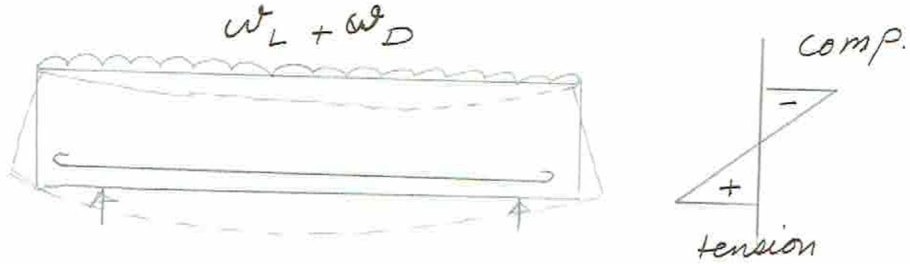
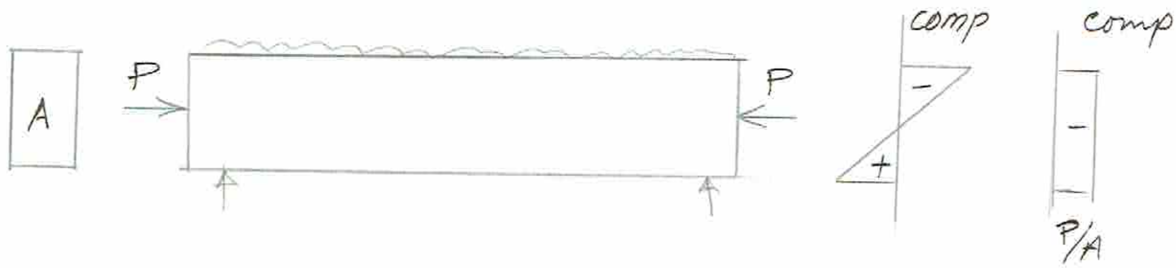
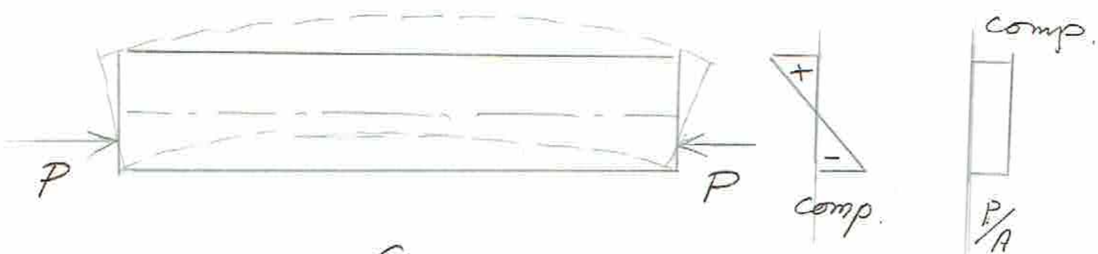
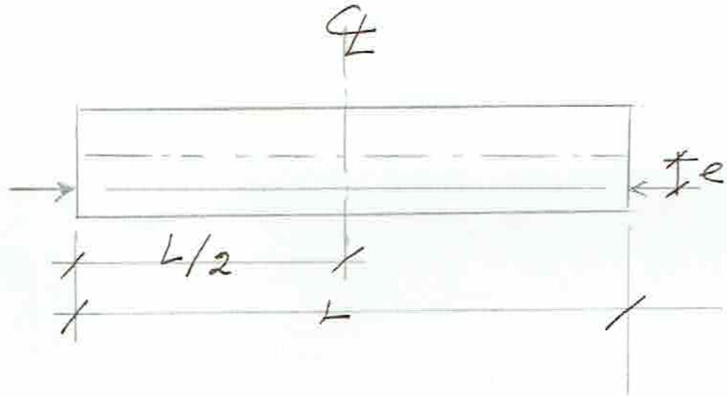
Prestressed Concrete
4th Year Civil
Engineering

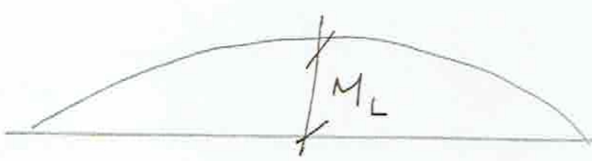




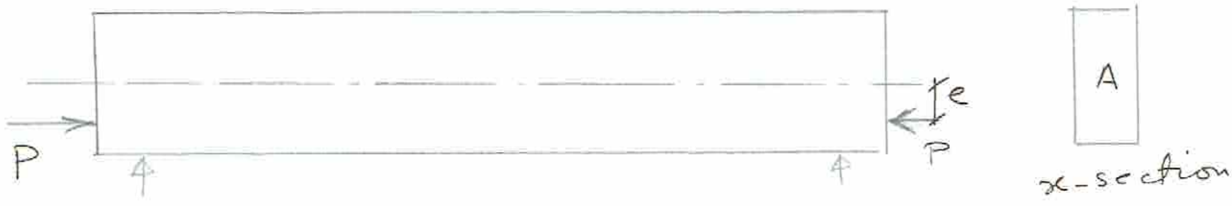


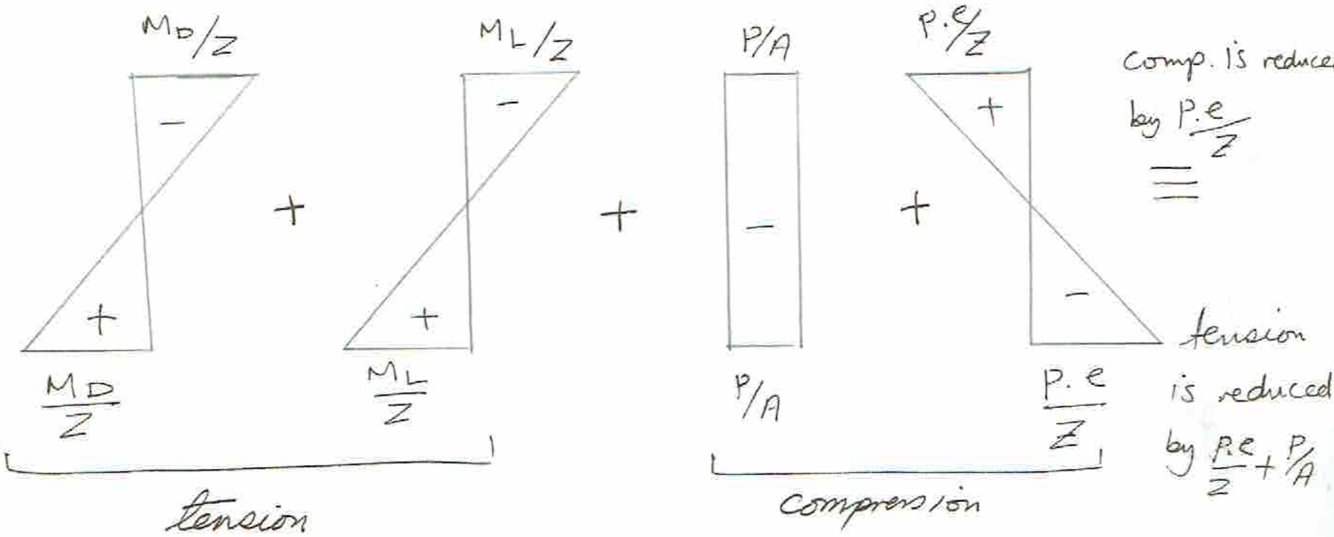


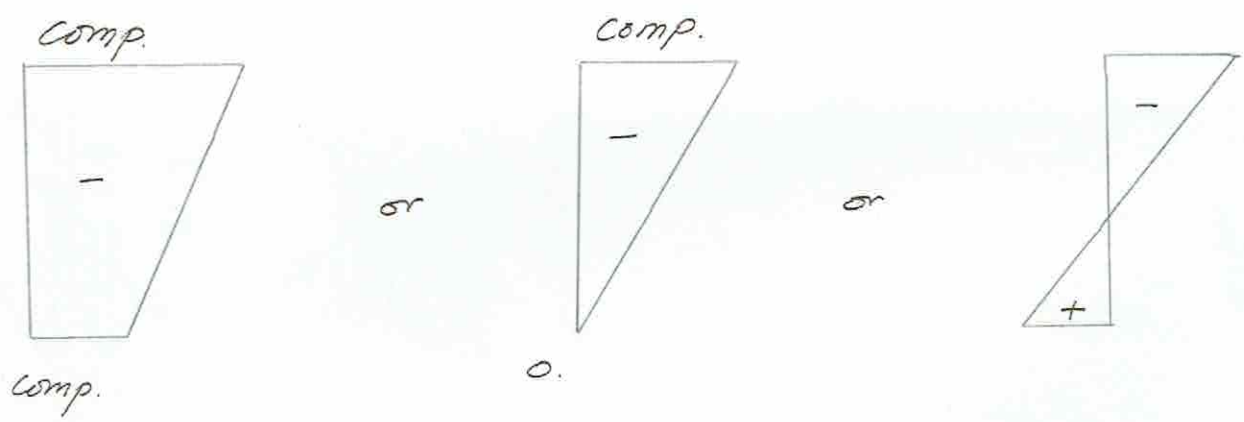
B.m. due to dead load.



B.m. due to live load.





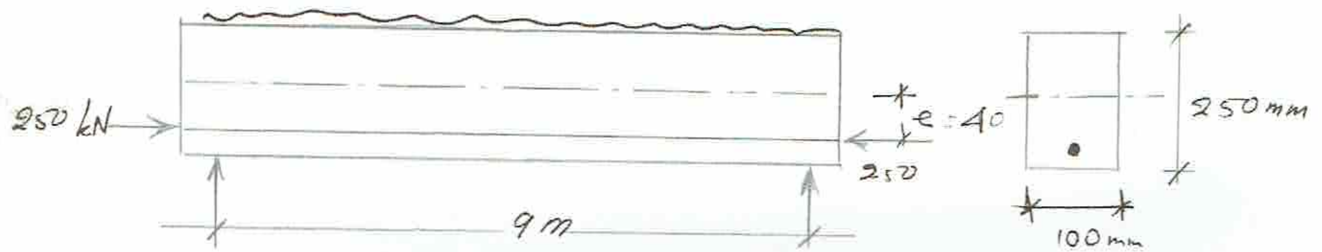


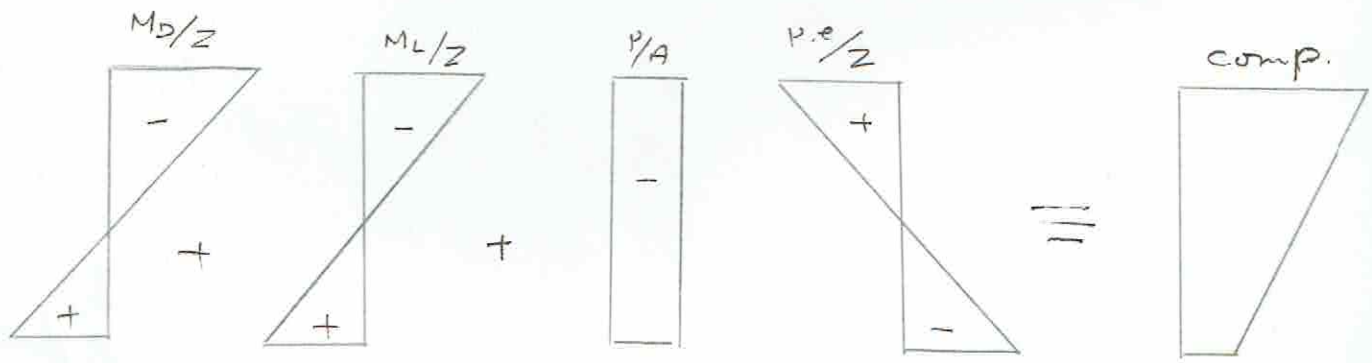
$$P/A + P.e/Z > \frac{M_L + M_D}{Z} \qquad \frac{P}{A} + \frac{P.e}{Z} = \frac{M_L + M_D}{Z} \qquad \frac{P}{A} + \frac{P.e}{Z} < \frac{M_L + M_D}{Z}$$

EX A simply supported prestressed beam of rectangular section $100 \times 250 \ mm^2$ and 9 m span. Prestressed by a straight cable with an effective force 250 kN. e = 40 mm below the centroid of the section. Live loads is 1.25 kN/m

① Find resultant stresses distribution at the centre of the ~~section~~ beam.

② Find the force to give zero stress at the bottom of the section at mid span.





$\frac{M_D}{Z}$ $\frac{M_L}{Z}$ P/A $\frac{P.e}{Z}$

bending stress due to L.L. | bending stress due to D.L. | direct stress due to prestress | bending stress due to prestressing

effect of prestressing

$P = 250\ kN \quad , e = 40\ mm$

$A = 250 \times 100 = 25 \times 10^3\ mm^2$

$I = \frac{1}{12} \times 100 \times 250^3 = 130.2 \times 10^6\ mm^4$

$y = \frac{250}{2} = 125\ mm$

$Z = I/y = \frac{130.2 \times 10^6}{125} = 1.04 \times 10^6\ mm^3$

$\omega_D = 0.1 \times 0.25 \times 24 = 0.6\ kN/m$

$M_D = \frac{\omega_D L^2}{8} = \frac{0.6 \times 9^2}{8} = 6.1\ kN.m$

$M_L = \frac{\omega_L L^2}{8} = \frac{1.25 \times 9^2}{8} = 12.6\ kN.m$

resultant stress at top fibre :

$= -\frac{P}{A} + \frac{P.e}{Z} - \frac{M_D}{Z} - \frac{M_L}{Z}$

$= \frac{-250 \times 10^3}{25 \times 10^3} + \frac{250 \times 10^3 \times 40}{1.04 \times 10^6} - \frac{6.1 \times 10^6}{1.04 \times 10^6} - \frac{12.6 \times 10^6}{1.04 \times 10^6}$

$= -10 + 9.6 - 5.86 - 12.1$

$= -18.36\ N/mm^2$ (compression)

resultant stress at bottom fibre :

$$= -P/A - \frac{P.e}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z}$$

$$= -10 - 9.6 + 5.86 + 12.1$$

$$= -1.64\ N/mm^2 \quad (\text{compression})$$

② $P = ?$ $\quad e = 40\ mm$

balanced stress i.e. tension = compression

stress at bottom fibre

$$= -P/A - \frac{P.e}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z} = 0.0$$

$$-\frac{P \times 10^3}{25 \times 10^3} - \frac{P \times 10^3 \times 40}{1.04 \times 10^6} + 5.86 + 12.1 = 0.$$

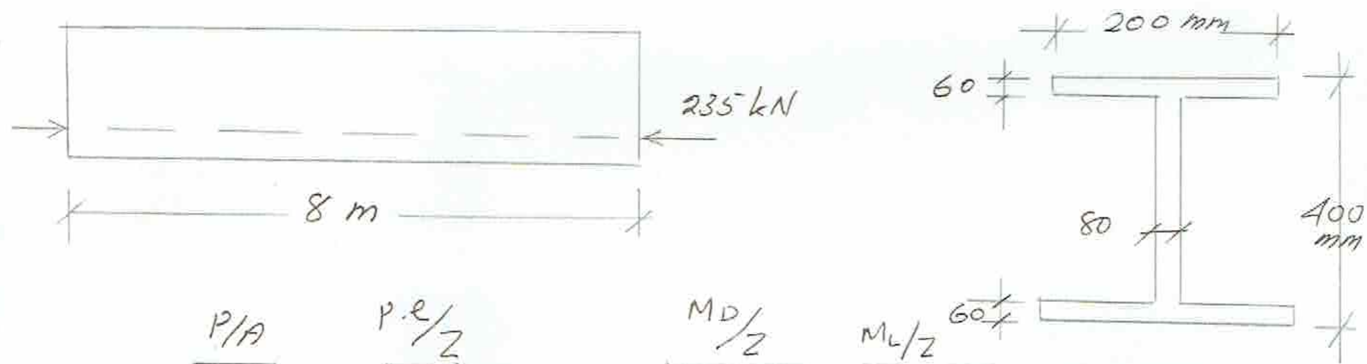
$$-0.04P - 0.038P + 18.00 = 0$$

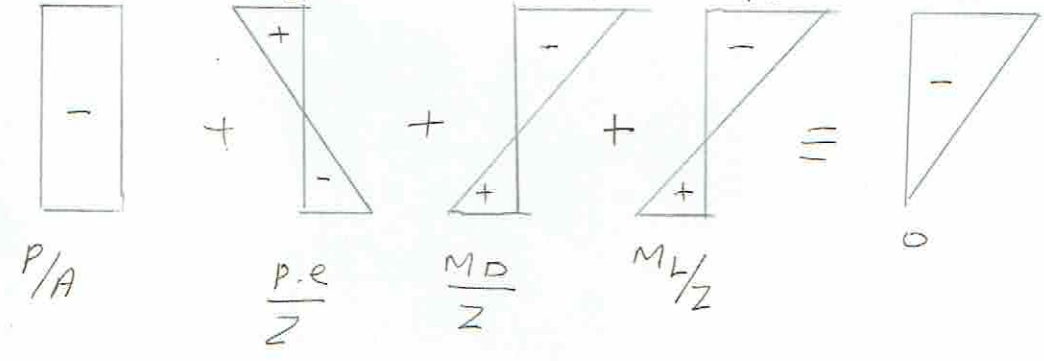
$$P = \frac{18.0}{0.078} = 231.0\ kN$$

EX: A prestressed simply supported beam 8 m span with an I_section and L.L. = 4 kN/m. The effective prestress force (prestress force) is 235 kN. The value of e makes the resultant stress at the bottom of the section at mid span equals to zero.

A) find the value of e.

B) If the tendon passes through the centroid of the section, find the prestress force to make the resultant stress at the bottom of the section at mid span equals to zero.





