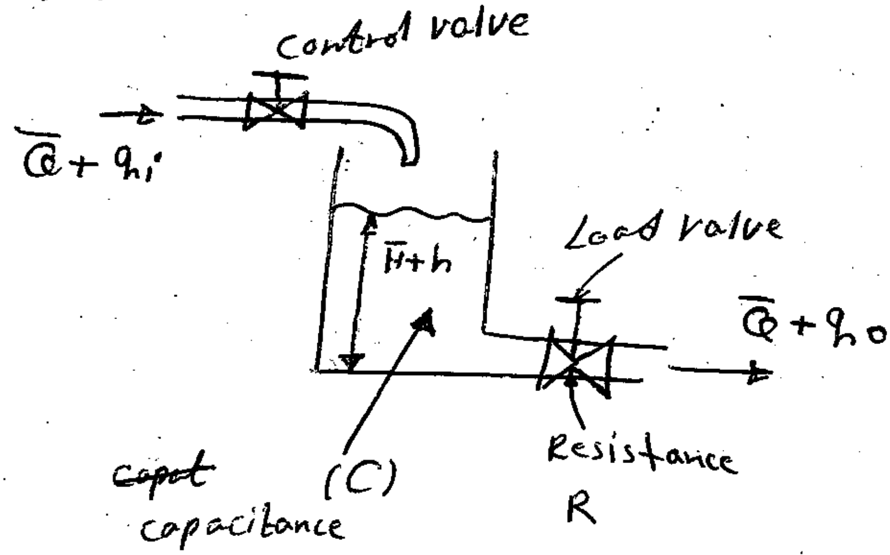
عماد عبد الكاظم ضيوث
دكتوراه هندسة ميكانيك

Chapter four

Mathematical Modeling of Fluid systems and Thermal systems

4.1 Liquid - Level systems



$$R = \frac{\text{Change in level difference , m}}{\text{Change in flow rate , } m^3/sec}$$

$$Q = KH$$

Q = steady-state Liquid flow rate , m^3/sec

K = coefficient , m^2/sec

H = steady-state head , m

* For Laminar flow :

$$R_L = \frac{dH}{dQ} = \frac{H}{Q}$$

(2)

* For turbulent flow

$$Q = K\sqrt{H}$$

Q = steady-state liquid flow rate, m^3/sec

K = coefficient, $m^{2.5}/sec$

H = stead-state head, m

$$R_t = \frac{dH}{dQ}$$

$$dQ = \frac{K}{2\sqrt{H}}\, dH$$

$$\frac{dH}{dQ} = \frac{2\sqrt{H}}{K} = \frac{2\sqrt{H}\sqrt{H}}{Q} = \frac{2H}{Q}$$

$$R_t = \frac{2H}{Q}$$

$$Q = \frac{2H}{R_t}$$

note:

$$C = \frac{\text{change in liquid stored, } m^3}{\text{change in head, } m}$$

السعة

* Liquid-Level systems

$$C\,dh = (q_i - q_o)\,dt$$

$$q_o = \frac{h}{R}$$

$$RC\frac{dh}{dt} + h = Rq_i$$

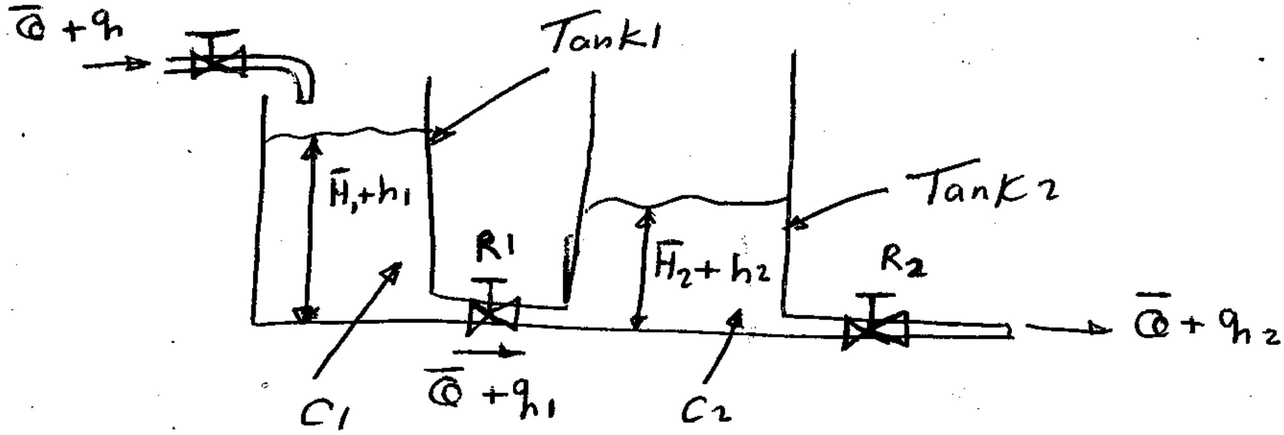
$$(RCs+1)H(s) = RQ_i(s)$$

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs+1}$$

$$\frac{Q_o(s)}{Q_i(s)} = \frac{1}{RCs+1}$$

$$Q_o(s) = \frac{1}{R}H(s)$$

* Liquid-level systems with interaction



$\bar{Q}$: steady-state flow rate

$\bar{H}_1$: steady-state Liquid level of tank 1

$\bar{H}_2$: " " " " 2

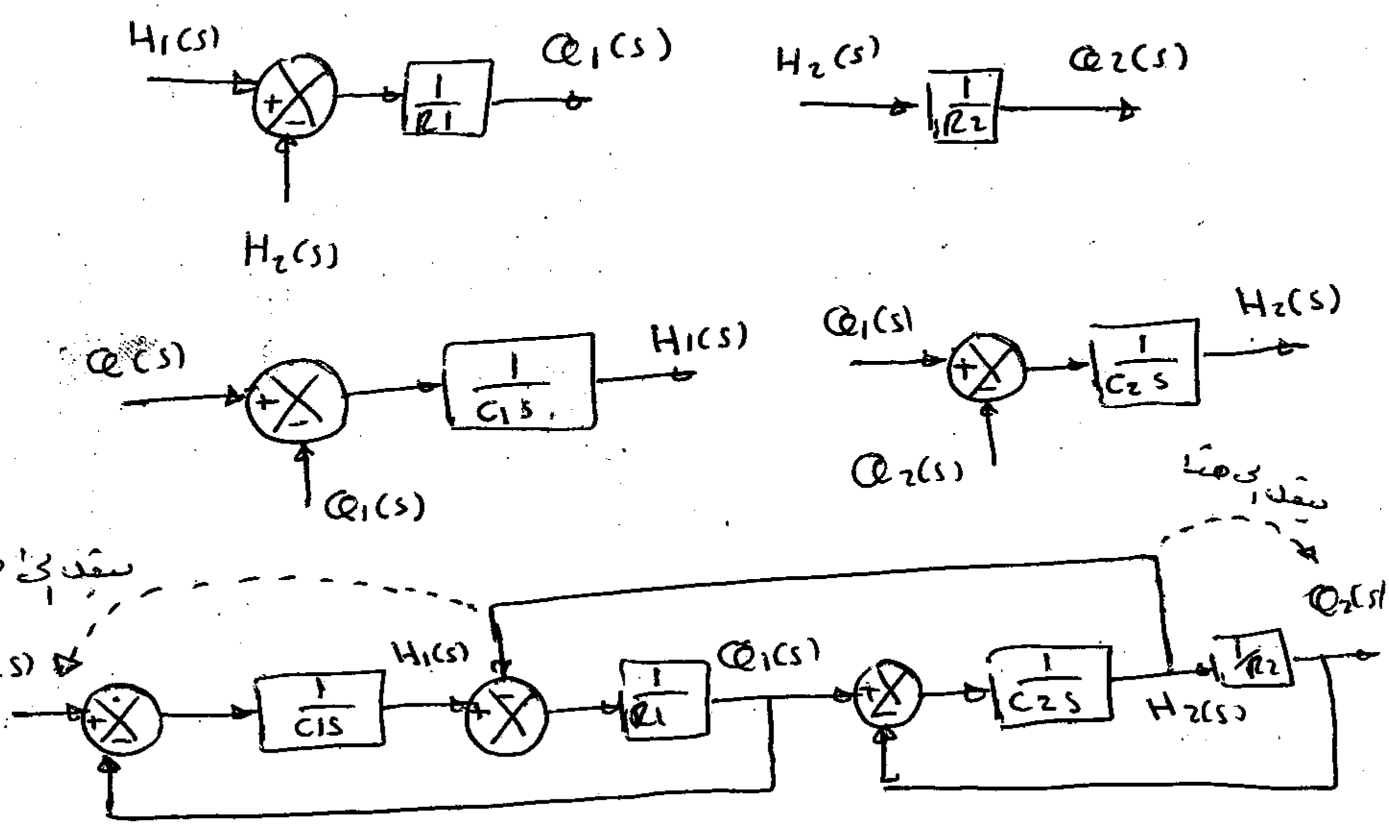
$$\frac{h_1 - h_2}{R_1} = q_1$$

$$c_1 \frac{dh_1}{dt} = q - q_1$$

$$\frac{h_2}{R_2} = q_2$$

$$c_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) s + 1}$$



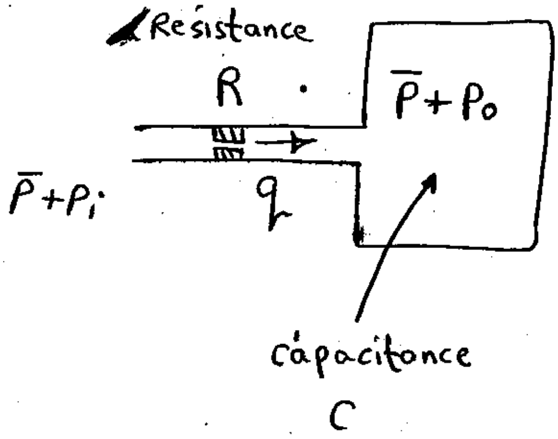
4.2 Pneumatic systems

$$R = \frac{\text{change in gas pressure difference, } lb_f/ft^2}{\text{change in gas flow rate, } lb/sec}$$

$$R = \frac{d(\Delta P)}{dq}$$

$$C = \frac{\text{change in gas stored, } lb}{\text{chang in gas pressure, } lb_f/ft^2}$$

