1

امثلة محلولة

حول signal + Block

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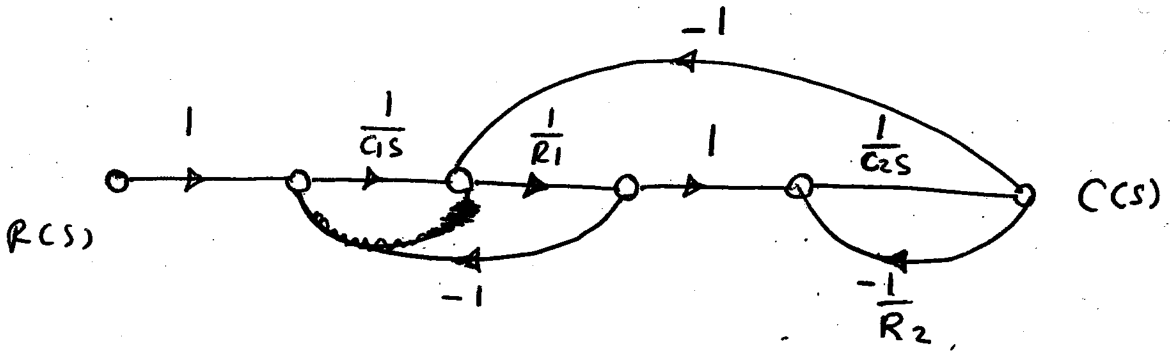
هذه الامثلة مأخوذة من كتاب

Modern Control Engineering
by
Katsuhiko Ogata

example: consider the system shown in the following figure. Obtain the closed-loop transfer function $C(s)/R(s)$.



solution:

$$P_1 = \frac{1}{C_1 s} \cdot \frac{1}{R_1} \cdot \frac{1}{C_2 s}$$

$$L_1 = -\frac{1}{C_1 s} \cdot \frac{1}{R_1}$$

$$L_2 = -\frac{1}{C_2 s} \cdot \frac{1}{R_2}$$

$$L_3 = -\frac{1}{R_1} \cdot \frac{1}{C_2 s}$$

$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2)$$

$$= 1 + \frac{1}{R_1 C_1 s} + \frac{1}{R_2 C_2 s} + \frac{1}{R_1 C_2 s} + \frac{1}{R_1 C_1 R_2 C_2 s^2}$$

(2)

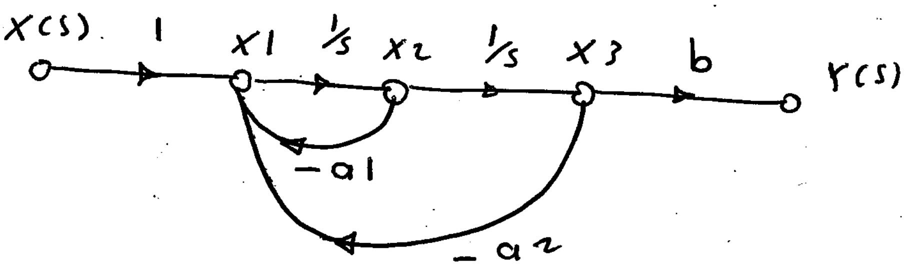
$\Delta_1 = 1$ وذلك لأن P_1 يمس كل من (L_1, L_2, L_3)

$$\therefore \frac{C(s)}{R(s)} = \frac{P_1 \Delta_1}{\Delta}$$

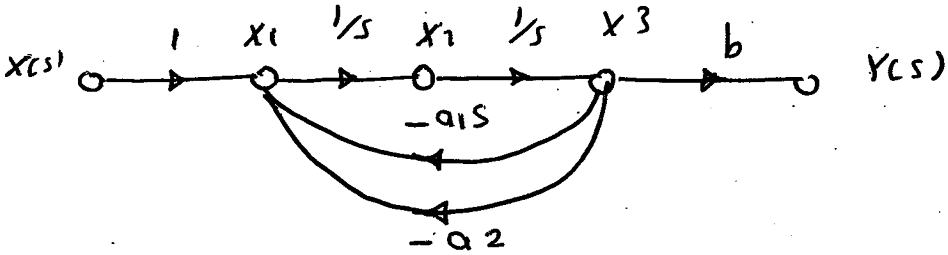
$$= \frac{\dfrac{1}{R_1 C_1 C_2 s^2}}{1 + \dfrac{1}{R_1 C_1 s} + \dfrac{1}{R_2 C_2 s} + \dfrac{1}{R_1 C_2 s} + \dfrac{1}{R_1 C_1 R_2 C_2 s^2}}$$

$$= \frac{R_2}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_2 C_1) s + 1}$$

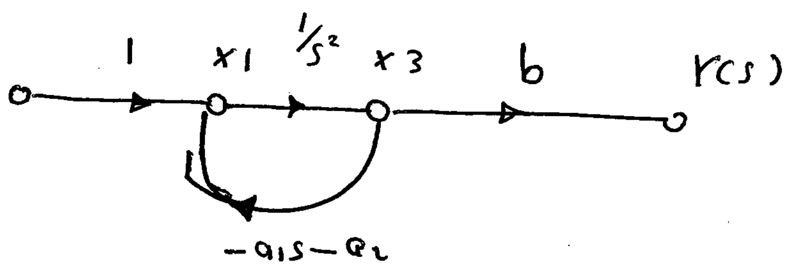
example: obtain the transfer function $Y(s)/X(s)$ of the system shown in the following figure.

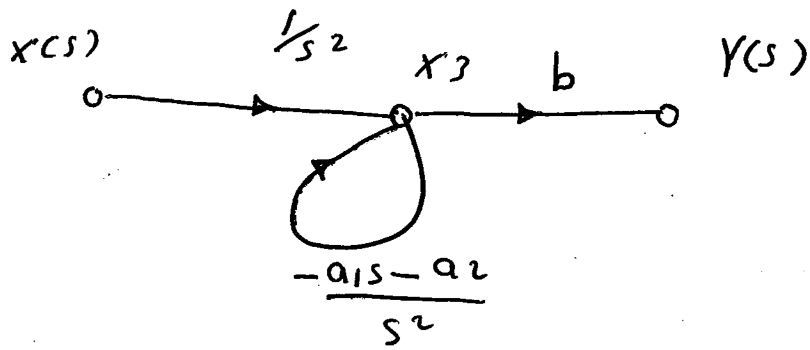


solution



(3)





$$X_3 = \frac{1}{s^2} X - \left(\frac{a_1 s + a_2}{s^2}\right) X_3$$

$$\therefore \quad (s^2 + a_1 s + a_2) X_3 = X$$

$$\frac{Y(s)}{X(s)} = \frac{b X_3}{X} = \frac{b}{s^2 + a_1 s + a_2} \quad \underline{\text{Ans}}$$

طريقة اخرى

$$P_1 = \frac{1}{s} \cdot \frac{1}{s} \cdot b = \frac{b}{s^2}$$

$$L_1 = -\frac{a_1}{s}$$

$$L_2 = -\frac{a_2}{s^2}$$

$$\Delta = 1 - \left(-\frac{a_1}{s} - \frac{a_2}{s^2}\right) = 1 + \frac{a_1}{s} + \frac{a_2}{s^2}$$

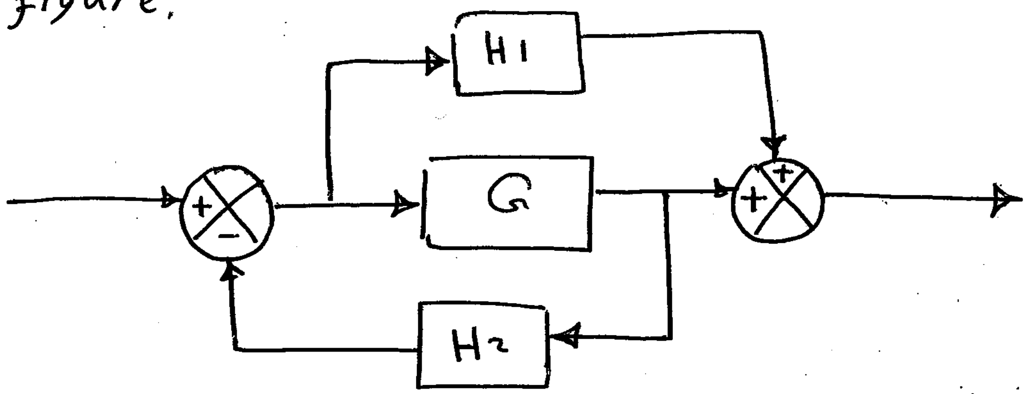
$$\Delta_1 = 1$$

نفس النتائج

$$\therefore \frac{Y(s)}{X(s)} = \frac{P_1 \Delta_1}{\Delta} = \frac{\frac{b}{s^2}}{1 + \frac{a_1}{s} + \frac{a_2}{s^2}} * \frac{s^2}{s^2} = \frac{b}{s^2 + a_1 s + a_2} \quad \underline{\text{Ans}}$$

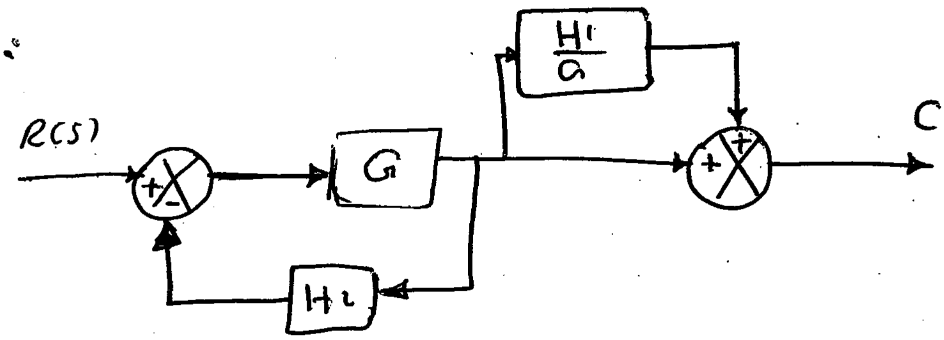
(4)

example: simplify the block diagram of the following figure.

