(1)

from (1) to (9)

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دكتوراه هندسه ميكانيك

Chapter Three

Mathematical Modeling of Dynamic systems

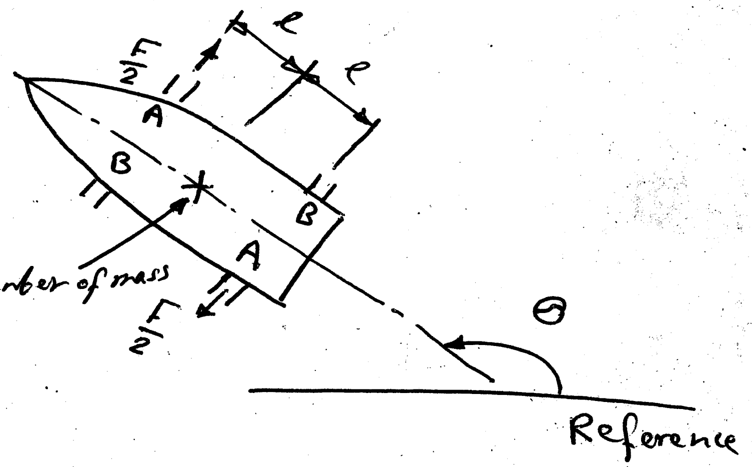
$$\text{transfer function} = G(s) = \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}$$

$$= \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

$n \geq m$

example:

(schematic diagram of a satellite abbitude) control system



$$J \frac{d^2\theta}{dt^2} = T$$

$$J s^2 \Theta(s) = T(s)$$

$$\frac{\Theta(s)}{T(s)} = \frac{1}{J s^2}$$ ⟵ Transfer function

T: Torque

θ: angular displacement

J: moment of inertia

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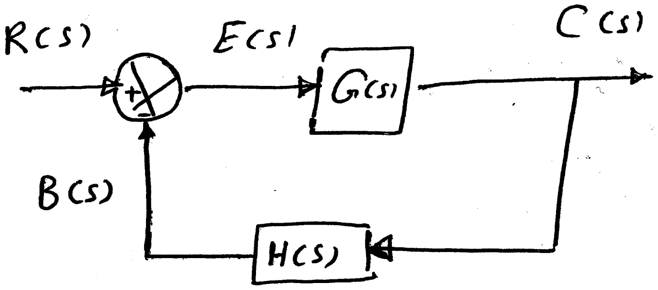
(2)

* open-Loop Transfer function and Feedforward Function.



$$\text{open-loop transfer function} = \frac{B(s)}{E(s)} = G(s)H(s)$$

$$\text{Feedforward transfer function} = \frac{C(s)}{E(s)} = G(s)$$

* Closed-Loop Transfer function.

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)\,C(s)$$

$$\therefore C(s) = G(s)\left[R(s) - H(s)\,C(s)\right]$$

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

$$C(s) + G(s)H(s)\,C(s) = G(s)R(s)$$

$$C(s)\left[1 + G(s)H(s)\right] = G(s)R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$C(s) = \frac{G(s)}{1 + G(s)H(s)}R(s)$$

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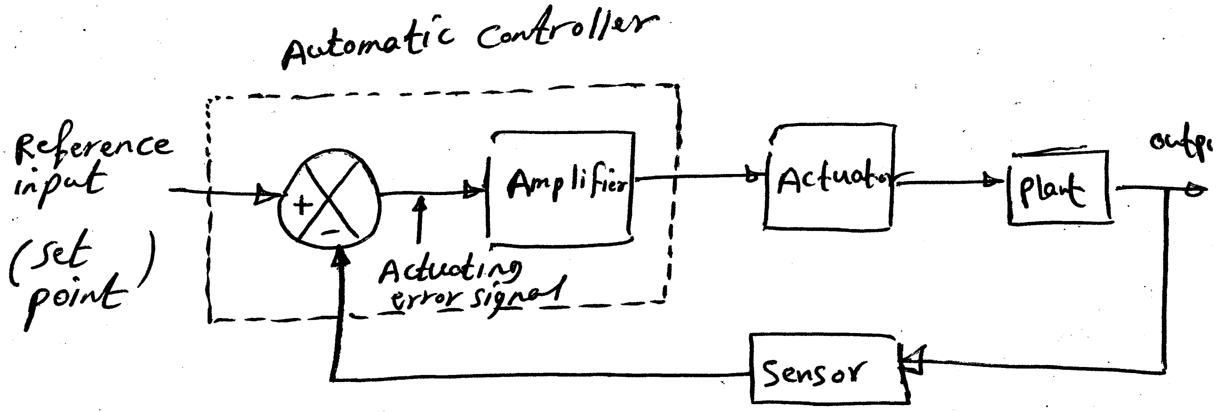
(3)

* Automatic controllers



* Block diagram of an industrial control system, which consists of an automatic controller, an actuator, a plant, and a sensor (measuring element).

* Classifications of industrial controlers.

1- Two-position or on-off controllers
2- proportional controllers
3- Integral controllers
4- proportional-plus- ~~integr~~ integral controllers
5- proportional-plus- derivative controllers
6- proportional-plus- integral-plus- derivative controllers

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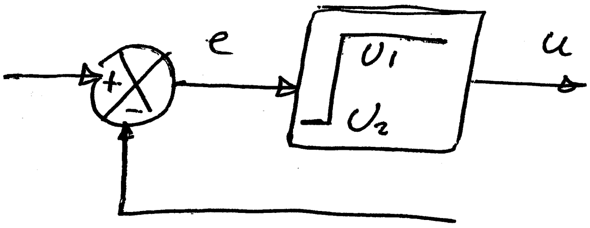
(4)

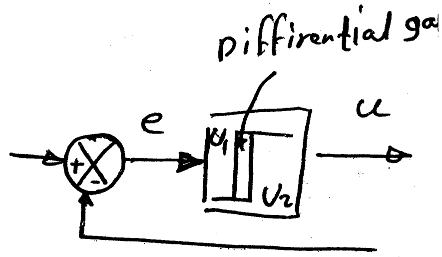
* Two-position or on-off Control Action.

$$u(t) = U_1 \quad , \quad \text{for } e(t) > 0$$
$$\quad = U_2 \quad , \quad \text{for } e(t) < 0$$



(a)

on-off Controller.



(b)

on-off controller with differential gap.

* proportional control Action.

$$u(t) = K_p \, e(t)$$

$$\frac{U(s)}{E(s)} = K_p$$

K_p : proportional gain.

* Integral Control Action.

$$\frac{du(t)}{dt} = K_i \, e(t)$$

$$u(t) = K_i \int_0^t e(t)\, dt$$

$$\frac{U(s)}{E(s)} = \frac{K_i}{s}$$

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(5)

* proportional-plus-Integral control Action.