(a) $A = 200 \times 60 \times 2 + 80\,(400 - 2 \times 60)$

$= 46.4 \times 10^3 \ mm^2$

$I = \frac{1}{12} \times 200 \times 60^3 \times 2 + 200 \times 60 \times 170^2 \times 2 + \frac{1}{12} \times 80 \times \overset{\overset{400-120}{6}}{280}{}^3$

$= 847 \times 10^6 \ mm^4$

$y = \frac{400}{2} = 200 \ mm$

$Z = I/y = \frac{847 \times 10^6}{200} = 4.24 \times 10^6 \ mm^3$

$w_D = \frac{46.4 \times 10^3}{10^6} \times 24 = 1.11 \ kN/m$

$M_D = \frac{1.11 \times 8^2}{8} = 8.88 \ kN.m$

$M_L = \frac{4 \times 8^2}{8} = 32 \ kN.m$

resultant stress at bottom fibre $= 0$

$0 = -\frac{235 \times 10^3}{46.4 \times 10^3} - \frac{235 \times 10^3 \times e}{4.24 \times 10^6} + \frac{8.88 \times 10^6}{4.24 \times 10^6} + \frac{32 \times 10^6}{4.24 \times 10^6}$

$e = 84 \ mm$

(b) $0 = -\frac{P \times 10^3}{46.4 \times 10^3} - 0 + 2.1 + 7.55$

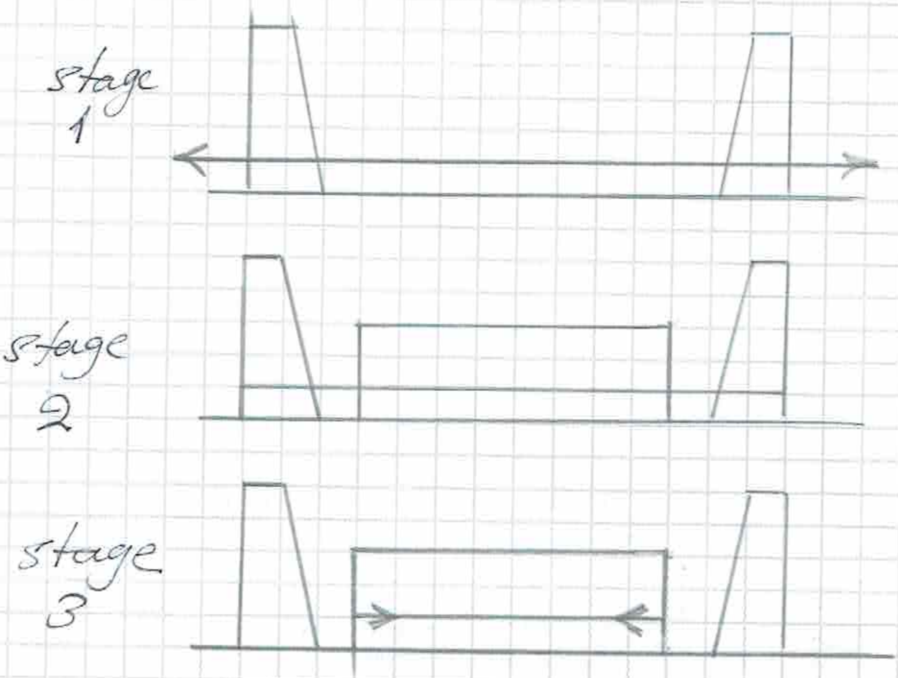
$P = 448 \ kN.$

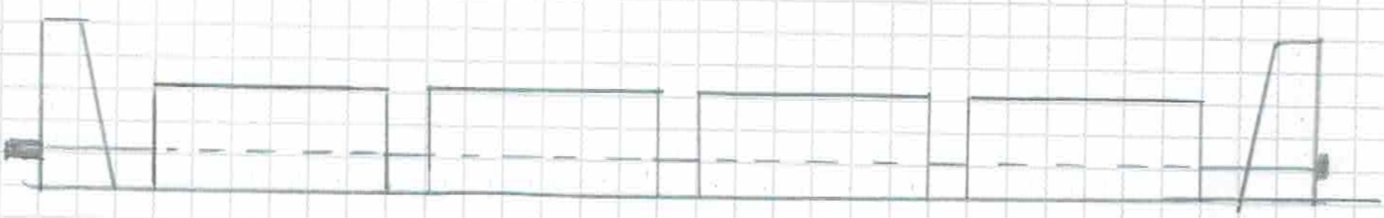
Methods :-

1- Pretensioning :

First The steel is tensioned between abutments and then The concrete is placed in moulds around it. When the concrete has achieved sufficient compressive strength, the steel is released from the abutment, transferring The force to the concrete through the bond that now exists between the steel and the concrete.

Pretensionsing of a single unit.





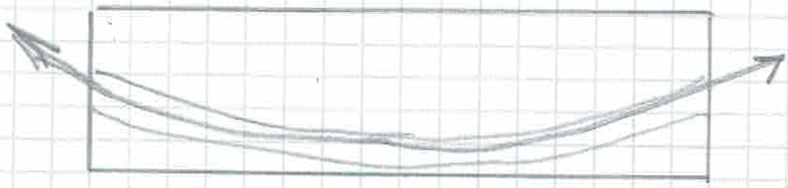
Pretensioning of mass production

2- Post-Tensioning :

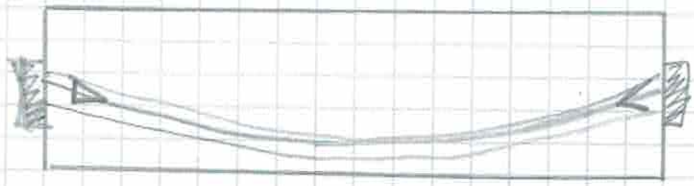
The concrete is cast first in the mould and allowed to harden before The prestress is applied. The steel may be placed in position to a predetermined profile and cast into the concrete, bond being prevented by enclosing the steel in a protective metal sheathing (Ducts). Or ducts may be formed in the concrete and the steel passed through after hardening has taken place. When the required concrete strength has achieved, The steel is stressed against The ends of the units and anchored off, thus putting The concrete into compression.

stage 1

stage 2



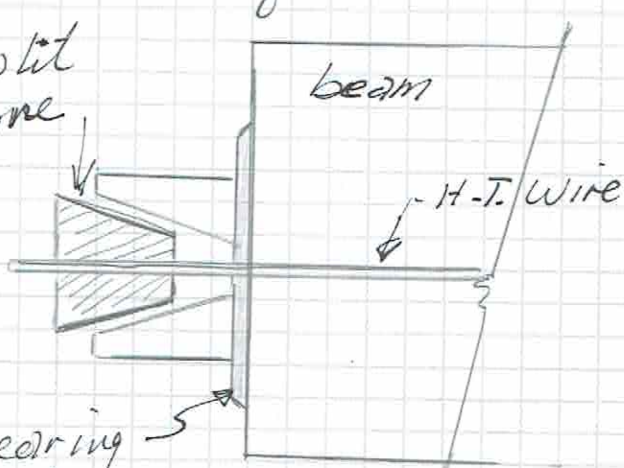
stage 3



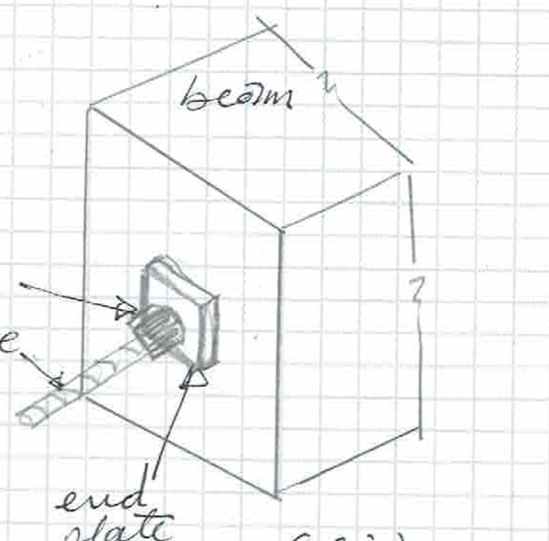
Post-tensiong can be done by two methods :-

i- Wedge action producing a frictional grip on The wire.

ii- Direct bearing from a bolt head formed at the ends of the wire.



(i)



(ii)

Material :

1 Concrete = Concrete of high compressive strength must be used 35 - 50 Mpa and above.

A- High value of (E) to

1- reduce the initial elastic strain under the influence of a prestress force.

2- reduce the creep strain which is proportional to the elastic strain.

This will reduce the loss in prestress forces.

B- High bearing stress. In post tensioned members high bearing stresses at the anchorage fittings.

C- High bond stress in pretenssiong.

D- In the presence of a quality control, concrete of high strength can be achieved.

Note :

Concrete strain due to sustained loading for a short time is more important in prestressed structures than in ordinary concrete structures due to it's effect in the loss of prestress forces.

Concrete strain due to stress, shrinkage and thermal changes has an important influence on the prestressed structures.

Permissible stresses _ cl. 18.4

1- Flexural stresses immediately after transfer before loss

a- compression - - - - $0.6 f'_{ci}$

b- tension in members without bonded auxilary reinforcement unprestressed or prestressed in tension zone -- $0.25\sqrt{f'_{ci}}$

2- Stresses at service loads after allowance for all prestress losses;

a. Compression ------ $0.45 f'_c$

b- Tension ------- $0.5\sqrt{f'_c}$

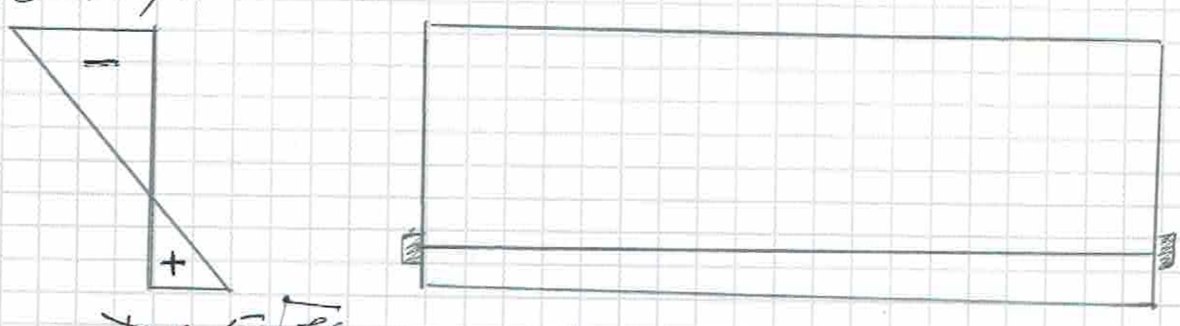
where;

f_{ci} = compressive strength of concrete at the time of initial prestress.

f'_c = Specified compressive strength of concrete

i.e.

$\ngtr 0.45 f'_c$



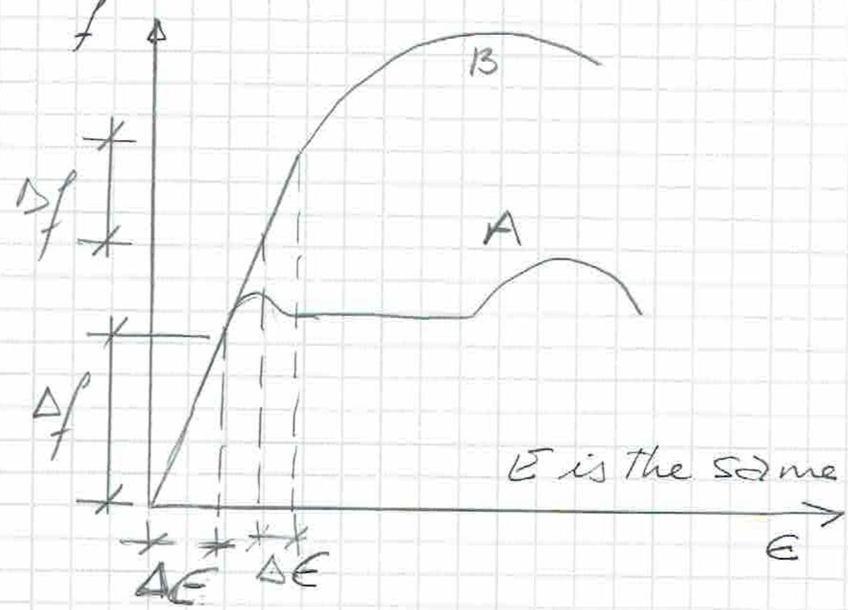
$\ngtr 0.5\sqrt{f'_c}$

at service load

2- Prestressing steel;

Prestressing is practical only when steels of a very high strength are used.

High tensile steel (H.T.S.) must be used.



A- Ordinary steel;

$$f_y = 275\ N/mm^2\ ,\ E_s = 2\times10^5\ N/mm^2$$

$$\epsilon_s = \frac{\Delta L}{L} = \frac{f_s}{E_s} = \frac{275}{2\times10^5} = 1.375\times10^{-3}$$

B- H.T.S.;

(Prestress bar) $f_y = 880\ N/mm^2$ (min. yield stress)

$$\epsilon_s = \frac{880}{2\times10^5} = 4.4\times10^{-3}$$

Assume long term strain in concrete due to shrinkage and creep alone $= 0.8\times10^{-3}$

net strain in steel after losses;

1- $(1.375 - 0.8)10^{-3} = 0.575\times10^{-3}$

2- $(4.4 - 0.8)10^{-3} = 3.6\times10^{-3}$

The corresponding stress after losses;

1- $f_s = E_s . \epsilon_s = 0.575 \times 10^{-3} \times 2 \times 10^5 = 115 \ N/mm^2$

2- $f_s = 3.6 \times 10^{-3} \times 2 \times 10^5 = 720 \ N/mm^2$

$\therefore$ % stress loss;

1- $\dfrac{275 - 115}{275} = 58\%$

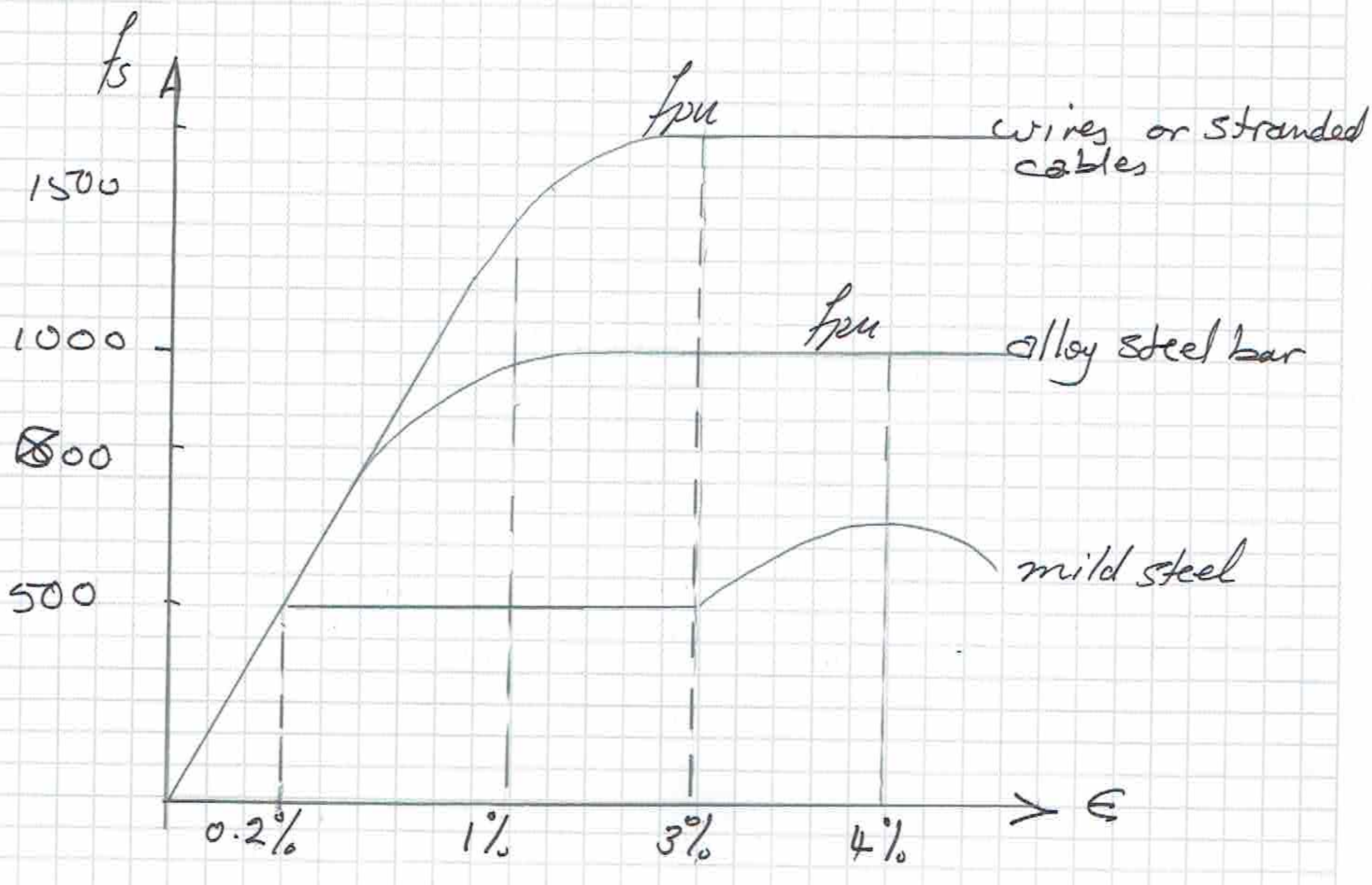
2- $\dfrac{880 - 720}{880} = 18\%$

Forms of steel:-

1- Individual wires, single, parallel bundles or cables having diameter 5 to 7 mm.

2- Stranded cable generally made of 7 wires, having dia 2 to 4 mm.

3- Alloy steel bars of dia. 10 to 32 mm.



Allowable stresses in steel;

Due to jacking force $0.8 f_{pu}$ or $0.9 f_{py}$

Pretensioning tendons
Immediately after transfer
or postensioning tendons immediately after anchoring } $0.7 f_{pu}$

where;

f_{pu} = ultimate strength of prestressing steel.

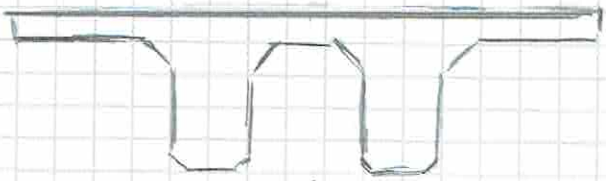
f_{py} : yield strength of prestressing steel.

Standarized Prestressing Steel:

Product	specification	Grade	min. yield strength N/mm²	min. tensile strength N/mm²
Prestressed wires	A421		1296 – 1330	1620 – 1725
Strands	A416	250	1465	1725
		270	1580	1860
Alloy steel bars	A722	type I	880	1034
		type II	827	1034

Shap Selection

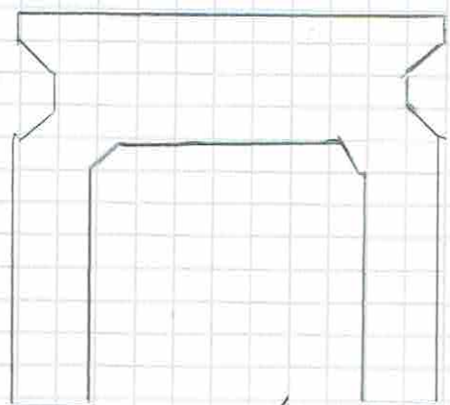
Several common shapes are used, some of them of standarized and mass produced, others are indivitually proportioned for large and important works.
Design and economical considerations must be taken into account in selecting the shape of a cross section.



