CoE 211 ELECTRONIC DEVICE PHYSICS

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Chapter Four

The Semiconductor in Equilibrium

This chapter aims to

- Study a semiconductor in equilibrium such that no external forces such as electric fields, magnetic fields, or temperature gradient are acting on the semiconductor.
- Address calculating the concentration for electrons and holes in the conduction and valence bands using the density of quantum states and Fermi-Dirac probability function.
- Discuss the properties of a semiconductor material when impurity atoms are added.

4.1 Equilibrium Distribution of Electrons and Holes

Semiconductor with $E_v < E_F < E_c$

- The figure shows:
  -) the density of quantum states in the conduction band, $g_c(E)$, (see (5) in ch. 3),
  -) the density of quantum states in the valence band, $g_v(E)$, (see (6) in ch. 3),
  -) the Fermi-Dirac distribution, $f_F(E)$, for $T > 0$, (see (7) in ch. 3).
- In the conduction band, if $E_c - E_F >> k_b T \rightarrow E - E_F >> k_b T$, then Fermi-Dirac distribution becomes

$$f_F(E) \approx \exp\left(-\frac{(E - E_F)}{k_b T}\right) .$$

- In the valence band, if $E_F - E_v >> k_b T \rightarrow E_F - E >> k_b T$, then the complement of Fermi-Dirac distribution becomes

$$1 - f_F(E) \approx \exp\left(-\frac{(E_F - E)}{k_b T}\right) .$$

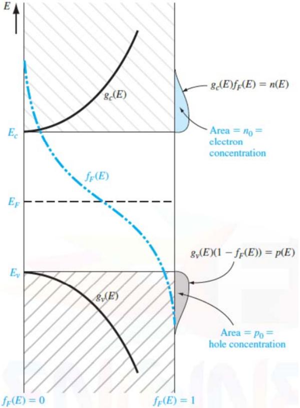


Figure: $g_c(E)$, $g_v(E)$, $f_F(E)$, and areas representing electron and hole concentrations for $E_v < E_F < E_c$.

- Approximated expressions for Fermi-Dirac distribution are used through out this chapter.

Semiconductor with $E_v < E_F < E_c$

- The thermal-equilibrium concentration of electrons in the conduction band is

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE = N_c \exp\left(-\frac{(E_c - E_F)}{k_b T}\right) , \tag{1}$$

-) $g_c(E)f_F(E)$ is the distribution of electrons (wrt energy) in the conduction band.
-) The above integration occurs when $E - E_F >> k_b T$.
-) N_c (in states/cm^3) is called the effective density of states in the conduction band

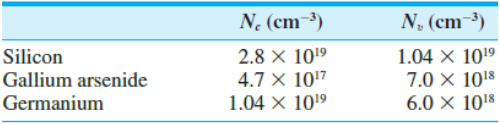
$$N_c = 2\left(\frac{2\pi m_n^\star k_b T}{h^2}\right)^{3/2} , \tag{2}$$

- The thermal-equilibrium concentration of holes in the valence band is

$$p_0 = \int_{-\infty}^{E_v} g_v(E)\,(1 - f_F(E)) = N_v \exp\left(-\frac{(E_F - E_v)}{k_b T}\right) , \tag{3}$$

-) N_v (in states/cm^3) is called the effective density of states in the valence band

$$N_v = 2\left(\frac{2\pi m_p^\star k_b T}{h^2}\right)^{3/2} . \tag{4}$$



	N_c (cm^{-3})	N_v (cm^{-3})
Silicon	2.8×10^{19}	1.04×10^{19}
Gallium arsenide	4.7×10^{17}	7.0×10^{18}
Germanium	1.04×10^{19}	6.0×10^{18}

Figure: Effective density of states function.

An Intrinsic Semiconductor

- An intrinsic (or pure) semiconductor is a semiconductor with NO impurity atoms or defects in the crystal.
- In an intrinsic semiconductor,

$$p_o = n_o \tag{5}$$

  -) Each elevated electron to the conduction band will create a hole in the valence band.
- Let $n_i = n_0$ and $p_i = p_0$ and $E_{Fi} = E_F$, where E_{Fi} is called intrinsic Fermi energy. Then, $n_i = p_i$.

$$n_i^2 = N_c N_v exp\left(-\frac{E_g}{k_b T}\right) . \tag{6}$$

  -) For a given intrinsic semiconductor, the value of n_i is a constant at a given T and does not depend on E_{Fi}.
- E.g., for Si: $E_g = 1.12$ eV, $m_n^\star = 1.08m$ and $m_p^\star = 0.56m$.
  Then, $n_i = 6.95 \times 10^9$ cm^{-3} at $T = 300$ K.
- n_i is a very strong function of temperature.

