1.5 Algebra of functions (sums, Differences, products, Quotients)

Let f(x) and g(x) be functions then:-

1. f+g is a function such that (f+g)(x)=f(x)+g(x)
2. f-g is a function such that (f-g)(x)=f(x)-g(x)
3. f.g is a function such that (f.g)(x)=f(x).g(x)
4. f/g is a function such that (f/g)(x)=f(x)/g(x), $g(x) \neq 0$

Notes: 1. $D_{f+g} = D_{f-g} = D_{f.g} = D_f \cap D_g$

2. $D_{f/g} = (D_f \cap D_g)/\{x \in D_g \,/\, g(x) = 0\}$

Ex(8) let $f(x)=\dfrac{x}{x+1}$ and $g(x)=\dfrac{x-1}{x}$ be functions find the domain of f(x), g(x) and D_{f+g}, D_{f-g}, $D_{f.g}$, $D_{f/g}$

Solu. $D_f = R/\{-1\}$, $D_g = R/\{0\}$

$D_f \cap D_g = R/\{0,1\}$

$$(f+g)(x) = \frac{x}{x+1} + \frac{x-1}{x} = \frac{x^2 + x^2 - 1}{x(x+1)} = \frac{2x^2 - 1}{x(x+1)}$$

$$(f-g)(x) = f(x) - g(x) = \frac{x}{x+1} - \frac{x-1}{x} = \frac{x^2 - x^2 + 1}{x(x+1)}$$

$$(f.g)(x) = f(x).g(x) = \frac{x}{x+1} \cdot \frac{x-1}{x} = \frac{x-1}{x+1}, \quad x \neq -1$$

$$D_{f+g} = D_{f-g} = D_{f.g} = R/\{0,-1\}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x}{x+1}}{\frac{x-1}{x}} = \frac{x^2}{x^2 - 1}$$

$$D_{f/g} = D_f \cap D_g /\{x; g(x) = 0\}$$

$$= R/\{0,-1\}/\{x; x^2 - 1 = 0\} = R/\{0,-1,1\}$$

Def **1-** f(x) is an odd function if f(-x)=-f(x) $\Rightarrow$ f(x) is symmetric about the origin

2- f(x) is an even function if f(-x)=f(x) $\Rightarrow$ f(x) is symmetric about the y-axis

Ex(9)

1. $f(x)=\dfrac{x^2-1}{\sin x}$

$$f(-x)=\frac{(-x)^2-1}{\sin(-x)}=\frac{x^2-1}{-\sin x}=\frac{x^2-1}{\sin x}=-f(x)$$

$\therefore$ f(x) is odd function

2. $f(x)=x^2\cos x$

$f(-x)=(-x)^2\cos(-x)$

$=x^2\cos x=f(x)$

$\therefore$ f(x) is even function

Ex(10) $f(x)=x^2+2$

$f(-x)=(-x)^2+2=x^2+2=f(x)$

f(x) is even function

Ex(11) $f(x)=x^3$

$f(-x)=(-x)^3=-x^3=-f(x)$

f(x) is odd function