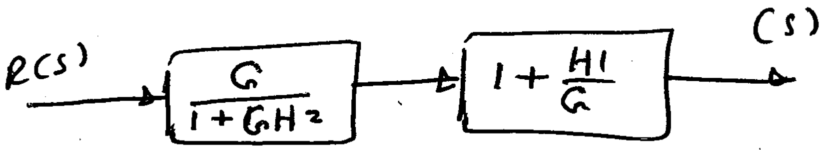


solution:

(a)



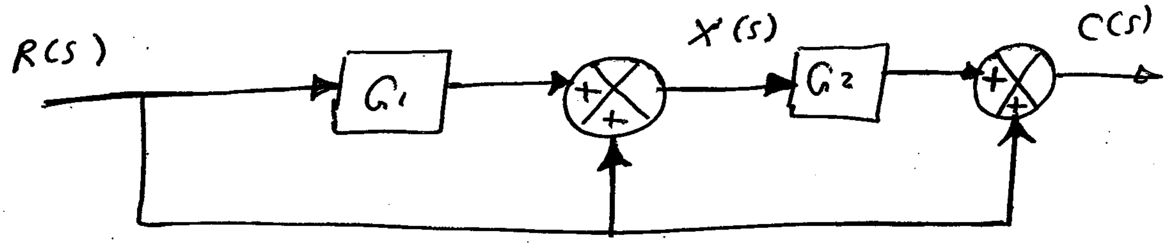
(b)



(c)

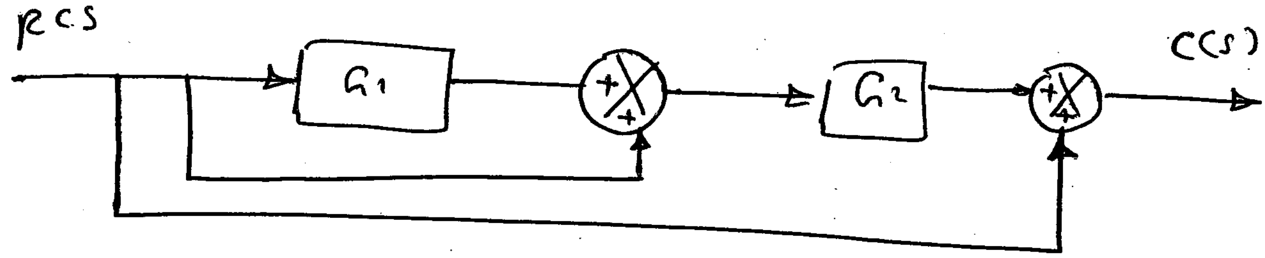
(5)

example: simplify the block diagram of the following figure. obtain the transfer function relating C(s) and R(s).

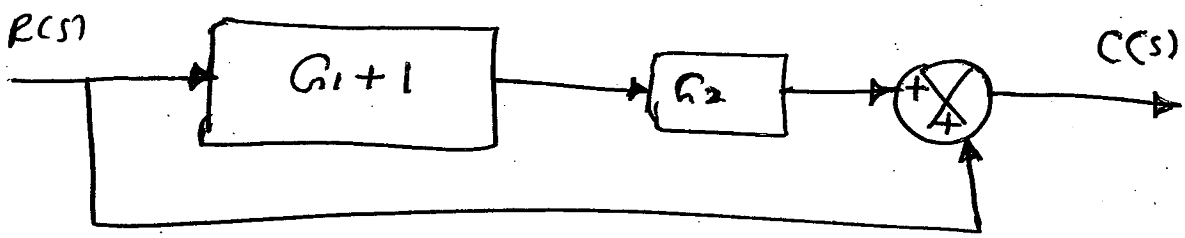


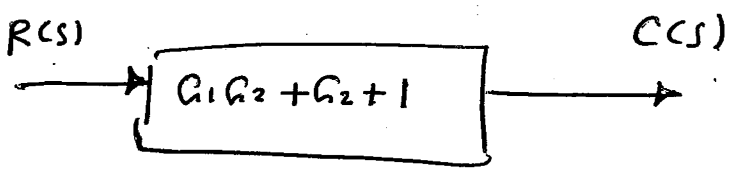
solution:

(a)



(b)

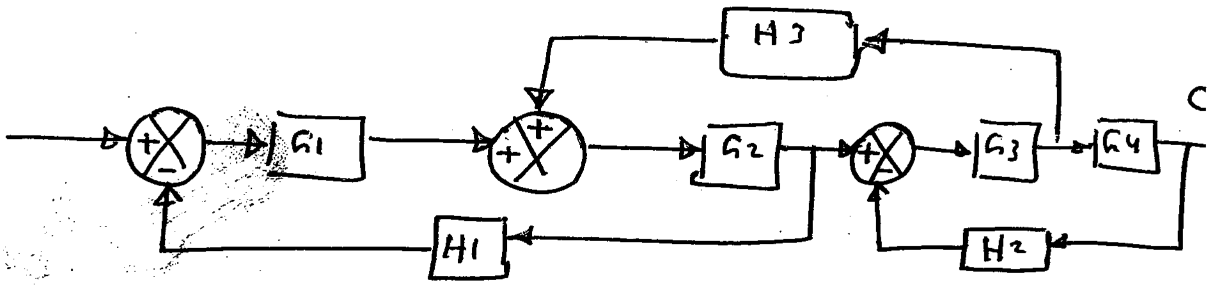




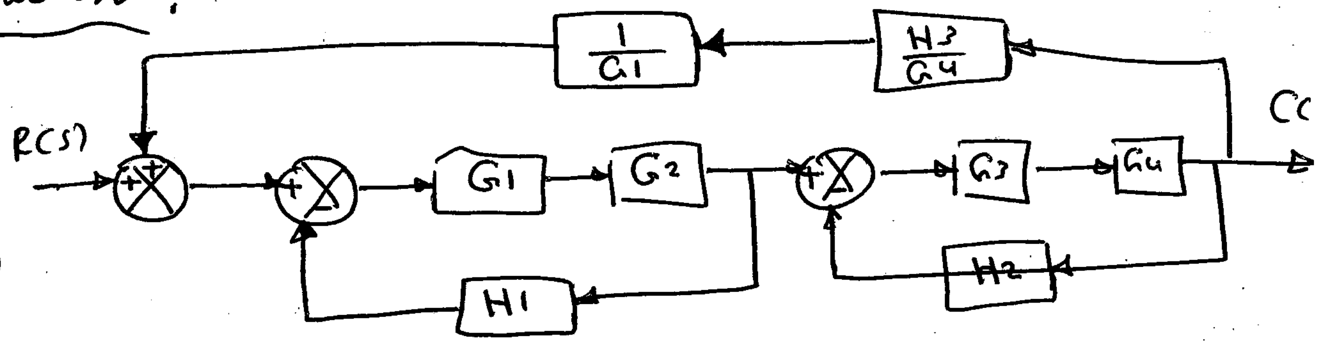
(6)

example :

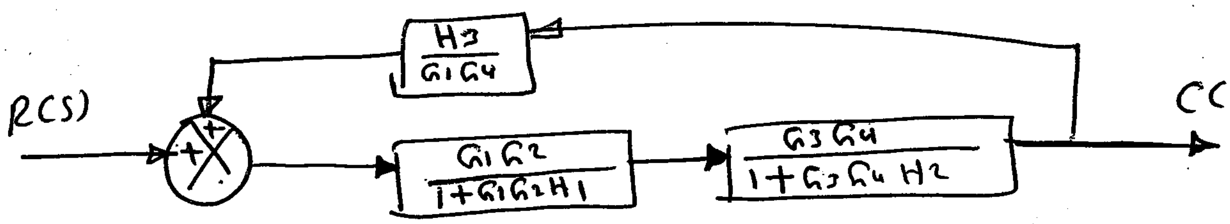
simplify the block diagram of the following figure. Then, obtain the closed-Loop transfer function $C(s)/R(s)$

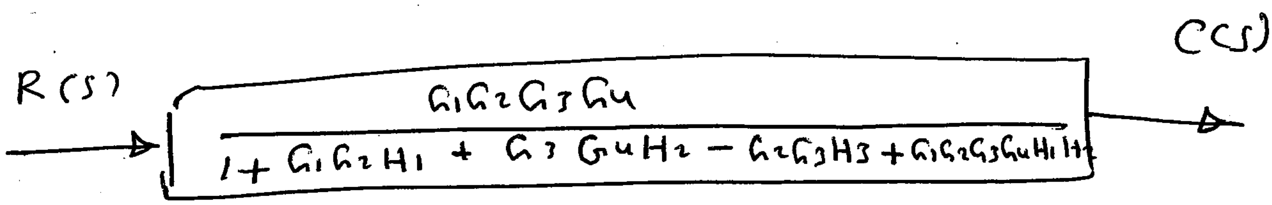


solution :



(a)





$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 - G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$$

(40)

1

تابع للفصل الثالث
امثلة كهربائية + ميكانيكية
(محلولة)

by
عماد عبد الناصر حنوت
دكتوراه هندسة ميكانيكية

33-3

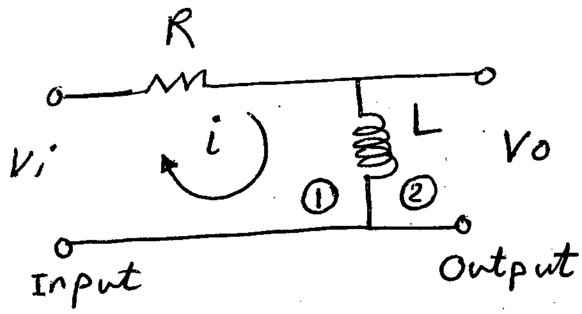
هذه الامثلة مأخوذة من كتاب (Electrical Mechanics and Automatic Control

by J.B. Gupta
S. Hasan Saeed

2010 الطبع الاولى
2011 + الطبع الثانية

فصل (5) في الكتاب)

example : Find the transfer function of the given network.