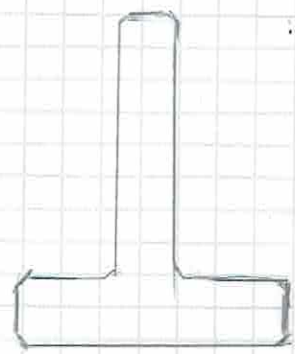
double T

single T

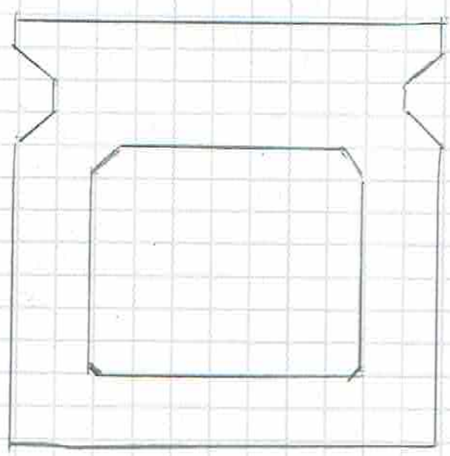
I-girder



channel slab



Inverted T



Box girder

Full Prestressing:

In which no tensile stress is allowed in the concrete at service loads.

Partially Prestressed:

In which some tensile stresses then cracking are permitted, provided that they are within the permissible limits. This is used when the live load is too high with respect to dead load.

Losses of Prestressing Forces:

1- Elastic shortening of concrete

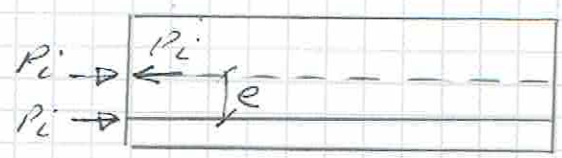
i- Pretensioned beams:

$$\Delta f_{S\,loss}(\text{elastic}) = E_c \frac{f_c}{E_c} = n \cdot f_c \qquad \frac{f_s}{E_s} = \frac{f_c}{E_c}$$

where; $n = E_c / E_s$

f_c = stress in concrete at the level of prestressing wire.

$$f_c = -\frac{P_i}{A} - \left(\frac{P_i \cdot e}{I/e}\right) + \frac{M_g}{I/e}$$



$$= -\frac{P_i}{A}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_g \cdot e}{I}$$

r = radius of gyration

P_i = approximately is taken equal to 10% less than jacking force (P_J).

ii- Post tensioned beam;

A- No loss, if all cables are simultaneously tensioned.

B- If the cables are tensioned successively, then the loss due to elastic shortening is equal to half the loss of the first cable.

2- Slip at anchorage:

- Occurs only in post tensioned members.
- due to slipping of wedges and also due to deformation of anchors.

$$\Delta f_s = \left(\frac{\Delta L}{L}\right) E_s$$

$\frac{\Delta L}{L} = \epsilon$, ΔL = shortening of wire
L = original length of wire.

3- Frictional losses:

1- Unintentional friction loss (wobble friction) due to variation of the tendon from its intended profile.

2- Intentional friction loss (curvature) friction due to intentional bends in the tendon profile as specified:

$$P_s = P_x \cdot e^{KL + \mu\alpha}$$

P_s = force at the jacking end of the tendons.

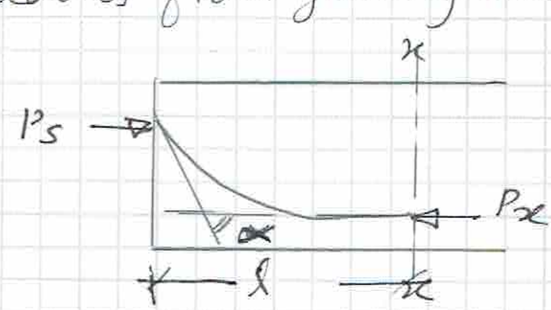
P_x = force at any section x along the tendons.

K = wobble friction coefficient per meter (0.003–0.0066)

l = tendon length from jacking end to the point x.

μ = Curvature friction coefficient (0.05–0.3)

α = angular change of tendons from jacking end to section x



simplified formula when $Kl + \mu\alpha \ngtr 0.3$

$$P_s = P_x\,(1 + Kl + \mu\alpha)$$

4- Creep of Concrete:

Shortening of concrete under sustained load is called creep.

$$\Delta f_s)_{creep} = C_c \cdot n \cdot f_c$$

C_c = creep coefficient (2 - 4) Nilson = 2.35

f_c = stress in concrete at the level of steel centroid, when the eccentrentic prestress force plus all sustained loads are acting.

Equation for calculating f_c in elastic shortening can be used except that M_g should be replaced by moment due to all dead loads plus that due to ~~any~~ any position of the live load that may be considered sustained.

- To account for this, it is taken 10% less than (P_i).

5- Shrinkage of concrete:

$$\Delta f_s)_{shrink} = \epsilon_{sh} \cdot E_s$$

$$\epsilon_{sh} = 0.0002 \rightarrow 0.0007$$

typical value = 0.0003

6- Relaxation of steel;

Elongation of steel under sustained loads is called relaxation.

$$\frac{f_s}{f_{si}} = 1 - \frac{\log t}{10}\left(\frac{f_{si}}{f_y} - 0.55\right) \qquad \text{when } \frac{f_{si}}{f_y} \not> 0.55$$

f_s : steel stress

t - hours after the application of initial stress f_{si} and $\log t$ is to the base 10.

Lump-Sum estimates of losses:

The following values were recommended for lump sum losses, including those due to elastic shortening, creep and relaxation but excluding those due to friction and ancherage slip.

For pretensioning — 241 N/mm^2

For post tensioning — 172 N/mm^2

Prestressing forces:

P_J = jacking force.

P_i = initial prestressing force after:

i- elastic shortening loss
ii- frictional loss
iii- slip of anchoring loss

} loss in P_J

P_e = effictive prestressing force after:

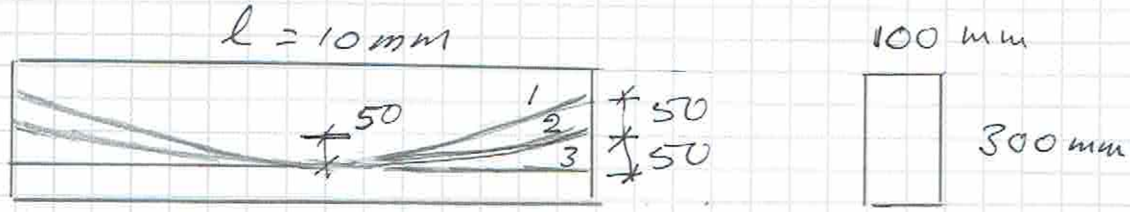
i- shrinkage loss
ii- creep loss
iii- relaxation

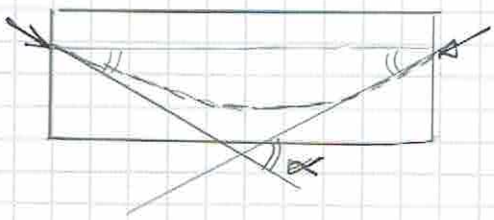
} loss in P_i

effectiveness ratio, $\boxed{R = \frac{P_e}{P_i}}$

EX1: A posttensioned concrete beam, 100 mm width, 300 mm depth tensioned by three cables and anchored. The cross sectional area of each cable is 200 mm^2 and initial stress in the cable is 1200 N/mm^2. Estimate the percentage friction loss in each cable

$l = 10\,m$, $\mu = 0.25$, $k = 0.0015$





Cable Profile: $y = \frac{4ex}{l^2}(l-x)$

$$y = \frac{4e}{l^2}(xl - x^2)$$

$$\frac{dy}{dx} = \frac{4e}{l^2}(l - 2x)$$

when $x = 0$, $\left.\frac{dy}{dx}\right)_{x=0} = \frac{4e}{l}$

For cable 1:

slope at one end $= \frac{4 \times 100}{10 \times 1000} = 0.04$ rad

cumulative angle between tangents α

$\alpha = 2 \times 0.04 = 0.08$ rad.

For cable 2: slope at one end $= 0.02$ rad.

$\alpha = 0.04$ rad.

cable 3 = $\alpha = 0$

Initial prestressing force in each cable at jacking end
$P_s = 200 \times 1200 = 240$ kN

If P_x is the prestressing force at the far end then;

$$kl + \mu\alpha = 0.0015 \times 10 + 0.25 \times 0.08$$
$$= 0.035 < 0.3$$

Hence $P_s = P_x(1 + kl + \mu\alpha) = P_x + P_x(kl + \mu\alpha)$

$\therefore$ loss of prestress $P_s - P_x = P_x(kl + \mu\alpha)$

Cable 1; $P_x = \dfrac{P_s}{1 + kl + \mu\alpha} = \dfrac{240 \times 10^3}{1.035} = 231.88 \times 10^3$ N

$\therefore$ loss of prestress $= 231.88 \times 10^3 \times (0.035) = 8.12$ kN

or $= 240 - 231.88 = 8.12$ kN

Cable 2; $P_x = \dfrac{P_s}{1 + 0.0015 \times 10 + 0.25 \times 0.04} = \dfrac{240 \times 10^3}{1.025}$

$= 234.15 \times 10^3$ N

$\therefore$ loss of prestress $= 234.15 \times 10^3 \times 0.25 = 5.85$ kN $\}$ 240

or $= 240 - 234.15 = 5.85$ kN

Cable 3: $P_x = \dfrac{P_s}{1 + 0.0015 \times 10 + 0.25 \times \text{zero}} = \dfrac{240 \times 10^3}{1.015} = 236.45$ kN

240

$\therefore$ loss of prestress $= 240 - 236.45 = 3.55$ kN

or $240 - 236.45 = 3.55$ kN

$\therefore$ % loss cable 1 $= 3.38\%$

cable 2 $= 2.43\%$

cable 3 $= 1.48\%$ wobble friction only

EX 2 - A simply supported concrete beam 100 x 300 mm cross section is posttensioned by three cables (straight). Estimate percentage loss due to elastic shortening of concrete when

1- The three cables are tensioned simultaneously.
2- The three cabes are tensioned successively.

$e = 50$ mm , initial stress $= 1200$ N/mm^2
cable cross section $= 50$ mm^2, $f'_c = 35$ N/mm^2

$$E_c = 4733\sqrt{f'_c} = 4733\sqrt{35} = 28000 \text{ N/mm}^2$$

cl. 8.5

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

$$n = \frac{E_s}{E_c} = \frac{2 \times 10^5}{28 \times 10^3} = 7.1 \simeq 7.0$$

case 1 = NO loss

case 2 = $P = 50 \times 1200 = 60$ kN

$A = 30 \times 10^3$, $I = 225 \times 10^6$ mm^4
$e = 50$, $y = 50$ mm

stress in concrete at level of steel:

$$f_c = -\frac{60 \times 10^3}{30 \times 10^3} - \frac{60 \times 10^3 \times 50 \times 50}{225 \times 10^6} = -2.67 \text{ N/mm}^2$$

cable 1: tensioned and anchored = No loss

cable 2 = tensioned and anchored - loss in cable 1
$= n \cdot f_c = 7 \times 2.67 = 18.69$ N/mm^2

cable 3 = Tensioned and anchored - loss in cable 1 & 2 given by =

cable 1 , loss in stress $= 7 \times 2.67 = 18.69$ N/mm^2 when 2 tensioned
cable 2 , " " " $= 7 \times 2.67 = 18.69$ N/mm^2 when 3 tensioned

%. total loss of stress due to elastic shortening of concrete is:

cable 1 = $18.69 + 18.69 = 37.38\ N/mm^2$

cable 2 = 18.69

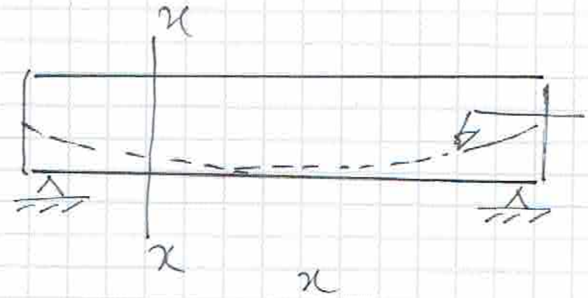
cable 3 = 0.

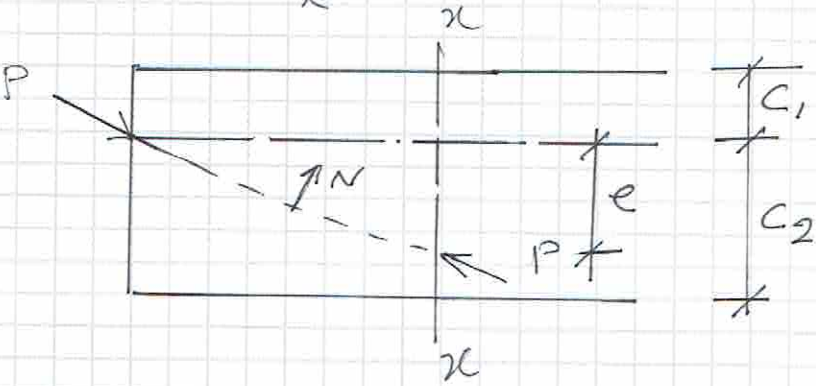
average loss of stress in all cables

$= 18.69\ N/mm^2$

% loss = 1.56%

Elastic Flexural Analysis:

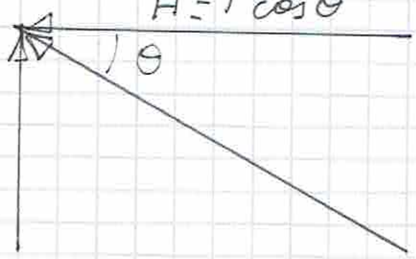


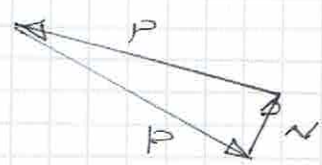


$H = P\cos\theta$

$V = H\tan\theta$

$= P\sin\theta$

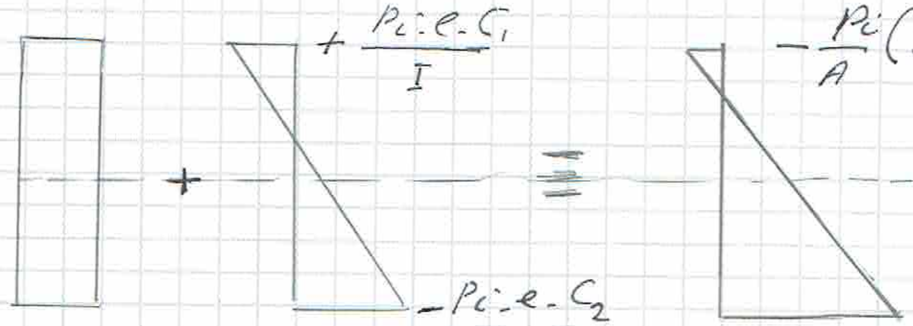




For small θ ; $\cos\theta \Rightarrow 1.0$

$\therefore H = P$

(a) Effect of Prestressing:

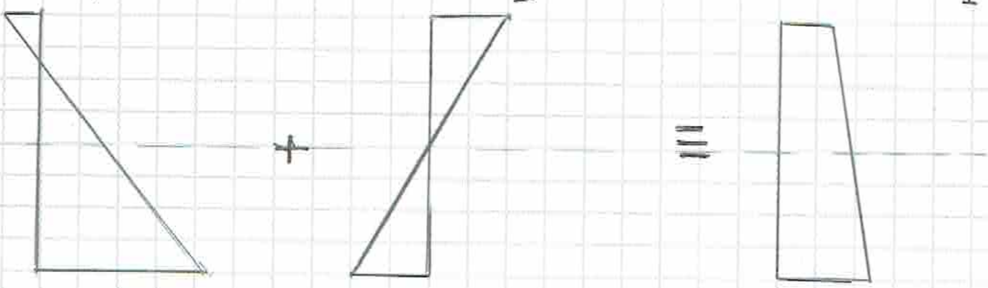


$$-\frac{P_i}{A} \quad + \quad +\frac{P_i \cdot e \cdot c_1}{I} \quad \equiv \quad -\frac{P_i}{A}\left(1 - \frac{e\,c_1}{r^2}\right)$$

$$-\frac{P_i}{A} \qquad -\frac{P_i \cdot e \cdot c_2}{I} \qquad -\frac{P_i}{A}\left(1 + \frac{e\,c_2}{r^2}\right)$$

(b) Effect of Prestress and beam dead load:

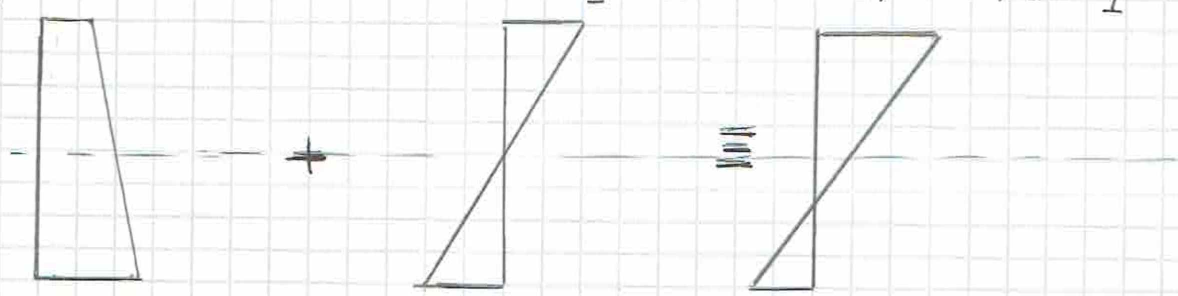
$-\frac{P_i}{A}\left(1-\frac{e c_1}{r^2}\right)$ $\qquad -\frac{M_g \cdot c_1}{I}$ $\qquad -\frac{P_i}{A}\left(1-\frac{e c_1}{r^2}\right)-\frac{M_g \cdot c_1}{I}$



$-\frac{P_i}{A}\left(1+\frac{e c_1}{r^2}\right)$ $\qquad +\frac{M_g \cdot c_2}{I}$ $\qquad -\frac{P_i}{A}\left(1+\frac{e c_1}{r^2}\right)+\frac{M_g \cdot c_1}{I}$