$$C = \frac{dm}{dp}$$



$$R = \frac{P_i - P_o}{q}$$

$$C = \frac{dm}{dp}$$

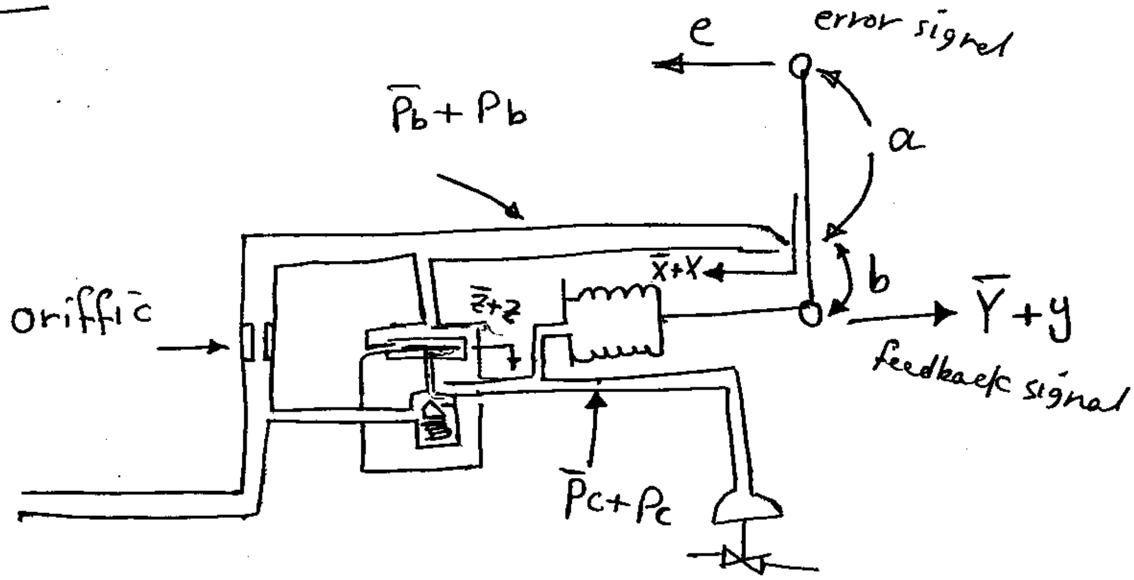
$$C\,dP_o = q\,dt$$

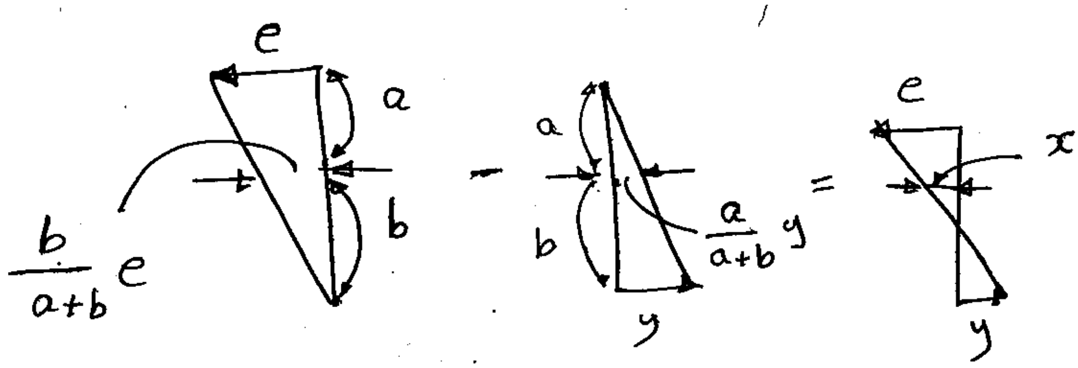
$$C\frac{dp_o}{dt} = \frac{P_i - P_o}{R}$$

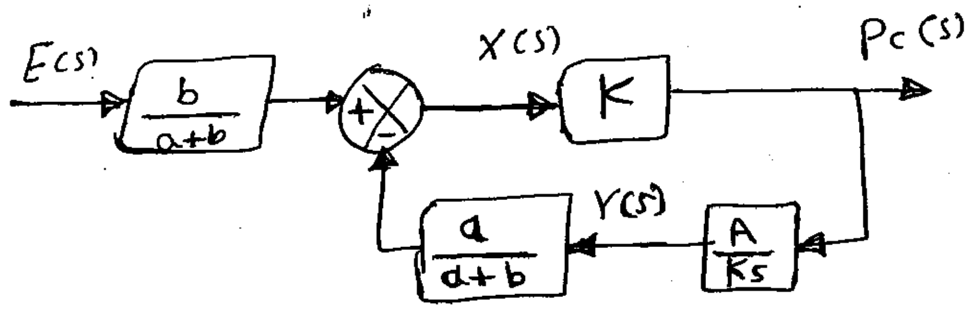
$$RC\frac{dP_o}{dt} + P_o = P_i$$

$$\frac{P_o(s)}{P_i(s)} = \frac{1}{RCs+1}$$

Example (Pneumatic Relay)







$$P_b = K_1 x$$

$$P_b = K_2 Z$$

$$P_c = K_3 Z$$

$$P_c = \frac{K_3 P_b}{K_2} = \frac{K_1 K_3}{K_2} x = K x$$

$$x = \frac{b}{a+b} e - \frac{a}{a+b} y$$

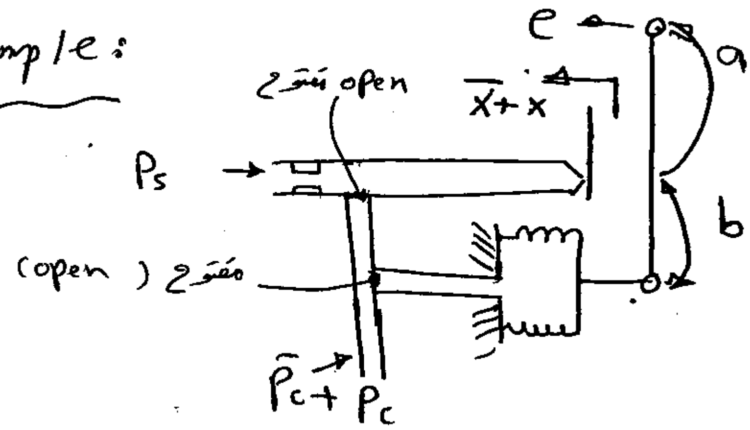
$$A P_c = K_s y$$

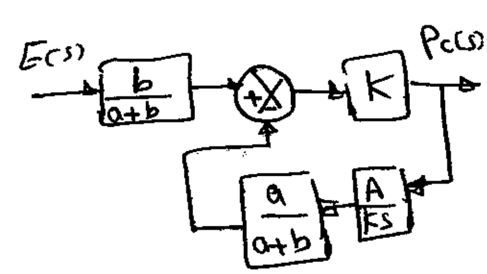
$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b} K}{1 + K \frac{a}{a+b} \cdot \frac{A}{K_s}} = K_p$$

(proportional controller)
≡ proportional action

* principle for obtaining Derivative control action.

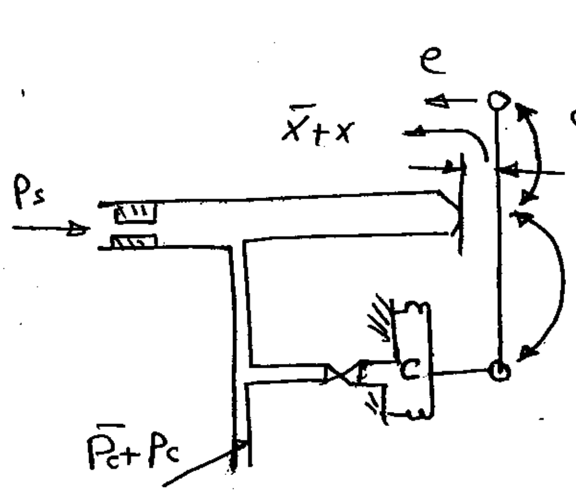
Example:

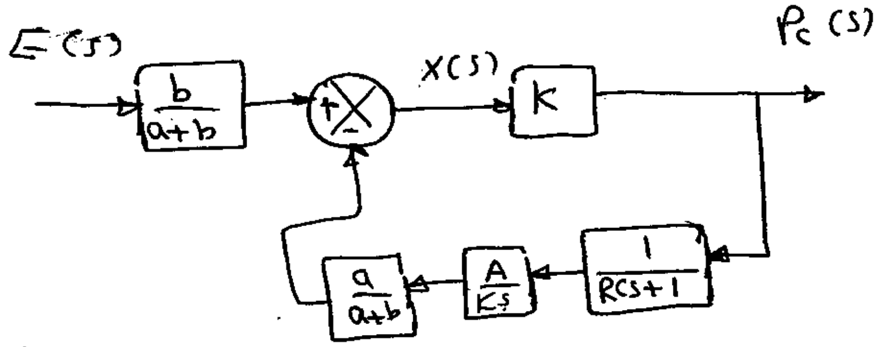




pneumatic proportional controller

(4)





proportional-plus-derivative controller

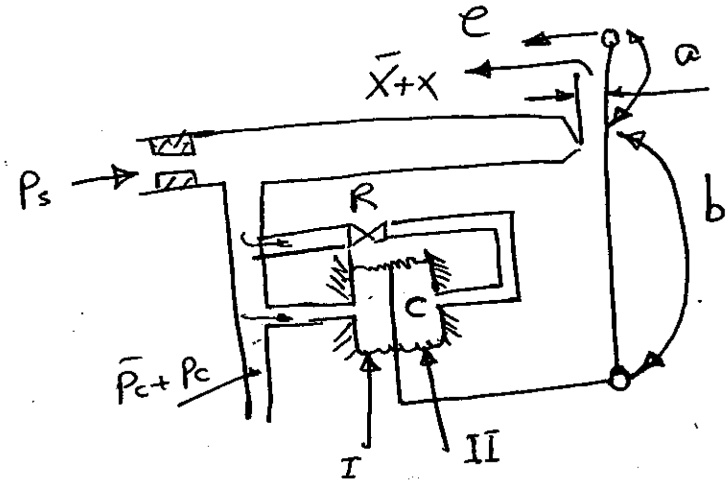
$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b} K}{1 + \frac{Ka}{a+b} \cdot \frac{A}{Ks} \cdot \frac{1}{RCs+1}}$$

In such a controller the loop gain [$KaA/(a+b)Ks(RCs+1)$ is made much greater than unity. Thus:

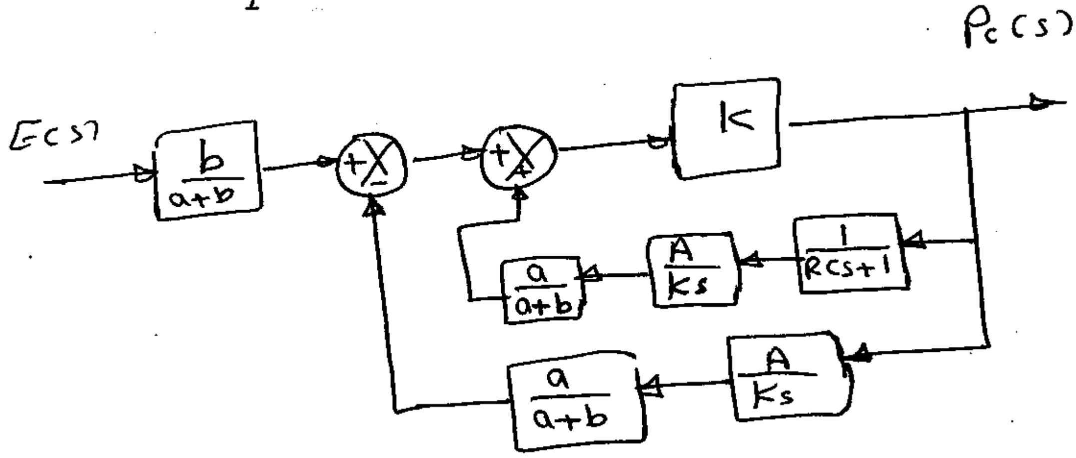
$$\frac{P_c(s)}{E(s)} = K_p(1 + T_d s) \Leftarrow (\text{PD controller})$$

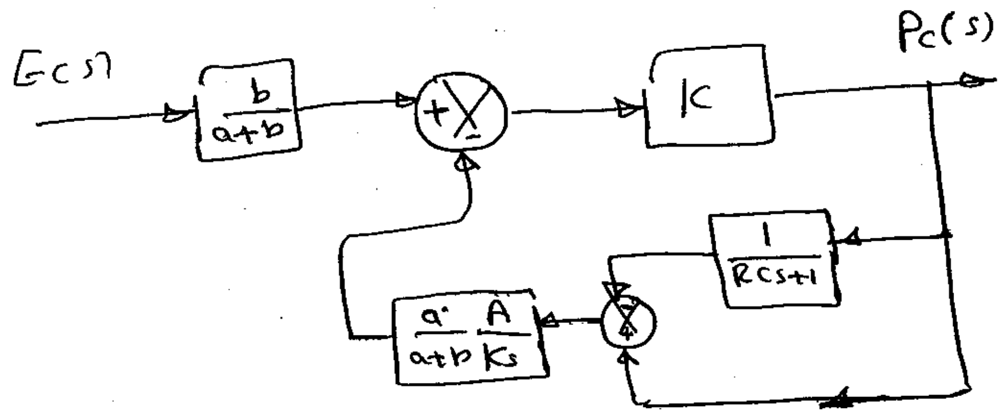
$$K_p = \frac{bKs}{aA} , \quad T_d = RC$$

* obtaining pneumatic proportional - plus - Integral control Action.



(PI controller)





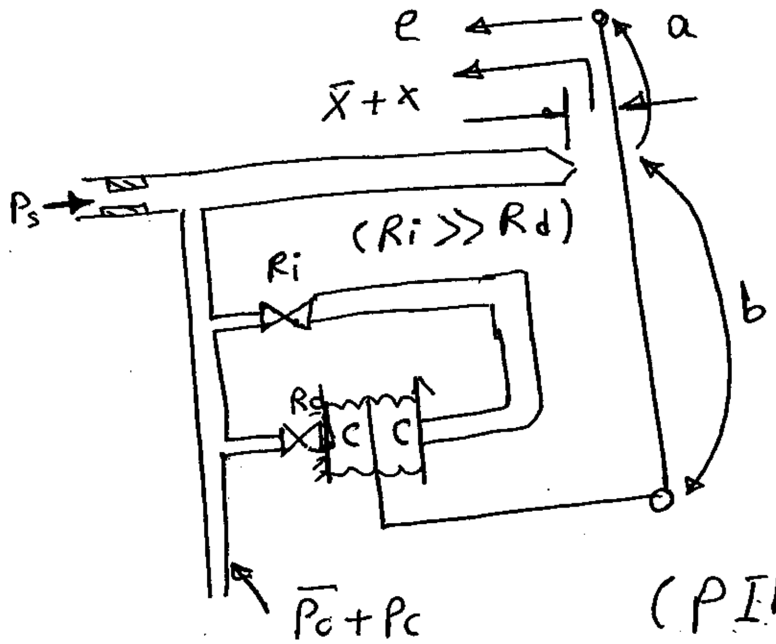
هذا خطأ ، أكرتس (1)