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t)\,dt$$

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s}\right)$$

where T_i is called the integral time.

* proportional-plus-Derivative control Action.

$$u(t) = K_p e(t) + K_p T_d \frac{de(t)}{dt}$$

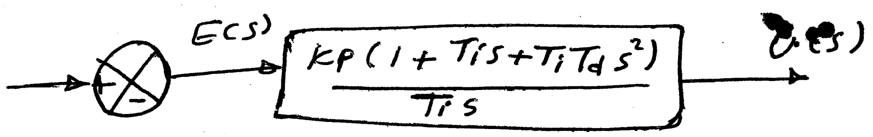
$$\frac{U(s)}{E(s)} = K_p (1 + T_d s)$$

where T_d is called the derivative time.

* proportional-plus-Integral-plus-Derivative Control Action.

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t)\,dt + K_p T_d \frac{de(t)}{dt}$$

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s\right)$$



PID controller

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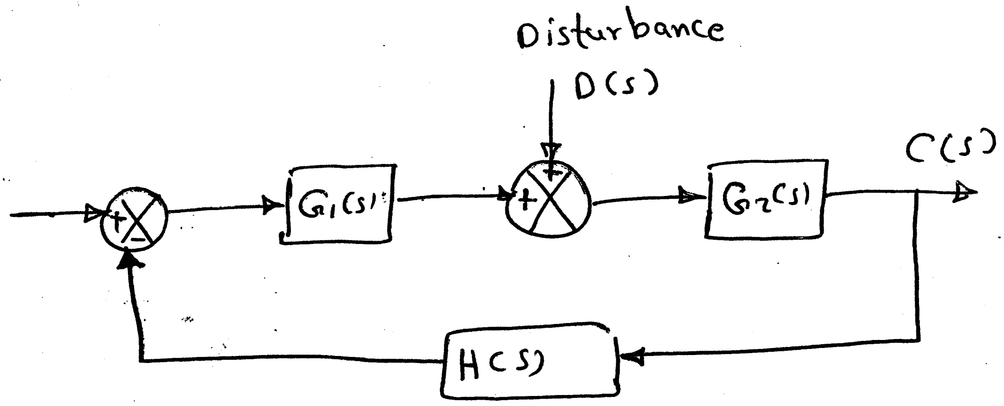
(6)

* Closed-Loop system subjected to a Disturbance.



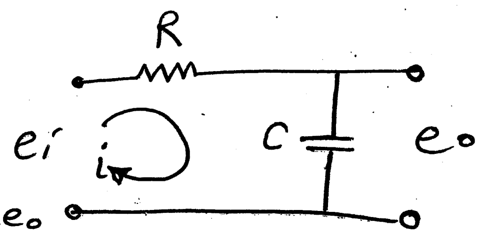
$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)\,G_2(s)\,H(s)}$$

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1 + G_1(s)\,G_2(s)\,H(s)}$$

$$C(s) = C_R(s) + C_D(s)$$

$$= \frac{G_2(s)}{1 + G_1(s)\,G_2(s)\,H(s)} \left[G_1(s) R(s) + D(s) \right]$$

* Procedures for Drawing a Block Diagram.



$$i = \frac{e_i - e_o}{R}$$

$$e_o = \frac{\int i\,dt}{C}$$

$$\left\{ \begin{aligned} I(s) &= \frac{E_i(s) - E_o(s)}{R} \\ E_o(s) &= \frac{I(s)}{Cs} \end{aligned} \right.$$

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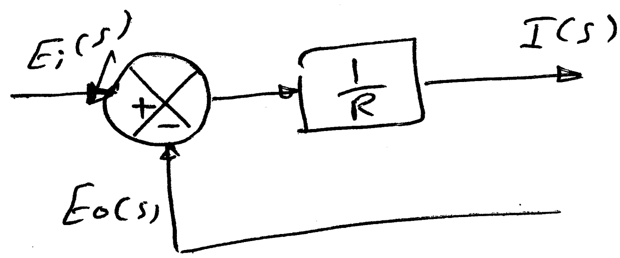
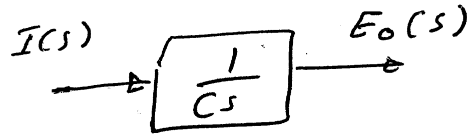
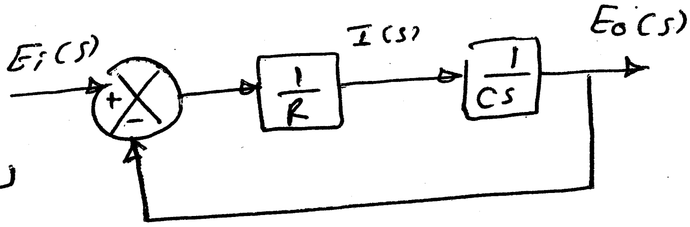
(7)







* لرسم block diagram

1. كتابة المعادلات التي تصف عمل كل جزء
2. ايجاد تحويل لابلاس لكل معادلة (Zero initial conditions)
3. وضع كل معادلة في Block مستقل
4. تجميع كل المعادلات والأجزاء في مخطط واحد

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دكتور داهشر ميا سليه

(8)

* Block Diagram Reduction.

لاختصار الـ Block diagram
يجب مراعاة مايلي :-

(1) يمكن وضع الـ Blocks كسلسلة (series)
عندما تكون الخرج للإشارة بالجزء اللاحقة

(2) عندما تكون هناك تحمل بين بعض الأجزاء يجب أن
توضع هذه الأجزاء كجزء واحد

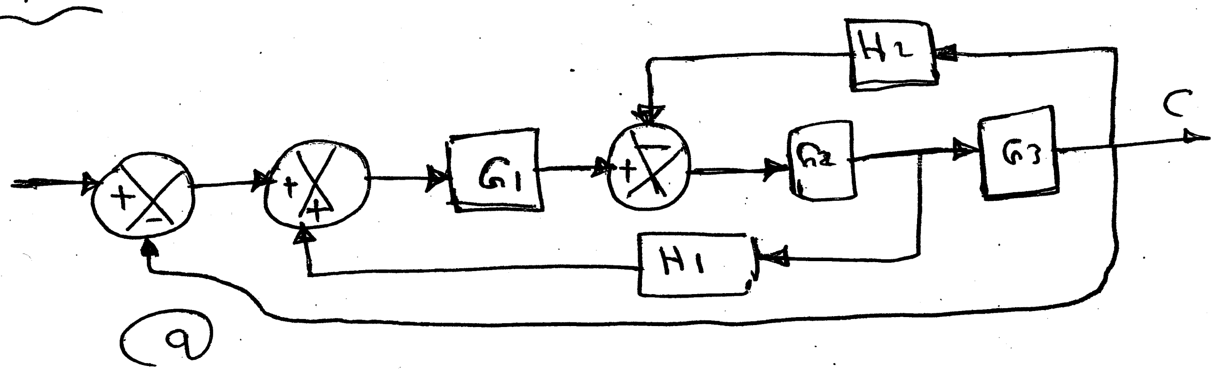
(3) اي مجموعة من الأجزاء الغير معرضة لحمل هو مشابه
يمكن وضعها كجزء واحد معادلة تمثل حاصل الضرب
للأجزاء جميعاً

(4) اي مخطط يحتوي على closed-loop متداخل
فيمكن اختصاره خطوة بعد خطوة.

