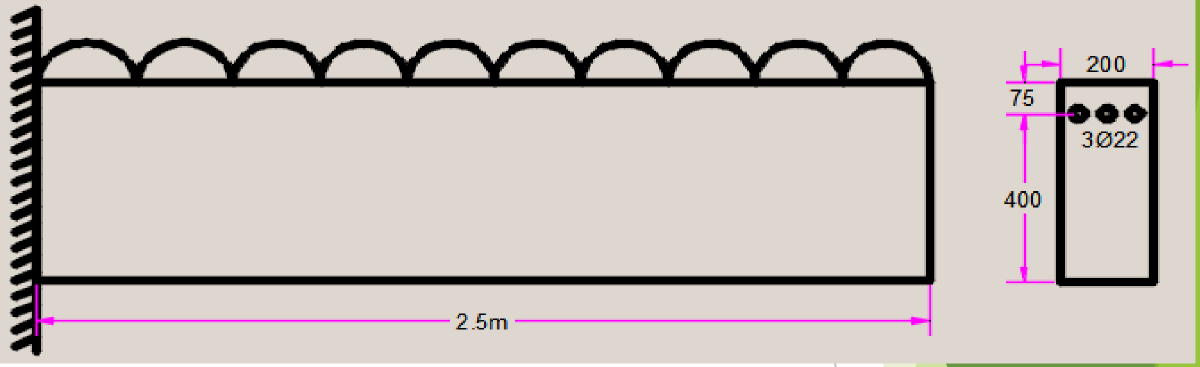
$$A_s = \frac{0.85\, f_c'\, a\, b}{f_y} = \frac{0.85 \times 28 \times 202.45 \times 400}{420} = 4589.02\, mm^2$$

C)

c=238.18 mm

a=202.45 mm

EX3: A 2.5m span cantilever beam has a rectangular section and reinforcement as shown in the figure. The beam carries a dead load including its own weight of 22 kN/m and a live load of 13 kN/m, using f'c=28 MPa, and fy=420 MPa, check if the beam is safe to carry the above loads.



$$A\text{Ø}22 = \frac{\pi}{4}(22)^2 = 380\ mm^2$$

$$As = 3 \times 380 = 1140\ mm^2$$

$$As_{min} = \begin{cases} \dfrac{1.4}{f_y} b_w d & for\ \ f_c' \leq 31.0\ MPa \\ \dfrac{0.25\sqrt{f_c'}}{f_y} b_w d & for\ \ f_c' > 31.0\ MPa \end{cases}$$

$$As_{min} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 200 \times 400 = 266.66\ mm^2 < 1140\ mm^2 \quad (OK)$$

$$w_u = 1.2\ D + 1.6\ L \quad \Rightarrow \quad w_u = 1.2\ \times 22 + 1.6\ \times 13 = 47.2\ kN/m$$

$$M_{u,max} = \frac{w_u \ell^2}{2} = \frac{47.2 \times 2.5^2}{2} = 147.5\ kN.m$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \Rightarrow \quad a = \frac{1140 \times 420}{0.85 \times 28 \times 200} = 100\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_n = 1140 \times 420 \left(400 - \frac{100}{2}\right) = 167.58\ kN.m$$

Check the Ductility:

$$c = \frac{a}{\beta_1} = \frac{100}{0.85} = 117.64\ mm$$

$$\varepsilon_s = 0.003 \left(\frac{d_t - c}{c}\right) = 0.003 \left(\frac{400 - 117.64}{117.64}\right) = 0.0072 > 0.005\ (OK)$$

$$\emptyset M_n = 0.9 \times 167.58 = 150.822\ kN.m > 147.5 \qquad (OK)$$

Thank you…

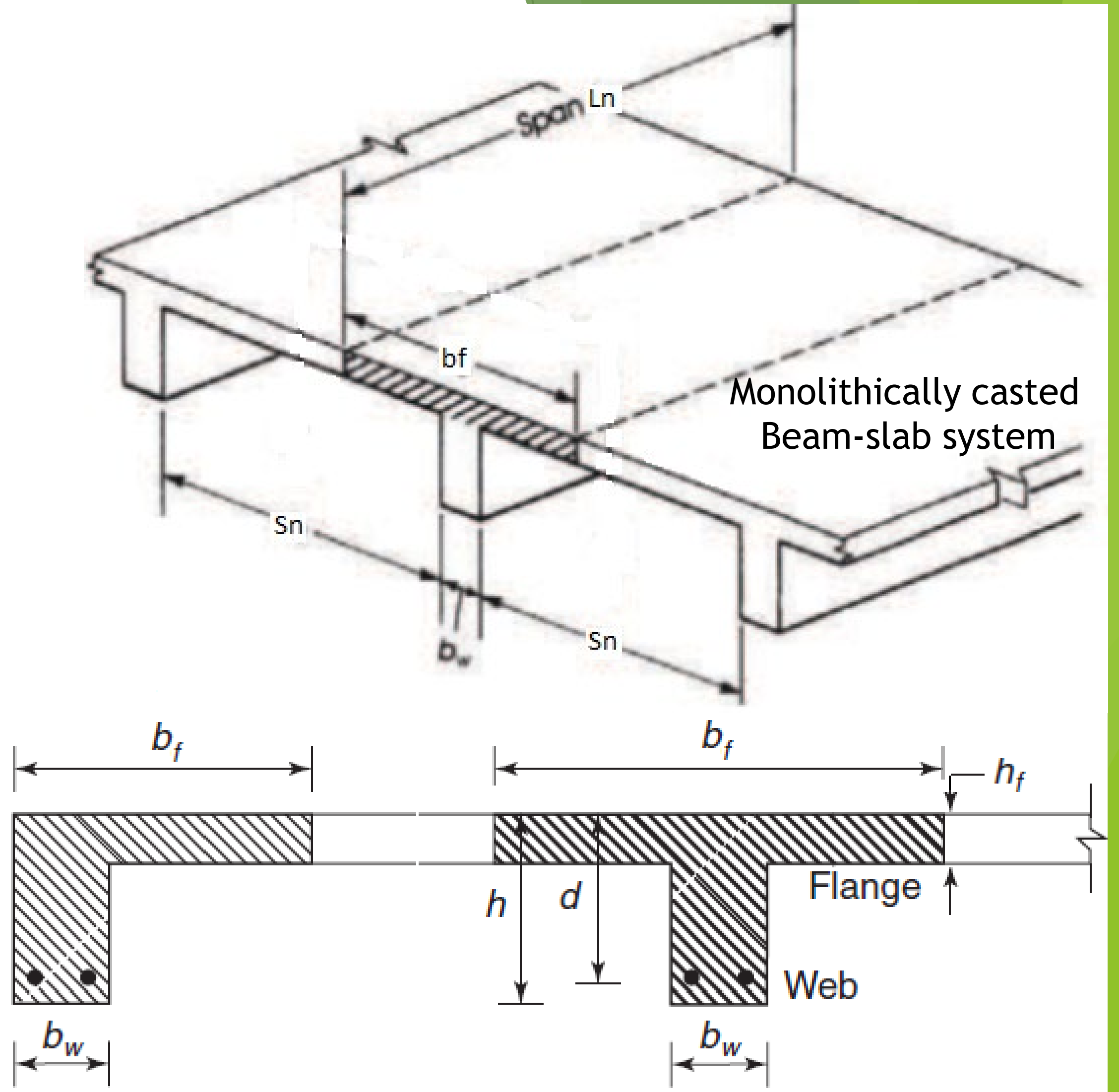
Flexural Analysis of T-Sections

By: Dr. Majed Ashoor

Flange Effective Width Limitations:

$$(T - Section):\ bf \leq \begin{cases} bw + 2(8hf) \\ bw + 2\left(\frac{Ln}{8}\right) \\ bw + 2\left(\frac{Sn}{2}\right) \end{cases}$$

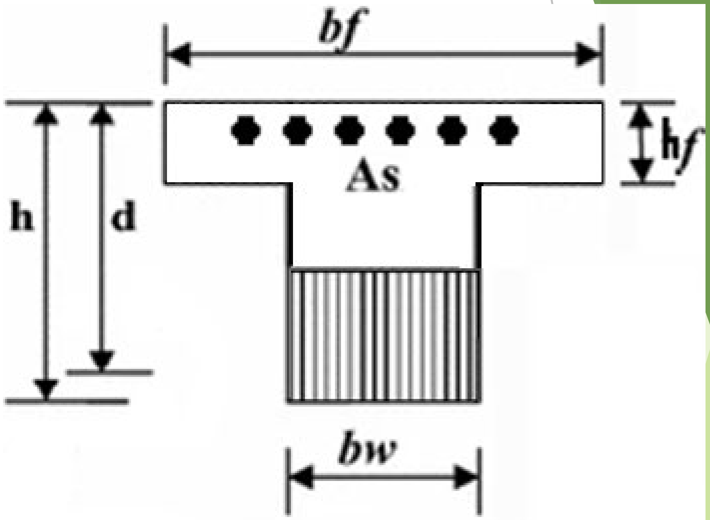
$$(L - Section):\ bf \leq \begin{cases} bw + (6hf) \\ bw + \left(\frac{Ln}{12}\right) \\ bw + \left(\frac{Sn}{2}\right) \end{cases}$$



$$Isolated\ T\ beam \begin{cases} bf \leq 4\ b_w \\ hf \geq 0.5\ b_w \end{cases}$$