$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b}K}{1+\left[\frac{Ka}{a+b}\frac{A}{Ks}\left(1-\frac{1}{RCs+1}\right)\right]}$$

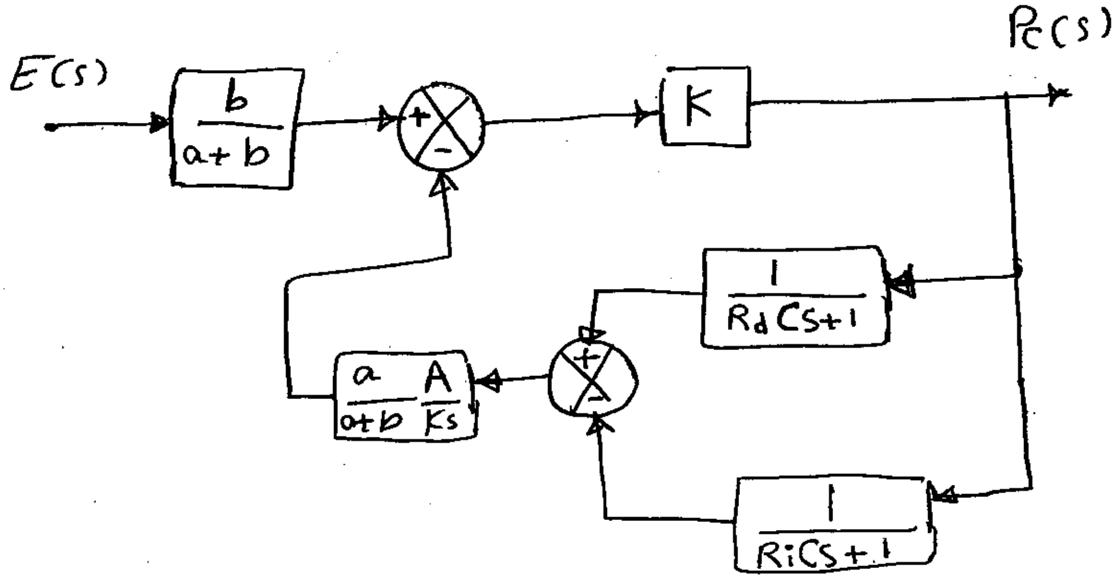
$\frac{P_c(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$, $K_p = bKs$, $T_i = RC$

(6)

* obtaining Pneumatic proportional-plus-integral-plus-Derivative Control Action:



(PID controller)



$$\frac{P_c(s)}{E(s)} = \frac{\frac{bK}{a+b}}{1+\frac{Ka}{a+b}\,\frac{A}{K_s}\,\frac{(R_iC-R_dC)s}{(R_dCs+1)(R_iCs+1)}}$$

هنا ١ عكس الاشارتين (1)

$K_p = \frac{bK_s}{aA}$

$T_i = R_i C$

$T_d = R_d C$

$T_i \gg T_d$

$$\Rightarrow \frac{P_c(s)}{E(s)} = \frac{bK_s}{aA}\,\frac{(T_d s+1)(T_i s+1)}{(T_i-T_d)s}$$

$$= K_p\left(1+\frac{1}{T_i s}+T_d s\right)$$

(3)

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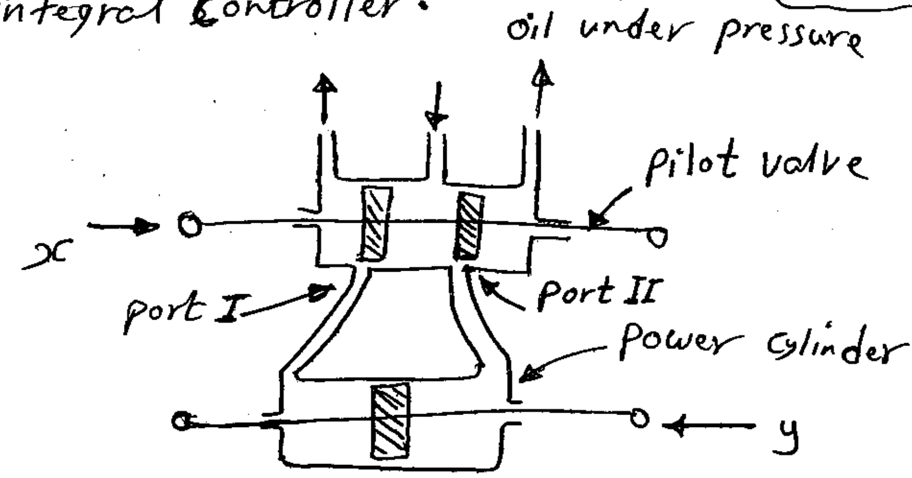
(1)

عماد عبد اللطيف حسين
دكتور داود طه مياتيلد

* Hydraulic systems

* Hydraulic integral Controller.