(5) حاصل ضرب المعادلات في الاتجاه الامامي
يجب ان تبقى كما هو

(6) حاصل ضرب المعادلات حول الـ Loop
يجب ان تبقى كما هو

example : simplify the following block diagram.



عرض
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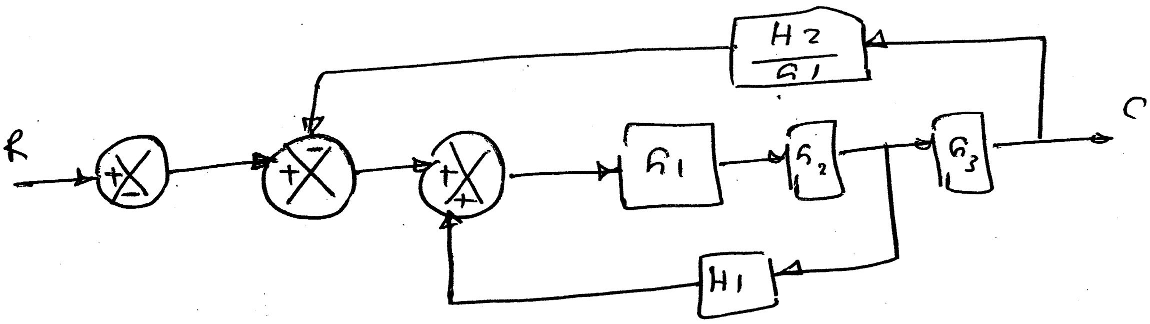
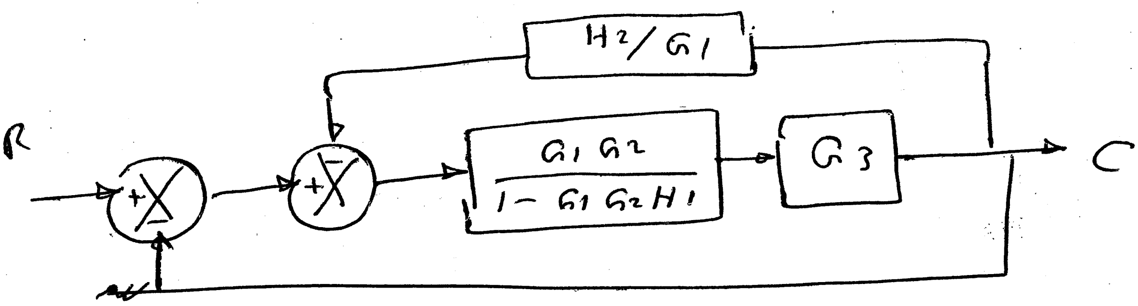
9-3

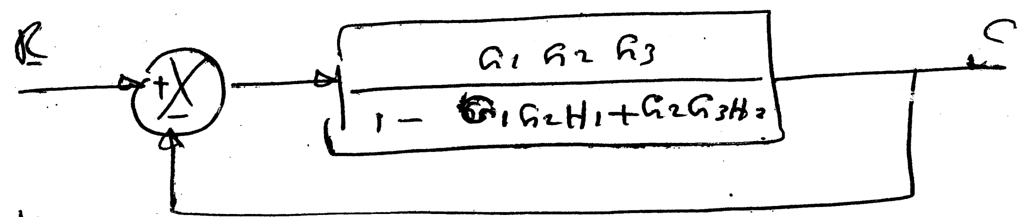
(9)



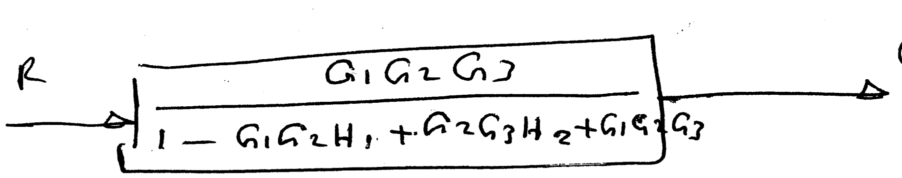
(b)



(C)



(d)



(e)

ملاحظة

١- البسط هو حاصل ضرب الدوال للاتجاه الامامي Feedforward

٢- المقام هو

$1-\sum$ (حاصل ضرب الدوال حول كل Loop)

$= 1-(G_1G_2H_1 - G_2G_3H_2 - G_1G_2G_3)$

$= 1-G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3$

(2)

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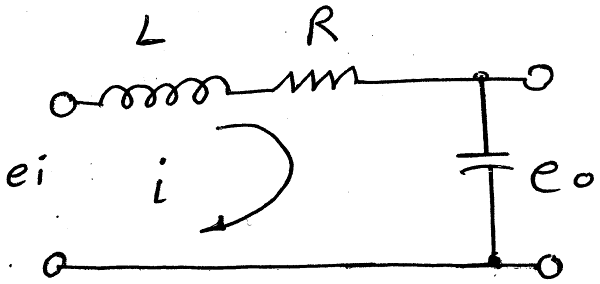
(10)

تكملة الفصل الثالث
from (10) to (16)

10-3

* LRC Circuit



$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = e_i$$

$$\frac{1}{C}\int i\,dt = e_o$$

Taking the Laplace transformation :

$$LS\,I(S) + R\,I(S) + \frac{1}{C}\frac{1}{S}I(S) = E_i(S)$$

$$\frac{1}{C}\frac{1}{S}I(S) = E_o(S)$$

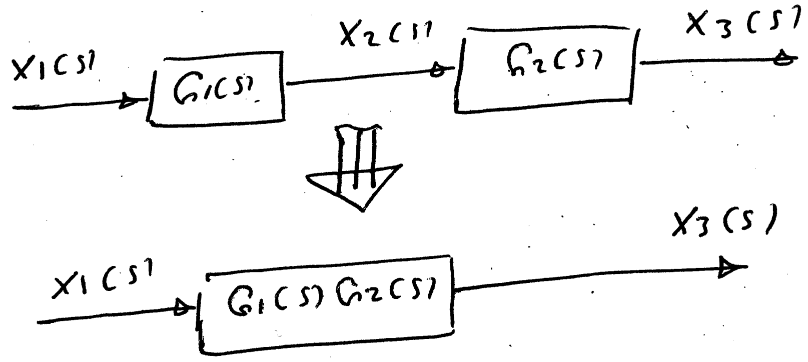
$$\frac{E_o(S)}{E_i(S)} = \frac{\frac{1}{\cancel{C}}\frac{1}{\cancel{S}}\cancel{I(S)}}{LS\cancel{I(S)} + R\cancel{I(S)} + \frac{1}{\cancel{C}}\frac{1}{\cancel{S}}\cancel{I(S)}} * \frac{\cancel{C}S/\cancel{I(S)}}{CS/I(S)}$$

$$\frac{E_o(S)}{E_i(S)} = \frac{1}{LCS^2 + RCS + 1}$$

where :
R : resistance (ohm)
L : inductance (henry)
C : capacitance (farad)