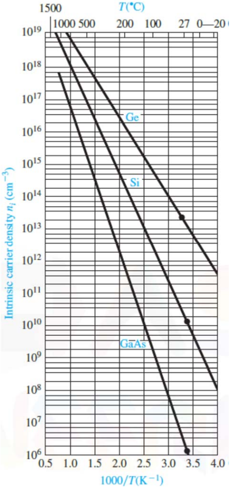
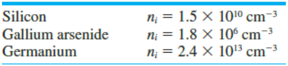


Figure: The intrinsic carrier concentration of Ge, Si, GaAs as a function of temperature.



Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

Figure: Accepted values of n_i at $T = 300$ K

Example 4.1:

(i) Determine the thermal threshold equilibrium electron and hole concentration in GaAs at $T = 400$ K for the case when the Fermi energy level is 0.3 eV above the valence band energy E_v.

(ii) Calculate the intrinsic concentration at (a) $T = 250$ K and (b) $T = 450$ K. (c) Determine the ratio of n_i at $T = 450$ K to that at $T = 250$ K.

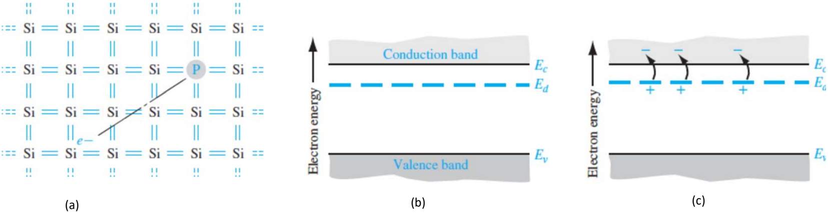
The values of N_c and N_v for GaAs at $T = 300$ K are $N_c = 4.7 \times 10^{17}$ cm^{-3} and $N_v = 7 \times 10^{18}$ cm^{-3}. Assume that $E_g = 1.42$ eV is constant over this temperature range.

4.2 Dopant Atoms and Energy Levels

- Extrinsic semiconductor is a semiconductor that contains a sufficient and controlled amount of impurity atoms called dopant to substantially change the electron or hole concentration.

n-type Semiconductor

- Consider adding an element of group V, such as P, to Si.
  -) Four electrons of P will contribute to the covalent bonding of Si.
  -) The 5th electron of P is called the donor electron. It has an energy level E_d and can be elevated to the conduction band using a small energy as compared with the electrons of the covalent band.
  -) The donors will not create holes in the valence band. It creates +ve ion, such as P^+, that will be fixed in the crystal.
- An n-type semiconductor is the resulting material from adding impurity atoms that will donates electron to the conduction band.



(a) (b) (c)

Figure: (a) 2D representation of the silicon lattice doped with a P atom. (b) The discrete donor energy state. (c) The effect of a donor state being ionized.

p-type Semiconductor

- Consider adding an element of group III, such as B, to Si.
- 3 electrons of B will contribute to the covalent bond.
- One covalent bond has an empty state. The energy of this state denoted by E_a is larger than E_v but much smaller than E_c.
- A valence electron can occupy this empty state and create a hole in the valence band.
- The acceptor will not generate electrons in the conduction band. It creates a -ve ion, such as B^-, that will be fixed in the crystal.
- A p-type semiconductor is the resulting material from adding impurity atoms that will accept an electron from the valence band.

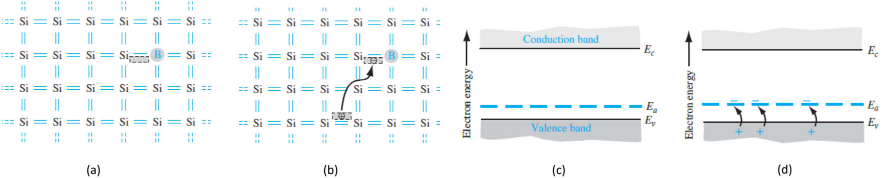
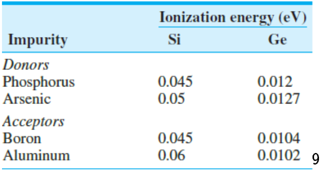


Figure: 2D representation of the silicon lattice (a) doped with a B atom, and (b) showing the ionization of the B atom resulting a hole. The energy band diagram showing (c) The discrete acceptor energy state, and (d) The effect of an acceptor state being ionized.