If flange located in Tension zone (Negative Moment), the section will be Rectangular with b=bw

If flange located in Compression zone, There will be two possibilities, depending on the concrete compression area required to satisfy the equilibrium:



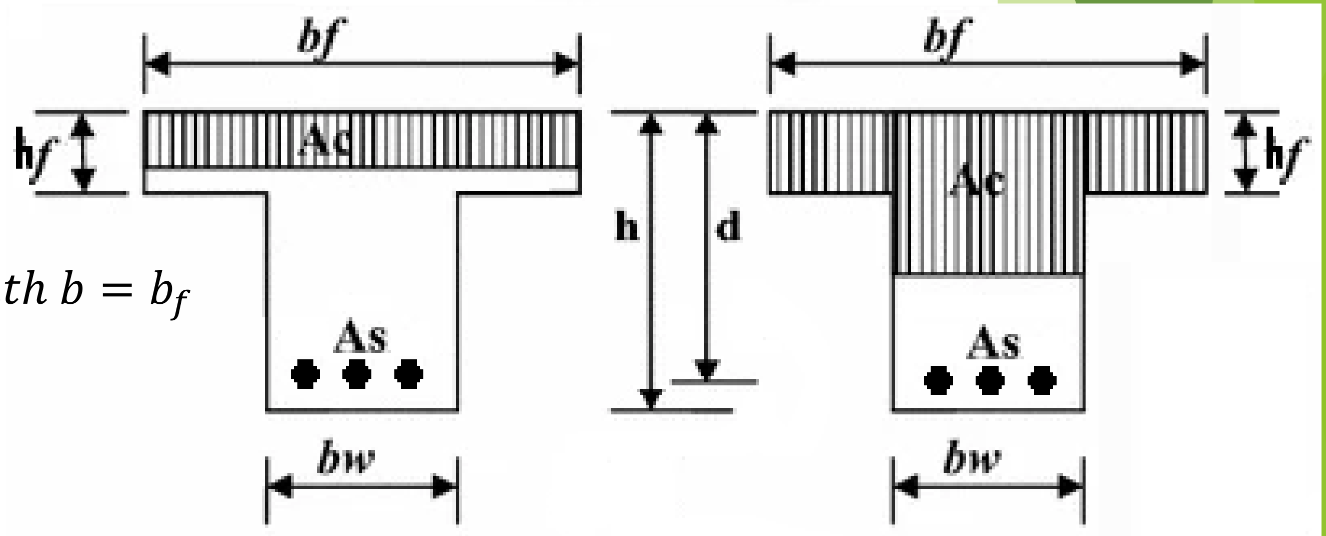
$$a \leq h_f \qquad (Rectangular\ Section\ with\ b = bf)$$

$$a > h_f \qquad (True\ T\ Beam)$$

$$0.85\ f'_c\ a\ b_f = A_s\ f_y$$

$$a = \frac{A_s\ f_y}{0.85\ f'_c b_f}$$

$$Check: \quad a \begin{cases} \leq h_f & \rightarrow\rightarrow\rightarrow \quad Rectangular\ Sec. with\ b = b_f \\ > h_f & \rightarrow\rightarrow\rightarrow \quad True\ T\ Section \end{cases}$$



For True T Section:

$$M_n = M_{n1} + M_{nov}$$

$$M_{nov} = 0.85\, f_c'\, h_f (b_f - b_w)\left(d - \frac{h_f}{2}\right)$$

$$A_{sov} = \frac{0.85\, f_c'\, h_f (b_f - b_w)}{f_y}$$

$$A_{s1} = A_s - A_{sov}$$

$$A_{s1}\, f_y = 0.85\, f_c'\, a\, b_w$$

$$a = \frac{A_{s1}\, f_y}{0.85\, f_c'\, b_w}$$

$$M_{n1} = 0.85\, f_c'\, a\, b_w \left(d - \frac{a}{2}\right)$$

$$M_n = M_{n1} + M_{nov}$$

$$c = \frac{a}{\beta_1}$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right)$$

$$M_u = \emptyset\, M_n$$

$$\rho_1 = \frac{A_{s1}}{b_w\, d}$$

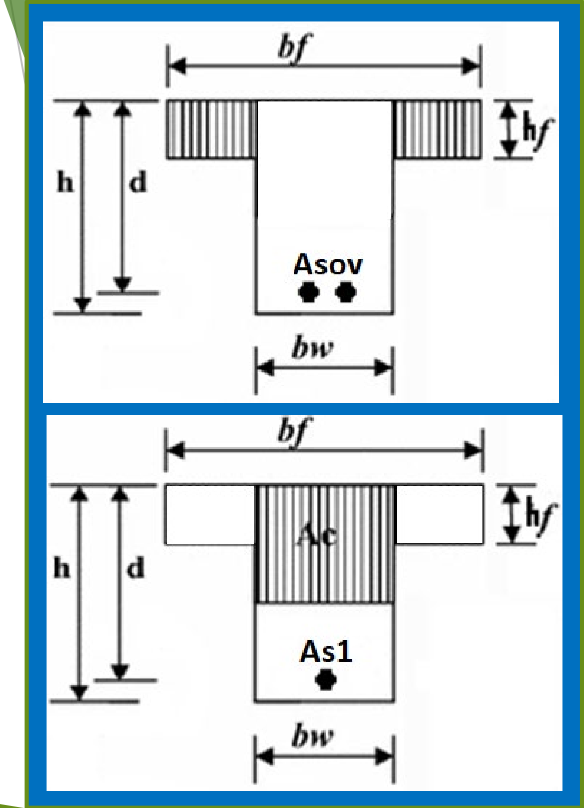
$$m = \frac{f_y}{0.85\, f_c'}$$

$$M_{n1} = \rho_1\, f_y b_w d^2 \left(1 - \frac{1}{2}\rho_1 m\right)$$

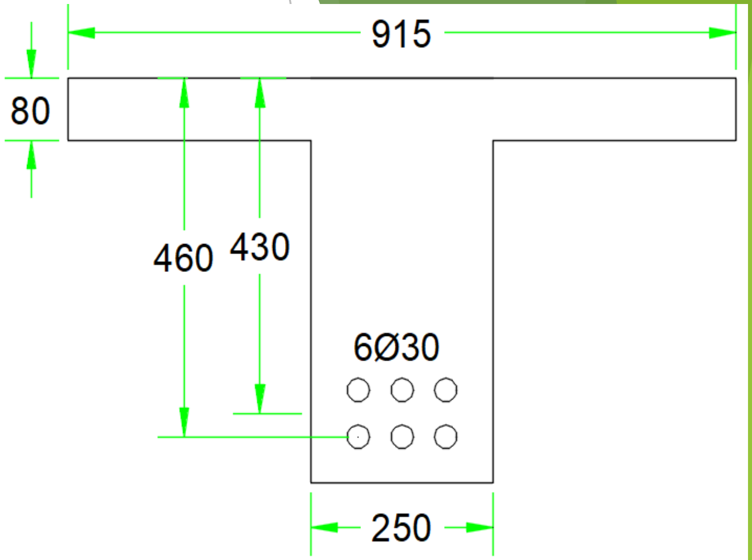
$$M_n = M_{n1} + M_{nov}$$

$$\rho_1 \le \rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \quad (for\ tension\ control)$$

$$M_u = \emptyset\, M_n$$



Ex1: Calculate the design moment strength of the T-section shown below, using f'c=24 MPa, fy=420 MPa