In mesh(1) we can write:

$$V_i = Ri + L\frac{di}{dt}$$

in mesh(2) :

$$V_o = L\frac{di}{dt}$$

Taking Laplace transform :-

$$V_i(s) = R\,I(s) + sL\,I(s)$$

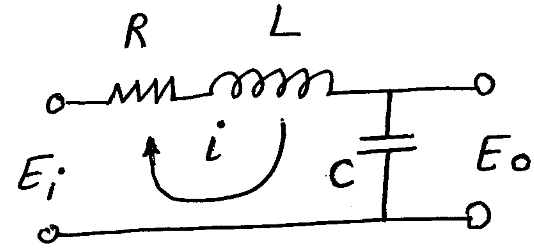
$$V_o(s) = sL\,I(s)$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{sL\,\cancel{I(s)}}{(R+sL)\,\cancel{I(s)}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{sL}{R+sL}$$

(2)

example: Determine the transfer function of the electrical network given below.



In both meshes:

$$E_i = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$E_o = \frac{1}{C}\int i\,dt$$

Taking Laplace transform

$$E_i(s) = R\,I(s) + sL\,I(s) + \frac{1}{Cs} I(s) = I(s)\left[R + sL + \frac{1}{Cs}\right]$$

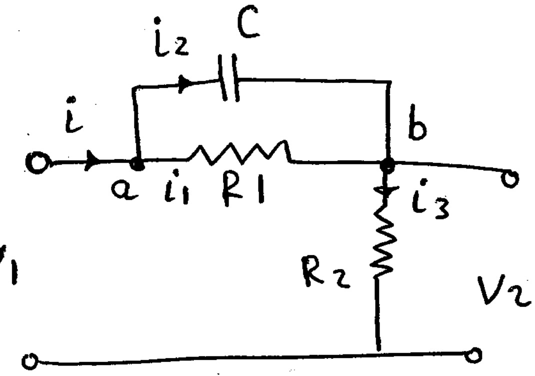
$$E_i(s) = I(s)\left[\frac{RCs + s^2LC + 1}{Cs}\right]$$

$$E_o(s) = \frac{1}{Cs} I(s)$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{\cancel{I(s)}}{\cancel{Cs}} \cdot \frac{\cancel{Cs}}{\cancel{I(s)}\,[s^2LC + sRC + 1]}$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{1}{s^2LC + sRC + 1}$$

example: obtain the transfer function $\frac{V_2(s)}{V_1(s)}$ for the figure shown below.



$$i = i_1 + i_2$$

$$i_1 = \frac{V_1 - V_2}{R_1}$$

$$i_2 = C\frac{d}{dt}(V_1 - V_2)$$

$$i = i_3 = \frac{V_2}{R_2}$$

$$\therefore \frac{V_2}{R_2} = \frac{V_1 - V_2}{R_1} + C\frac{d}{dt}(V_1 - V_2)$$

(3) Taking Laplace transform

$$\frac{V_2(s)}{R_2} = \frac{1}{R_1}V_1(s) - \frac{1}{R_1}V_2(s) + Cs\,V_1(s) - CsV_2(s)$$

$$\frac{V_2(s)}{R_2} + \frac{1}{R_1}V_2(s) + CsV_2(s) = \frac{1}{R_1}V_1(s) + CsV_1(s)$$

$$V_2(s)\left[\frac{1}{R_1} + \frac{1}{R_2} + Cs\right] = V_1(s)\left[\frac{1}{R_1} + Cs\right]$$

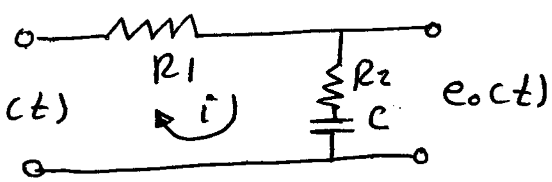
$$V_2(s)\left[\frac{R_1+R_2+R_1R_2Cs}{R_1R_2}\right] = V_1(s)\left[\frac{1+R_1Cs}{R_1}\right]$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R_2 + R_1R_2Cs}{R_1+R_2+R_1R_2Cs}$$

---

example: Find the transfer function of Lag network shown in Figure below.

In both meshes we can write:



$$e_i(t) = R_1 i(t) + R_2 i(t) + \frac{1}{c}\int i(t)\,dt$$

$$e_o(t) = R_2 i(t) + \frac{1}{c}\int i(t)\,dt$$

taking Laplace transform :-

$$E_i(s) = \left[R_1 + R_2 + \frac{1}{Cs}\right]I(s)$$

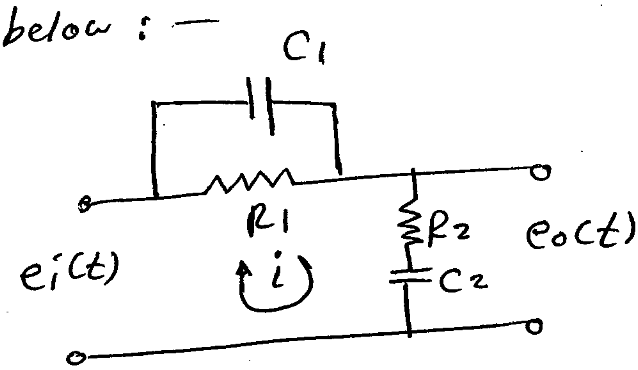
$$E_o(s) = \left[R_2 + \frac{1}{Cs}\right]I(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{\left[R_2 + \frac{1}{Cs}\right]\cancel{I(s)}}{\left[\frac{R_1Cs + R_2Cs + 1}{Cs}\right]\cancel{I(s)}}$$

$$= \frac{[R_2Cs+1]/\cancel{Cs}}{[R_1Cs+R_2Cs+1]/\cancel{Cs}} = \frac{1+R_2Cs}{1+R_1Cs+R_2Cs}$$

(4)

<u>example</u> : Determine the transfer function of the figure below :—



تحليل السؤال

نحدد له الممانعات (Z_1, Z_2)

وكما يلي

$$Z_1 = \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{R_1 C_1 s + 1}$$ (توازي)

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{R_2 C_2 s + 1}{s C_2}$$ (التوالي)

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

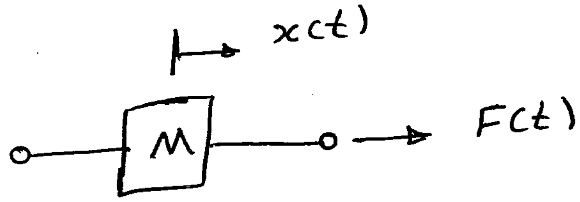
$$\frac{E_o(s)}{E_i(s)} = \frac{(1 + R_2 C_2 s)/sC_2}{\frac{R_1}{R_1 C_1 s + 1} + \frac{R_2 C_2 s + 1}{s C_2}}$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{(1 + R_1 C_1 s)(1 + R_2 C_2 s)}{(1 + R_1 C_1 s)(1 + R_2 C_2 s) + R_1 C_2 s}$$

* Mechanical system

* Translational systems

(1) Inertia Force



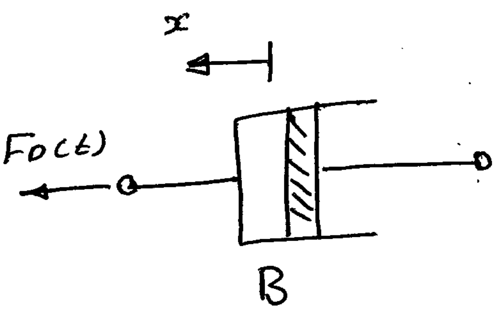
$$F_M(t) = M a(t)$$

$$F_M(t) = M \frac{dv(t)}{dt}$$

$$F_M(t) = M \frac{d^2x(t)}{dt^2}$$

F : Force.
M : mass
a : acceleration

(2) Damping Force

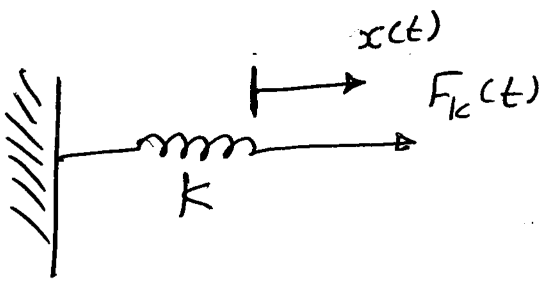


$$F_D(t) = B v(t)$$

$$= B \frac{d}{dt} x(t)$$

B : Damping coefficient (N/m/sec)

