

Functions

1.2 Def:- A function f from a set A to a set B "written $f: D \rightarrow R$ " is a rule which assigns a single element $y \in B$ to each element $x \in A$.

Notes:

The element $y \in B$ denoted $f(x)$.

The set A is called the domain of f .

The set R is called the range of f .

Ex(3) Given the function $f(x)=x^2-2x+3$. find $f(-1)$, $f(0)$, $f(2)$, $f(x+1)$, $f(f(x))$, $f(f(1))$

Solu. $f(x)=x^2-2x+3$

$$f(-1)=(-1)^2-2(-1)+3=1+2+3=6$$

$$f(0)=0-0+3=3$$

$$f(2)=4-4+3=3$$

$$f(x+1)=(x+1)^2-2(x+1)+3=x^2+2x+1-2x-2+3=x^2+2$$

$$f(f(x))=(f(x))^2-2(f(x))+3=(x^2-2x+3)^2-2(x^2-2x+3)+3$$

$$=x^4+4x^2+9-4x^3+6x^2-12x-2x^2+4x-6+3=x^4-4x^3+8x^2+8x+6$$

$$f(f(1))=1^4-4(1)^3+8(1)^2+8(1)+8=1-4+8-8+6=3$$

Ex(4) Find the domain and range of the following functions:-

1. $y=f(x)=x^2$, D : all x or $D: -\infty < x < \infty$

$$x = \pm\sqrt{y}, \quad R: y \geq 0$$

2. $y = \frac{x-1}{x-6}$, $D: x \neq 6$

$$yx-6y=x-1$$

$$yx-x=6y-1$$

$$x = \frac{6y-1}{y-1}, \quad R: y \neq 1$$

3. $y = \sqrt{1-x^2}$

$$1-x^2 \geq 0 \Rightarrow (1-x)(1+x) \geq 0$$

$$D: -1 \leq x \leq 1$$

$$y^2=1-x^2 \Rightarrow x^2=1-y^2 \Rightarrow x = \sqrt{1-y^2}$$

$$R: -1 \leq y \leq 1$$

4. $y = \sqrt{x-3}$

$$x-3 \geq 0 \Rightarrow x \geq 3$$

$$D: (3, \infty)$$

R: $(0, \infty)$

5. $y = \sqrt{x^2 - 4x + 3}$

$$x^2 - 4x + 3 \geq 0 \Rightarrow (x-3)(x-1) \geq 0$$

D: $x \leq 1$ or $x \geq 3$

$$y^2 = x^2 - 4x + 3 \Rightarrow x^2 - 4x + 3 - y^2 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 1(3 - y^2)}}{2 \cdot 1} \Rightarrow x = \frac{4 \pm \sqrt{16 - 12 + 4y^2}}{2}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{4 - 4y^2}}{2} \Rightarrow x = \frac{4 \pm 2\sqrt{1 - y^2}}{2} \Rightarrow x = 2 \pm \sqrt{1 - y^2}$$

R: all y or $-\infty < y < \infty$

6. $y = \sqrt{2 - \sqrt{x}}$

$$2 - \sqrt{x} \geq 0 \Rightarrow 2 \geq \sqrt{x} \Rightarrow 4 \geq x$$

D: $0 \leq x \leq 4$

$$y^2 = 2 - \sqrt{x} \Rightarrow \sqrt{x} = 2 - y^2 \Rightarrow x = (2 - y^2)^2$$

R: all v