$$A\rho\, dy = q\, dt$$

$q = kg/sec$

$dt = sec$

$dy = m$

$A = m^2$

$\rho = kg/m^3$

$$q = K_1 x$$

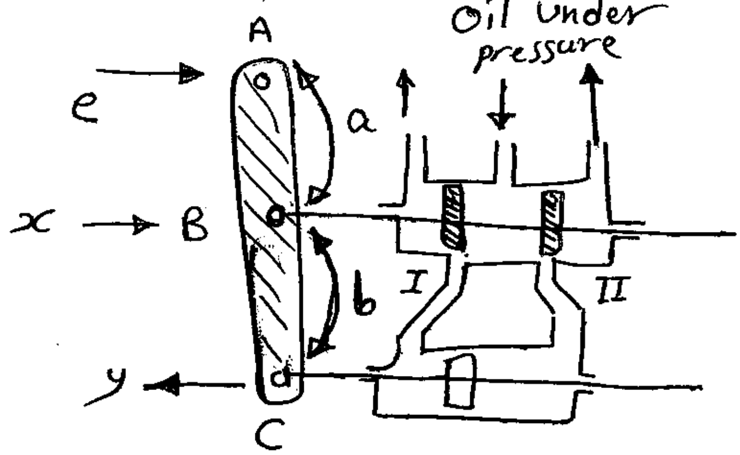
$$A\rho \frac{dy}{dt} = K_1 x$$

$$A\rho s Y(s) = K_1 X(s)$$

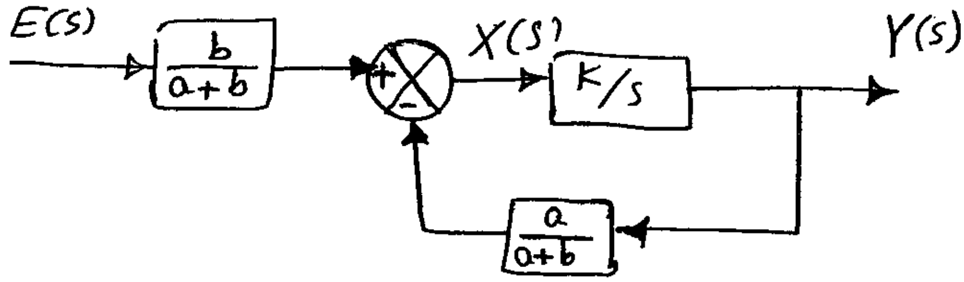
$$\frac{Y(s)}{X(s)} = \frac{K_1}{A\rho s} = \frac{K}{s}$$

$$K = \frac{K_1}{(A\rho)}$$

* Hydraulic proportional controller



Servomotor that acts as a proportional Controller

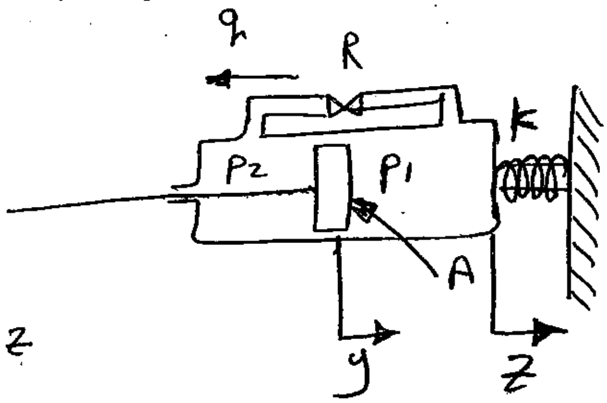


$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b}\,\frac{K}{s}}{1+\frac{K}{s}\,\frac{a}{a+b}}$$

Noting that under the normal operating conditions we have $|[Ka/[s(a+b)]]| >> 1$, this led to :

$$\frac{Y(s)}{E(s)} = \frac{b}{a} = K_p \quad \text{(proportional controller)}.$$

* Dashpots (damper)

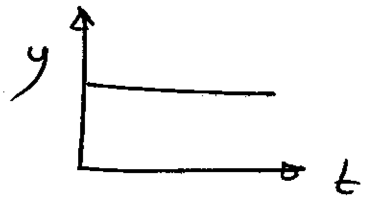


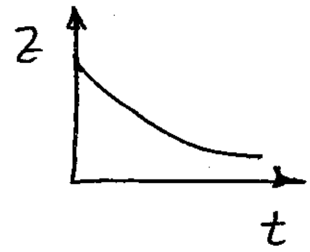
$$A(P_1 - P_2) = kz$$

$A = in^2$

$k = lbf/in$

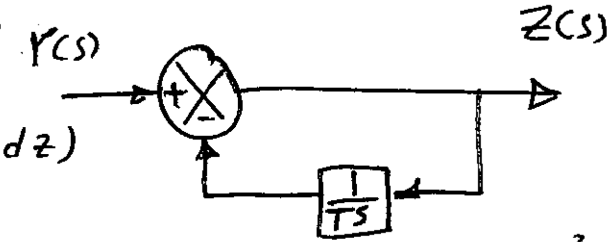
$$q = \frac{P_1 - P_2}{R}$$





$q = lb/sec$

$R = lbf\text{-}sec/in^2\text{-}lb$



$$q\,dt = A\rho(dy - dz)$$

$\rho = lb/in^3$

$dt = sec.$

$$T = \frac{RA^2\rho}{K}$$

$$\frac{dy}{dt} - \frac{dz}{dt} = \frac{q}{A\rho} = \frac{P_1 - P_2}{RA\rho} = \frac{kz}{RA^2\rho}$$

$$\frac{dy}{dt} = \frac{dz}{dt} + \frac{kz}{RA^2\rho}$$

$$sY(s) = sZ(s) + \frac{k}{RA^2\rho} Z(s)$$

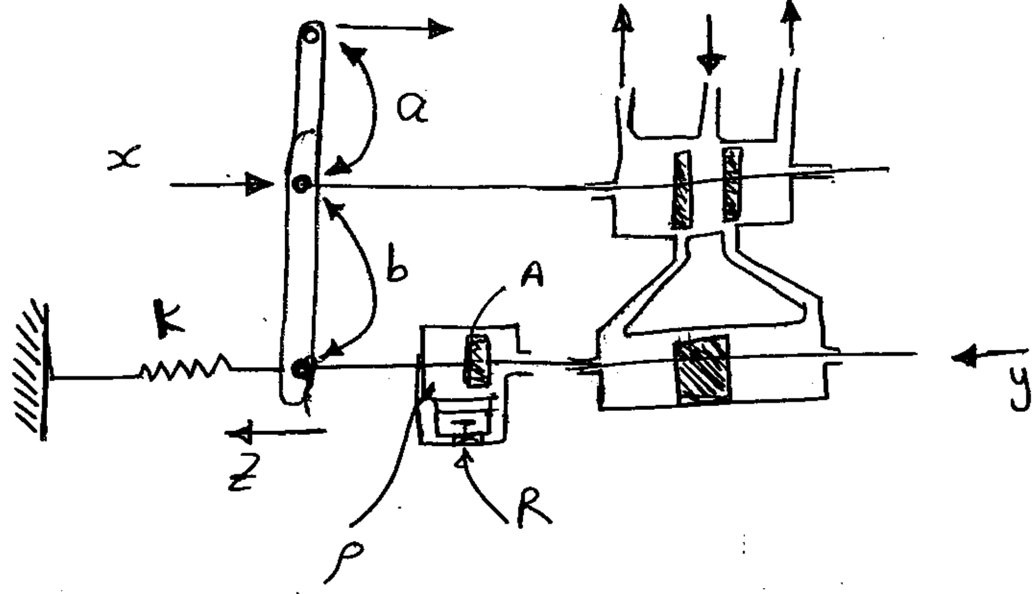
$$\frac{Z(s)}{Y(s)} = \frac{s}{s + \frac{K}{RA^2\rho}}$$