(3) spring Force



$$F_k(t) \propto x(t)$$

$$F_k(t) = k x(t)$$

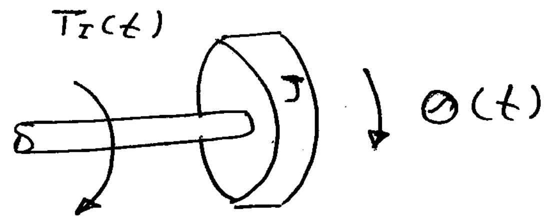
$$F_k(t) = K \int v(t)\, dt$$

k : spring constant or stiffness

(6)

* Rotational Systems

(1) Inertia Torque:



$$T_I(t) = J\alpha(t)$$

$$T_I(t) = J\frac{d}{dt}\omega(t)$$

$$T_I(t) = J\frac{d^2}{dt}\theta(t)$$

where,

$\omega(t)$ = angular velocity

$\theta(t)$ = angular displacement

$\alpha(t)$ = angular acceleration

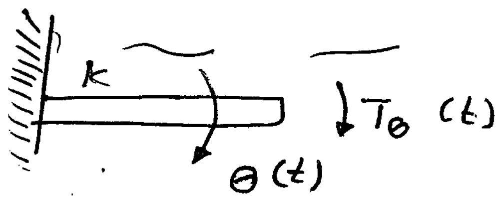
J = moment of inertia

(2) Damping Torque:

$$T_D(t) = B\omega(t)$$

$$T_D(t) = B\frac{d}{dt}\theta(t)$$

(3) spring Torque:



$$T_\theta(t) = K\theta(t)$$

K = N-m/rad

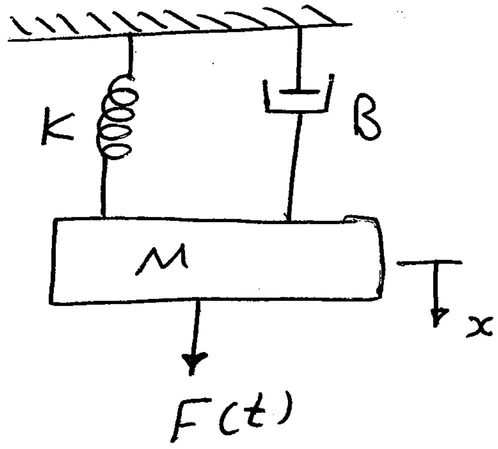
The analogous quantities are:

No.	Translational	Rotational
1	Force, F	Torque, T
2	Acceleration, a	Angular acceleration, α
3	velocity, v	Angular velocity, ω
4	Displacement, x	Angular displacement, θ
5	mass, M	Moment of Inertia, J
6	Damping coefficient, B	Rotational damping Co., B
7	stiffness, K.	Torsional stiffness, K

(7) *D'Alembert's principle

example: (translational)

External force = $F(t)$



Resisting Forces:

(1) Inertia Force

$$F_m(t) = -M \frac{d^2}{dt^2} x(t)$$

(2) Damping Force

$$F_D(t) = -B \frac{d}{dt} x(t)$$

(3) spring Force

$$F_K(t) = -K x(t)$$

According to D'Alembert's principle :-

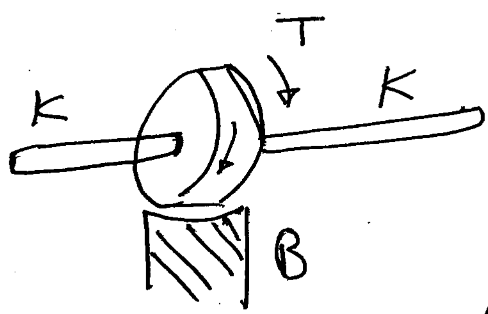
$$F(t) + F_m(t) + F_D(t) + F_K(t) = 0$$

$$F(t) - M \frac{d^2}{dt^2} x(t) - B \frac{d}{dt} x(t) - K x(t) = 0$$

or

$$F(t) = M \frac{d^2}{dt^2} x(t) + B \frac{d}{dt} x(t) + K x(t)$$

example: (torsional)



External Torque = $T(t)$

Resisting Torque;

(1) Inertia Torque

$$T_I(t) = -J \frac{d\omega(t)}{dt}$$

(2) Damping Torque

$$T_D(t) = -B \frac{d}{dt} \theta(t)$$

(3) spring Torque

$$T_K(t) = -K\theta$$

According to D'Alembert principle

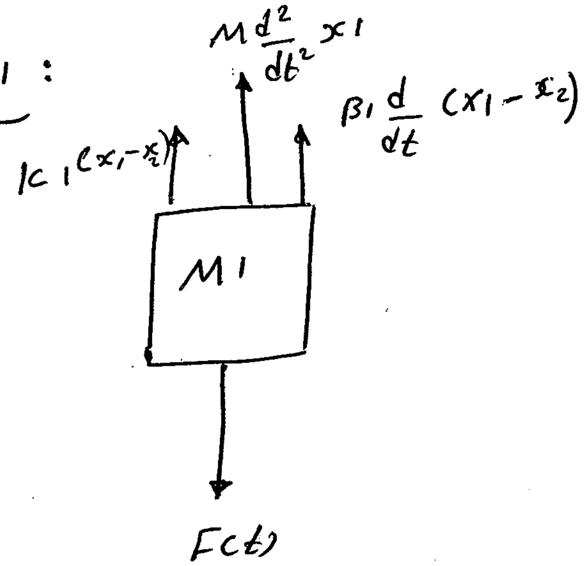
$$T(t) + T_I + T_D + T_K = 0$$

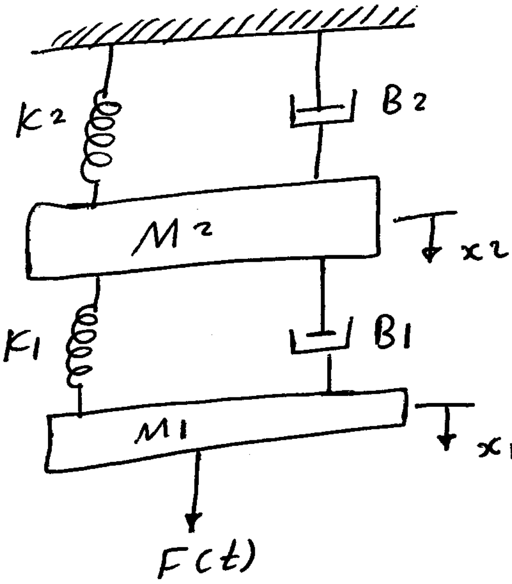
$$T(t) - J \frac{d}{dt} \omega(t) - B \frac{d}{dt} \theta(t) - K\theta(t) = 0$$

$$T(t) = J \frac{d}{dt} \omega(t) + B \frac{d}{dt} \theta(t) + K\theta(t)$$

(8) **Example**: Draw the free body diagram and write the differential equation of the given system shown below.

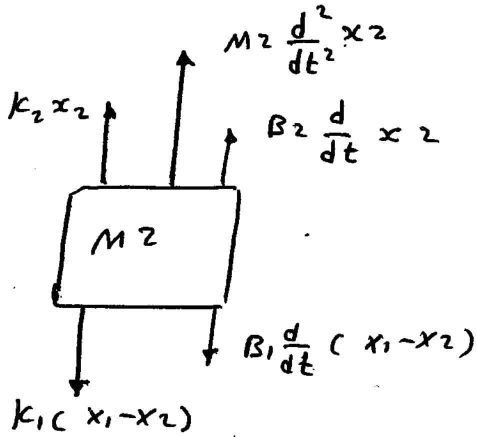
For M1 :





$$F(t) = M_1\frac{d^2}{dt^2}x_1 + B_1\frac{d}{dt}(x_1-x_2) + K_1(x_1-x_2)$$

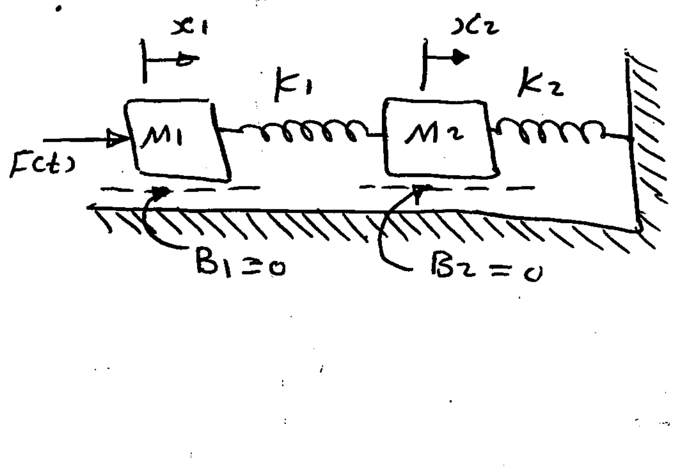
For M2 :



$$K_1(x_1-x_2) + B_1\frac{d}{dt}(x_1-x_2) =$$
$$M_2\frac{d^2x_2}{dt^2} + B_2\frac{dx_2}{dt} + K_2x_2$$

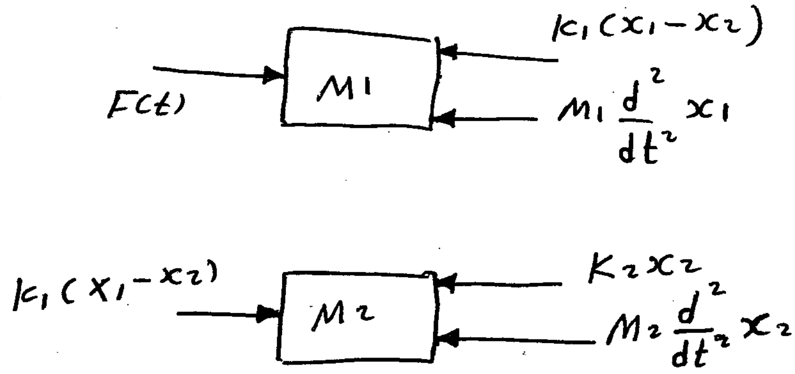
Example:

Write the differential equations describing the dynamics of the system shown in the following figure and find the ratio $\frac{X_2(S)}{F(S)}$.



