Numbers

1.1 Inequalities

If a and b are real numbers then one of the following is true. a>b or a=b or a<b

Notes:

1. If a>b then –a<-b

    4>2 then –4<-2

2. If a>b then 1/a<1/b

    4>2 then 1/4<1/2

Intervals

An interval is aset of numbers x having one of the following forms:-

1. Open interval: a<x<b ≡ (a, b)

2. Close interval: $a \leq x \leq b \equiv [a,b]$

3. Half open from the left or half close from the right: $a < x \leq b \equiv (a,b]$

4. Half close from the left or half open from the right: $a \leq x < b \equiv [a,b)$

Notes:

1. a<x<∞ ≡ a<x ≡ (a,∞)

2. $a \leq x < \infty \equiv a \leq x \equiv [a,\infty]$

3. -∞<x<a ≡ x<a ≡ (-∞,a)

4. $-\infty < x \leq a \equiv x \leq a$

5. -∞<x<∞ ≡ (-∞,∞)

Ex(1) Find the solution set of the following inequalities:-

1. 12x-21<27+4x

    12x-4x<27+21

    8x<48

    x<48/8=6

    solution set is -∞<x<6

2. x^2-x-12<0

    (x+3)(x-4)<0

    solution set is -3<x<4

3. $2x^2$+5x+2>0

    (2x+1)(x+2)>0

    solution set is x<-2 or x>-1/2

4. x^3-x$\leq$0

    x(x^2-1) $\leq$0

    x(x-1)(x+1) $\leq$0

    solution set is x$\leq$-1 or 0$\leq$x$\leq$1

5. $x^2+2x+2>0$

$x^2+2x+1+1>0$

$(x+1)^2+1>0$

solution set is $-\infty<x<\infty$

6. $\dfrac{x-1}{x^2+x-6}<0$

$\dfrac{x-1}{(x+3)(x-2)}<0$

solution set is: $x<-3$ or $1<x<2$

Absolute Value

The absolute value of the real number x is defined as:-

$$|x|=\begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$$

$$|3|=3$$

$$|-3|=-|-3|=3$$

In general

$$|u(x)|=\begin{cases} u(x), & u(x)\geq 0 \\ -u(x), & u(x)<0 \end{cases}$$

Properties of the absolute value

1. $|x\cdot y|=|x|\cdot|y|$

2. $|x|=|-x|$

3. $|x+y|\leq|x|+|y|$

4. $|x|<a$ mean $-a<x<a$

5. $|x|\leq a$ mean $-a\leq x\leq a$

6. $|x|>a$ mean $x<-a$ or $x>a$

7. $|x|\geq a$ $\qquad x\leq -a$ or $x\geq a$

Ex(2) Find the solution set of the following inequalities:-

a. $\left|\dfrac{3x+1}{2}\right|<1$

solu.

$$-1<\dfrac{3x+1}{2}<1\Rightarrow-2<3x+1<2\Rightarrow-2-1<3x<2-1\Rightarrow-3<3x<1$$

$$\Rightarrow-1<x<1/3$$

b. $|x-1|\geq 5$

solu.

$$|x-1|\geq 5\Rightarrow x-1\leq-5 \text{ or } |x-1|\geq 5\Rightarrow x-1\geq-5\Rightarrow x\leq-4 \text{ or } x\geq 6$$