Solution: $A_b = 706\ mm^2$

$A_s = 6 \times 706 = 4236\ mm$

$0.85\ f_c'\ a\ b_f = A_s\ f_y$

$$a = \frac{A_s\ f_y}{0.85\ f_c' b_f} = \frac{4236 \times 420}{0.85 \times 24 \times 915} = 95.31 > 80\ \ (T.T.S)$$

$$M_n = M_{n1} + M_{nov}$$

$$M_{nov} = 0.85\ f_c'\ h_f(b_f - b_w)\left(d - \frac{h_f}{2}\right)$$

$$M_{nov} = 0.85 \times 24 \times 80\ (915 - 250)\left(430 - \frac{80}{2}\right)$$

$$M_{nov} = 423.26\ kN.m$$

$$A_{sov} = \frac{0.85\ f_c'\ h_f\left(b_f - b_w\right)}{f_y}$$

$$A_{sov} = \frac{0.85 \times 24 \times 80(915 - 250)}{420} = 2584\ mm^2$$

$$A_{s1} = A_s - A_{sov}$$

$$A_{s1} = 4236 - 2584 = 1652\ mm^2$$

$$A_{s1}\ f_y = 0.85\ f_c'\ a\ b_w$$

$$a = \frac{A_{s1}\ f_y}{0.85\ f_c'\ b_w}$$

$$a = \frac{1652 \times 420}{0.85 \times 24 \times 250} = 136.04\ mm$$

$$M_{n1} = 0.85\ f_c'\ a\ b_w \left(d - \frac{a}{2}\right)$$

$$M_{n1} = 0.85 \times 24 \times 136.04 \times 250 \left(430 - \frac{136.04}{2}\right)$$

$$M_{n1} = 251.14\ kN.m$$

$$M_n = M_{n1} + M_{nov}$$

$$M_n = 251.14 + 423.26 = 674.4\ kN.m$$

$$c = \frac{a}{\beta_1} = \frac{136.04}{0.85} = 160.04$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right)$$

$$\varepsilon_t = 0.003\left(\frac{460 - 160.04}{160.04}\right) = 0.0056 > 0.005\ (Tension\ Control)$$

$$M_u = \emptyset\ M_n$$

$$M_u = 0.9 \times 674.4 = 606.96\ kN.m$$

Thank you…

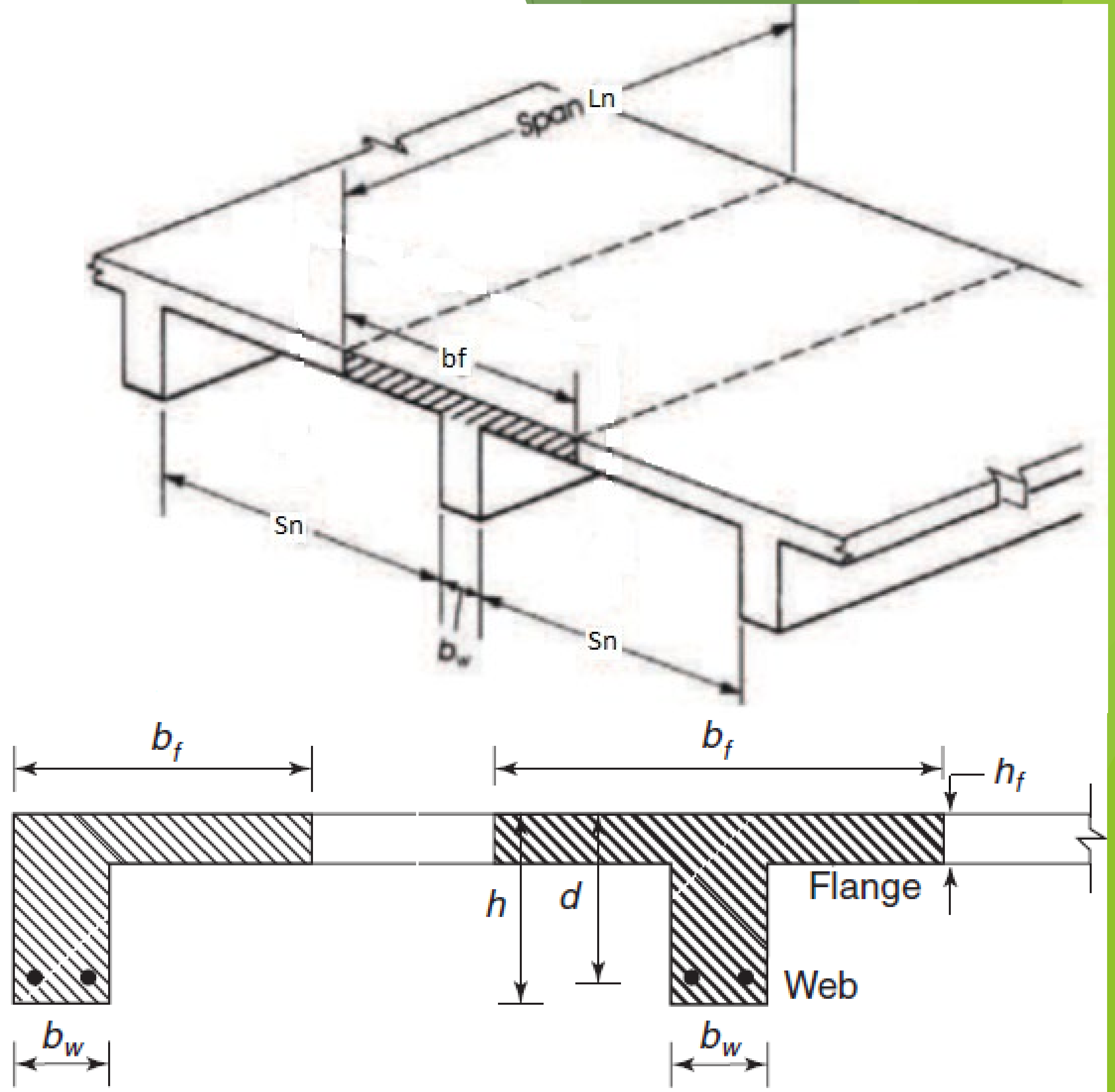
Flexural Design of T-Sections

By: Dr. Majed Ashoor

$$(T - Section):\ bf \leq \begin{cases} bw + 2(8hf) \\ bw + 2\left(\frac{Ln}{8}\right) \\ bw + 2\left(\frac{Sn}{2}\right) \end{cases}$$

$$(L - Section):\ bf \leq \begin{cases} bw + (6hf) \\ bw + \left(\frac{Ln}{12}\right) \\ bw + \left(\frac{Sn}{2}\right) \end{cases}$$

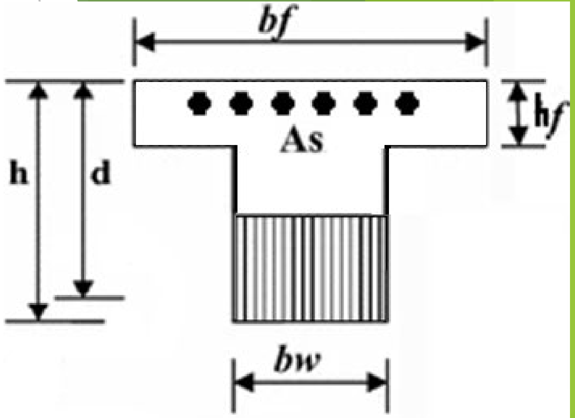
$$Isolated\ T\ beam \begin{cases} bf \leq 4\ b_w \\ hf \geq 0.5\ b_w \end{cases}$$



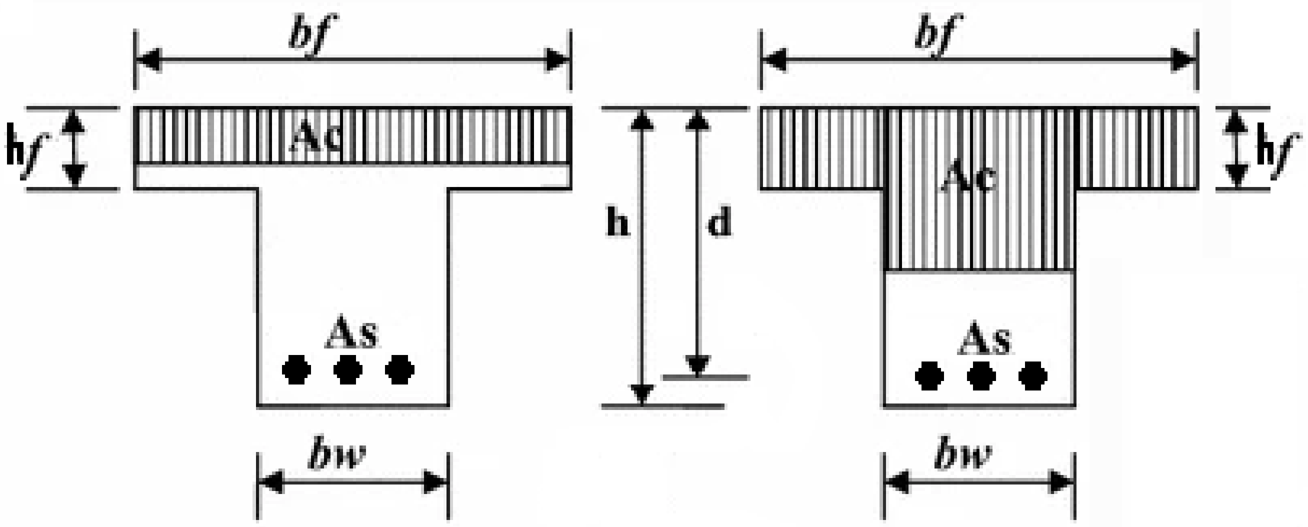
Generally, the depth of the beam is not determined depending on the T-section at the Mid-span, instead, the depth is calculated depending on the rectangular section at the support or depending on the max. Shear.