(9)

Solution :



For M1 :

$$F(t) = M_1 \frac{d^2}{dt^2} x_1 + k_1(x_1 - x_2)$$

For M2 :

$$k_1(x_1 - x_2) = k_2 x_2 + M_2 \frac{d^2}{dt^2} x_2$$

Taking Laplace :

$$F(s) = M_1 s^2 x_1(s) + k_1 X_1(s) - k_1 X_2(s) \quad \text{--- (a)}$$

$$k_1 X_1(s) - k_1 X_2(s) = K_2 X_2(s) + M_2 s^2 X_2(s) \quad \text{--- (b)}$$

From (b) :

$$X_1(s) = \frac{X_2(s)}{k_1}\left[s^2 M_2 + k_1 + K_2\right] \quad \text{--- (c)}$$

sub (c) in (a)

$$F(s) = \frac{X_2(s)}{k_1}\left[s^2 M_2 + k_1 + k_2\right]\left[s^2 M_1 + K_1\right] - K_1 X_2(s)$$

$$\therefore \frac{X_2}{F(s)} = \frac{k_1}{(s^2 M_2 + k_1 + k_2)(s^2 M_1 + k_1) - K_1^2}$$

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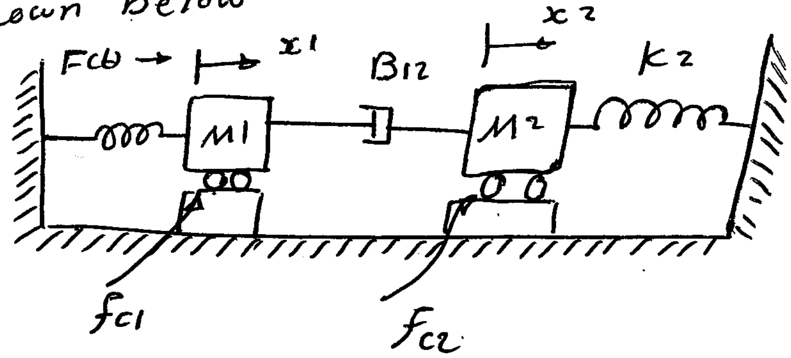
Example :

~~Deri~~
Drive the system equations and find the value of $X_2(s)/F(s)$ for the system shown below.

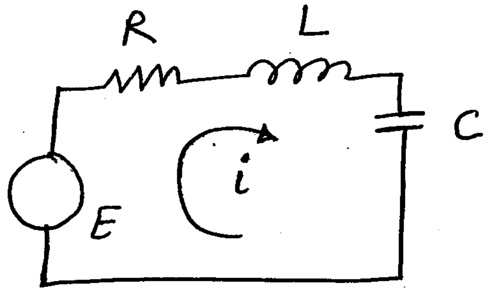


from free body diagrams we can write :

$$\frac{X_2(s)}{F(s)} = \frac{s B_{12}}{(s^2 M_1 + s B_{12} + s f_{c1} + k_1)(s^2 M_2 + s B_{12} + s f_{c2} + k_2) - s^2 B_{12}^2}$$

* Analogous System

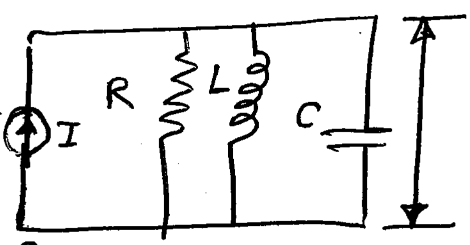
series RLC



$$E = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \quad \text{----(S1)}$$

$$I = \frac{E}{R} + \frac{1}{L}\int E\,dt + C\frac{dE}{dt} \quad \text{---(P1)}$$

parallel RLC



eq (S1) can be write in terms of charge (q) as :

$$E = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}q \quad \text{-----(S2)}$$

(11)

equation (P1) can be wribe in terms of magnetic flux linkage "

$$I = \frac{1}{R}\left(\frac{d\phi}{dt}\right) + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2} \quad \text{---- (P2)}$$

where $\phi = \int E dt$, $E = d\phi/dt$

from eq. ($E = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}q$) and

eq. ($F(t) = M\frac{d^2}{dt^2}x(t) + B\frac{d}{dt}x(t) + Kx(t)$)

we can write the following table of (force-voltage analogy)

No.	Mechanical Translational system	Electrical system
1	Force (F)	Voltage (E)
2	Mass (M)	Inductance (L)
3	Stiffness (K) (Elastance, 1/K	Reciprocal of capacitance (1/C) capacitance (C)
4 5	Displacement (X)	charge (q)

* from eq. ($F(t) = M\frac{d^2}{dt^2}x(t) + B\frac{d}{dt}x(t) + Kx(t)$) and

eq. ($I = \frac{1}{R}\left(\frac{d\phi}{dt}\right) + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2}$)

The force - current analogy can be written as :

No.	Mechanical Translational system	Electrical system
1	Force (F)	Current (I)
2	mass (M)	Capacitance (C)
3	Damping coeff. (B)	Reciprocal of resistance (1/R) (i.e. conductance (G))
4	Stiffness (K) (Elastance (1/K))	Reciprocal of inductance (1/L) (inductance (L))
5	Displacement (X)	Flux linkage (ϕ)
	Velocity ($\dot{X}$)	Voltage (E)

(12) From eq. ($T(t) = J \frac{d}{dt} \omega(t) + B \frac{d}{dt} \theta(t) + K\theta(t)$) and eq. ($E = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}q$),

The Torque-voltage Analogy can be written as:

No.	Mechanical Rotational system	Electrical system
1	Torque (T)	Voltage (E)
2	Moment of inertia (J)	inductance (L)
3	Damping coeff. (B)	Resistance (R)
4	Stiffness (K) (Elastance, 1/K)	Reciprocal of capacitance (1/C) (capacitance (C))
5.	Angular displacement (θ)	Charge (q)
6.	Angular velocity (ω)	current (i)

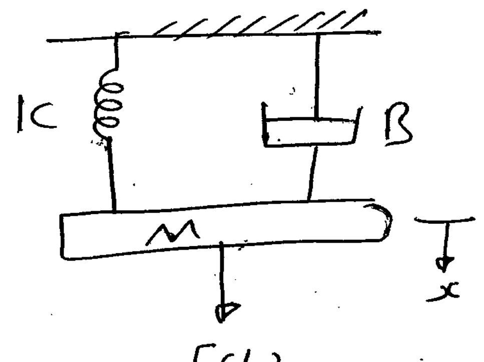
From eq. ($T(t) = J\frac{d}{dt}\omega(t) + B\frac{d}{dt}\theta(t) + K\theta(t)$) and eq. ($I = \frac{1}{R}(\frac{d\phi}{dt}) + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2}$),

The Torque - Current analogy can be written as:-

No.	Mechanical Rotational system	Electrical system
1	Torque (T)	Current (I)
2	Moment of inertia (J)	Capacitance (C)
3	Damping coeff. (B)	Reciprocal of Resistance (1/R) (i.e. conductance (G)).
4	Stiffness (K) (Elastance, 1/K)	Reciprocal of Inductance (1/L) (Inductance (L))
5	Angular displacement (θ)	Flux linkage (ϕ)

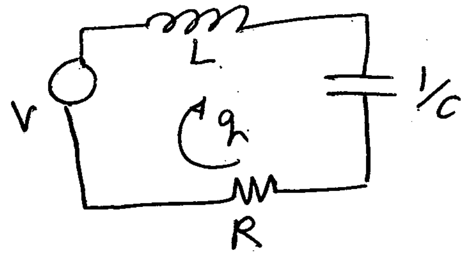
(13) Example:

Draw the analogous electrical network of the given system. (use f-v analogous).