(11)

* Transfer functions of Nonloading cascaded elements.



$$G_1(s) = \frac{X_2(s)}{X_1(s)}$$

$$G_2(s) = \frac{X_3(s)}{X_2(s)}$$

$$G(s) = \frac{X_3(s)}{X_1(s)} = \frac{X_2(s)\, X_3(s)}{X_1(s)\, X_2(s)} = G_1(s)\, G_2(s)$$

* Signal Flow graphs

* Definitions.

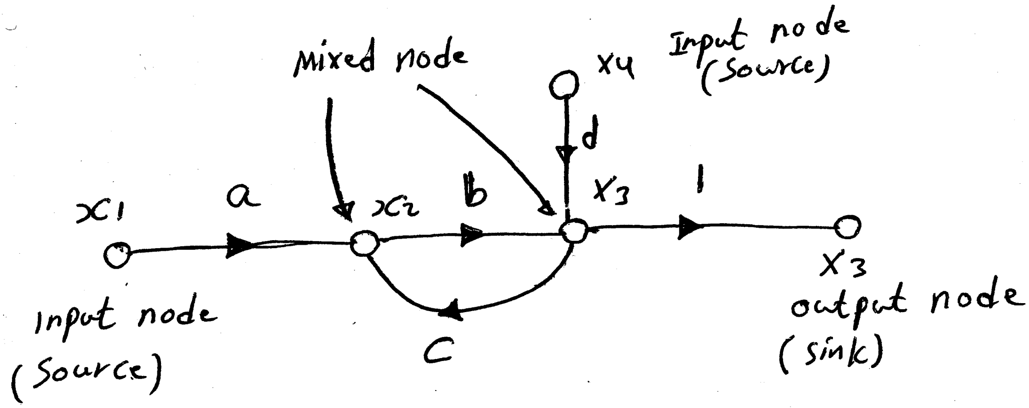
- Node: A node is a point representing a variable or signal.
- Transmittance: The transmittance is a real gain or complex gain between two nodes.
- Branch: A branch is a directed line segment joining two nodes. The gain of a branch is a transmittance
- Input node or Source: An input node or source is a node that has only outgoing branches. This corresponds to an independent variable.
- output node or sink: An output node or sink is a node that has only incoming branches. This corresponds to a dependent variable
- mixed node: A mixed node is a node that has both incoming and outgoing branches.

(12)

- path : A path is a traversal of connected branches in the direction of the branch arrows.
- Loop : A Loop is a closed path
- Nontouching Loops : Loops are nontouching if the do not possess any common nodes.

- Loop gain : The Loop gain is the product of the branch transmittances of a Loop.
- Forward path : A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes more than once.

- Forward path gain : A forward path gain is the product of the branch transmittances of a forward path.



Signal flow graph

13 أكثض

* properties of signal flow graphs.

1- A branch indicabesk the functional dependence of one signa[l] on another.

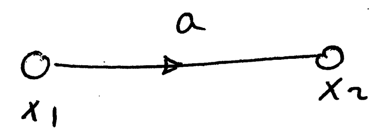
2- A node adds the signals of all incoming branches and transmits this sum to all outgoing branches

3- A mixed node, which has both incoming and outgoing branc[hes] may be treated as an output node (sink) by adding an outgoing branch of unity transmittance.

4- For a given system, a signal flow graph is not unique. many different signal flow graphs can be drawn for a given system by writing the system equations differently.

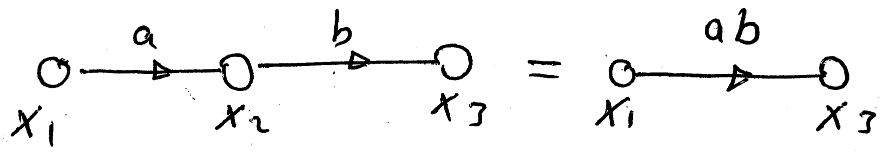
* Signal flow graph algebra.

1- The value of a node with one incoming branch is



$$x_2 = a x_1$$

2- The total transmittance of cascaded branches is equal to the product of all the branch transmittanc[es]. Cascaded branches can thus be combined into a ~~sign~~ single branch by multiplying the transmittances.



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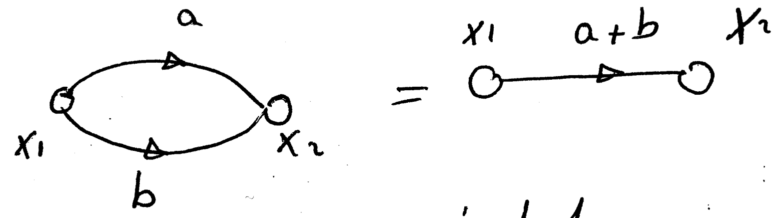
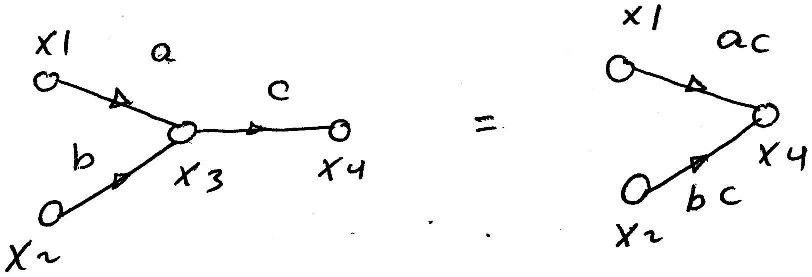
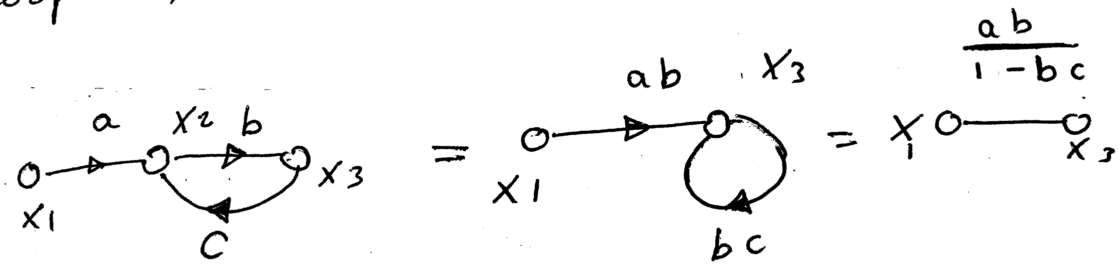
3- parallel branches may be combined by adding the transmittances.