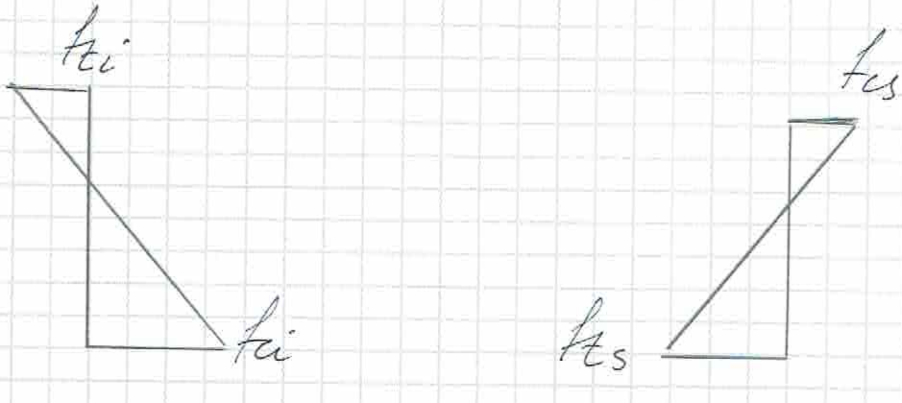
(c) Effect of Prestress, beam dead load and service load:

$-\frac{P_e}{A}\left(1-\frac{e c_2}{r^2}\right)-\frac{M_g \cdot c_1}{I}$ $\qquad -\frac{M_s \cdot c_1}{I}$ $\qquad -\frac{P_e}{A}\left(1-\frac{e c_1}{r^2}\right)-\frac{c_1}{I}(M_g+M_s)$



$-\frac{P_e}{A}\left(1+\frac{e c_2}{r^2}\right)+\frac{M_g \cdot c_2}{I}$ $\qquad +\frac{M_s \cdot c_2}{I}$ $\qquad -\frac{P_e}{A}\left(1+\frac{e c_2}{r^2}\right)+\frac{c_2}{I}(M_g+M_s)$

The resultant stresses should be within the permissible limits.



prestress (P_i)
+ selfwt (M_g)

prestress (P_e)
+ full service load

EX3: A post tensioned simply supported concrete beam, tensioned by single tendon of parabolic profile.

beam section : $b = 280\ mm$, $h = 710\ mm$

cable profile : $e_{max} = 200\ mm$, $e_{min} = 0.0$

initial prestressing force $(P_i) = 1500\ kN$

effectiveness ratio $R = 0.84$

span $= 12\ m$

$LL = 15\ kN/m$, $DL = 4.4\ kN/m$

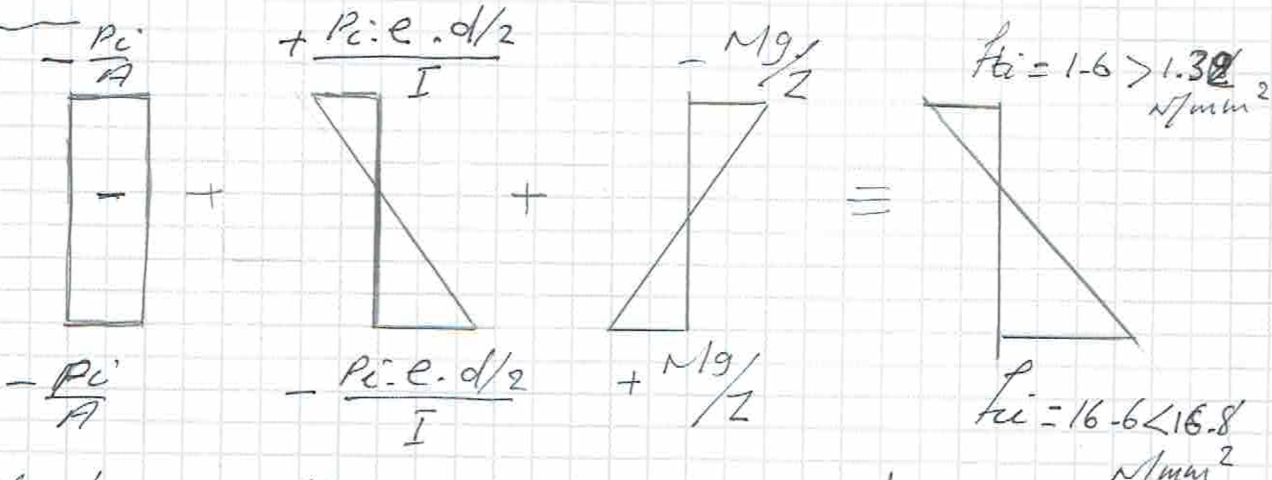
$f'_c = 34\ N/mm^2$, $f_{ci} = 28\ N/mm^2$

Find; bending stress distribution at mid span section for the following cases:

1- at initial condition before applying imposed loads
2- at full service loads.

Check results with the ACI_Code requirements.

Case 1;



initial condition = prestress + selfwt.

$$A = 280 \times 710 = 198.8 \times 10^3\ mm^2$$

$$I = \frac{1}{12} \times 280 \times 710^3 = 8351 \times 10^6\ mm^4$$

$$Z = I/y = \frac{8351 \times 10^6}{710/2} = 23.5 \times 10^6\ mm^3$$

$$w_g = 0.28 \times 0.71 \times 24 = 4.77\ kN.m$$

$$M_g = \frac{wl^2}{8} = \frac{4.77 \times 12^2}{8} = 85.86 \text{ kN.m}$$

stress at top fibre (f_{ti}):

$$= -\frac{P_i}{A} + \frac{P_i \cdot e}{Z} - \frac{M_g}{Z} = -\frac{1500 \times 10^3}{198.8 \times 10^3} + \frac{1500 \times 10^3 \times 200}{23.5 \times 10^6} - \frac{85.65 \times 10^6}{23.5 \times 10^6}$$

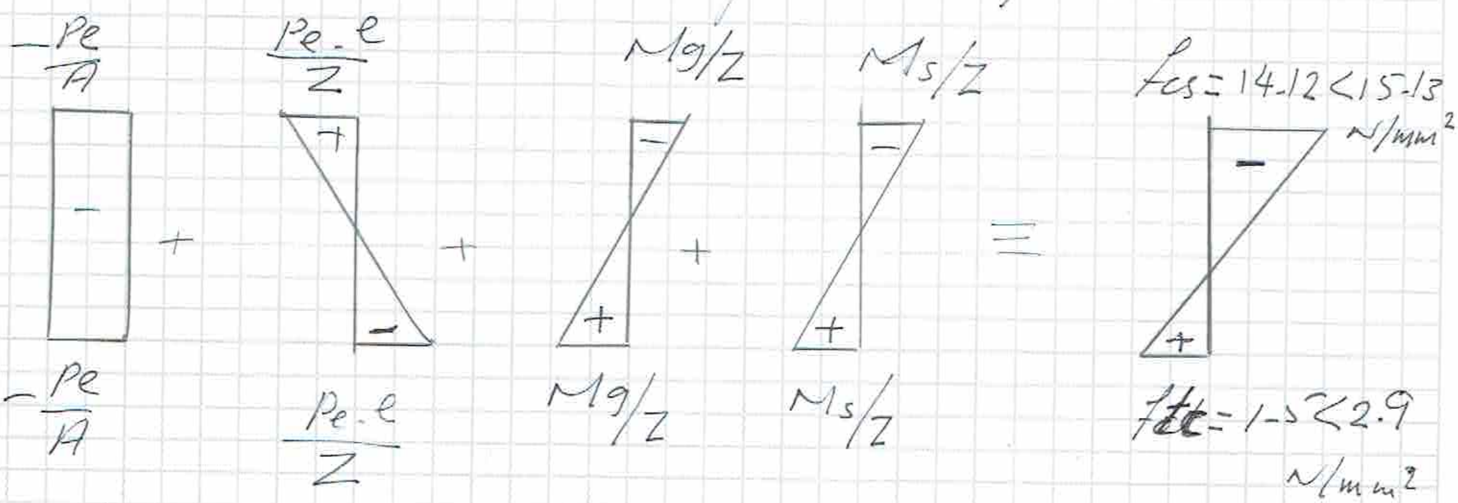
$$= -7.5 + 12.76 - 3.65$$

$$= +1.60 \text{ N/mm}^2 \text{ (tension)}$$

stress at bottom fibre (f_{ci})

$$= -7.5 - 12.76 + 3.65 = -16.61 \text{ N/mm}^2 \text{ (compression)}$$

② At full service load (prestress + beam deadload + super imposed load)



$$f_{cs} = -\frac{P_e}{A} + \frac{P_e \cdot e}{Z} - \frac{M_g}{Z} - \frac{M_s}{Z}$$

$$M_s = \frac{(4.4 + 1.5)12^2}{8} = 349.2 \text{ kN.m}$$

$$R = \frac{P_e}{P_i} \Rightarrow P_e = 0.84 \times 1500 = 1260 \text{ kN}$$

$$\therefore f_{cs} = -\frac{1260 \times 10^3}{198.8 \times 10^3} + \frac{1260 \times 10^3 \times 200}{23.5 \times 10^6} - \frac{85.86 \times 10^6}{23.5 \times 10^6} - \frac{349.2 \times 10^6}{23.5 \times 10^6}$$

$$= -14.12 \text{ (comp.)}$$

$$f_{ct} = -0.84 \times 20.26 + \frac{(85.86 + 349.2) \times 10^6}{23.5 \times 10^6} = +1.5 \text{ tension N/mm}^2$$

Allowable stresses (ACI-code):

1- At transfer

a- Compression $= 0.6 \times 28 = 16.8\ N/mm^2$

b- tension $= 0.25 \sqrt{28} = 1.32\ N/mm^2$

2- At service load

a- Compression $= 0.45 \times 34 = 15.3\ N/mm^2$

b- tension $= 0.5 \sqrt{34} = 2.9\ N/mm^2$

$f_{ci} = 16.61 < 16.8\ N/mm^2 \quad \therefore$ o.k.

$f_{ti} = 1.6 > 1.32\ N/mm^2$ <u>NOT o.k.</u>

$f_{cs} = 14.12 < 15.3\ N/mm^2 \quad \therefore$ o.k.

$f_{ts} = 1.5 < 2.9\ N/mm^2 \quad \therefore$ o.k.

Ex 4: A posttensioned concrete simply supported beam of 10 m span having:

midspan cross section; $b = 150$ mm, $h = 500$ mm

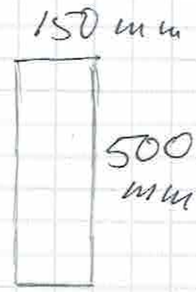
$f'_c = 27\ N/mm^2$, at transfer $f'_{ci} = 21\ N/mm^2$

$R = 0.8$ (Effectiveness ratio).

Find; 1- Values of (P_i) and (e) for maximum bending moment that can be applied due to service loading $(M_s = M_d + M_l)$.

2- Maximum UDL, the beam can carry.

Allowable stresses:



$f_{cs} = 0.45 f'_c = -12.15\ N/mm^2$

$f_{ts} = 0.5\sqrt{f'_c} = +2.59\ N/mm^2$

$f_{ci} = 0.6 f'_{ci} = -12.6\ N/mm^2$

$f_{ti} = 0.25\sqrt{f'_{ci}} = +1.14\ N/mm^2$

$A_c = 150 \times (500) = 75 \times 10^3\ mm^2$ (section area)

$$Z = \frac{\frac{1}{12} \times 150 \times (500)^3}{250} = 6.25 \times 10^6\ mm^3$$

$w_{self\,wt.} = 0.15 \times 0.5 \times 24 = 1.8\ kN/m$

$$M_g = \frac{1.8 \times 10^2}{8} = \underline{22.5}\ kN.m$$

(1) $$f_{cs} = -\frac{P_e}{A_c} + \frac{P_e . e}{Z} - \frac{M_g}{Z} - \frac{M_s}{Z} \quad \text{--- (1)}$$

$$f_{ts} = -\frac{P_e}{A_c} - \frac{P_e . e}{Z} + \frac{M_g}{Z} + \frac{M_s}{Z} \quad \text{--- (2)}$$

adding eqs 1 & 2

$$-12.15 + 2.59 = -\frac{2Pe}{A_c}$$

$$P_e = \frac{9.56 \times 75 \times 10^3}{2 \times 10^3} = 358.5 \text{ kN}$$

$$\therefore P_i = \frac{P_e}{0.8} = \frac{358.5}{0.8} = 448 \text{ kN} \qquad R = \frac{P_e}{P_i}$$

From $$f_{ti} = -\frac{P_i}{A_c} + \frac{P_i \cdot e}{Z} - \frac{M_g}{Z}$$

or $$1.14 = -\frac{448 \times 10^3}{75 \times 10^3} + \frac{448 \times 10^3 \times e}{6.25 \times 10^6} - \frac{22.5 \times 10^6}{6.25 \times 10^6}$$

$$e = 149.4 \text{ mm}$$

From $$f_{ci} = -\frac{P_i}{A_c} - \frac{P_i \cdot e}{Z} + \frac{M_g}{Z}$$

$$-12.6 = -\frac{448 \times 10^3}{75 \times 10^3} - \frac{448 \times 10^3 \times e}{6.25 \times 10^6} + \frac{22.5 \times 10^6}{6.25 \times 10^6}$$

or $e = \underline{\underline{142.6}}$ mm

Hence $e_{max} = \underline{\underline{142.6}}$ mm

Ans.

$$f_{cs} = -\frac{P_e}{A_c} + \frac{P_i \cdot e}{Z} - \frac{M_g}{Z} - \frac{M_s}{Z}$$

$$-12.15 = \frac{358 \times 10^3}{75 \times 10^3} + \frac{358 \times 10^3 \times 142.6}{6.25 \times 10^6} - \frac{22.5 \times 10^6}{6.25 \times 10^6} - \frac{M_s}{6.25 \times 10^6}$$

$$-12.15 = -4.78 + 8.18 - 3.6 - \frac{M_s}{6.25 \times 10^6}$$

$$\therefore M_s = \underline{\underline{74.75}} \text{ kN.m}$$

$$M_s = M_d + M_l = 74.75 \text{ kN.m}$$

$$w_s = \frac{M_s \times 8}{100} = \frac{74.75 \times 8}{100} = 5.97 \text{ kN/m}$$

EX5: Design a posttensioned simply supported concrete beam having the following design data:

$LL = 14.6$ kN/m , $D.L = 7.3$ kN/m

$f_c' = 40$ N/mm^2 , $f_{ci}' = 29$ N/mm^2

$f_{pu} = 1034$ (alloy steel) N/mm^2

$R = 0.85$, span $= 12$ m , e = variable eccentricity.

Permissible stresses according to ACI code are:

comp. at transfer, $f_{ci} = -0.6 \times 29 = -17.4$ N/mm^2

tension " " , $f_{ti} = 0.25\sqrt{29} = 1.34$ N/mm^2

comp. at service load, $f_{cs} = -0.45 \times 40 = -18$ N/mm^2

tension at service load, $f_{ts} = 0.5 \times \sqrt{40} = 3.16$ N/mm^2

Assume beam selfwt $= 6$ kN/m

$$M_g = \frac{6 \times 12^2}{8} = 108 \text{ kN.m}$$

$$M_s = M_l + M_d = \frac{(7.3 + 14.6)12^2}{8} = 394.2 \text{ kN.m}$$

At transfer

$$f_{ti} = -\frac{P_i}{A} + \frac{P_i \cdot e}{Z_1} - \frac{M_g}{Z_1} \quad \text{---(1) top}$$

$$f_{ci} = -\frac{P_i}{A} - \frac{P_i \cdot e}{Z_2} + \frac{M_g}{Z_2} \quad \text{---(2) bottom}$$

after all losses:

$$f_{ts} = -\frac{P_e}{A} - \frac{P_e \cdot e}{Z_2} + \frac{M_g}{Z_2} + \frac{M_s}{Z_2} \quad \text{---(3) bottom}$$

$$f_{cs} = -\frac{P_e}{A} + \frac{P_e \cdot e}{Z_1} - \frac{M_g}{Z_1} - \frac{M_s}{Z_1} \quad \text{---(4) top}$$

$$1.34 = -\frac{P_i}{A} + \frac{P_i \cdot e}{Z_1} - \frac{108 \times 10^6}{Z_1} \quad --(1)$$

$$-18 = -R\frac{P_i}{A} + \frac{RP_i}{Z_1} - \frac{108 \times 10^6}{Z_1} - \frac{394.2 \times 10^6}{Z_1} \quad -(4)$$

eq. (1) x R and subtract (4) from (1).

$$1.34\,R = -\frac{RP_i}{A} + \frac{RP_i e}{Z_1} - \frac{R \times 108 \times 10^6}{Z_1}$$

$$-18.0 = -\frac{RP_i}{A} + \frac{RP_i \cdot e}{Z_1} - \frac{108 \times 10^6}{Z_1} - \frac{394.2 \times 10^6}{Z_1}$$

$+ \qquad + \qquad - \qquad + \qquad +$

$$\therefore 19.14 = \frac{108 \times 10^6}{Z_1}(1 - 0.85) + \frac{394.2 \times 10^6}{Z_1}$$

$$\therefore Z_1 = 21.44 \times 10^6 \ mm^3$$

From (2) & (3)

$$R \times -17.4 = -\frac{RP_i}{A} - \frac{RP_i \cdot e}{Z_2} + \frac{108 \times 10^6}{Z_2} \times R \quad ---(2)$$

$$3.16 = -\frac{RP_i}{A} - \frac{RP_i \cdot e}{Z_2} + \frac{108 \times 10^6}{Z_2} + \frac{394.2 \times 10^6}{Z_2} \quad --(3)$$

$- \qquad + \qquad + \qquad - \qquad -$

$$-17.94 = -\frac{\overbrace{0.15}^{(1-0.85)} \times 108 \times 10^6}{Z_2} - \frac{394.2 \times 10^6}{Z_2}$$

or $Z_2 = 22.87 \times 10^6 \ mm^3$ $\quad \therefore$ take $Z = 22.87 \ mm^3$

Assume $b = h/2$, $Z = \frac{1}{6} bh^2$

$$\therefore h^3/12 = 22.87 \times 10^6 \Rightarrow$$

$h = 650 \ mm$

$b = 325 \ mm$

$$Z = \frac{bh^2}{6} = \frac{1}{6} \times 325 \times 650^2 = 22.88 \times 10^6 \ mm^3$$

to find P_i

$$+1.34 = -\frac{P_i}{A} + \frac{P_i . e}{Z} - \frac{M_g}{Z} \quad — (1)$$

$$-17.4 = -\frac{P_i}{A} - \frac{P_i . e}{Z} + \frac{M_g}{Z} \quad — (2)$$

$$1.34 - 17.4 = -\frac{2P_i}{A}$$

or $P_i = 1696$ kN

From (1);

$$1.34 = \frac{1696 \times 10^3}{325 \times 650} + \frac{1696 \times 10^3 \times e}{22.88 \times 10^6} - \frac{108 \times 10^6}{22.88 \times 10^6}$$

$e = 190$ mm

eq.(2) gives $e = 190$ mm (as a check.)