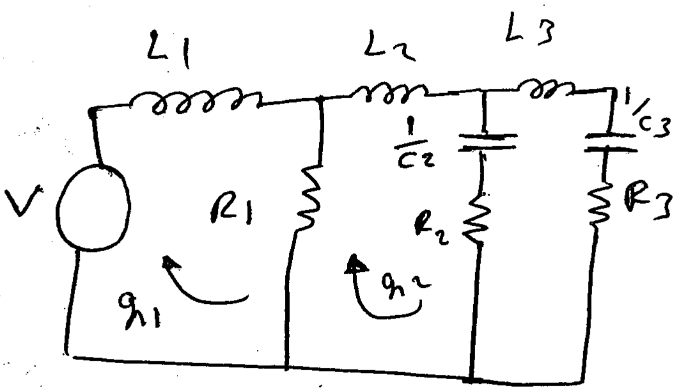
$$f = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$

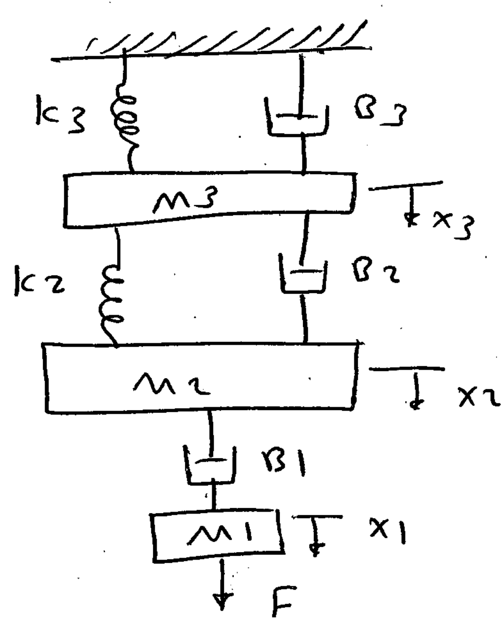
$$v = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q$$



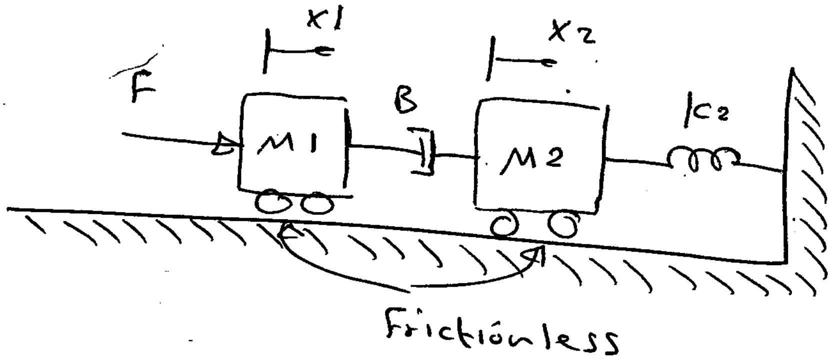
series RLC circuit

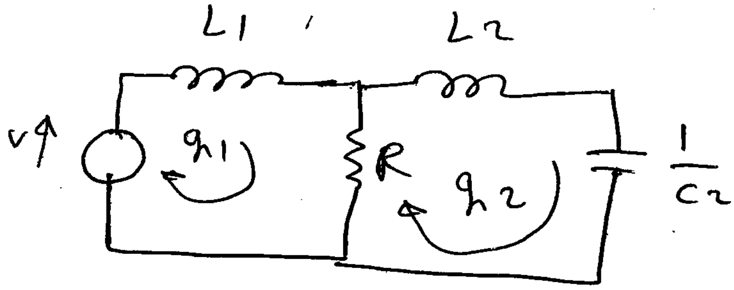
Example: Draw the analogous electrical network of the given system. (use f-v analogy).



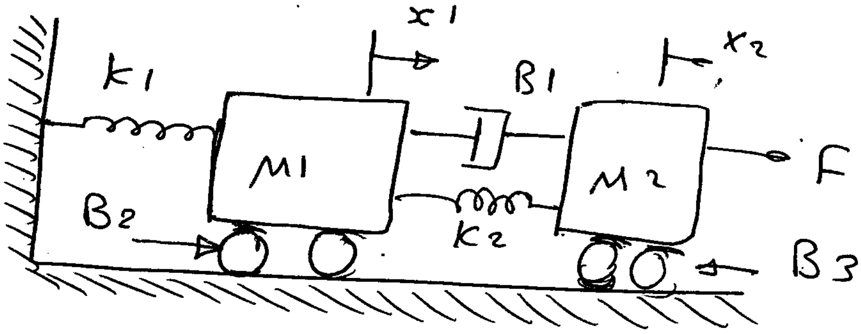


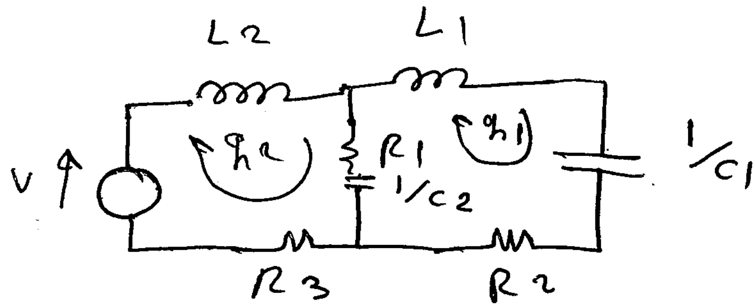
(14) Example: Draw the electrical analogous circuit of the following system.



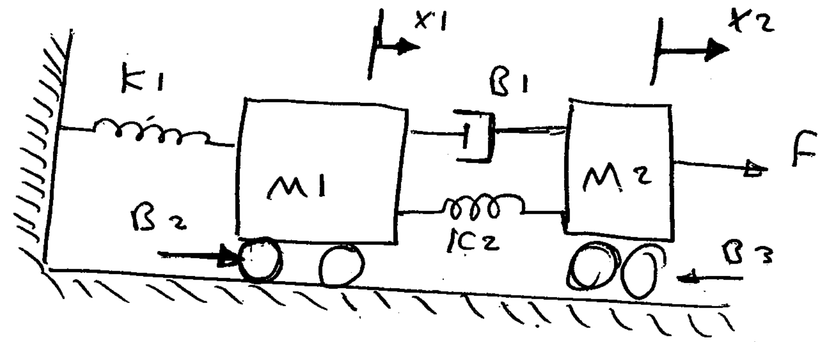


Example: Draw the analogous electrical circuit of the system shown below (use f-v) ~~and~~

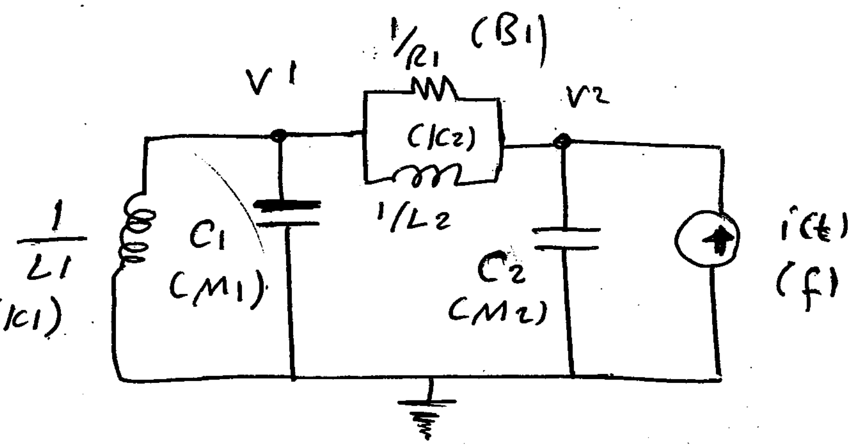




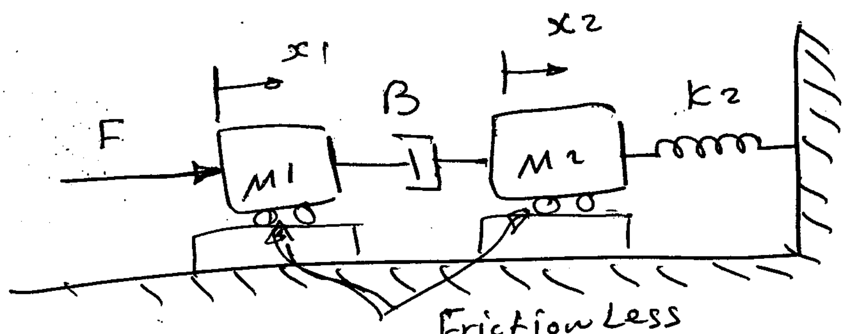
Example: Draw the analogous electrical circuit of the given system (use f-i analogy. Assume frictionless wheels)

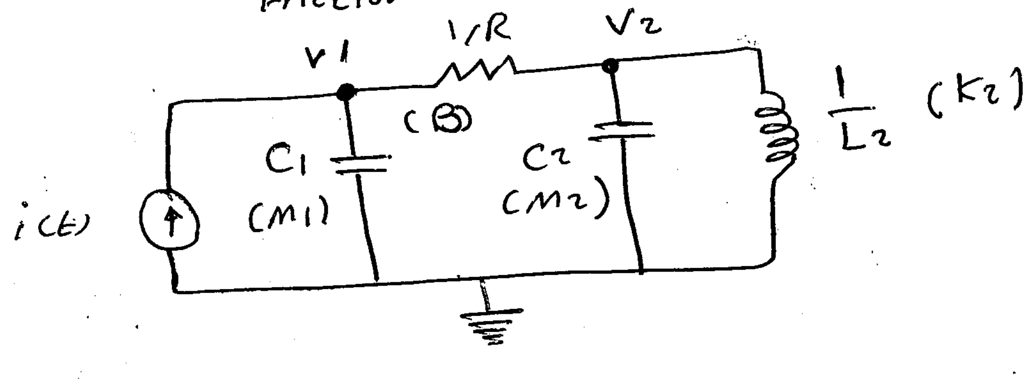


$B_2 = B_3 = 0$



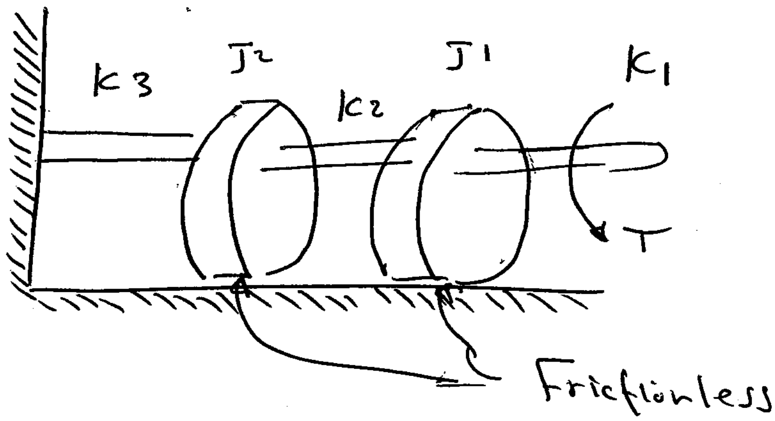
Example: Draw the analogous electrical circuit of the system shown in the figure below (use f-i analogy).