There are two cases for design of T-Section:

1. The T-Section is subjected to Negative B.M, “near the supports”.
   For this case the flange is subjected to tension, and will be not effective in resisting B.M, so the section will be a rectangular section with b=bw.



2. The T-Section is subjected to Positive B.M, “near midspan”. For this case there are two possibilities:
   a. The dimensions of the sections are known, “h, bw, bf, & hf” and the unknown is only As.
   b. The depth of the section “h” or “d” are unknowns as well as the As.



Design for case "a" the unknow is "As" only

1. Calculate the effective depth 'd' by assuming one layer or two layers of tension reinforcement:

$$d = h - 65 \qquad (one\ layer\ rein.)$$

$$d = h - 90 \qquad (two\ layers\ rein.)$$

2. Assume a=hf and calculate the Mnf

$$M_{nf} = 0.85\ f_c'\ b_f\ h_f\left(d - \frac{h_f}{2}\right)$$

$$if\ M_{nf}\begin{cases} \geq \dfrac{M_u}{\emptyset} & Case\ I: rectangular\ section \\ < \dfrac{M_u}{\emptyset} & Case\ II: True\ T\ section \end{cases}$$

For Case I: Design as a rectangular section with b=bf

For Case II (T.T.S): Calculate Mnov

$$M_{nov} = 0.85\ f_c'\ h_f\ (b_f - b_w)\left(d - \frac{h_f}{2}\right)$$

$$A_{sov} = \frac{0.85\, f_c'\, h_f\left(b_f - b_w\right)}{f_y}$$

$$M_{n1} = \frac{M_u}{\emptyset} - M_{nov}$$

$$R_{n1} = \frac{M_{n1}}{b_w\, d^2}$$

$$\rho_1 = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2m\, R_{n1}}{f_y}}\right)$$

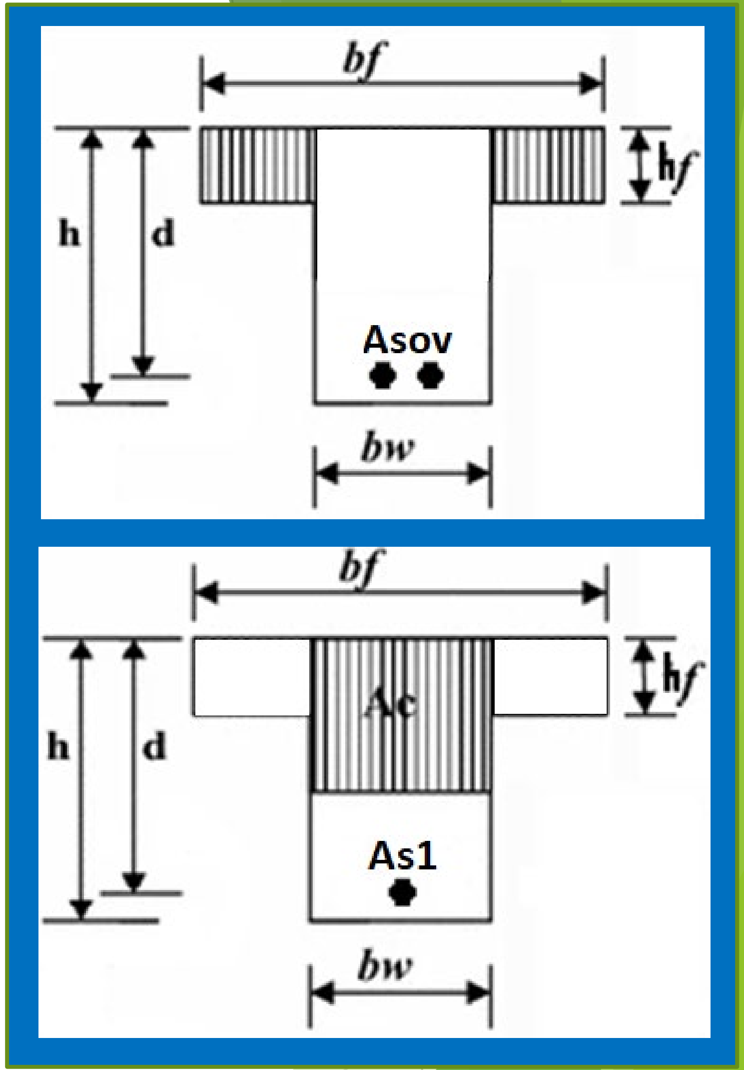
$$A_{s1} = \rho_1\, b_w\, d$$

$$A_s = A_{s1} + A_{sov}$$

$$\rho_1 \le \rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \quad (for\ tension\ control)$$

$$M_n = M_{n1} + M_{nov}$$

$$M_u = \emptyset\, M_n$$



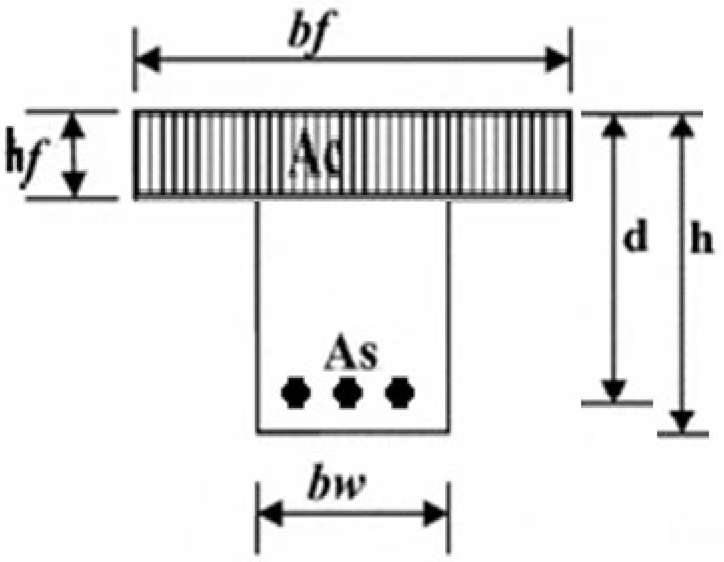
3. For the case that (h , d & As) all are unknowns:

$$Assume\ a = h_f \quad (a\ rectangular\ section\ with\ b = bf)$$

$$A_s = \frac{0.85\ f_c'\ b_f\ h_f}{f_y}$$

$$M_n = \frac{M_u}{\emptyset} = A_s\ f_y \left(d - \frac{h_f}{2}\right)$$

$$d = \frac{M_n}{A_s\ f_y} + \frac{h_f}{2}$$



If value of d is acceptable then:

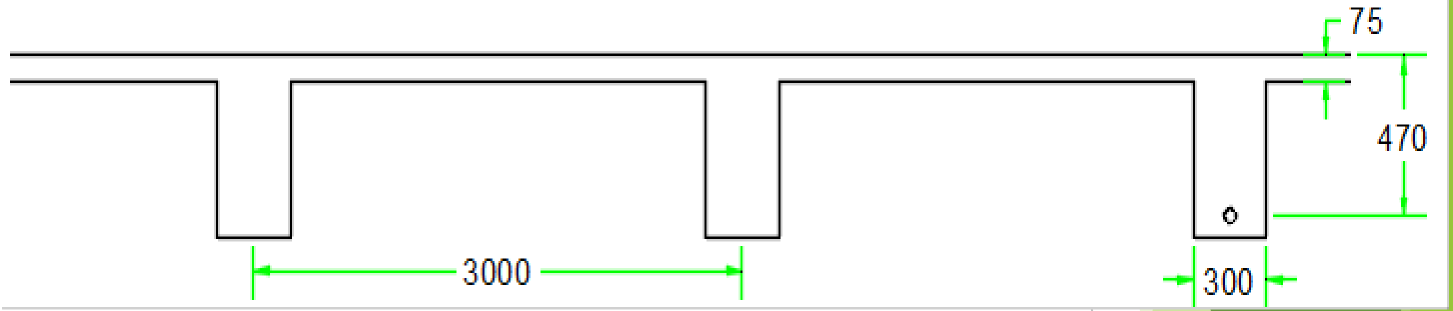
$$h = d + 65 \qquad (one\ layer\ rein.)$$

$$h = d + 90 \qquad (two\ layers\ rein.)$$

If we choose d(new) greater than the d calculated above, the section need to be redesign as a (rectangular section) with d=dnew

If we choose d(new) smaller than the d calculated above, the section need to be redesign as a (True T section) with d=dnew

Ex1: The floor system shown in figure, consist of 75mm slab supported by 4 m clear span beams, spaced at 3.0m on center. The beams have a web width bw=300mm, and an effective depth d=470mm. Calculate the necessary reinforcement for a typical interior beam if the factored applied moment=720 kN.m.
use fc'=21 MPa, fy=420MPa.