For alloy steel bars, $f_{pu} = 1034$ N/mm^2

permissible stress immediately after transfer;

$= 0.7 \times f_{pu} = 0.7 \times 1034 = 723.8$ N/mm^2

$$A_s = \frac{P_i}{f_s} = \frac{1696 \times 10^3}{723.8} = 2344 \text{ mm}^2$$

provide 5 – ϕ 25 mm, $A_{st} = 2455$ mm^2

check for selfwt = $w_g = 0.325 \times 0.65 \times 24 = 5.07$ < 6.0 o.k

$$M_g = \frac{5.07 \times 12^2}{8} = 91.26 \text{ kN.m}$$

check for stresses:

$$f_{ti} = \frac{1696 \times 10^3}{325 \times 650} + \frac{1696 \times 10^3 \times 190}{22.88 \times 10^6} - \frac{91.26 \times 10^6}{22.88 \times 10^6}$$

$= -8.03 + 14 - 3.98 = 2.0 > 1.34$ Not o.k.

take $e = 180$ mm

$$f_{ti} = -8.03 + \frac{1696 \times 10^3 \times 180}{22.88 \times 10^6} - 3.98$$

$$= -8.03 + 13.34 - 3.98 = 1.33 < 1.34 \ N/mm^2$$

$\therefore$ O.K

$$f_{ci} = -8.03 - 13.34 + 3.98$$

$$= (-)17.38 < (-)17.4 \ N/mm^2 \quad \therefore \text{o.k.}$$

$$f_{cs} = -0.85 \times 8.03 + 0.85 \times 13.34 - \frac{(91.26 + 394.2) \times 10^6}{22.88 \times 10^6}$$

$$= -6.82 + 11.34 - 21.2$$

$$= (-)16.68 < (-)18 \ N/mm^2 \quad \therefore \text{o.k.}$$

$$f_{ts} = -6.82 - 11.34 + 21.2$$

$$= 3.04 < 3.16 \ N/mm^2 \quad \therefore \text{o.k.}$$

EX6: Design the beam in Ex5 using cables with constant eccentricity.

NOTE: The allowable concrete tensile stress in the ends of simply supported members is twice of what it is elsewhere in the beam.

- -

1- The critical section is at the supports at transfer:

$$f_{ti} = -\frac{P_i}{A} + \frac{P_i \cdot e}{Z_1} \quad \text{—(1)}$$

$$f_{ci} = -\frac{P_i}{A} - \frac{P_i \cdot e}{Z_2} \quad \text{—(2)}$$

2- The critical section is at mid-span at service load:

$$f_{ts} = -\frac{P_e}{A} - \frac{P_e \cdot e}{Z_2} + \frac{M_g}{Z_2} + \frac{M_s}{Z_2} \quad \text{—(3)}$$

$$f_{cs} = -\frac{P_e}{A} + \frac{P_e \cdot e}{Z_1} - \frac{M_g}{Z_1} - \frac{M_s}{Z_1} \quad \text{—(4)}$$

From eq (1) & (4):

$$R\,f_{ti} = -R\frac{P_i}{A} + R\frac{P_i \cdot e}{Z_1} \qquad \text{eq (1)} \times R$$

$$- \; f_{cs} = \overset{+}{-} \frac{RP_i}{A} \overset{-}{+} \frac{RP_i \cdot e}{Z_1} \overset{+}{-} \frac{M_g}{Z_1} \overset{+}{-} \frac{M_s}{Z_1} \quad \text{—(4)}$$

Substract

code

$$0.85(2 \times 1.34) + 18 = \frac{(108 + 394.2)}{Z_1} \times 10^6$$

$$Z_1 = 24.77 \times 10^6 \ mm^3$$

From 2 & 3

$$R f_{ci} = -R\frac{P_i}{A} - \frac{R P_i \cdot e}{Z_2} \qquad eq.\,2 \times R$$

$$- \quad f_{ts} = -R\frac{P_i}{A} - R\frac{P_i \cdot e}{Z_2} + \frac{M_g + M_s}{Z_2} \quad \text{— eq(3)}$$

Subtract

$$-0.85 \times 17.4 - 3.16 = -\frac{(108 + 394.2)}{Z_2} \times 10^6$$

$$Z_2 = 27.99 \times 10^6$$

$$\frac{1}{6} b h^2 = \frac{h^3}{12} = 27.99 \times 10^6 \Rightarrow h = 695 \text{ mm}$$

take $h = 700$ mm , $b = 350$ mm

$$A_c = 350 \times 700 = 245 \times 10^3 \text{ mm}^2$$

$$Z = \frac{1}{6} b h^2 = 28.58 \times 10^6 \text{ mm}^3$$

to find P_i ; $\quad f_{tc} = -\frac{P_i}{A_c} + \frac{P_i \cdot e}{Z}$ — (1)

$$f_{ci} = -\frac{P_i}{A_c} - \frac{P_i \cdot e}{Z} \quad \text{— (2)}$$

add

$$2 \times 1.34 - 17.4 = -2\frac{P_i}{A_c}$$

$$P_i = \frac{(2.68 - 17.4)\, 245 \times 10^3}{2 \times 10^3} = 1808 \text{ kN}$$

Subtracting (2) from (1)

$$2.68 + 17.4 = \frac{2 \times 1808 \times e \times 10^3}{28.58 \times 10^6} \Rightarrow e = \underline{158} \text{ mm}$$

$$A_s = \frac{1808 \times 10^3}{0.7 \times 1034} = 2498 \text{ mm}^2 \Rightarrow \underline{5 - 25\,\phi} \text{ alloy steel bars}$$

check stress at service loads:

$$f_{cs} = -\frac{P_e}{A} + \frac{P_e \cdot e}{Z} - \frac{(M_g + M_s)}{Z} = -6.27 + 8.5 - 17.5$$

$$= -15.26 < 18 \text{ N/mm}^2$$

$$f_{ts} = -6.27 - 8.5 + 17.5$$

$$= 2.73 < 3.16 \text{ N/mm}^2 \quad \therefore \underline{O.K}$$

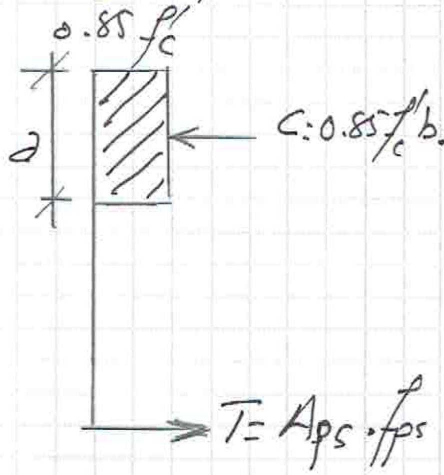
Flexural Strength :

ACI, Code : Rectangular Sections

1-) If $\rho_p \frac{f_{ps}}{f'_c} < 0.3$, the strength of prestressed beams can be found from the following expression :

$$M_u = \phi\, T\,(d - a/2)$$

$$M_u = \phi\, A_{ps} \cdot f_{ps}\,(d - a/2) \quad \text{— (1)}$$



where $a = \dfrac{A_{ps} \cdot f_{ps}}{0.85 f'_c\, b}$

eq.(1) becomes :

$$M_u = \phi\, A_{ps} \cdot f_{ps}\, d \left(1 - \frac{0.59\, \rho_p \cdot f_{ps}}{f'_c}\right)$$

The values of steel stress f_{ps} can be calculated from :

For bonded members : $f_{ps} = f_{pu}\left(1 - \dfrac{0.5\, \rho_p\, f_{pu}}{f'_c}\right)$

For unbonded members = $f_{ps} = f_{se} + 70 + \dfrac{f'_c}{100\, \rho_p}$

but not more than f_{py} or $f_{se} + 400$ MPa

where : f_{se} = effective steel prestress after losses (not to be less than $0.5 f_{pu}$)

2-) If $\rho_p \frac{f_{ps}}{f'_c} > 0.3$, The ultimate moment can be taken as : $M_u = \phi\,(0.25 f'_c\, b d^2)$

Flexural Strength of flanged sections:

For flanged sections such as I, T, the flexural strength of the beams, ~~the~~ can be calculated using the same method that is used in ordinary beams.

The total steel area is divided into two parts for computational purposes. The first part balances the compression in the overhanging portion of the flange, the second part balances the compression in the web.

ρ_p ; is taken as the steel ratio of tension steel area which is required to develope the compression in the web.

1- If $\rho_p \frac{f_{ps}}{f'_c} < 0.3$ then underreinforced.

$$M_u = \phi\left[A_{pw} f_{ps} d\left(1 - \frac{0.59 A_{pw} \cdot f_{ps}}{b_w \cdot d \cdot f'_c}\right) + 0.85 f'_c (b - b_w) h_f (d - h_f/2)\right]$$

where $A_{pw} = A_{ps} - A_{pf}$

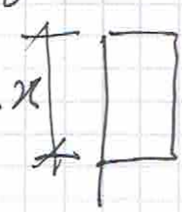
$$A_{pf} = 0.85 \frac{f'_c}{f_{ps}} (b - b_w) h_f$$

2- If $\rho_p \frac{f_{ps}}{f'_c} > 0.3$ then overreinforced

$$M_u = \phi\left[0.25 f'_c b_w \cdot d^2 + 0.85 f'_c (b - b_w) h_f \left(d - \frac{h_f}{2}\right)\right]$$

code: $\rho_p \frac{f_{ps}}{f'_c} \leq 0.36 \beta_1$ instead of 0.3

e.g : $0.36 \beta_1 = 0.288$ for $f'_c = 34$ MPa $\quad a = \beta_1 . x$



$x = 0.423 d$

overreinforced $x > 0.423 d$

EX: 7 A post tensioned bonded prestressed concrete beam with a section shown in the figure. What is the design strength of the beam?

$f'_c = 34$ mpa , $f_{pu} = 1725$ MPa

Tensile steel ratio, $\rho_p = \dfrac{A_{ps}}{bd} = 4.6 \times 10^{-3}$

$\rho_p = \dfrac{1484}{508 \times 632} = 4.6 \times 10^{-3}$

Steel stress at the design load:-

$f_{ps} = f_{pu}\left(1 - \dfrac{0.5 \rho_p f_{pu}}{f'_c}\right)$

$= 1725\left(1 - \dfrac{0.5 \times 4.6 \times 1725}{34 \times 10^3}\right) = 1524\ N/mm^2$

Check ratio $\rho_p \dfrac{f_{ps}}{f'_c} = \dfrac{4.6 \times 1524}{34 \times 10^3} = 0.206 < 0.3$

$\therefore$ underreinforced section

$M_u = \phi A_{ps} f_{ps} \cdot d\left(1 - \dfrac{0.59 \rho_p f_{ps}}{f'_c}\right)$

$= 0.9 \times 1484 \times 1524 \times 632 \times (1 - 0.59 \times 0.206)$

$M_u = \underline{\underline{1130}}$ kN.m

EX 8! A posttensioned, prestressed beam with a cross section shown below, having the following data:

$f'_c = 35$ Mpa , $f_{pu} = 1350$ Mpa

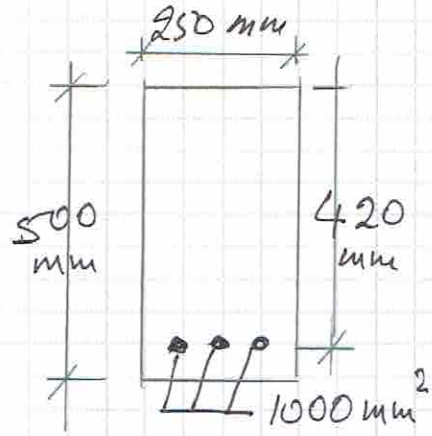
$M_d = 80$ kN.m , $M_L = 90$ kN.m , $f_{se} = 930\ N/mm^2$

total loss = 170 Mpa.

Find the ultimate flexural strength for the following cases:

1- Well grouted

2- Grouting is Omitted



① $\rho_p = \dfrac{1000}{250 \times 420} = 0.0095$

$f_{ps} = f_{pu}\left(1 - 0.5\ \dfrac{\rho_p f_{pu}}{f'_c}\right)$

$= 1350\left(1 - 0.5\ \dfrac{0.0095 \times 1350}{35}\right) = 1102$ MPa

check ratio, $\rho_p \dfrac{f_{ps}}{f'_c} = 0.0095 \times \dfrac{1102}{35} = 0.299 < 0.3$

$\therefore$ underreinforced

$a = \dfrac{1000 \times 1102}{0.85 \times 35 \times 250} = 149$ mm

$M_u = \phi A_{ps} f_{ps}\,(d - a/2) = 0.9 \times 1000 \times 1102\left(420 - \dfrac{149}{2}\right)$

$= \underline{342}$ kN.m.

Required moment = $1.4 \times 80 + 1.7 \times 90 = 265 < 342$ kN.m

reserve = 23%

② Grouting is Omitted

$\rho = 0.0095$

$f_{ps} = f_{se} + 70 + \dfrac{f'_c}{100\rho_p}$

$$f_{se} = (930 - 170) + 70 + \frac{35}{100 \times 0.0095}$$

$f_{ps} = 867$ MPa $< 760 + 400$ $\therefore$ o.k.

$$\rho_p \frac{f_{ps}}{f_c'} = 0.0095 \times \frac{867}{35} = \cancel{0.2035}\ 0.235 < 0.3$$

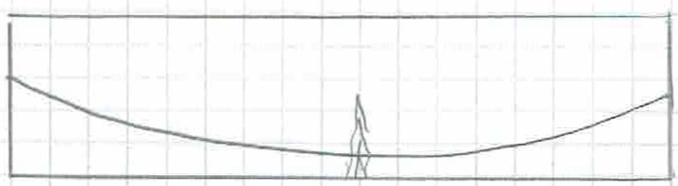
$\therefore$ underreinforced

$$a = \frac{1000 \times 867}{0.85 \times 35 \times 280} = 116 \text{ mm}$$

$$M_u = 0.9 \times 1000 \times 867 \left(420 - \frac{116}{2}\right)$$
$$= 282 \text{ kN.m} > 265 \text{ kN.m}$$

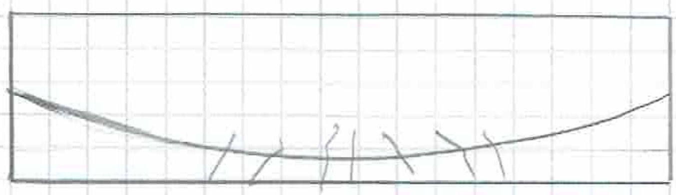
reserve = 6%

ungrounted



one or few wide cracks

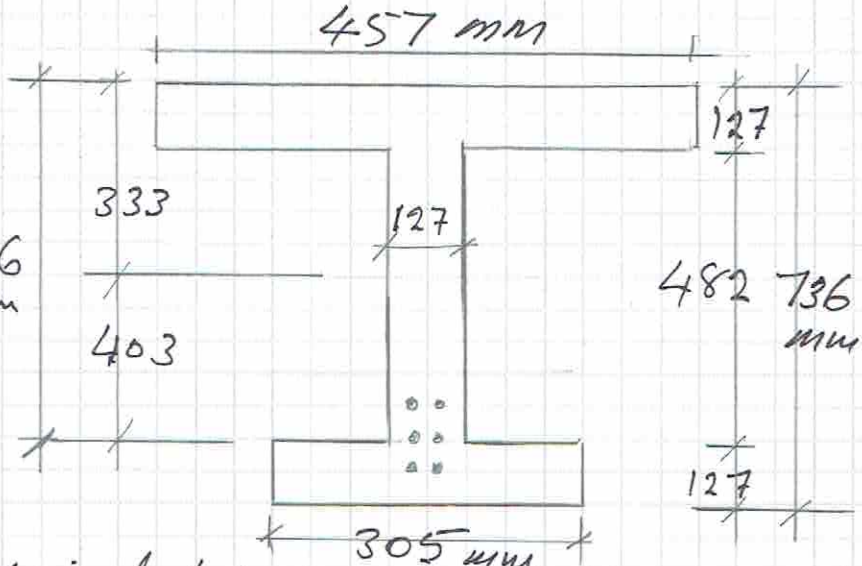
Grouted



spread of cracks

Ex9: A posttensioned, bonded, prestressed concrete beam having the cross section shown in the figure. What is the ultimate bending moment that the beam can carry. $f'_c = 34$ MPa, $f_{pu} = 1897$ MPa, $A_{ps} = 1129\ mm^2$

$A_c = 157.6 \times 10^3\ mm^2$



— — -

tensile steel ratio

$$\rho_p = \frac{A_{ps}}{bd}$$

$$= \frac{1129}{457 \times 623} = 0.00397$$

The steel stress at the design load is:

$$f_{ps} = f_{pu}\left(1 - 0.5\frac{\rho_p f_{pu}}{f'_c}\right)$$

$$\therefore f_{ps} = 1897\left(1 - 0.5 \times \frac{0.00397 \times 1897}{34}\right)$$

$$= 1687\ N/mm^2 \qquad f_{ps} \text{ approximitly } \simeq 0.9 f_{py}$$

check location of the neutral axis to decide whether a flanged section or not.

(winter)

$$1.4 d \rho_p \frac{f_{ps}}{f'_c} = 1.4 \times 623 \times 0.00397 \times \frac{1687}{34}$$

$$= 171\ mm > 127\ mm$$

$\therefore$ N.A. is below the underside of the flange, then flanged section.

$$A_{pf} = 0.85\frac{f'_c}{f_{ps}}(b - b_w)h_f$$

$$= \frac{0.85 \times 34}{1687}(457 - 127)(127) = 718.0\ mm^2$$

$A_{pw} = A_{ps} - A_{pf}$

$= 1129 - 718 = 411 \ mm^2$

ratio, $\rho_p \dfrac{f_{ps}}{f'_c} = \dfrac{411}{127 \times 623} \times \dfrac{1687}{34} = 0.257 < 0.3$

steel ratio to balance comp. in the web.