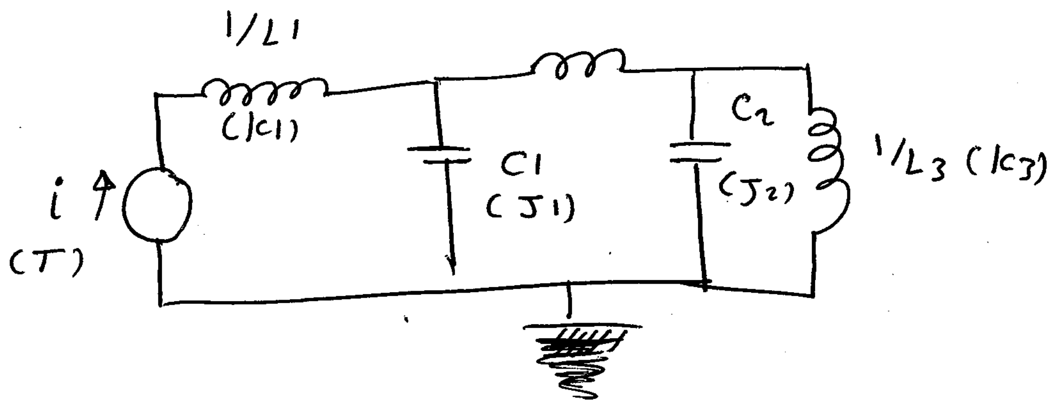




(16)

example : Draw the analogous electrical circuit of a rotational mechanical system shown in the following figure (use f-i analogy).

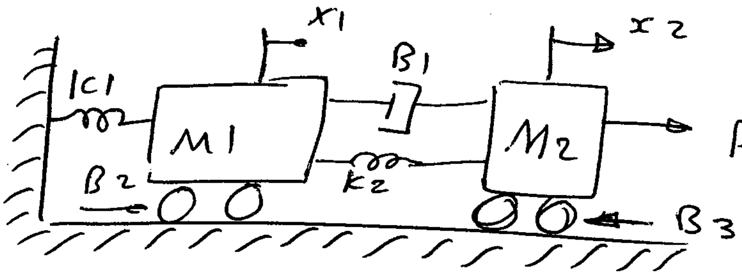


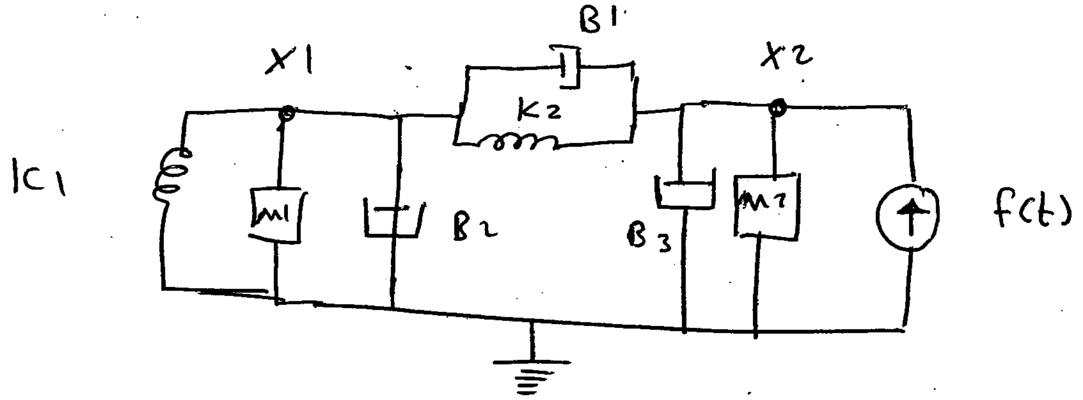


(17)

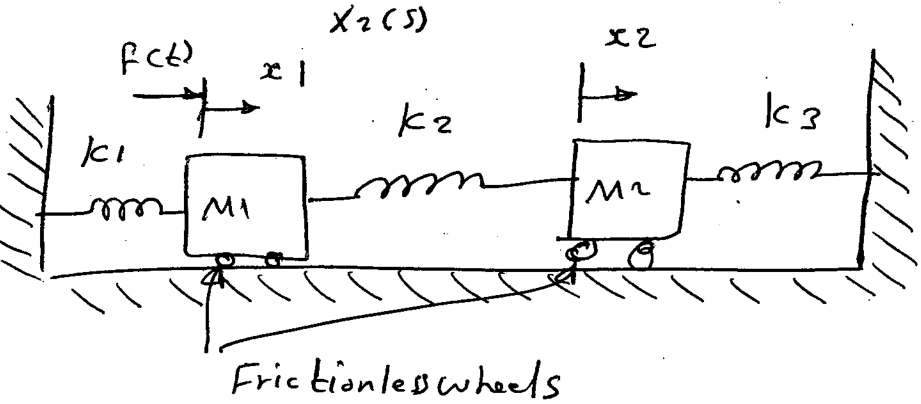
* Mechanical Equivalent Network.

example: Draw the mechanical network of the sytem shown In the following figure

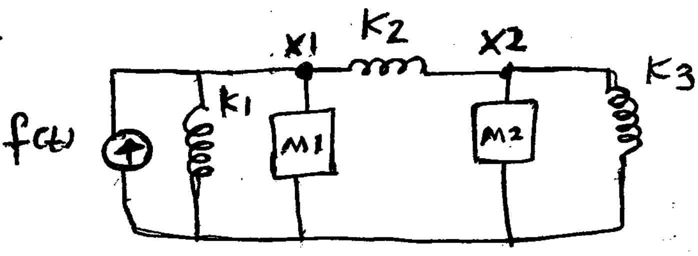




example: Draw the mechanical equivalent network, write the system equation and find $\frac{F(s)}{X_2(s)}$.



$$\frac{F(s)}{X_2(s)} = \frac{(s^2 M_1 + K_1 + K_2)(s^2 M_2 + K_2 + K_3) - K_2^2}{K_2}$$



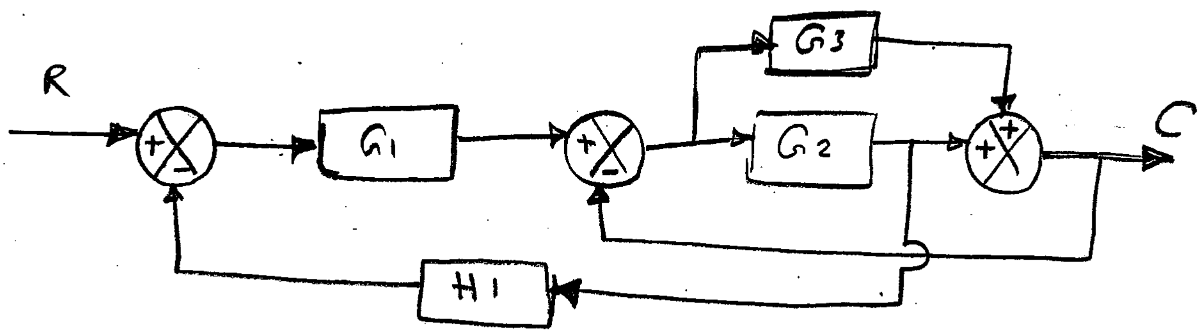
(1)

50 - 3

عماد عبيد الماضي
دكتوراه هندسة ميكانيكية

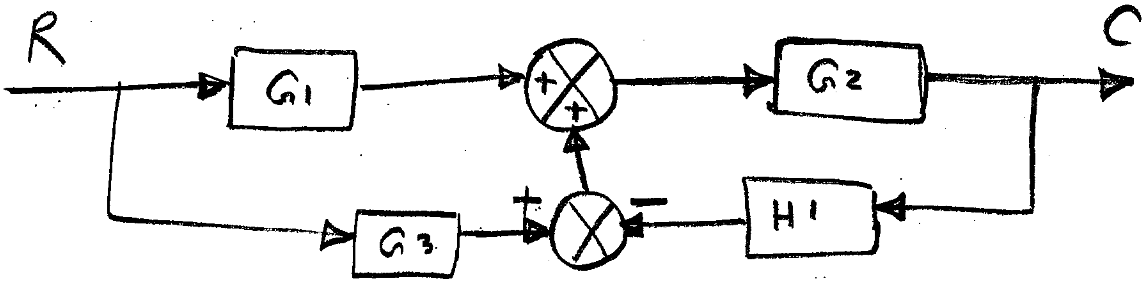
<u>Sheet no. 1</u> Chapter Three

Q1/ Determine the transfer function C/R from the block diagram shown in the fig. 1. (Ans. $\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_2 + G_3 + G_1G_2H_1}$