Solution:

$$(T - Section):\ bf \le \begin{cases} bw + 2(8hf) \\ bw + 2\left(\dfrac{Sn}{2}\right) \\ bw + 2\left(\dfrac{Ln}{8}\right) \end{cases}$$

$$(T - Section):\ bf \le \begin{cases} 300 + 2(8 \times 75) = 1500 \\ 300 + 2\left(\dfrac{2700}{2}\right) = 3000 \\ 300 + 2\left(\dfrac{4000}{8}\right) = 1300 \end{cases}$$

Then: bf=1300mm, d=470, Mu=720 kN.m

$$M_n = \frac{M_u}{\emptyset} = \frac{720}{0.9} = 800\ kN.m$$

assume a=hf

$$M_{nf} = 0.85\ f_c'\ b_f\ h_f\ \left(d - \frac{h_f}{2}\right)$$

$$M_{nf} = 0.85 \times 21 \times 1300 \times 75\ \left(470 - \frac{75}{2}\right) = 752.71 < 800\ (T.T.S)$$

$$M_{nov} = 0.85\ f_c'\ h_f(b_f - b_w)\left(d - \frac{h_f}{2}\right)$$

$$M_{nov} = 0.85 \times 21 \times 75 \times (1300 - 300)\left(470 - \frac{75}{2}\right)$$

$$M_{nov} = 579\ kN.m$$

$$A_{sov} = \frac{0.85 \times 21 \times 75(1300 - 300)}{420} = 3187.5\ mm^2$$

$$M_{n1} = \frac{M_u}{\emptyset} - M_{nov}$$

$$M_{n1} = 800 - 579 = 221\ kN.m$$

$$R_{n1} = \frac{M_{n1}}{b_w\ d^2}$$

$$R_{n1} = \frac{221 \times 10^6}{300 \times 470 \times 470} = 3.335$$

$$\rho_1 = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2m\ R_{n1}}{f_y}}\right)$$

$$m = \frac{f_y}{0.85\ f_c'} = \frac{420}{0.85 \times 21} = 23.53$$

$$\rho_1 = \frac{1}{23.53}\left(1 - \sqrt{1 - \frac{2 \times 23.53 \times 3.335}{420}}\right)$$

$$\rho_1 = 0.00886$$

$$\rho_{max} = \frac{3}{8}\ \frac{\beta_1}{m}\left(\frac{d_t}{d}\right) = \frac{3}{8} \times \frac{0.85}{23.53} \times (1) = 0.0135 > 0.00886\ (ok.T.C)$$

$$A_{s1} = \rho_1\ b_w\ d = 0.00886 \times 300 \times 470 = 1249.26\ mm^2$$

$$A_s = A_{s1} + A_{sov}$$

$$A_s = 1249.26 + 3187.5 = 4436.76\ mm^2$$

$$A_{smin} = \frac{1.4}{f_y}\ b_w\ d$$

$$A_{smin} = \frac{1.4}{420}\ \times 300 \times\ 470 = 470\ mm^2 \ll 4436.76\ (ok)$$

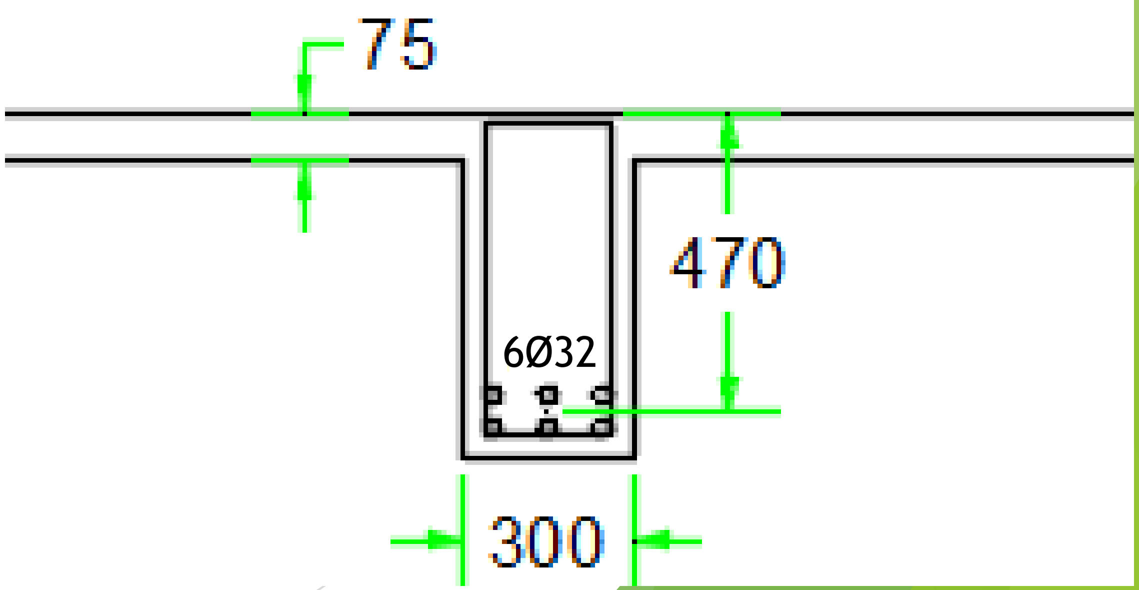
Use 6Ø32 in two layers:

$$As = 6 \times 804 = 4824\ mm^2 > 4436.76$$

$$As1(provided) = 4824 - 3187.5 = 1636.5\ mm^2$$

$$\rho_{1(provided)} = \frac{1636.5}{300 \times 470} = 0.0116 < 0.0135\ (ok. T. C)$$

$$s = \frac{200 - 3 \times 32}{2} = 52 > 32\ (ok)$$



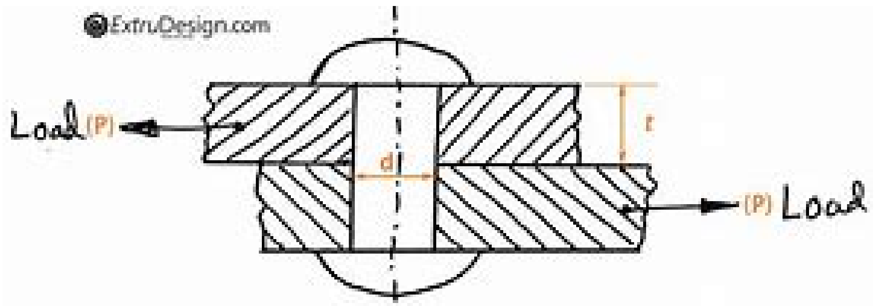
Thank you…

Shear Design for Beams

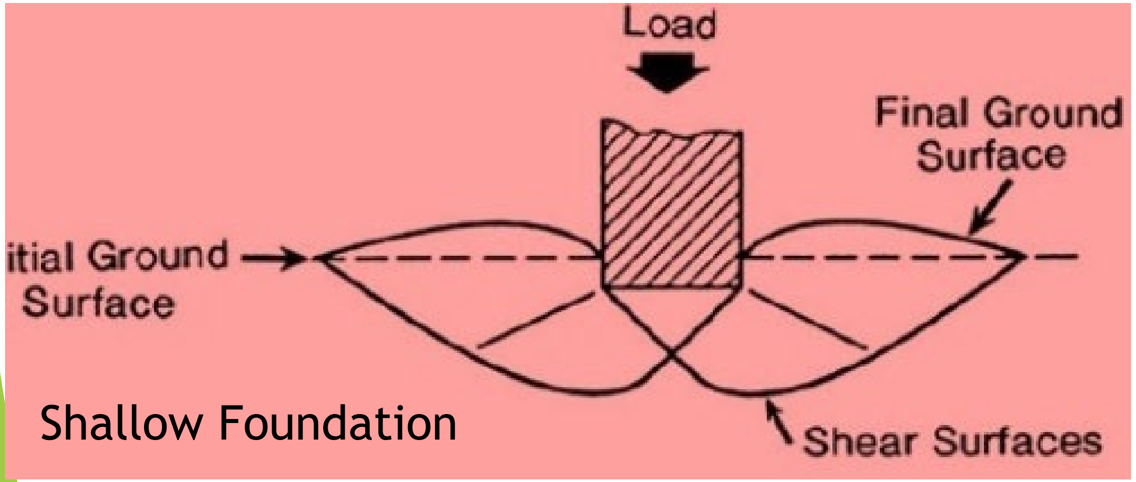
(One Way Shear)