Figure: Impurity ionization energies in silicon and germanium.



Impurity	Ionization energy (eV) Si	Ionization energy (eV) Ge
Donors		
Phosphorus	0.045	0.012
Arsenic	0.05	0.0127
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06	0.0102

4.3 Equilibrium Distribution for Extrinsic Semiconductor

- Adding donor or acceptor to a semiconductor would change the electron and hole distribution.
- If $E_F > E_{Fi}$, electrons are the majority carrier and holes are the minority carrier. This corresponds to an n-type semiconductor.
- If $E_F < E_{Fi}$, holes are the majority carrier and electrons are the minority carrier. This corresponds to a p-type semiconductor.
- Still n_0 in (1) and p_0 in (3) are valid.
- Adding $\pm E_{Fi}$ in the exp argument of (1), n_0 can be written alternatively as

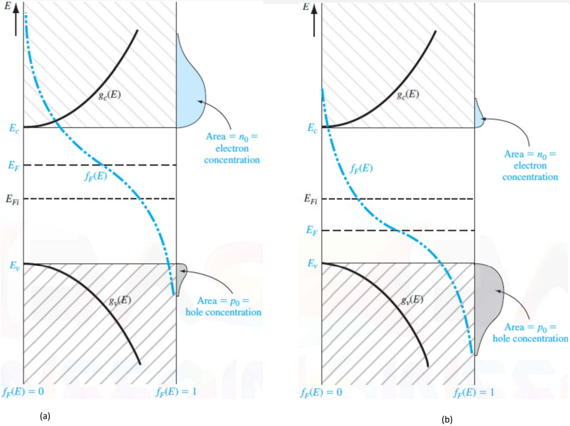
$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{k_b T}\right) . \qquad (7)$$

-) For an n-type semiconductor, $n_0 > n_i$ and $E_F > E_{Fi}$.

- Similarly, adding $\pm E_{Fi}$ in the exp argument of (3), p_0 can be written alternatively as

$$p_0 = n_i \exp\left(-\frac{(E_F - E_{Fi})}{k_b T}\right) . \qquad (8)$$

-) For a p-type semiconductor, $p_0 > n_i$ and $E_F < E_{Fi}$.



(a)

(b)

Figure: Density of states function, Fermi-Dirac probability function, and areas representing the electron and hole concentration for (a) $E_F > E_{Fi}$ and (b) $E_F < E_{Fi}$.

- We can show that product $n_0 p_0$ is

$$n_0 p_0 = n_i^2 . \qquad (9)$$

-) $n_0 p_0$ is a constant for a given T and a given semiconductor.

4.4 Statistics of Donors and Acceptors

- The number or concentration of electrons remaining in (or occupying) the donor state is

$$\begin{aligned} n_d &= \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{k_b T}\right)} \\ &= N_d - N_d^+ \, . \end{aligned} \tag{10}$$

  -) N_d is the concentration of donor atoms,
  -) N_d^+ is the concentration of the ionized donors.

- The number or concentration of holes in the acceptor state is

$$\begin{aligned} p_a &= \frac{N_a}{1 + \frac{1}{4}\exp\left(\frac{E_F - E_a}{k_b T}\right)} \\ &= N_a - N_a^- \, . \end{aligned} \tag{11}$$

  -) N_a is the concentration of acceptor atoms,
  -) N_a^- is the concentration of the ionized acceptors.

- The ratio of electrons in the donor state to the total number of electrons when $E_d - E_F >> k_b T$ and $E_c - E_F >> k_b T$ is

$$\frac{n_d}{n_d + n_0} \approx \frac{1}{1 + \frac{N_c}{2N_d}\exp\left(-\frac{(E_c - E_d)}{k_b T}\right)} \, . \tag{12}$$

  -) $(E_c - E_d)$ is the ionization energy required to elevate donor electrons from E_d band to E_c band.
  -) As $\frac{n_d}{n_d + n_0} \to 0$, *complete ionization* occurs, where almost all donor atoms have donated electrons to the conduction band.