4- A mixed node may be eliminated.



5- A loop may be eliminated



$X_3 = bX_2$

$X_2 = aX_1 + cX_3$

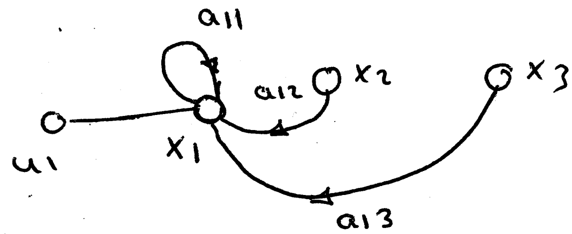
$\therefore X_3 = abX_1 + bcX_3$

$$X_3 = \frac{ab}{1-bc} X_1$$

15

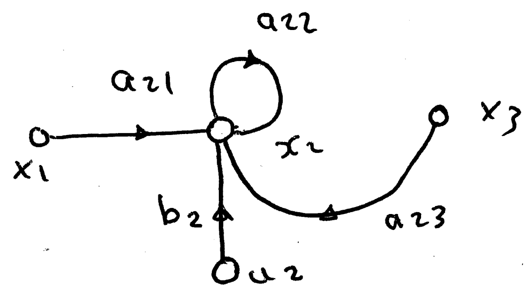
* signal flow graph representation of Linear systems.

$$X_1 = a_{11} X_1 + a_{12} X_2 + a_{13} X_3 + b_1 u_1$$



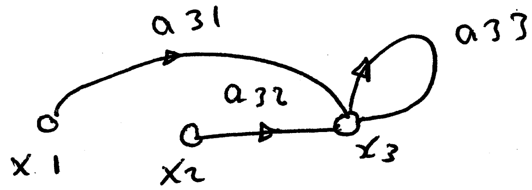
U1 : input

$$X_2 = a_{21} X_1 + a_{22} X_2 + a_{23} X_3 + b_2 u_2$$



U2 : input

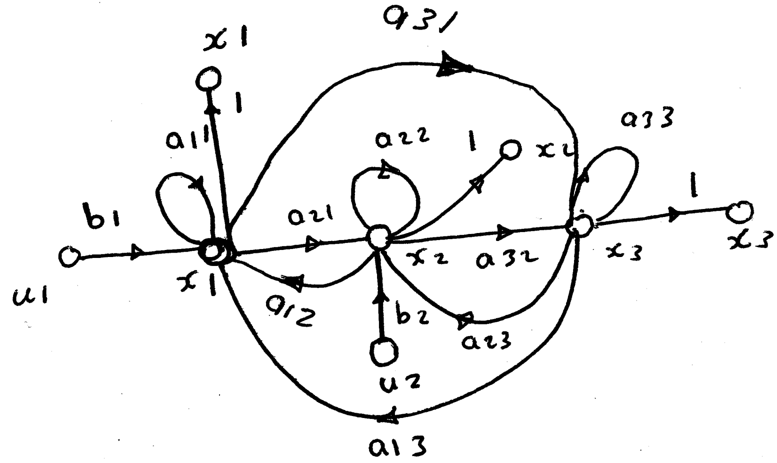
$$X_3 = a_{31} X_1 + a_{32} X_2 + a_{33} X_3$$



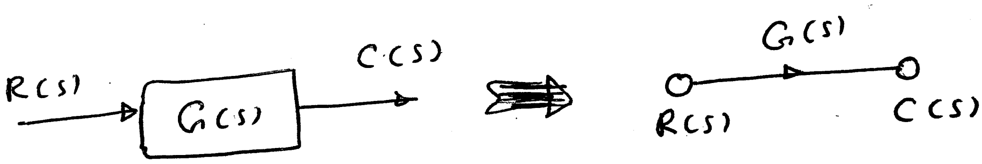
16

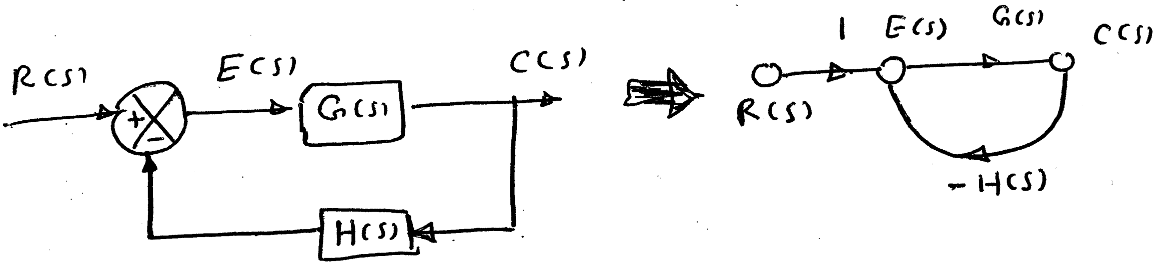
where

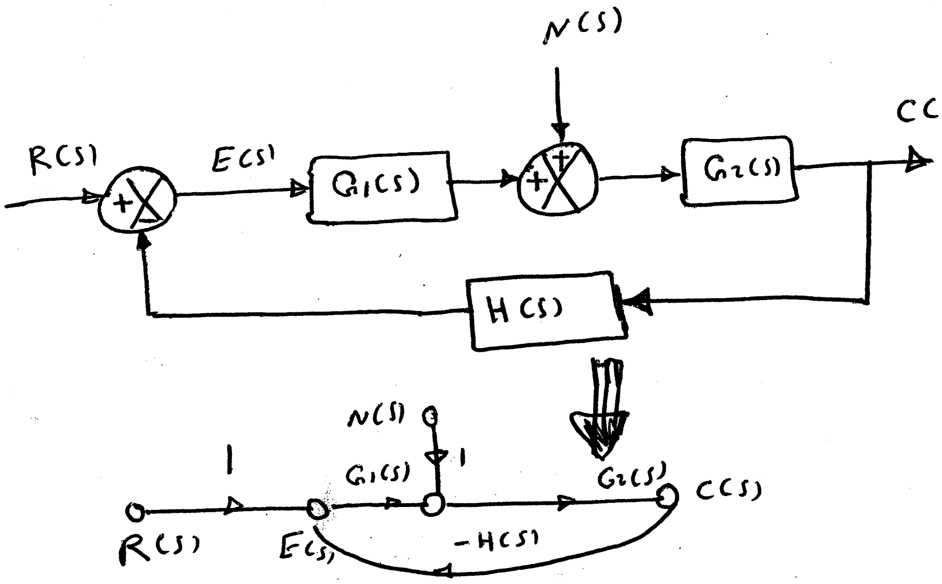
X_1, X_2 and x_3 are output variables.