By: Dr. Majed Ashoor

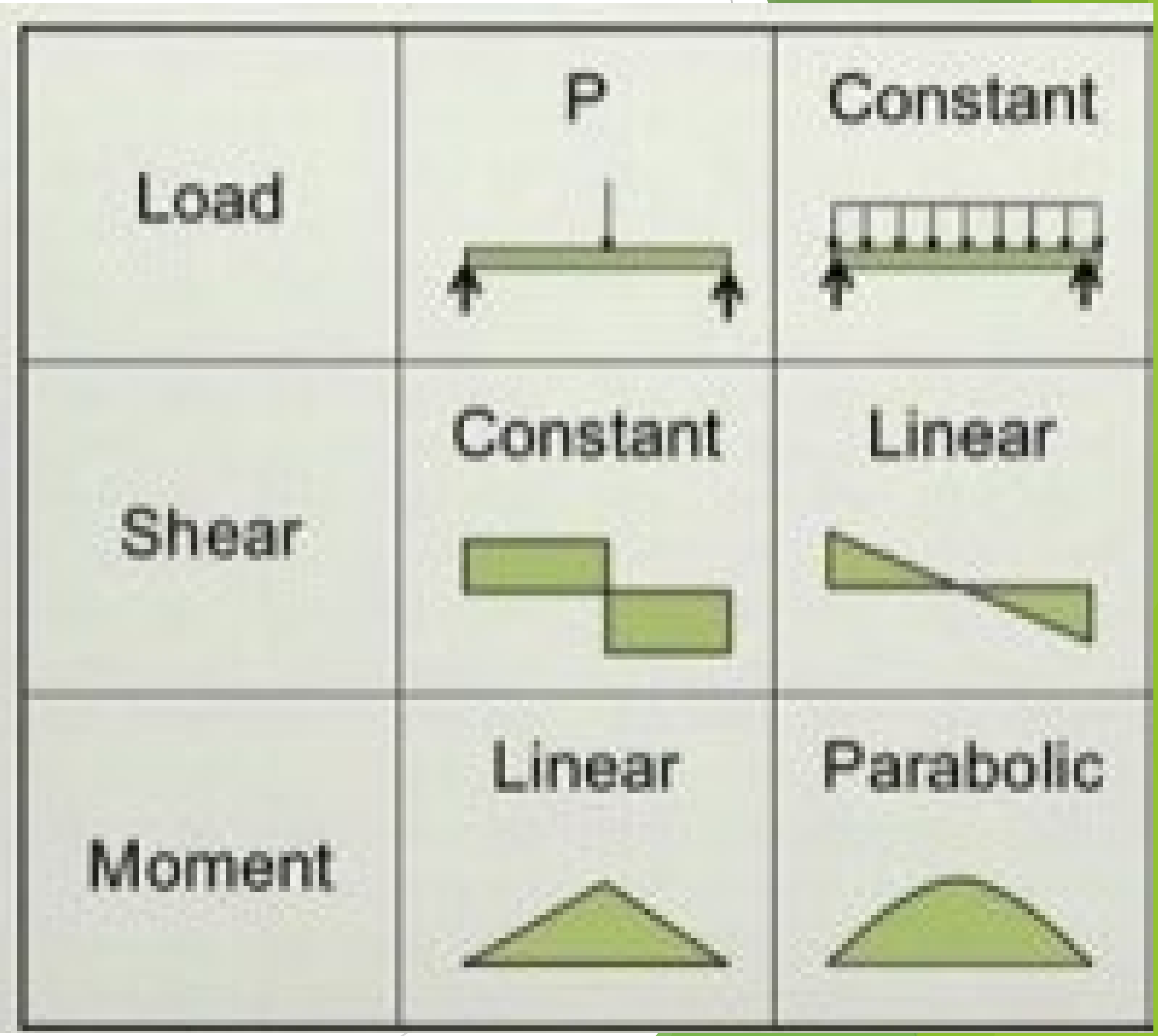
Direct Shear & Shear Force Diagram



Steel Bolted Connection



Shallow Foundation

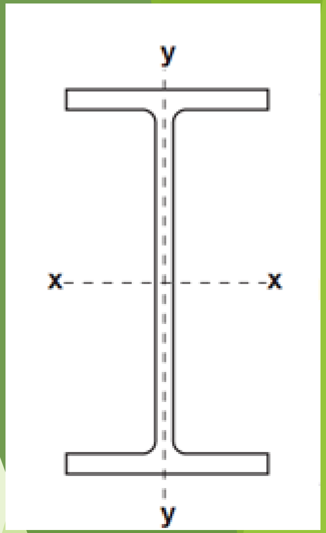


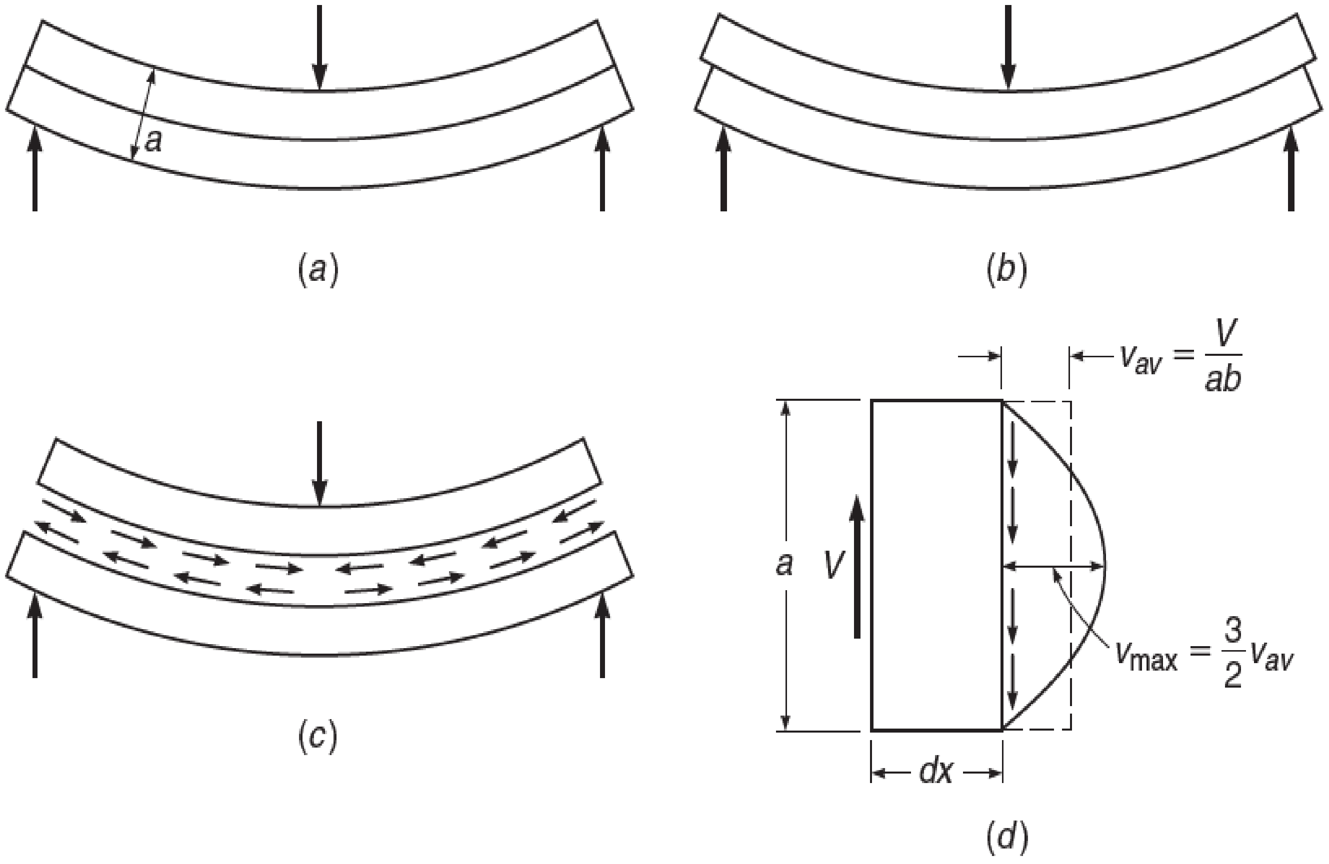
Load	P	Constant
Shear	Constant	Linear
Moment	Linear	Parabolic

Shear Stress Formulae

The shear theory is not completely rigorous like flexure theory, because the shear strain is not clearly defined, and there are many variables effect the RC response to shear forces.