$\therefore$ underreinforced section

$$\therefore M_u = \phi\left[A_{pw} \cdot f_{ps} \cdot d\left(1 - \frac{0.59 A_{pw} \cdot f_{ps}}{b_w \cdot d \cdot f'_c}\right) + 0.85 f'_c (b - b_w) h_f \left(d - \frac{h_f}{2}\right)\right]$$

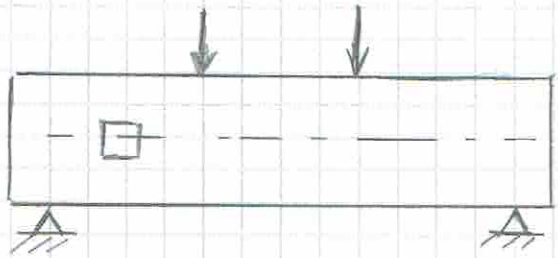
$$= 0.9\left[411 \times 1687 \times 623\left(1 - \frac{0.59 \times 411 \times 1687}{127 \times 623 \times 34}\right) + 0.85 \times 34 \times 330 \times 127\left(623 - \frac{127}{2}\right)\right]$$

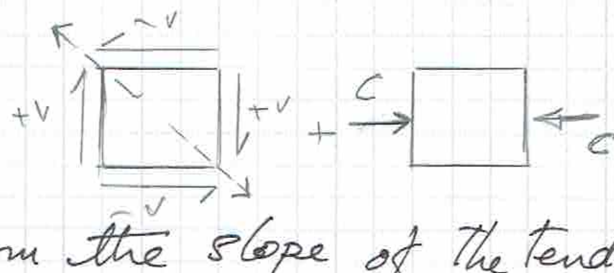
$$= 0.9\left[366 \times 10^6 + 677.6 \times 10^6\right]$$

$$= 939 \ kN.m.$$

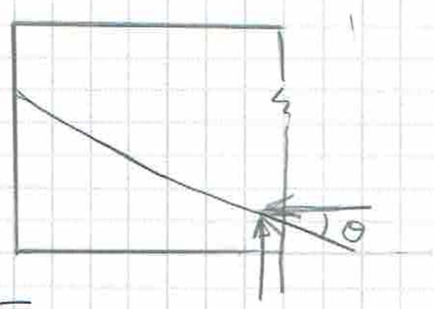
Shear in Prestressed Concrete Beams :

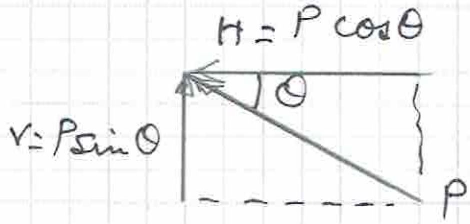
The density of diagonal tensile stress is reduced by two factors;
The first one results from the combination of longitudinal compression stress and shear ing stress.





The scond factor results from the slope of the tendons.





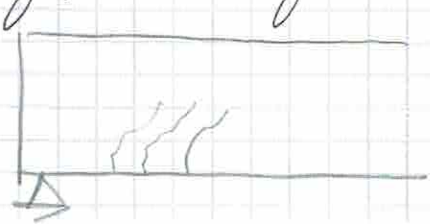
The magnitude of counter shear is $V = P_e \sin\theta$

NOTE : The two factors are not considered as as safety measures or additional factors of safety.

Types of shear cracks:

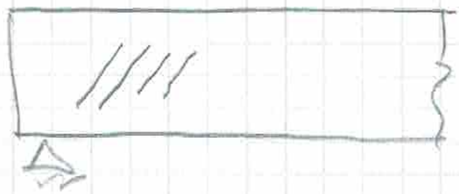
Two types of diagonal cracks that can bo observed in prestressed concrete beams.

1- Flexural - Shear cracks:
starts nearly vertical at the tension face of the beam and epread diagonally upwards. Often found in beams with low value of prestress force.
occuring at V_{ci}.



2- Web-shear cracks:

Start in the web due to high diagonal tension, then spread diagonally both upwards and downwards. Often found in beams with thin webs with high prestress force. Occuring at v_{cw}.



ACT code:

small amount of shear to cause shear crack

$$v_{ci} = 0.05\sqrt{f'_c} + \frac{V_d + V_l \cdot M_{cr}/M_l}{b_w \cdot d} \qquad V_{cr} = \frac{V_l}{M_l} \cdot M_{cr}$$

$$\text{but } v_{ci} \nless 0.14\sqrt{f'_c}$$

V_d: factored total dead load shear at the section (beam dead load + superimposed dead load).

M_l, V_l = are computed from live load combination creating maximum moment.

M_{cr} = moment externally applied causing flexural cracking at the section.

modulus of rupture

$$M_{cr} = \left(\frac{I}{y_t}\right)(0.5\sqrt{f'_c} + f_{pe} - f_d)$$

I = second moment of area

y_t = distance from centroid of gross section (neglecting steel) to tension face of the member.

f_{pe} = compressive stress in concrete due to effective prestress (after losses) at extreme fibre where external loads cause tension.

f_d = unfactored dead load stress at the same extreme fibre as f_{pe}. (beam D.L + superimposed D.L) in case of UDL type of loading then;

$$v_{ci} = 0.05\sqrt{f'_c} + \frac{V_u . M_{cr}}{M_u . b_w . d}$$

$$M_{cr} = \frac{I}{C_2}\left(0.5\sqrt{f'_c} + f_{pe}\right)$$

Code : allows

$$v_{cw} = 0.3\left(\sqrt{f'_c} + f_{pe}\right) + \frac{V_p}{b_w . d}$$

V_p : is the component of prestress acting perpendicular to the axis of the member (vertical component of the effective prestress force).

f_{pc} = compressive stress in the concrete after losses at the centroid of the concrete section (or the juction of the web and the flange when the centroid lies in the flange).

ACI code ; equation for calculating the concrete shear resistanse (v_c) directly.

$$v_c = 0.05\sqrt{f'_c} + 5\,\frac{V_u}{M_u}.d$$

M_u : is the bending moment occuring simultaneously with shear force (V_u).

$$\frac{V_u}{M_u}.d \ngtr 1.0$$

$$0.17\sqrt{f'_c} < v_c < 0.42\sqrt{f'_c}$$

For uniformly loaded simply supported beams

$$\frac{V_u}{M_u} = \frac{l - 2x}{x(l-x)}$$

x : is the distance from the support centre line along a beam of span (l).

Shear Reinforcement for Prestressed Concrete Beams:

The same method that is used in ordinary beams can be followed to provide shear reinforcement in prestressed beams. Note that factored loads are to be used in calculating shear stresses.

Shear stress at factored $v_u = \dfrac{V_u}{\phi b_w \cdot d}$

Since (d) might be variable (variable eccentricity) then $d \not< 0.8h$

Spacing of vertical web reinforcement;

$$S = \frac{A_v \cdot f_y}{\underbrace{(v_u - v_c)}_{v_s} b_w}$$

v_c = shear stress carried by concrete at factored load.

Code: If v_s i.e. $(v_u - v_c) > 0.33\sqrt{f'_c}$ then max. spacing should be reduced to half.

v_s; in any case should not be taken greater than $0.66\sqrt{f'_c}$

Max. Spacing:

3/4 h and not $>$ 600 mm

Code also specifies that sections located at a distance less than $h/2$ from the face of the support may be designed from the shear computed at $h/2$.

Min. amount of steel:

ACI Code: A minimum amount of shear reinforcement must be provided whenever

$$\phi V_c > V_u > \phi \frac{V_c}{2}$$

given by: $A_v = 0.34 \frac{b_w . S}{f_y}$

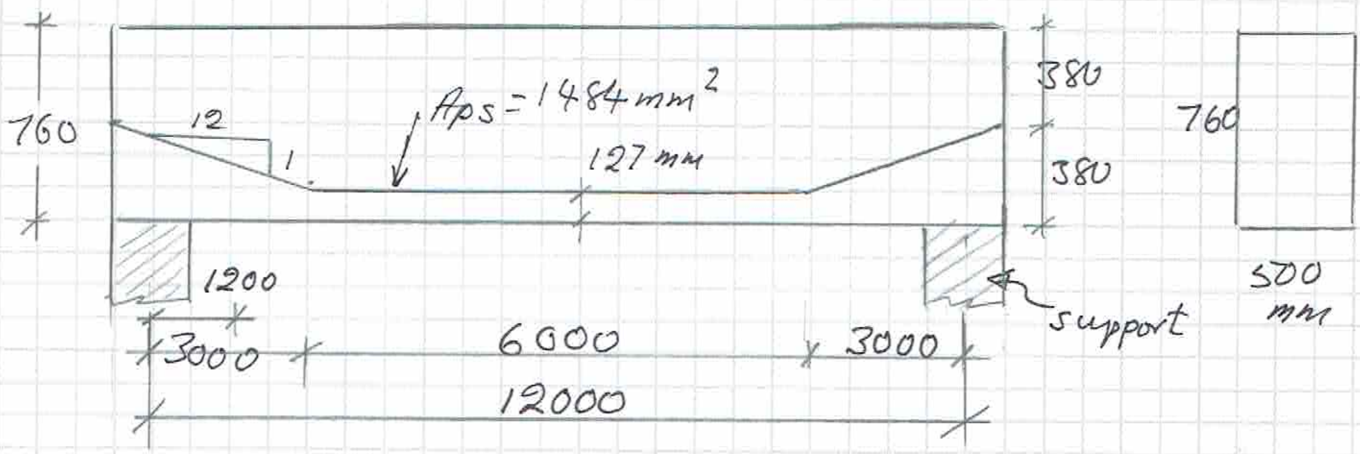
A_v = area of two legs.

In prestress members, where the effective prestress force greater than ($>$ 40%) of the tensile strength of the steel, The min. area may be taken as:

$$A_v = \frac{A_{ps}}{80} \left(\frac{f_{pu}}{f_y}\right) \frac{s}{d} \sqrt{\frac{d}{b_w}}$$

or $A_v = 0.34 \frac{b_w . s}{f_y}$

Ex: 10



A simply supported prestressed concrete beam shown in the fig. above; find the value of $v_n = v_c$ at section 1200 mm from the support centre. Also find the maximum stirrups spacing at the same section.

$P_e = 1449$ kN

$LL = 20$ kN/m , beam selfwt. $= 9$ kN/m

$f'_c = 34$ N/mm² , $f_{pu} = 1725$ N/mm², $f_y = 275$ N/mm²

U_ stirrups of 10 mm dia. are to be used.

(1)

(a) $w_u = 9 \times 1.4 + 20 \times 1.7 = 46.6$ kN/m

$V_u = 46.6\,(6 - 1.2) = 224$ kN

$M_u = 46.6 \times 1.2\,(6 - 0.2) = 302$ kN.m

$d = 380 + \underset{\text{from slope}}{100} = 480$ mm

Code : $d = 0.8 \times h = 0.8 \times 760 = 608$ mm

$\therefore$ use $d = \underline{608}$ mm

$$\frac{V_u \cdot d}{M_u} = \frac{224 \times 480}{302 \times 1000} = 0.36 < 1.0 \quad \therefore \text{o.k.}$$

ACI formula for nominal stress:

$$v_c = 0.05\sqrt{f_c'} + 5\ \frac{V_u \cdot d}{M_u}$$

$$= 0.05\sqrt{34} + 5 \times 0.36 = 2.09\ N/mm^2$$

lower limit $= 0.17\sqrt{f_c'} = 0.99\ N/mm^2$

upper limit $= 0.42\sqrt{f_c'} = 2.45$

$\therefore$ take $v_c = \underline{\underline{2.09}}\ N/mm^2$

(b) Flexural shear cracking strength using ACI code exact formula:

$$v_{ci} = 0.05\sqrt{f_c'} + \frac{\frac{V_l}{M_l} M_{cr} + V_d}{b_w \cdot d}$$

$$M_{cr} = \left(\frac{I}{y_t}\right)\left(0.5\sqrt{f_c'} + f_{pe} - f_d\right)$$

$$I = \frac{1}{12} \times 500 \times 760^3 = 18.3 \times 10^9\ mm^4$$

$y_t = 380\ mm$, $e = 100\ mm$ from slope

f_{pe} = compressive stress due to effective prestress =

$$= -\frac{1449}{500 \times 760} - \frac{1449 \times 100 \times 380 \times 10^3}{18.3 \times 10^9}$$

$$= -3.8 - 3.0 = 6.8\ N/mm^2$$

f_d = unfactored dead stress

$$f_d = \frac{My}{I} = \frac{58.33 \times 380 \times 10^6}{18.3 \times 10^9} = 1.2\ N/mm^2$$

$$\therefore M_{cr} = \frac{18.3 \times 10^9}{380 \times 10^6}\left(0.5\sqrt{34} + 6.8 - 1.2\right)$$

$$= 410\ kN.m$$

Factored live load moment and shear:

$$M_\ell = 20 \times 1.7 \times 1.2\,(6 - 0.6) = 220 \text{ kN.m}$$

$$V_\ell = 20 \times 1.7\,(6 - 1.2) = 163 \text{ kN}$$

$$\frac{V_\ell}{M_\ell} \cdot M_{cr} = \frac{410 \times 163}{220} = 303.8 \text{ kN}$$

or check with $\dfrac{V_\ell}{M_\ell} = \dfrac{l - 2x}{x(l - x)} = \dfrac{12 - 2 \times 1.2}{1.2(6 - 1.2)}$

$= 0.74$

$$\frac{V_\ell}{M_\ell} = \frac{163}{220} = 0.74$$

V_d = dead load shear

$= 9 \times 1.4\,(6 - 1.2) = 60.48$ kN

$d = 608$ mm   code $= 0.8h$

$$\therefore v_{ci} = 0.05\sqrt{34} + \frac{(303.8 + 60.48) \times 10^3}{500 \times 608}$$

$= 1.43 \text{ N/mm}^2$

$\min v_{ci} = 0.14\sqrt{f_c'} = 0.8 < 1.43 \text{ N/mm}^2$ $\therefore$ o.k.

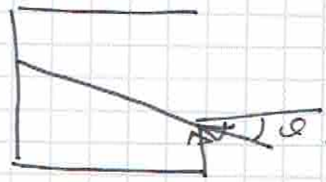
(c) Web shear cracking strength using ACI formula (exact):

$$v_{cw} = 0.3\left(\sqrt{f_c'} + f_{pc}\right) + \frac{V_p}{b_w . d}$$

f_{pc} = Compressive ~~cracking~~ ~~strength~~ ~~using~~ stress in concrete at centroid of section resisting live load only ($e = 0$)

$$f_c = \frac{1449 \times 10^3}{500 \times 760} = 3.8 \text{ N/mm}^2$$

$d = 608$ mm



for small value of θ
$\sin\theta = \tan\theta$

$\therefore V_p = \frac{1}{12} \times 1449 = 121.75$ kN

$\therefore v_{cw} = 0.3(5.8 + 3.8) + \frac{121.75 \times 10^3}{500 \times 608}$

$= \underline{\underline{3.28}}$ N/mm^2

$v_{ci} < v_{cw}$ $\quad\therefore$ v_{ci} controls $= 1.43$ N/mm^2

[or take v_c directly $= 2.09$ N/mm^2]

(2) Shear stress at section considered (at factored load) is v_u.

$$v_u = \frac{224 \times 10^3}{500 \times 608} = 0.736 \text{ N/mm}^2$$

$$\phi\, v_c/2 = \frac{0.85 \times 1.43}{2} = 0.607$$

$v_u > \phi \frac{v_c}{2}$ $\quad\therefore$ nominal web reinforcement is required

$A_v = 0.34 \frac{b_w . s}{f_y}$ $\Rightarrow$ $f_y < 410$ code

two legs

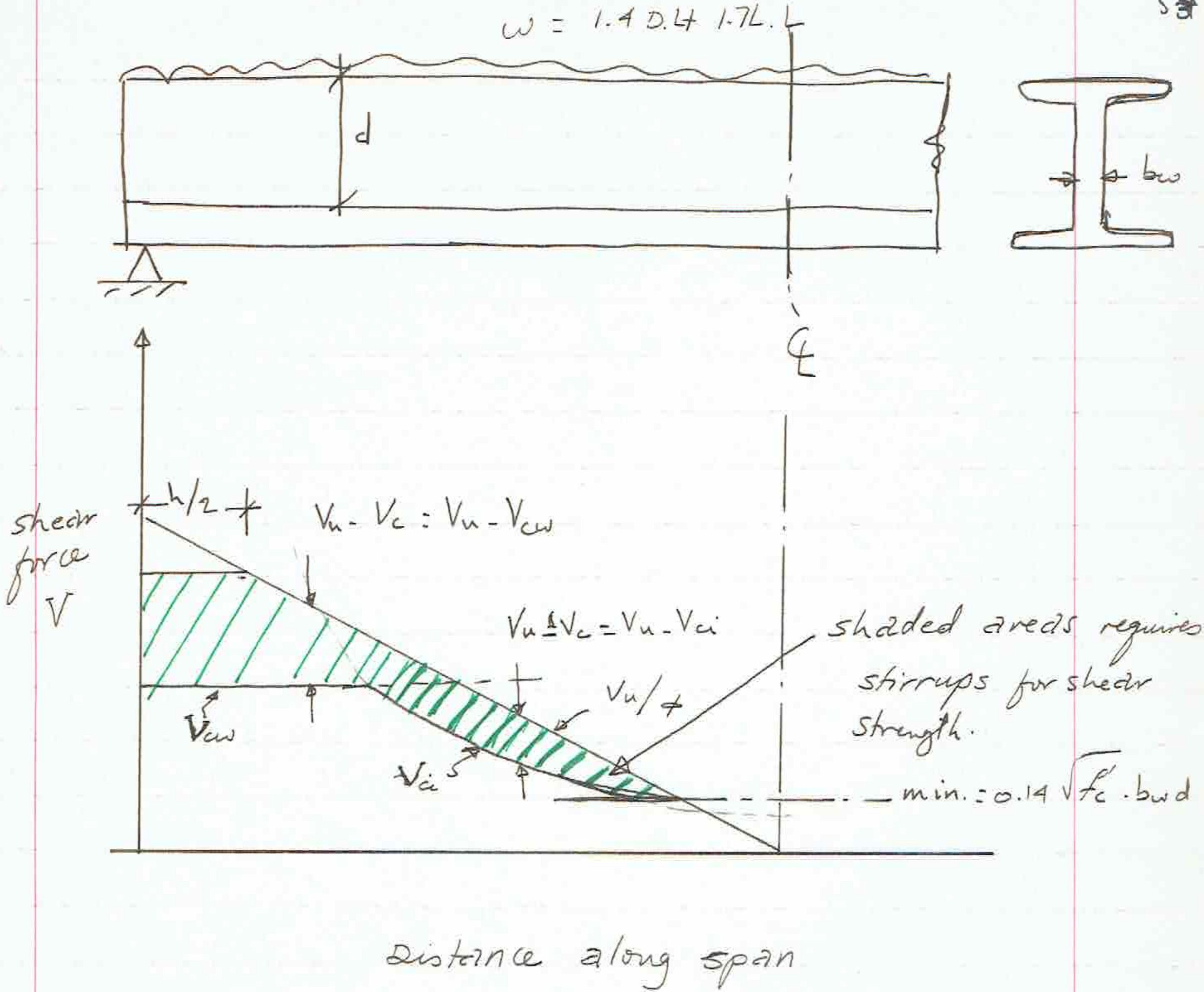
$2 \times 78.5 = 0.34 \frac{500 \times s}{275}$

$\therefore s = 254$ mm

max. spacing $= 3/4\, h = 0.75 \times 760 = 570$ mm

or 600 mm

$\therefore$ take $s = 250$ mm

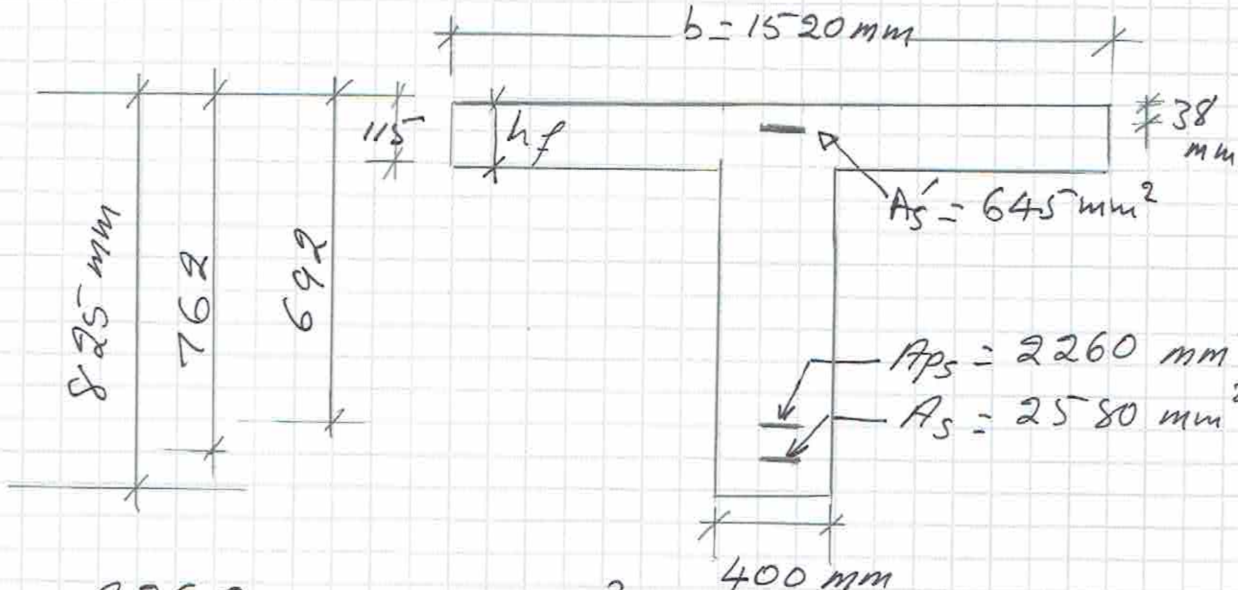


ACI analysis for shear strength - distribution of shears along span.