* signal flow graphs of control systems.







(3)

تابع للعقد الثالث

17-3

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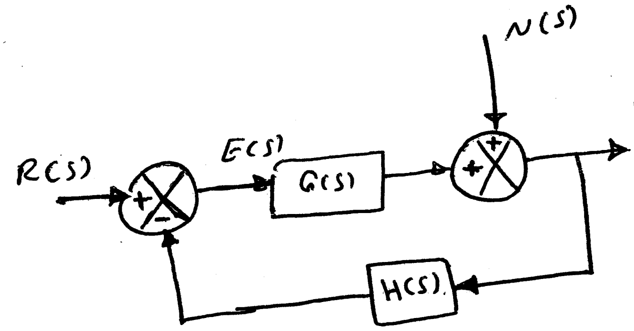
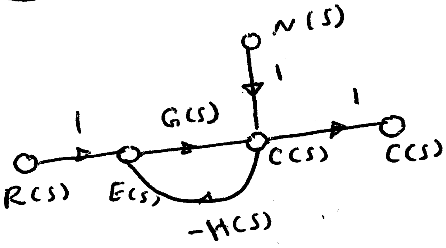
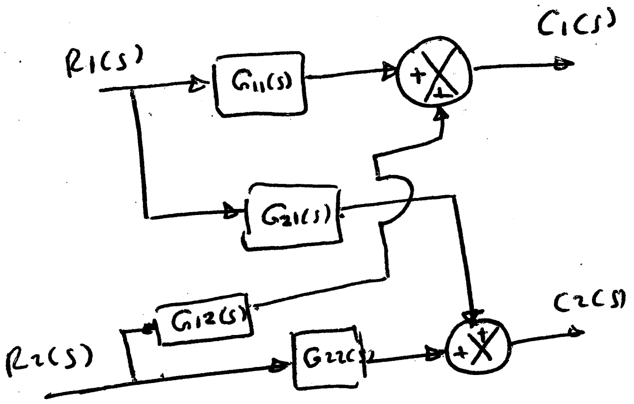
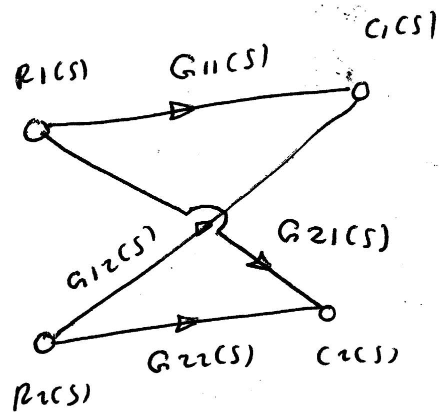
(17)



$\Rightarrow$







* Mason's Gain Formula for Signal Flow Graphs.

$$P = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

where,

P_K = Path gain or transmittance of Kth forward path

Δ = determinant of graph

= 1 − (sum of all individual Loop gains) + (sum of gain products of all possible combinations of two nontouching Loops) − (sum of gain products of all possible combinations of three nontouching Loops) + ⋯

$$= 1 - \sum_a L_a + \sum_{b,c} L_b L_c - \sum_{d,e,f} L_d L_e L_f + \cdots$$

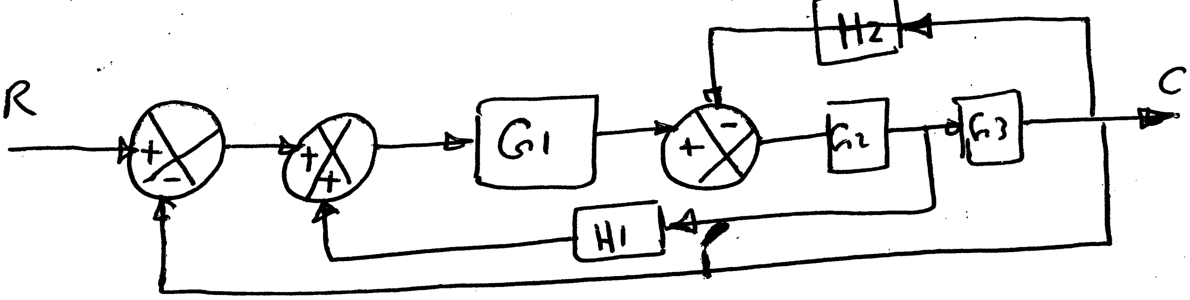
(18) $\sum_a L_a$ = sum of all individual Loop gains

$\sum_{b,c} L_b L_c$ = sum of gain products of all possible combinations of two nontouching loops

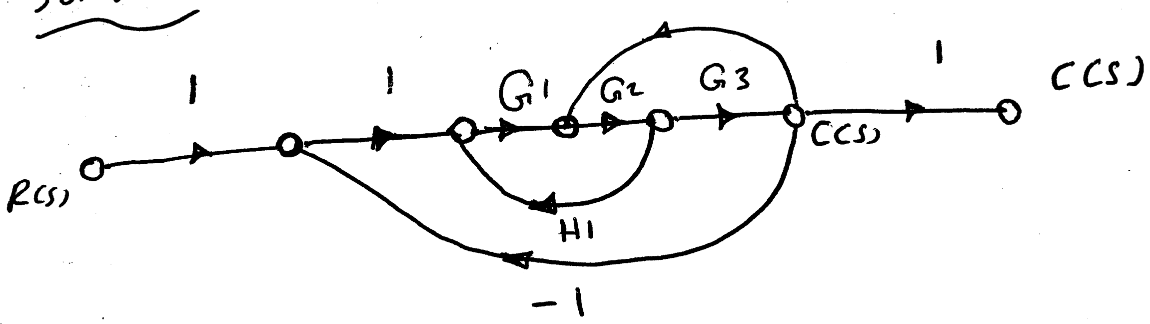
$\sum_{d,e,f} L_d L_e L_f$ = Sum of gain products of all possible combinations of three nontouching Loops

Δ_K = cofactor of the K_{th} forward path determinant of the graph with the loops touching the K_{th} forward path removed, that is, the cofactor Δ_K is obtained from Δ by removing the Loops that touch path P_K.

example : Consider the system shown in the following figure. ~~A signal flow graph for this system is shown in~~ determine $C(s)/R(s)$ by use of Mason's gain formula.



solution :



$P_1 = G_1 G_2 G_3$

(19)

$L_1 = G_1 G_2 H_1$

$L_2 = -G_2 G_3 H_2$

$L_3 = -G_1 G_2 G_3$

$\Delta = 1 - (L_1 + L_2 + L_3)$

$= 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3$

$\Delta_1 = 1$

$$\frac{C(s)}{R(s)} = P = \frac{P_1 \Delta_1}{\Delta}$$

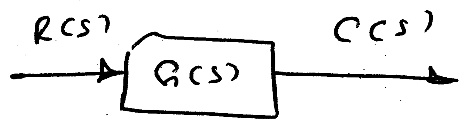
$$= \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