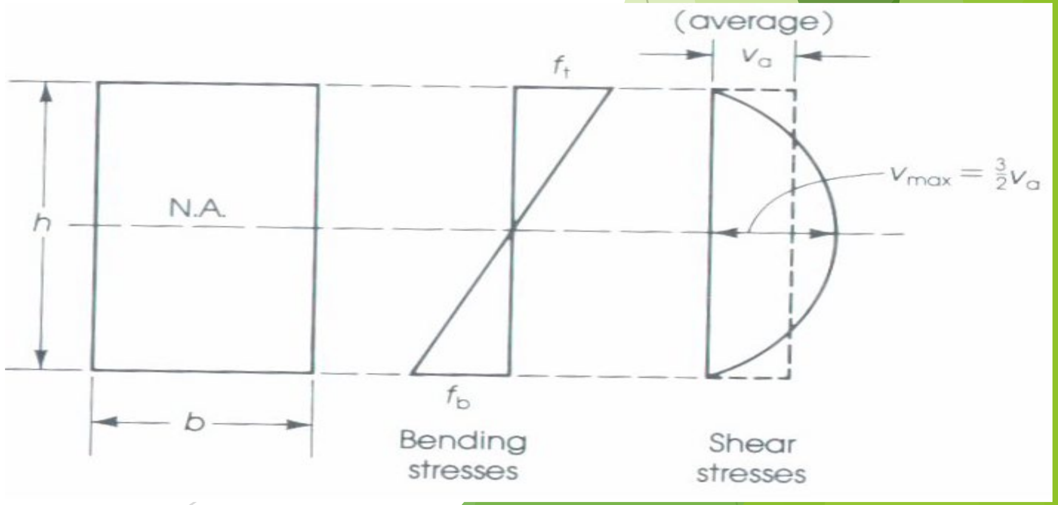
The shear stress distribution in any section of the beam is derived from the longitudinal shear force required to satisfy the equilibrium of forces as shown in the figure below:



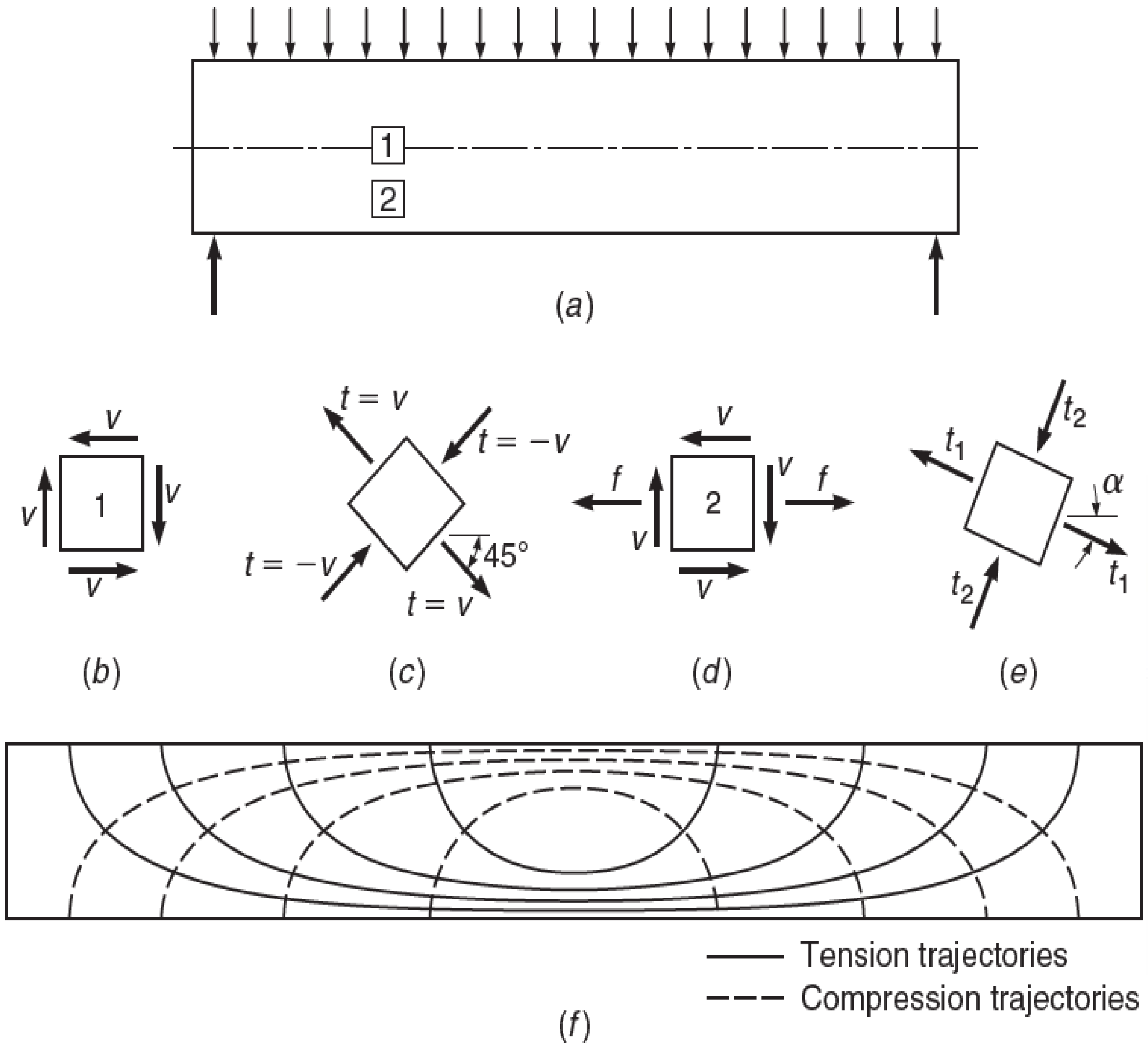


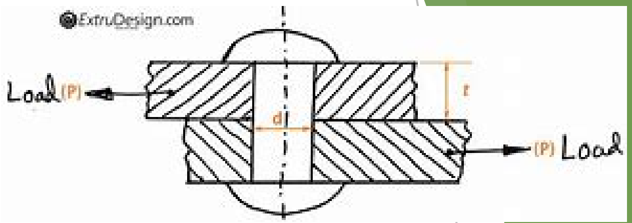
$$\sigma = \frac{M\,y}{E\,I} \qquad (Flexure\ Formulae)$$