E11 :

Find the ultimate flexural capacity of the bonded post tensioned beam shown below.

$f'_c = 27$ MPa , $f_{pu} = 1863$ MPa , $f_y = 410$ MPa



$$\rho_p = \frac{2260}{1520 \times 692} = 2.4 \times 10^{-3}$$

$$f_{ps} = 1863\left(1 - \frac{0.5 \times 2.4 \times 10^{-3} \times 1863}{27}\right)$$

$$= 1725 \text{ MPa}$$

Average depth of tensile reinforcement:

$$= \frac{2260 \times 1725 \times 692 + 2580 \times 410 \times 762}{2260 \times 1725 + 2580 \times 410} = \frac{3503.8 \times 10^6}{4.96 \times 10^6}$$

$d = 706$ mm

check ratio $\rho_p \dfrac{f_{ps}}{f'_c} = \dfrac{2260 \times 1725}{1520 \times 706 \times 27} = 0.13$

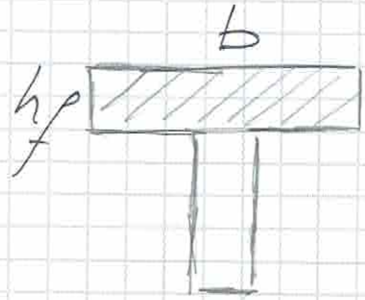
$$\rho \frac{f_y}{f_c} = \frac{2580 \times 410}{1520 \times 706 \times 27} = 0.036$$

$$\rho' \frac{f_y}{f'_c} = \frac{645 \times 410}{1520 \times 706 \times 27} = 0.009$$

$$\rho_p \frac{f_{ps}}{27} + \rho \frac{f_y}{f'_c} - \rho' \frac{f_y}{f'_c} = 0.13 + 0.036 - 0.009 = 0.157 < 0.3$$

∴ underreinforced section

compressive force in the flange:



$= 0.85 f'_c b h_f + A'_s f_y$

$= 0.85 \times 27 \times 1520 \times 115 + 645 \times 410$

$= \underline{4276}$ kN

Total Tensile force $= A_{ps} \cdot f_{ps} + A_s f_y$

$= 2260 \times 1725 + 2580 \times 410$

$= 4960$ kN > 4276 kN

$\therefore$ flanged section

compressive force in the overhanging portion of the flange and compression steel:

$= 0.85 \times 27 (1520 - 400) 115 + 645 \times 410$

$= \underline{3220}$ kN

Force to be developed by the web:

$= (A_{ps} f_{ps} + A_s f_y) - 3220$

$= 4960 - 3220 = 1740$ kN

check steel ratio of the web (in this case $b = 400$ mm)

$$\rho_p \frac{f_{ps}}{f'_c} + \rho \frac{f_y}{f'_c} - \rho' \frac{f_y}{f'_c} = \frac{1740 \times 10^3}{400 \times 706 \times 27}$$

$= 0.23 < 0.3$

$\therefore$ web is underreinforced

$$M_u = \phi\left[A_{pw} \cdot f_{ps} \cdot d\left(1 - \frac{0.59\, A_{pw} \cdot f_{ps}}{b_w \cdot d\; f'_c}\right) + 0.85 f'_c (b - b_w) h_f \left(d - h_f/2\right) + A'_s f_y (d - d')\right]$$

$$= 0.9\left[1740 \times 10^3 \times 706 \left(1 - \frac{0.59 \times 1740 \times 10^3}{400 \times 706 \times 27}\right) + 0.85 \times 27\,(1520 - 400) \times 115 \times \left(706 - \frac{115}{2}\right) + 645 \times 410\,(706 - 38)\right]$$

$$= 0.9\left[1063 + 1916.9 + 176.65\right]$$

$$= \underline{\underline{2841}} \quad kN.m$$

Advantages of prestressed concrete:

1- Crack free under service loads

* a structure exposed to weather, elimination of cracks prevents corrosion.

* a crack free prestressed member has greater stiffness under loads because it's entire section is effective.

2- Prestressed concrete permits accommodation of both shrinkage and creep reasonably well.

3. Precompression of the concrete reduces the tendency for inclined cracking.

4 The use of curved tendons provides a vertical component to aid in carrying the shear. Shear strength is more consistent than ordinary reinforced concrete.

5- High ability to absorb energy (impact resistance)

6- High fatigue resistance due to the low steel stress variation resulting from the high initial pretension.

7- High live load capacity.

8- Use of prestressed concrete also permits partial testing of both steel and concrete through application of prestress.

Some of disadvantages:

1. The stronger materials used have higher unit cost.

2. More complicated formwork may be required.

3. End anchorage and bearing plates are usually required.

4. Labor cost are greater.

5. More conditions must be checked in design and closer control of every phase of construction is required.

Short span members and single units application of any kind are likely to be uneconomical in prestress concrete. However economy is usually achieved when units can be standardized and the same units repeated many times. For many situations, the desirability of achieving a certain advantage is sufficient to justify a higher initial cost.

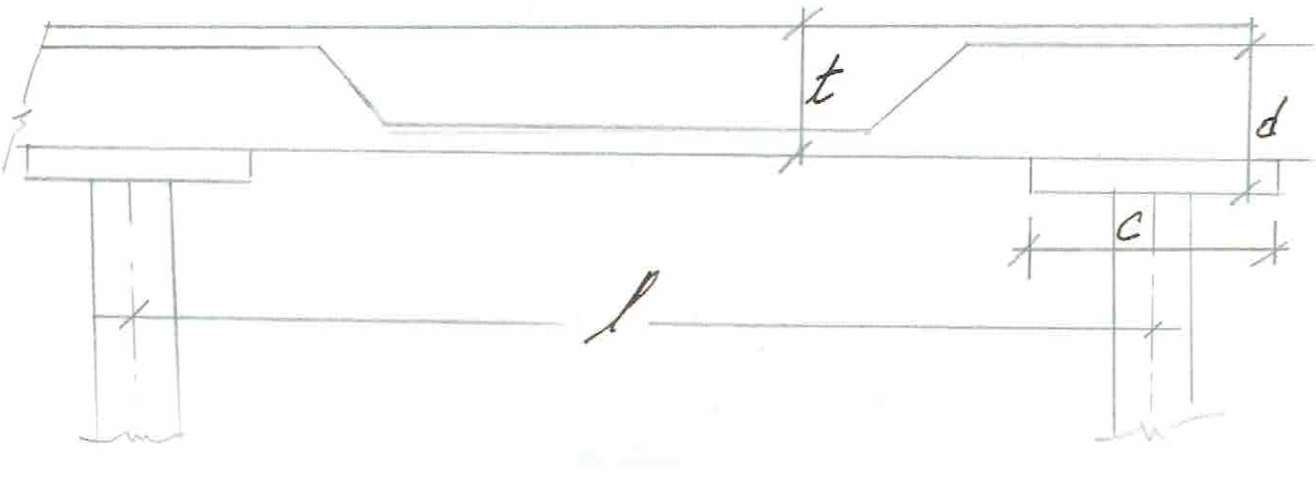
Prestressing need not create a crack free structure at service load. The level of prestressing can be used that will acomplish the desired crack control of stiffness objective. So called partial prestressing has become common in construction.

FLAT SLAB

A concrete flat slab is a particular type of two way slab, reinforced in two or more direction generally without beams or girders to transfer the loads to supporting members....

Advantages:

1. Lesser construction depth required for floors system and subsequent reduction in story height.
2. Reduction of dead load and foundation loads due to decrease in overall weight and height of structure.
3. Simpler formwork required for construction.
4. Improved fire resistance as compared to other types.
5. Easier to illuminate due to absence of beams.
6. Easier to install sprinkler systems and other piping and utilities.
7. Capability of resisting concentrated loads.
8. Easier to paint or apply acoustical treatment to underside.

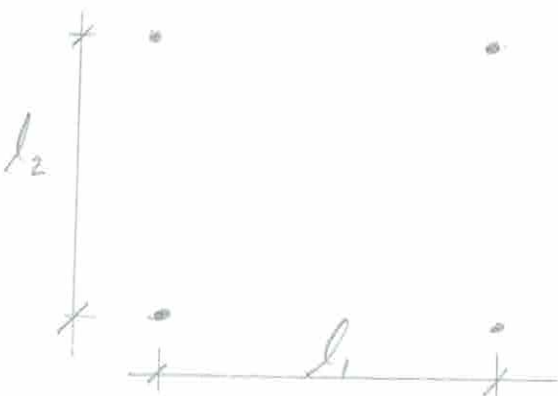


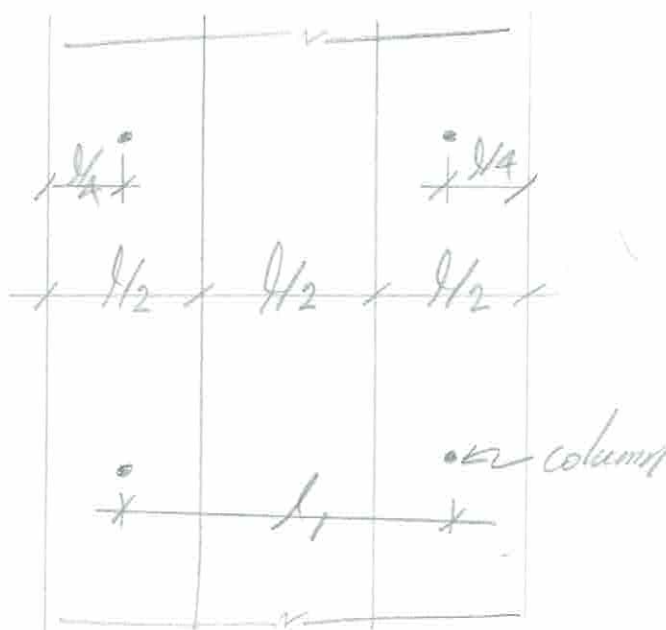
N.B:

1- Total thickness of slab and drop panel used in calculating the negative steel area should not exceed 1.5 the slab thickness i.e. in any case ($d \leq 1.5\, t$)

2- Side of drop panel shall be at least 0.33 panel length (i.e. $\geq 0.33\, l$)

3- Drop thickness $\leq$ (Projection of drop/4)





Three assumptions are made for the application of this method:

1- Longer dimension / short dimension ≤ 1.33

2. Any two adjacent spans should not differ by more than 20%.

3. There should be at least 3 panels in each direction.

$M_o = w l_2 l_1^2 / 8$ (without drop)

M_o (for slab) $= WL^2/8 = (M_{pos.} + M_{neg.})$

Westergard proposition:

1- Column strip:

$M_{neg} = 0.48\, M_o$

$M_{pos.} = 0.21\, M_o$

2. Middle strip:

$M_{neg} = 0.17\, M_o$

$M_{pos} = 0.14\, M_o$

ACI Moment ratio

Strip	without drop		with drop	
	Neg.	Pos.	Neg.	Pos.
column strip	0.46	0.22	0.5	0.2
Middle strip	0.16	0.16	0.15	0.15

* $M = C \cdot M_o$
* for interior panels
* Min slab thickness for use with ultimate strength design:

f_y N/mm^2	with drop panel	without drop panel
275	L/40 or 100 mm	L/36 or 125 mm
340	L/36 or 100 mm	L/33 or 125 mm
410	L/33 or 100 mm	L/30 or 125 mm

ACI Moments (for panel with drop)

$$M_o = 0.1\, WLF\left(1 - \frac{2c}{3L}\right)^2 \quad \text{(square panel)}$$

where $F = 1.15 - c/L \geq 1.0$

W = total load on panel (slab) with drop panel if any.

$W = w \times$ area of panel. = kN

L = span of strip c/c of columns.

C = diameter of column capital and equal to zero ($C=0$) if does not exists

If panel is rectangular then $l_1 \neq l_2$ or $L_x \neq L_y$

$$M_{ox} = 0.1\, WL_x F\left(1 - \frac{2c}{3L_x}\right)^2$$

$$M_{oy} = 0.1\, WL_y F\left(1 - \frac{2c}{3L_y}\right)^2$$

Ex: Design an interior panel of flat slab construction (5×5 m) in dimension without drop panel to carry a live load of 5 kN/m^2. Given that $f_y = 410$ N/mm^2.

Solution:

1- Sellection of thickness:

$L/30$ or 125 mm for $f_y = 410$ N/mm^2

$L/30 = 5000/30 = 166.7$ mm > 125 mm

use $t = 200$ mm

2. Moments:

$w = 1.4 D.L + 1.7 L.L. = 15.5$ kN/m^2

$$M_o = \frac{w l_2 l_1^2}{8} = \frac{15.5 \times 5 \times 25}{8} = 242.2 \text{ kN.m}$$

3. Moment distribution:

a- Column strip: $M_{pos.} = 0.22 M_o = 53.28$ kN.m

$M_{Neg} = 0.46 M_o = 111.4$ kN.m

b- Middle strip: $M_{pos.} = 0.16 M_o = 38.75$ kN.m

$M_{neg} = 0.16 M_o = 38.75$ kN.m

≤ 242.2 kN.m

Shear in flat slab:

The ACI building code specified that for shear calculation two checks should be made:

First: at a critical section at a distance (d/2) from the column capital or drop panel which is ~~along~~ along a periphery disignated.

Second: at a distance (d) from the support considering the slab as a wide beam, seldom if ever will the second check be critical.

$$v_{cu} \ngtr \phi\, 0.33\sqrt{f'_c} \quad \text{no shear reinf.}$$

$$v_{cu} \ngtr \phi\, 0.5\sqrt{f'_c} \quad \text{with web reinf.}$$

* Reinforcement bars may not be used to resist shear in slab less than 250 mm thick.

Ex: An (5.5 x 5.5 m) flat slab must resist a live load of $4\ kN/m^2$ and supported on 400 mm round columns. Design the required depth to satisfy shear. $f'_c = 30$ MPa, $\gamma_c = 24\ kN/m^3$.

Assume slab thickness = 200 mm

then $D.L = \frac{200}{1000} \times 24 = 4.8\ kN/m^2$

$L.L. = 4.0\ kN/m^2$

$w_u = 1.4 \times 4.8 + 1.7 \times 4.0 = 13.52\ kN/m^2$

1

1- The critical section is at $d/2$ around the column, then the critical shear periphery:

$b_o = \pi(400+d)$

take $d = t - 35 = 165$ mm

$\therefore b_o = \pi(400+165) = 1776$ mm

$V_u = w\left[5.5\times5.5 - \frac{\pi}{4}(0.165+0.4)^2\right]$

$= 405.59$ kN

$v_u = \frac{V_u}{bd} = \frac{405.59\times10^3}{1776\times165} = 1.38\ N/mm^2$

all. $v_{cu} = \phi\, 0.33\sqrt{f_c'} = 0.85\times0.33\sqrt{30}$

$= 1.53 > 1.38\ N/mm^2$

$\therefore$ o.K.

2. Assume the slab acts as a wide beam, the shear along one side of the panel at a distance (d) from the columns, then

$V_u = (5.5 - 0.4 - 2d)^2 \frac{W}{4}$

$= (5.5 - 0.4 - 2\times0.165)^2 \frac{13.52}{4} = 77$ kN

$v_u = \frac{77\times10^3}{4.77\times10^3\times165} = 0.01 < 1.53\ N/mm^2$

0.2 0.76

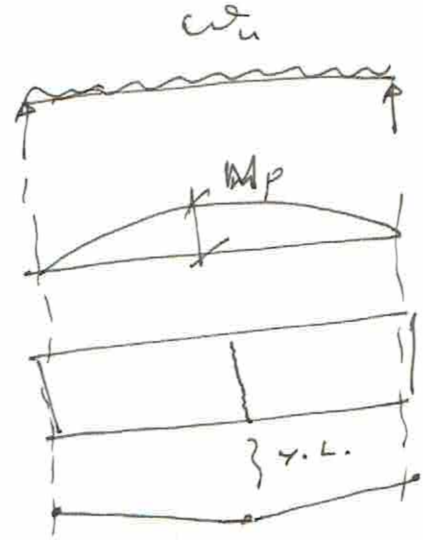
$\therefore$ slab thickness is o.K.

$(5.5 - 0.4 - 2\times0.165) = 4.77$

* A flat slab system should be used only in structures of relatively short spans and light live loads.
* The max. story height shall not exceed 12.5 feet. 3.7 m
* The building shall not have a height exceeding 125 feet.
* Columns shall not be offset more than 10 percent of the span.

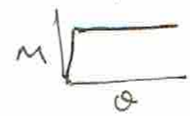
Yield - line Theory of Slab Analysis

- Moment analysis of reinforced concrete slab based on inelastic considerations is known as Y. L. theory.
- first proposed by K.W. Johansen.
- it permits the determination of failure moments in the slabs of irregular as well as rectangular shapes for a variety of support conditions.
- slabs are ~~all~~ underreinforced ($\rho < \rho_b$).