- Similarly, the ratio of holes in the acceptor state to the total number of holes when $E_a - E_F >> k_b T$ and $E_F - E_v >> k_b T$ is

$$\frac{p_a}{p_a + p_0} \approx \frac{1}{1 + \frac{N_v}{4N_a} \exp\left(-\frac{(E_a - E_v)}{k_b T}\right)} . \tag{13}$$

  -) As $\frac{p_a}{p_a+p_0} \rightarrow 0$, *complete ionization* occurs, where almost all acceptors atoms have accepted electrons from the valence band.
- Complete ionization for the donor and acceptor atoms can occur for $T > 300$ K.
- For an n-type semiconductor, freeze out occurs at $T = 0$ when all electrons are in the donor state. This means $N_d^+ = 0$ and $E_F > E_d$.
- For a p-type semiconductor, freeze out occurs at $T = 0$ when there are no electrons in the acceptor atoms. This means $N_a^- = 0$ and $E_F < E_a$.

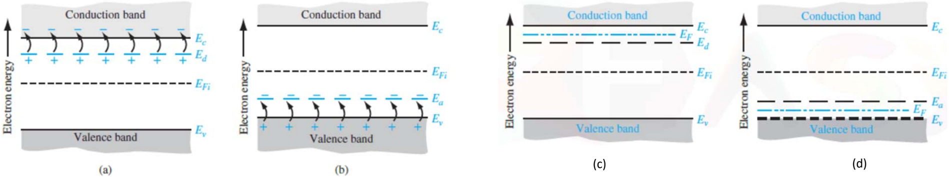


Figure: (a) Energy band diagram for the donor states at $T = 300$ K. (b) Energy band diagram for the acceptor states at $T = 300$ K. (c) Energy band diagram for the donor states at $T = 0$ K. (d) Energy band diagram for the acceptor states at $T = 0$ K

Example 4.2:

Determine the temperature at which 90% of acceptor atoms are ionized. Consider p-type silicon doped with boron at a concentration of $N_a = 10^{16}$ cm^{-3}, the value of N_v for Si is $N_v = 1.04 \times 10^{19}$ cm^{-3} and the ionization energy $E_a - E_v = 0.045$ eV.

4.5 Charge Neutrality

- In thermal equilibrium, the net charge density of charged particles is zero. That is, the semiconductor crystal is electrically neutral.
- In thermal equilibrium, a uniformly doped semiconductor is neutrally charged when

$$n_0 + \overbrace{(N_a - p_a)}^{N_a^-} = p_0 + \overbrace{(N_d - n_d)}^{N_d^+},$$

-) Assume complete ionization and express $p_0 = n_i^2/n_0$, then

$$n_0^2 - (N_d - N_a)n_0 - n_i^2 = 0,$$

-) Solving the above 2nd order linear equation gives the electron concentration n_0

$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}. \tag{14}$$

-) (14) is used to calculate the electron concentration for an n-type semiconductor or when $N_d > N_a$.

- Similarly,

$$p_0 = \frac{n_i^2}{n_0}, \tag{15}$$

$$\text{or} \quad p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}. \tag{16}$$

-) (16) is used to calculate the hole concentration for a p-type semiconductor or when $N_a > N_d$.

Example 4.3:

A compensated semiconductor is the one that contains both donor and acceptor impurity atoms in the same region. Assume that a Silicon semiconductor material at $T = 300$ K is doped with Arsenic atoms to a concentration of 2×10^{15} cm^{-3} and with Boron atoms to a concentration of 1.2×10^{15} cm^{-3}. Consider $n_i = 1.5 \times 10^{10}$ cm^{-3} for Si.
(a) Is the material n-type or p-type?
(b) Determine n_0 and p_0.
(c) Additional Boron atoms are to be added such that the hole concentration is 4×10^{15} cm^{-3}. What concentration of Boron atoms must be added and what is the new value of n_0?

Example 4.4:

A silicon device is doped with donor impurity atoms at a concentration of 10^{15} cm^{-3}. For the device to operate properly, the intrinsic carriers must contribute no more than 5% to the total electron concentration.

(a) What is the maximum temperature that the device may operate? *Hint: You shall end up with a nonlinear equation of temperature, and you'll need to write a code to solve it.*

(b) What is the change in $E_c - E_F$ from the $T = 300$ K value to the maximum temperature value determined in part (a).

Use the following parameters for Si at $T = 300$ K: $N_c = 2.8 \times 10^{19}$ cm^{-3}, $N_v = 1.04 \times 10^{19}$ cm^{-3}, $n_i = 1.5 \times 10^{10}$ cm^{-3}, and $E_g = 1.12$ eV. Assume that E_g does not change over this range of temperature.

End of Chapter Four