let $T = RA^2\rho/K$

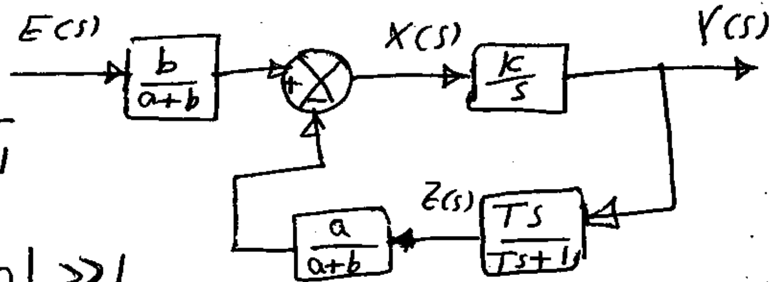
$$\frac{Z(s)}{Y(s)} = \frac{s}{s + \frac{1}{T}} = \frac{TS}{TS+1} = \frac{1}{1 + \frac{1}{TS}}$$

* The dashpot is a differentiating element.

* Obtaining hydralic proportional-plus-Integral Control Action.



$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b}\,\frac{k}{s}}{1+\frac{ka}{a+b}\cdot\frac{T}{Ts+1}}$$



$$\left|kaT/[(a+b)(Ts+1)]\right| \gg 1$$

$$\frac{Y(s)}{E(s)} = \frac{b}{a}\left(\frac{Ts+1}{Ts}\right)$$

$$= K_p\left(1+\frac{1}{T_i s}\right) \qquad \text{(PI controller)}$$

$$T_i = T = \frac{RA^2\rho}{k}$$

* Obtaining hydraulic proportional-plus-Derivative Control Action.

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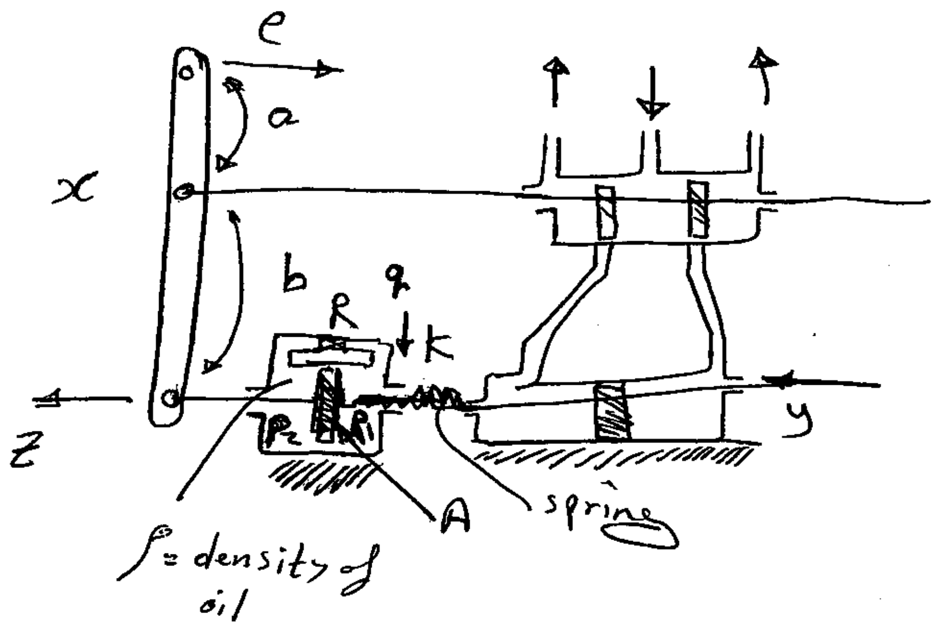