$$\tau = \frac{V\,Q}{I\,b} \qquad (Shear\ Formulae)$$

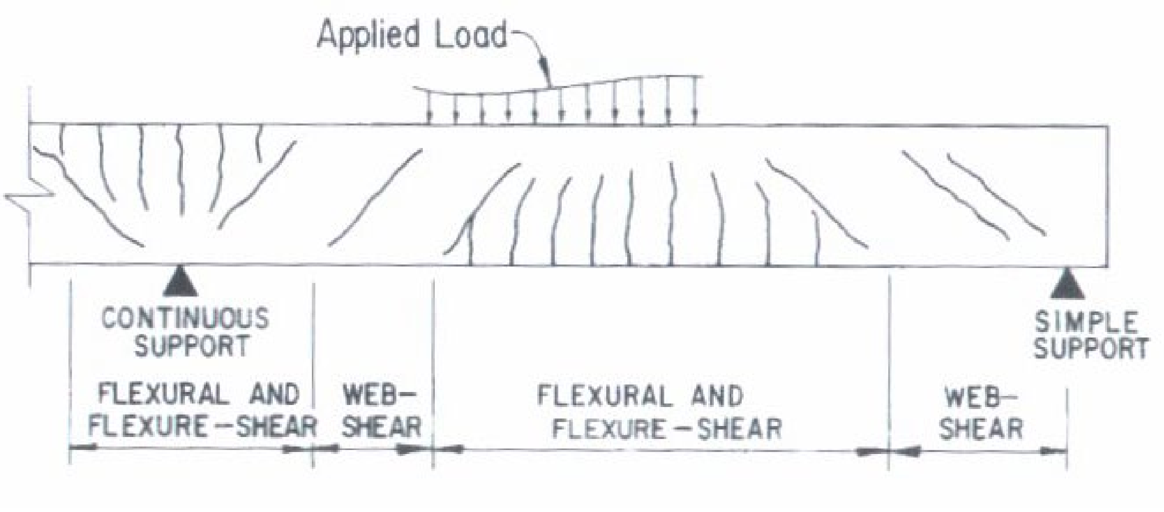


Diagonal Tension

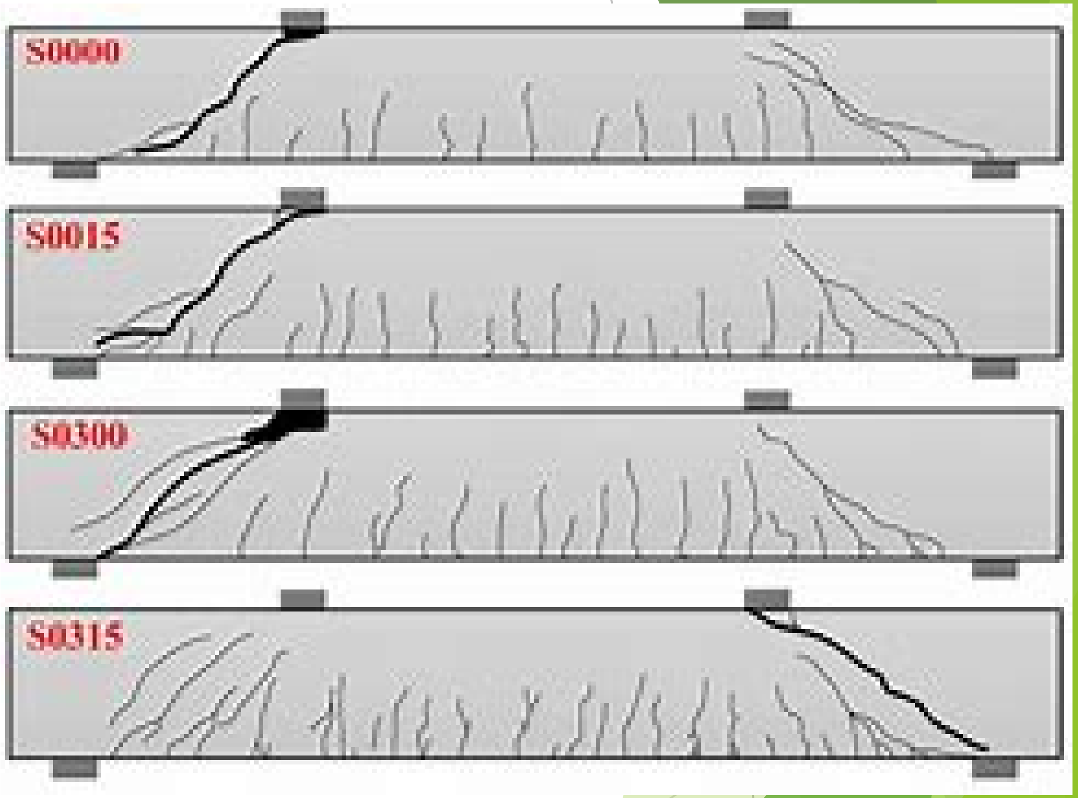


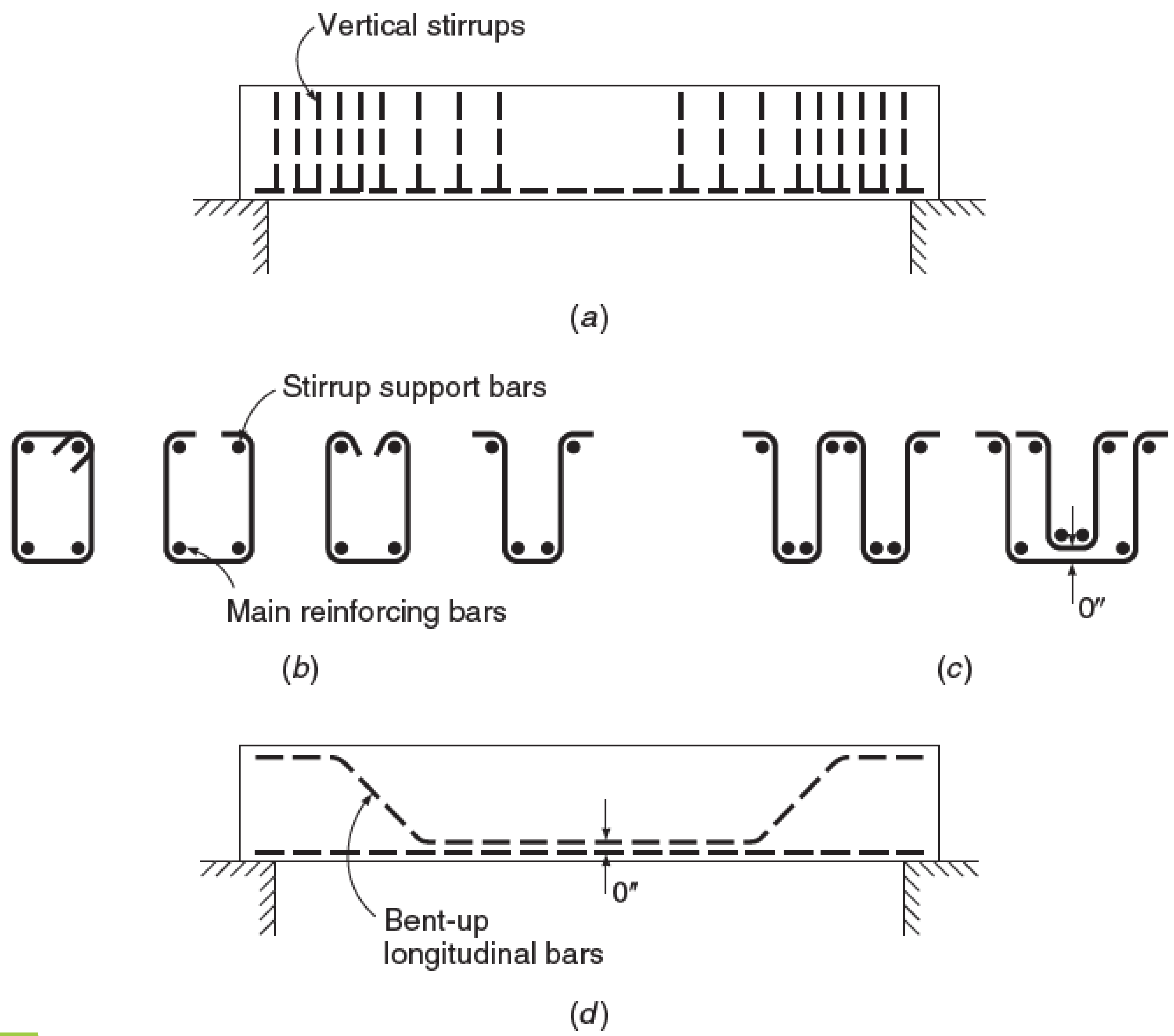


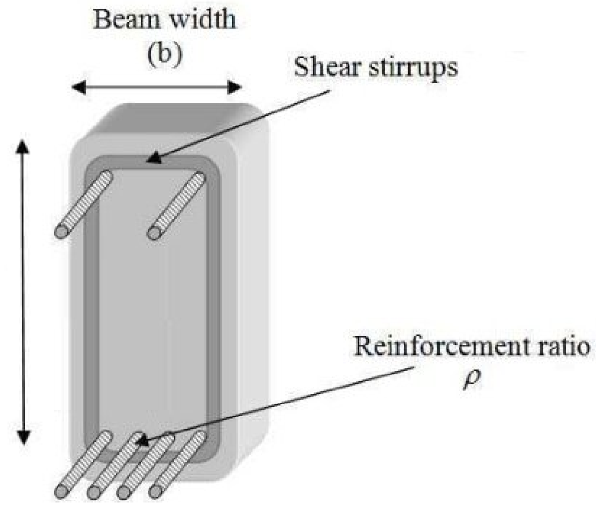
Pure shear is rarely the dominated case in the reinforced concrete members, it is mostly combined with bending moment.
And even when the stress inside the member is pure shear, the failure would be caused by the diagonal tension stress rather than the shear stress.



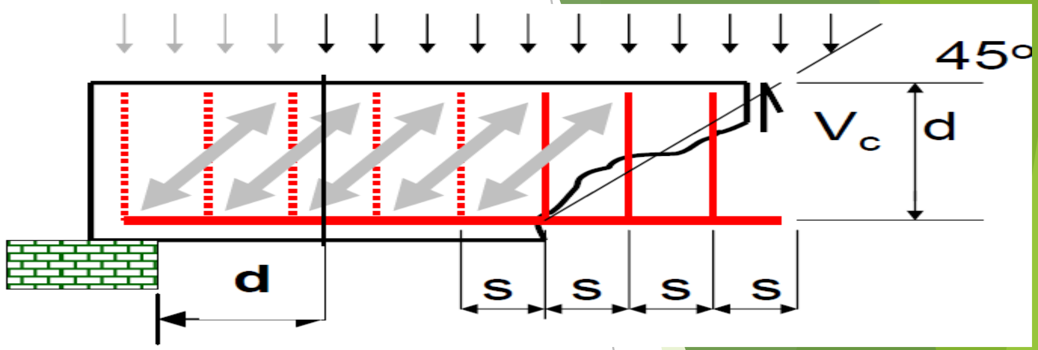
Shear (Diagonal Tension) Cracks and Failure




Vertical stirrups
(a)
Stirrup support bars
Main reinforcing bars
(b)
0″
(c)
0″
Bent-up
longitudinal bars
(d)


Beam width
(b)
Shear stirrups
Reinforcement ratio
ρ

ACI Provisions for Shear



$$V_n \geq \frac{V_u}{\phi}$$

$$V_u = 1.2V_D + 1.6V_L \quad \text{and } \phi = 0.75$$

$$V_n = V_c + V_s$$

$$V_c = 0.17\sqrt{f_c'}\, b_w\, d$$

$$V_s = n\, A_v\, f_{yt}$$

$$n \times s = d$$

n ⟷ s

$$V_s = A_v\, f_{yt}\, \frac{d}{s}$$

$$s = \frac{A_v\, f_{yt}\, d}{V_s}$$

n: number of stirrups intersect a crack of 45° (no. of stirrups of a distance d)

Av: area of all vertical legs of one stirrup (depends on the stirrup shape)

fyt: yield strength of stirrups steel should be ≤ 420 MPa

Thank you…