عماد عبد الكاظم حسون
دكتوراه هندسة ميكانيك

(1) Block diagram ملاحظات حول

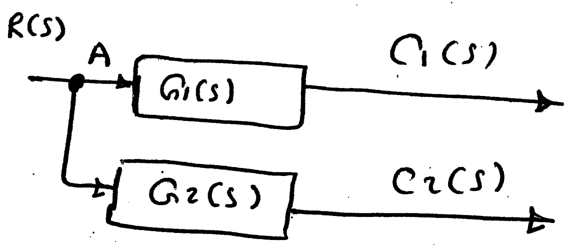
(Linear control systems with matlab applications by B.S. Manke) هذه الملاحظات مأخوذة من كتاب

(1)



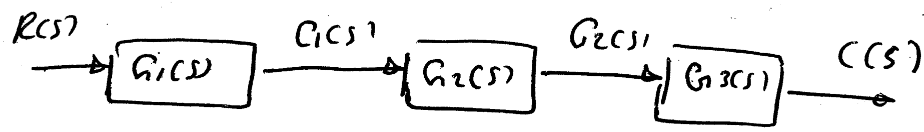
$$C(s) = R(s)\cdot G(s)$$

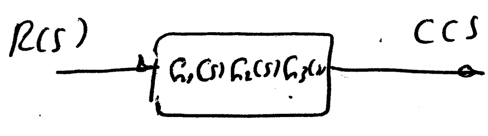
(2)



$$C_1(s) = R(s)\,G_1(s)$$
$$C_2(s) = R(s)\,G_2(s)$$

(3)

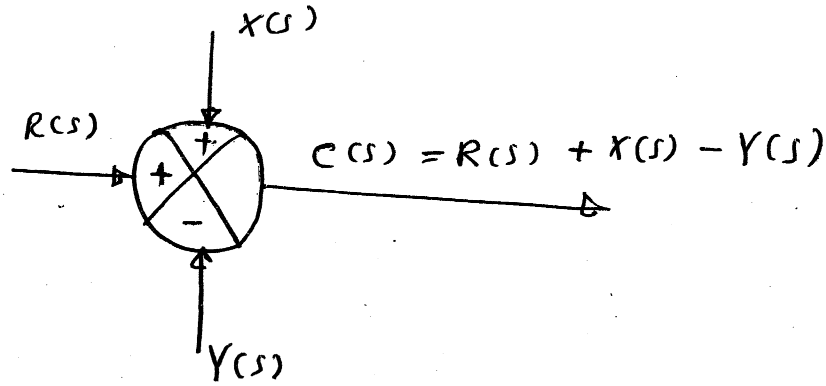




$$\frac{C(s)}{R(s)} = G_1(s)\,G_2(s)\,G_3(s)$$

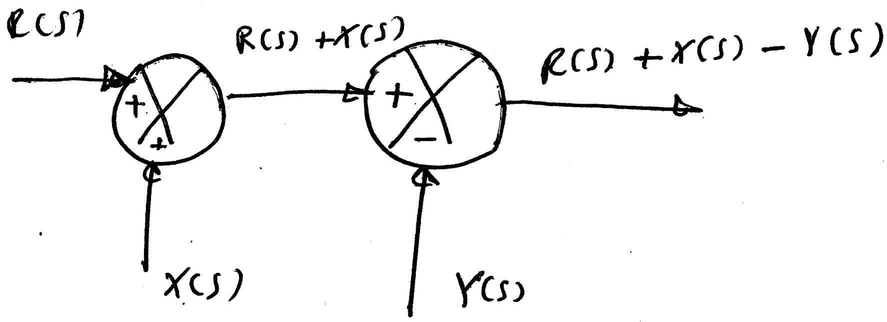
(2)

(4)



(5)

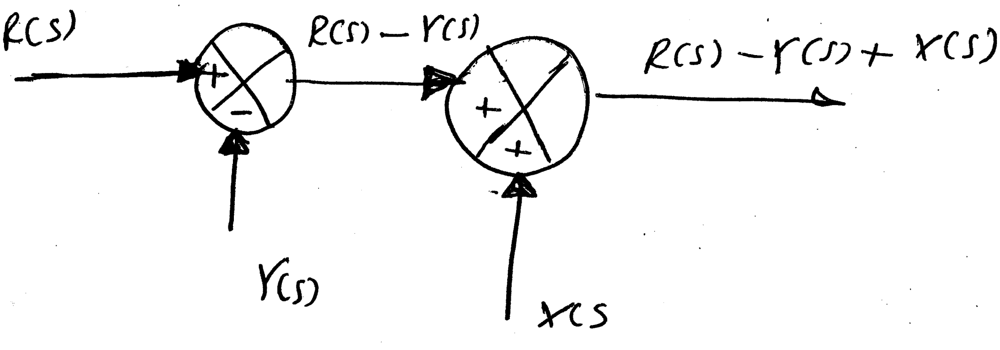
(a)



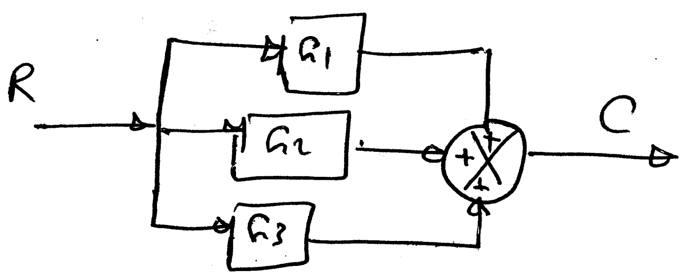
ممكن ننقل

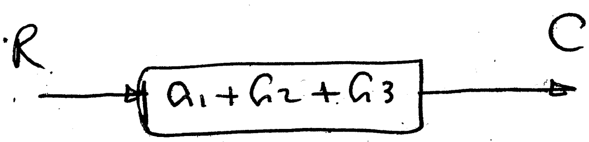
بدل

(b)



(6)

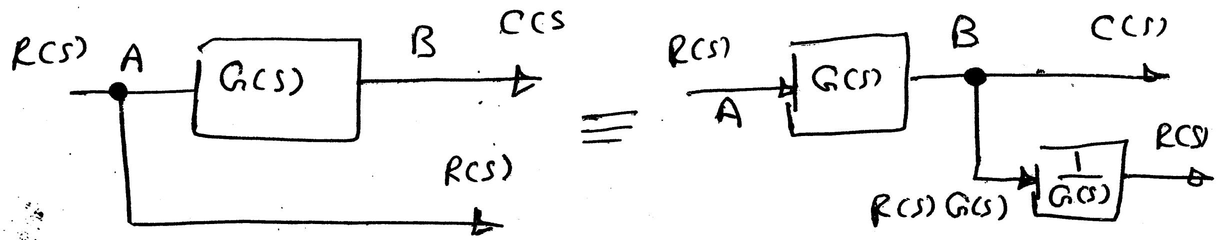




$$\frac{C}{R} = G_1 + G_2 + G_3$$

(3)