(Fig. 1)

Q2/ For the block diagram shown in the figure below, determine the overall transfer function. (Ans. $\frac{C}{R} = \frac{G_1G_2 + G_2G_3}{1 + G_2H_1}$)

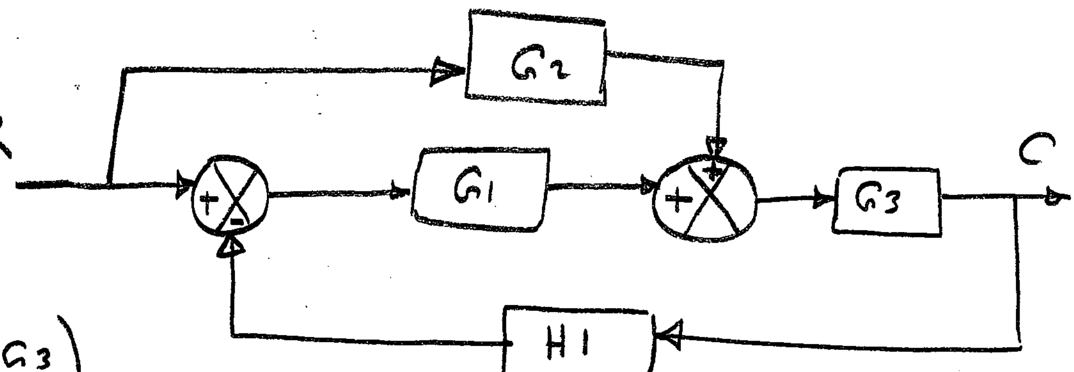


(Fig. 2)

Q3/ Reduce the block diagram shown in (Fig.3) into:

(i) a form having one block in the forward path and one in feedback path

(ii) single block representation form.

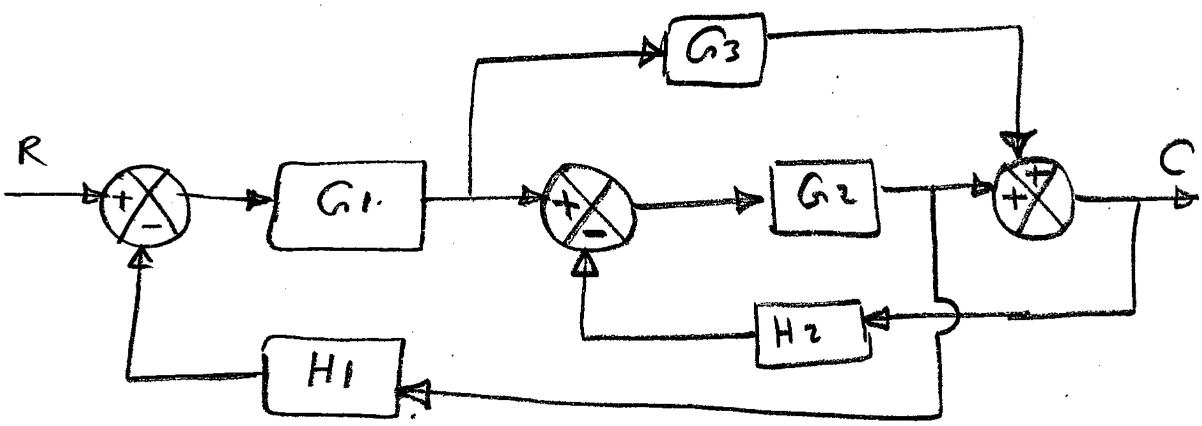


Ans.

$$\frac{C}{R} = \frac{G_1G_3 + G_2G_3}{1 + G_1G_3H_1}$$

(Fig.3)

Q4/ Determine the overall transfer function relating C and R for the system whose block diagram is shown in Fig.4. (Ans. $\frac{C}{R} = \frac{G_1G_2 + G_1G_3}{1 + G_2H_2 + G_1G_2H_1 - G_1G_2G_3H_1H_2}$



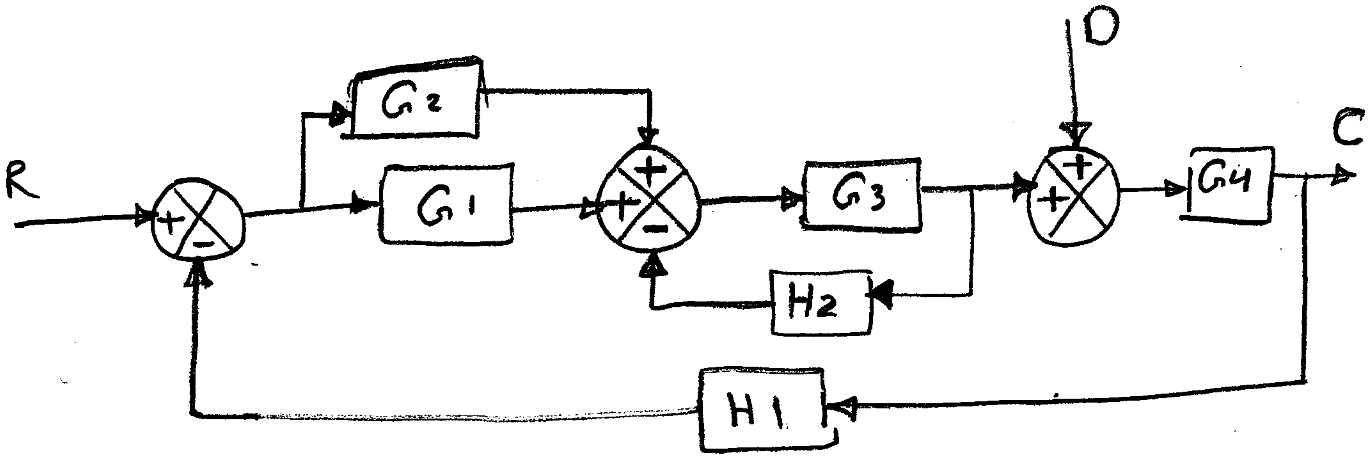
(Fig.4)

Q5/ Determine the ratio C/R , C/D and the total output for the system whose block diagram is shown in Fig. 5 . (Ans.

$$\frac{C}{R} = \frac{G_1 G_3 G_4 + G_2 G_3 G_4}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1}$$

$$\frac{C}{D} = \frac{G_4 (1 + G_3 H_2)}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1}$$

$$\text{Total output} = \frac{G_1 G_3 G_4 + G_2 G_3 G_4}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1} \cdot R + \frac{G_4 (1 + G_3 H_2)}{1 + G_3 H_2 + G_1 G_3 G_4 H_1 + G_2 G_3 G_4 H_1} \cdot D$$



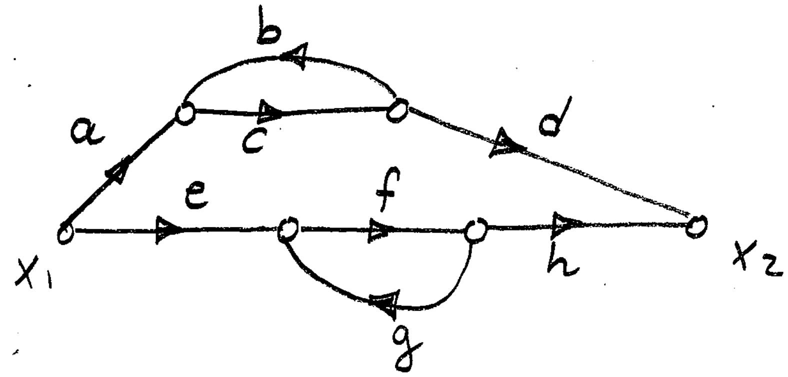
(Fig. 5)

هذه الأسئلة مأخوذة من كتاب
(Linear control systems) by Manke)

عماد عبدالستار كاظم خيوت
دكتوراه هندسة ميكانيكية

① Sheet no. 2 — chapter Three

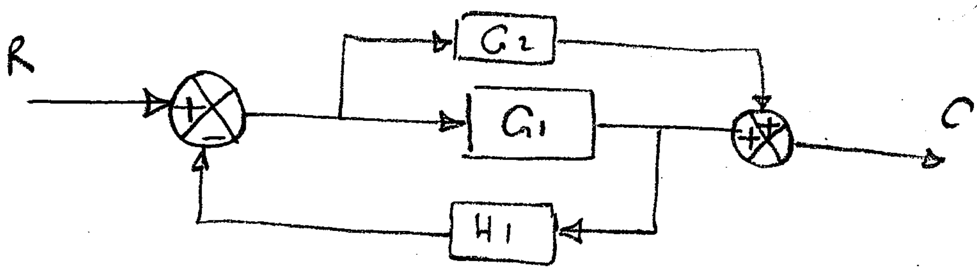
Q1/ Determine the overall transmittance relating x_2 and x_1



(Ans. $\dfrac{x_2}{x_1} = \dfrac{acd(1-gf) + efh(1-bc)}{1-(bc+gf)+bcfg}$

~~~~~~~~~~

Q2/ Draw a signal flow graph for the system whose block diagram is shown in the following figure. Determine the overall transmittance.



(Ans. $\dfrac{C}{R} = \dfrac{G_1 + G_2}{1 + G_1 H_1}$)
~~~~~~~~~~