$$k(y-z) = A(P_2 - P_1)$$

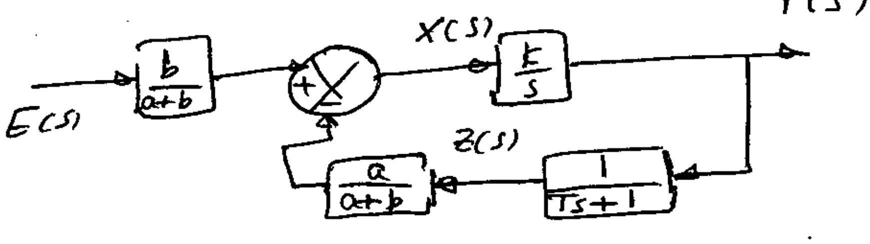
$$q = \frac{P_2 - P_1}{R}$$

$$q\,dt = \rho A\,dz$$

$$y = z + \frac{A}{k} qR = z + \frac{RA^2\rho}{k}\frac{dz}{dt}$$

$$\frac{Z(s)}{Y(s)} = \frac{1}{Ts+1}$$

$$T = \frac{RA^2\rho}{k}$$



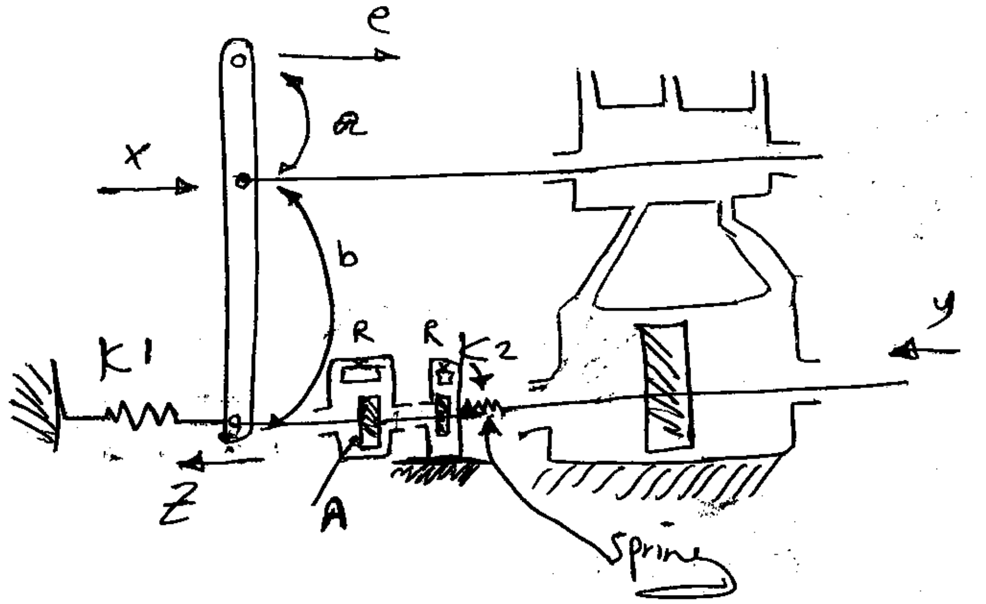
$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b}\frac{k}{s}}{1 + \frac{a}{a+b}\frac{k}{s}\frac{1}{Ts+1}} \Longrightarrow \frac{Y(s)}{E(s)} = K_p(1+Ts)$$

(PD Controller)

where $K_p = \frac{b}{a}$, $T = \frac{RA^2\rho}{K}$

* Obtaining hydraulic proportional-plus-Integral-plus-Derivative control Action. (6)

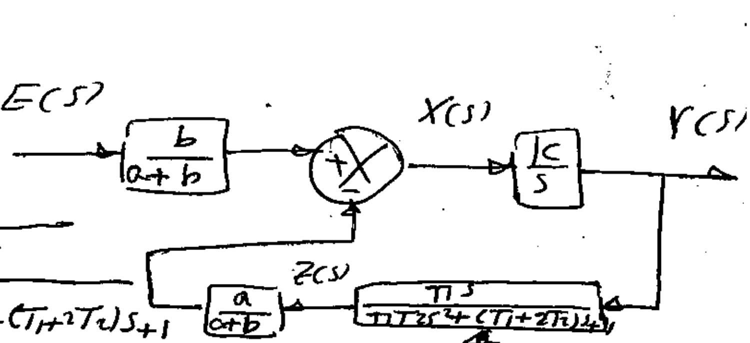
16-4



يرجى مراجعة الكتاب للاطلاع على الرسم بدقة Ogata

$$\frac{Z(s)}{Y(s)} = \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}$$

هذا اشتقاقه سوف نعطيه في المحاضرة



$$\frac{Y(s)}{E(s)} = \frac{b}{a+b} \cdot \frac{\frac{k}{s}}{1 + \frac{a}{a+b}\frac{k}{s} \cdot \frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}}$$

توضيح

$$\boxed{\frac{T_1 s}{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}}$$

$$\frac{Y(s)}{E(s)} = \frac{b}{a} \frac{T_1 T_2 s^2 + (T_1 + 2T_2)s + 1}{T_1 s}$$

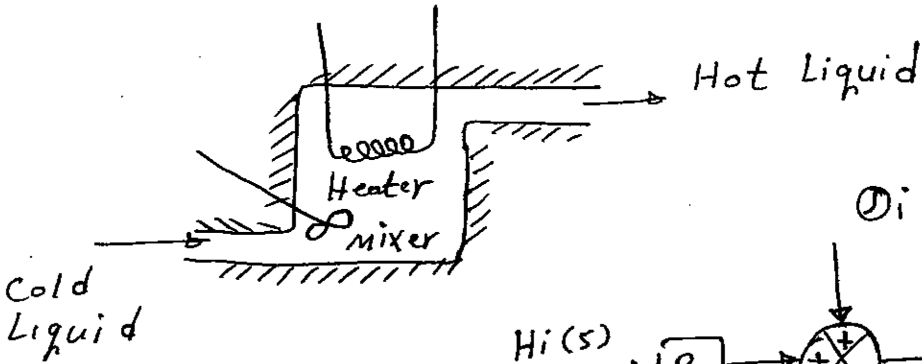
$$= K_p + \frac{K_i}{s} + K_d s$$

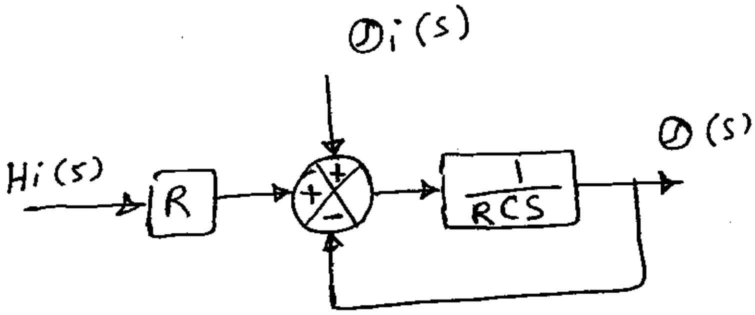
(PID) Controller

where

$$K_p = \frac{b}{a}\frac{T_1 + 2T_2}{T_1}, \quad K_i = \frac{b}{a}\frac{1}{T_1}, \quad K_d = \frac{b}{a}T_2$$

* Thermal systems





$$R = \frac{\Theta}{h_o}$$

$$c\,d\Theta = (h_i - h_o)\,dt$$

$$c\frac{d\Theta}{dt} = h_i - h_o$$

$$RC\frac{d\Theta}{dt} + \Theta = Rh_i$$

$$\frac{\Theta(s)}{H_i(s)} = \frac{R}{RCs+1}$$ ← (لتغير كمية الحرارة)

$$RC\frac{d\Theta}{dt} + \Theta = \Theta_i$$ ← (لتغير درجة الحرارة)

$$\frac{\Theta(s)}{\Theta_i(s)} = \frac{1}{RCS+1}$$

$$RC\frac{d\Theta}{dt} + \Theta = \Theta_i + Rh_i$$ (يستعمل لكلا الحالتين)

حيث

R : thermal Resistance
C : thermal capacitance
Θ : temerature
H : heat rate

i : للدخول
o : للخروج
t : الزمن

نهاية الفصل الرابع