As the load is increased gradually the steel will yield and the concrete above the N.A. will crush along the line of maximum moment. This line is called "Yield Line"

In this slab the formation of one yield line will make the slab to collapse because at yield line the slab will go on rotating - the deflection will go on increasing under constant load.



Location of Yield line:

As it is evident that for analysis by yield line theory, the first thing to know is the location of yield lines and axes of rotations.

In cases such as given earlier, the ~~not~~ location of Y.L. and axes of rotation were simple. In other cases a set of guide lines can be derived on the principle that Y. lines and axes of rotations should be such as to convert the slab into a mechanism of ultimate load.

Guidelines are as follows:

i. Yield lines are generally straight.

ii. Axes of rotation generally lie along lines of supports (the support line may be a real hinge, or it may establish the location of a yield line which acts as a plastic hinge).

iii. Axes of rotation pass over columns.

iv. A yield line passes through the intersection of the axes of rotation of adjacent slab segments.

Notations used:

free edge

s. s. edge

fixed or continuous edge.

Y. L. (+ ve)

Y. L. (- ve)

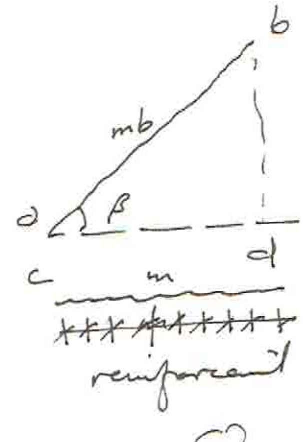
axes of rotation

m +ve moment per unit length of Y. L.

m′ -ve " " " " " "

Yield moment:

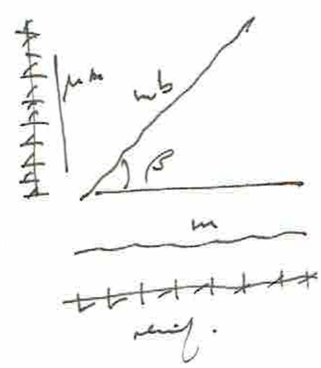
$$m = M_u = \phi \rho f_y b d^2 \left(1 - 0.59 \rho \frac{f_y}{f_c'}\right)$$



(a)

If a yield line, ultimate moment m_b per unit length makes an angle β with the yield line at right angles to the reinforcement, i.e. angle $(90-\beta)$ with the reinforcement as shown in fig (a), the yield moment, m_b can be derived as follows;

The component in the direction of the yield line ab of the moment m to the steel is $m \cos\beta \times cd$. This must be equal to the total moment along the yield line ab, i.e. equal to $m_b \times ab$



(b)

or $\quad m \cos\beta \times cd = m_b \times ab$

or, $\quad m_b = m \cos^2\beta$

If the reinforcement is provided in two directions at right angles and if the yield moments are m and μm as shown in (b) then

$$m_b = m \cos^2\beta + \mu m \cos^2(90-\beta)$$
$$= m(\cos^2\beta + \mu \sin^2\beta)$$

If the reinforcement arranged in two directions at right angles are unequal the slab is said to be orthotropically reinforced In this case:

$$m_b = m(\cos^2\beta + \mu \sin^2\beta)$$

but if the reinforcement arranged in two directions at right angles ~~equal to~~ are equal the slab is said to be Isotropically reinforced. In this case $\mu = 1$

$$m_b = m(\cos^2\beta + \sin^2\beta) = m$$

<u>Methods of analysis</u>

virtual work method, Equilibrium method:

<u>Virtual Work method</u>:

- Based on the principle that the external work done by the applied loads ~~is~~ causing a small virtual displacement is equal to the internal work done in rotation along the yield lines.
- Having postulated a yield line pattern of failure, the first step is to give any convenient point in the slab a vertual deflection δ. In terms of δ the corresponding deflection of all parts of this slab may be calculated:

Exernal work done $= \Sigma \int\int w.\delta.dx.dy$

which is ~~suffice~~ simplified to external load $= \Sigma(w.\delta)$

where $w =$ distributed load on the slab at collapse.

Internal work done $= \Sigma(m\, l\, \theta)$

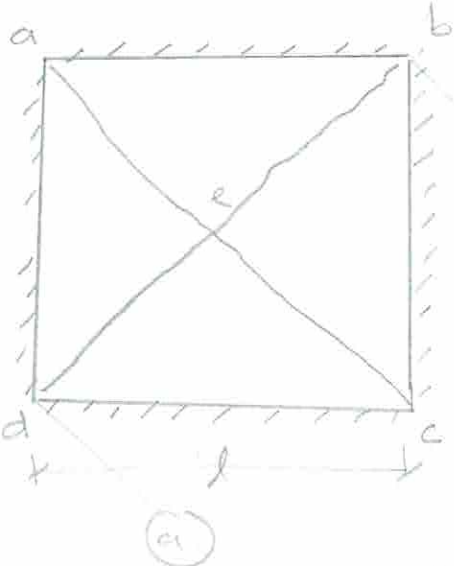
where $m =$ ultimate moment per unit of yield line length

$l =$ yield line length

$\theta =$ rotation along yield line.

$$\Sigma(w\delta) = \Sigma(m\, l\, \theta)$$

Ex: Isotropically reinforced square slab, simply supported and uniformly loaded. Find w_u. (5)



Assuming the yield pattren as shown above, the yield moment along the diagonal yield line will be (m) since the slab is equally reinforced in both directions. For small deflection (δ) at (e) the rotation at yield lines are shown in fig (b) which is a view of diagonal yield line deb. The C.G. of the load on each ~~triagonal~~ triangular segment of the slab deflects by $\delta/3$, hence

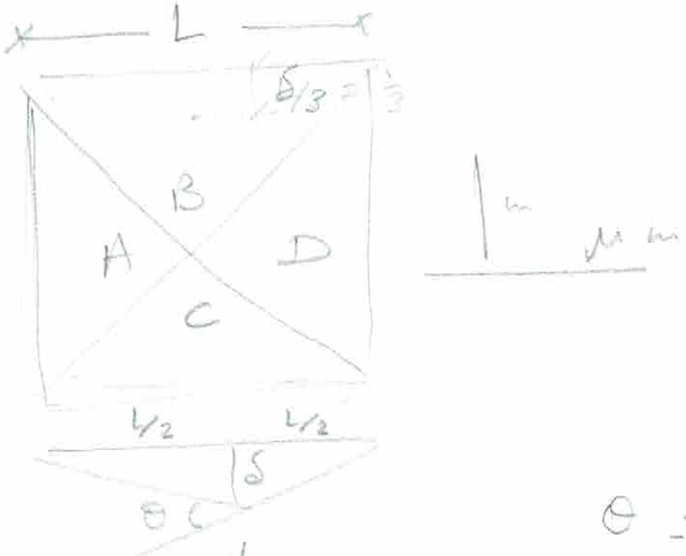
External work done $= \Sigma(w\cdot\delta) = 4 \times \frac{1}{4} w_u l^2 (\delta/3) = \frac{1}{3} w_u l^2 \delta$

from fig (b) the rotation at yield line ae $= 2\theta = \frac{2\delta}{l/\sqrt{2}} = \frac{2\sqrt{2}\,\delta}{l}$.

Internal work done on aec $= m.\theta.l\sqrt{2} = m\left(\frac{2\sqrt{2}\,\delta}{l}\right)(l\sqrt{2}) = 4m\delta$

internal work done on deb $= 4m\delta$

$\therefore \frac{1}{3} w_u l^2 \delta = 8m\delta$ or $w_u = \frac{24m}{l^2}$



$\delta = 1$ unit

$$\theta = \frac{1}{L/2} = \frac{2}{L} \text{ for all elements}$$

Load $\frac{wL^2}{4} \times (\delta = \frac{1}{3}) = \frac{1}{12} wL^2$

total $= 4 \times \frac{1}{12} wL^2 = \frac{1}{3} wL^2$

$$\text{Total Ext. work} = \frac{wL^2}{3}$$

Internal wk.. Elements A+D $= \Sigma m l \theta = 2\mu m L . \frac{2}{L} = 4\mu m$

Elements B+C $= 2 m L . \frac{2}{L} = 4m$

Int. wrk. Element A+D $= 4\mu m$

Element B+C $= 4m$

$$\left.\begin{array}{l} 4\mu m \\ 4m \end{array}\right\} = 4m(1+\mu)$$

Int. wrk = Exter wrk.

$$\frac{wL^2}{3} = 4m(1+\mu)$$

$$m = \frac{wL^2}{12(1+\mu)} = \frac{wL^2}{24} \qquad \mu = \mu m$$

Design a square slab 5m x 5m simply supported on all four edges carrying a live load of 6 kN/m².

Depth of slab: $h = 90$ mm

or $\frac{\text{clear parimeter}}{180} = \frac{4 \times 5000}{180} = 111.1 \simeq 120$ mm

$\therefore$ take $h = 120$ mm

$\text{selfwt} = 0.12 \times 1 \times 1 \times 24 = 2.88 \text{ kN/m}^2$

$\text{finishing} = 0.025 \times 1 \times 1 \times 24 = 0.60 \text{ kN/m}^2$

$\text{total D.L.} = 3.48 \text{ kN/m}^2$

$W_u = 1.4 \text{ D.L.} + 1.7 \text{ L.L.}$

$= 1.4 \times 3.48 + 1.7 \times 6 = 15.576 \text{ kN/m}^2$

$w_u = \frac{24\,m}{l^2}$

l = Clear span + 2h or c/c span

$l = 5 + 2 \times 0.12 = 5.24$ m

or $m = \frac{w_u l^2}{24} = \frac{15.576 \times 5.24^2}{24} = 17.82$ kN.m

with max. steel ratio $= 0.027$

$17.82 \times 10^6 = 0.9 \times 0.027 \times \overset{275\times}{1000} \times d^2 (1 - 0.59 \times 0.027 \times \frac{275}{20})$

or $d^2 = 58.43$ $\quad \therefore h = 58.43 + 20 + 12 + 6 = 96.43 < 120$ mm

with $h = 120$ m $\therefore d = 120 - 20 - 6 - 12 = 82$ mm

$M_u = \phi \rho f_y b d^2 (1 - 0.59 \rho \frac{f_y}{f'_c})$

$$= 0.9 \times \rho \times 275 \times 1000 \times 82^2 \left(1 - 0.59 \rho \times \frac{275}{20}\right)$$

$$17.82 \times 10^6 = 1664.19 \times 10^6 \rho - 13500.7 \times 10^6 \rho^2$$

or $\rho^2 - 0.123266 \rho + 0.00132 = 0$

or $\rho = \dfrac{0.123266 \pm \sqrt{0.0152 - 4 \times 1 \times 0.00132}}{2}$

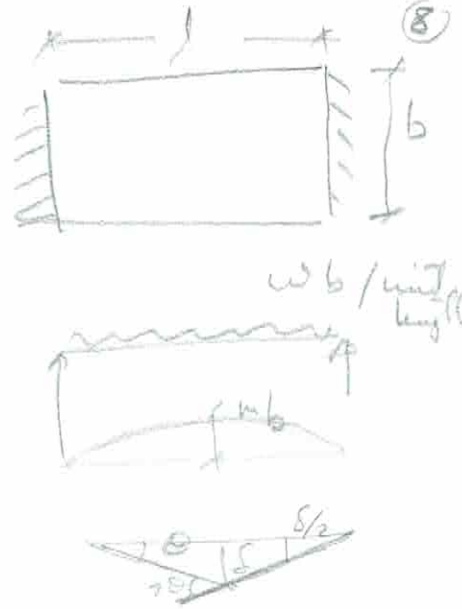
$= 0.0118$

$As = 0.0118 \times 1000 \times 82^2 = 968 \text{ mm}^2$

$S = \dfrac{113 \times 1000}{968} = 116.74 \text{ mm } c/c$

Hence provide 12 mm ϕ @ 115 c/c both ways.

8



Ex: One way slab simply supported on two short edges and free on longer edges as shown, with yield moment / unit length = m.

Find the value of UDL, w_u per unit length the slab can carry.

$$\text{Ext. work} = 2 \times \left(w_b \times l/_2 \times \frac{\delta}{2} \right)$$

$$= \frac{w_b \cdot l \cdot \delta}{2}$$

$$\text{Int. work} = m_b \times 2\theta$$

$$\text{Ext. work} = \text{Int. work}$$

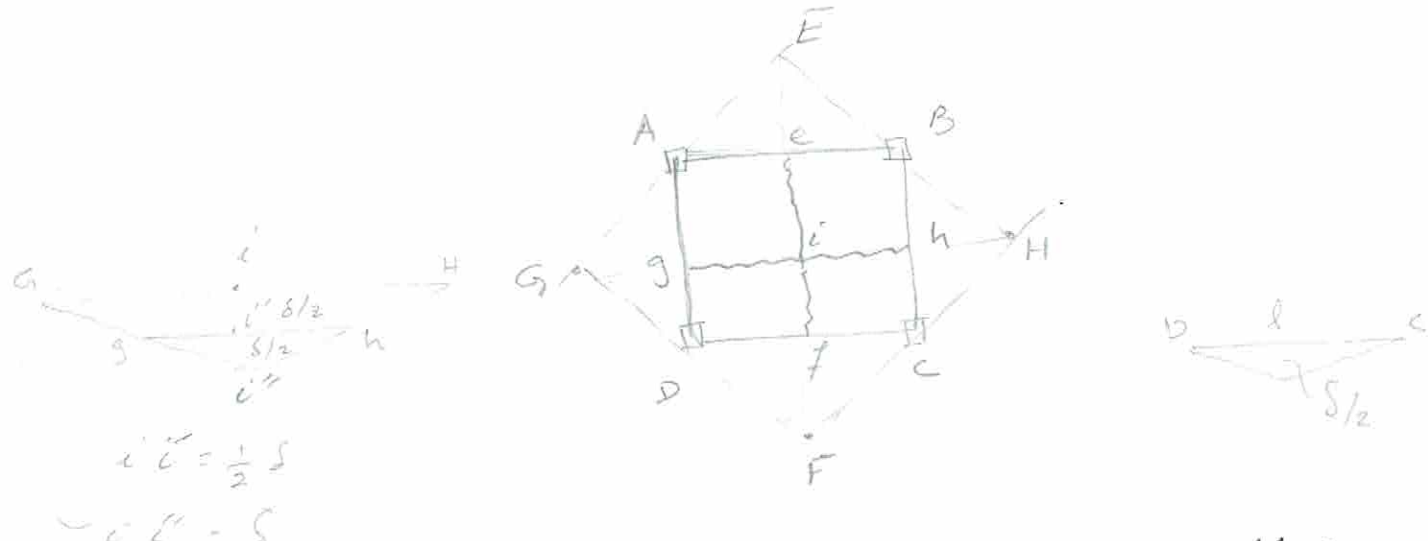
$$\frac{mb\, l\, \delta}{2} = m_b \times 2\theta$$

$$\theta = \delta/l/_2 = \frac{2\delta}{l}$$

$$w_b\, l\, \frac{\delta}{2} = mb \times 2 \times \frac{2\delta}{l}$$

$$\text{or } w_b = \frac{8m}{l^2}$$

Ex: Isotropically reinforced square panel (slab), supported at corners only and uniformly loaded. Find m. (9)



$$\text{Ext. work} = 4\left(l/_2 \times l/_2 \times w \times \delta/_2\right) = \frac{w l^2 \delta}{2}$$

$$\text{Int. work} : 2\left(m l \times \frac{\delta}{l/_2}\right) = 4 m \delta$$

$$\therefore \frac{w l^2}{2} = 4m \quad \text{or} \quad m = \frac{w l^2}{8}$$

Ex: Isotropically reinforced circular slab, simply supported all round, and uniformly loaded. Find m.

$$\text{Ext. work} : w \cdot \pi r^2 \times \frac{1}{3} = \frac{w}{3}\pi r^2$$

$$\text{Int. work} = (2\pi r\, m)\frac{1}{r} = 2\pi m$$

$$\text{Hence} \quad \frac{w}{3}\pi r^2 = 2\pi m$$

$$\text{or} \quad m = \frac{w r^2}{6}$$

Ex: Isotropically reinforced circular slab, continuous all round the circumference, and uniformly loaded.

$$\frac{1}{3} w \pi r^2 = 2\pi m + 2\pi m'$$

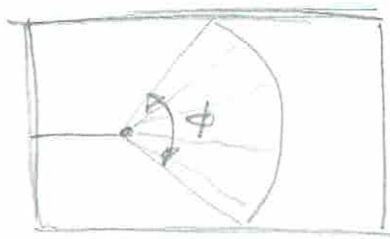
$$= 2\pi (m + m')$$

$$\text{or} (m + m') = \frac{w r^2}{6}$$

An.

Analysis for Concentrated load:

When a point load acts on a slab of any shape, collapse may occure by formation of a cone of yield lines, or, in certain cases, part of a cone, which is sometimes called a "fan". As shown in the figure.



The internal work done on the positive yield line making up the fan is $\Sigma(m l \theta) = \phi m$.

where ϕ is the angle subtended or the centre of the fan.

Ex: Isotropically reinforced square slab simply supported and loaded with centrally point load, W. Find W.

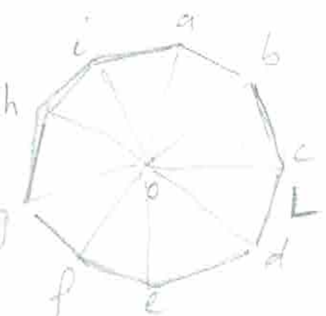
selfwt is neglected.

Ext. Work $= W \times 1 = W$

Int. work $= m\, 2\pi$ — $\phi = 360° = 2\pi$

$\therefore W = 2\pi m$.

Ex: Find the ultimate load for a polygon of (n) sides, continuously supported on all edges and subjected to a uniformly distributed load. Each side of the polygon is L. Find w_u.



$cd = L$

$oq = p$

Considering the Δ ocd

$$\measuredangle cod = 2\pi/n$$

$$\text{Ext. Work} = \frac{1}{2} \times cd \times oq \times w_u \times \frac{1}{3}\delta$$

$$= \frac{w_u}{6}\, cd \times od \times \delta = \frac{w_u}{6} . L . P . \delta$$

$$\text{Int. Work} = m . cd . \theta = m . L . \frac{\delta}{P}$$

$$\therefore \frac{w_u}{6} . \cancel{L} . P . \cancel{\delta} = m . \cancel{L} . \cancel{\delta}/p$$

$$\text{or } w_u = \frac{6m}{p^2}$$

Note: If the slab is circular i.e. $n \Rightarrow \infty$ $p = r$

$$\text{or } w_u = \frac{6m}{r^2}$$