(7)



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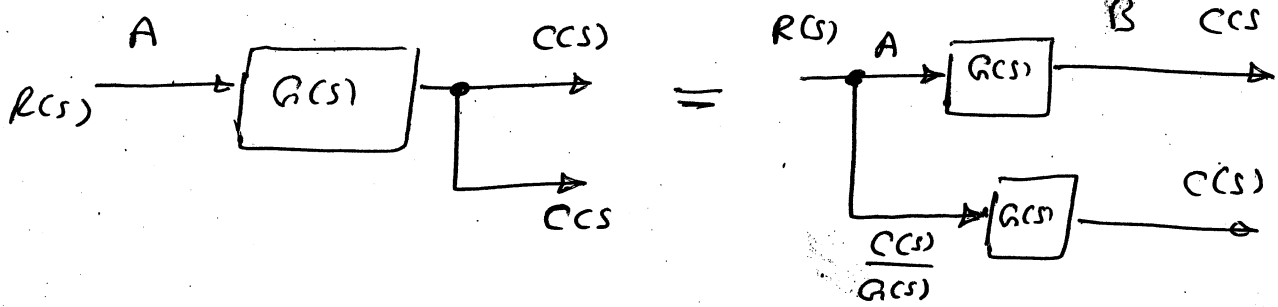

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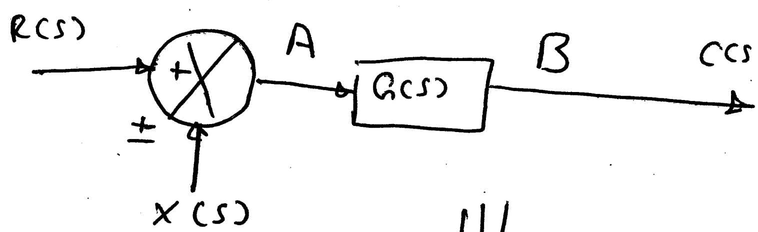
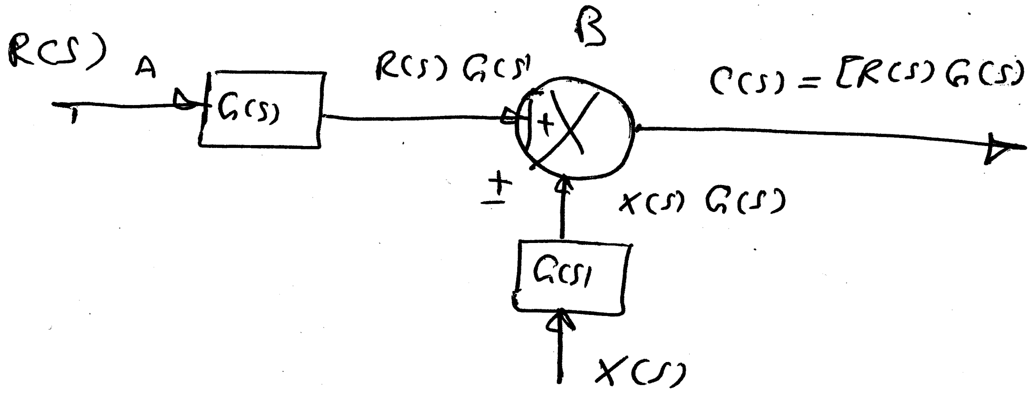


$$C(s) = [R(s) \pm X(s)]\,G(s)$$
$$= [R(s)G(s) \pm X(s)G(s)]$$

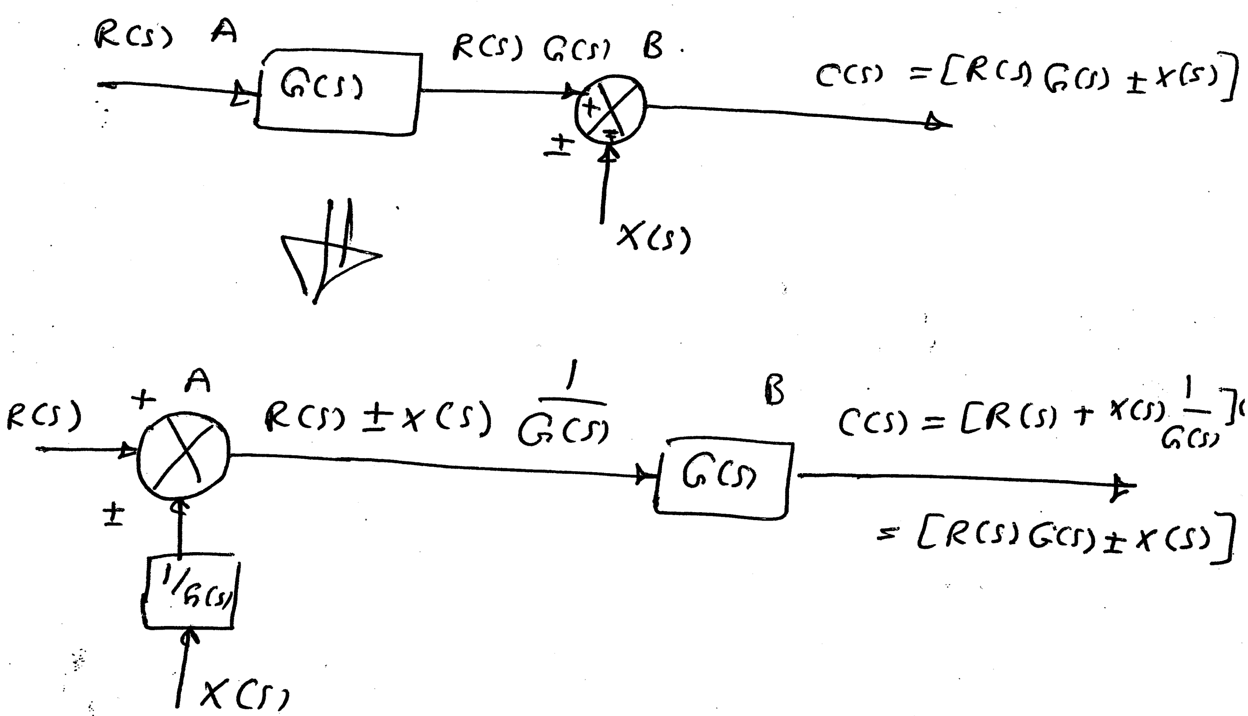
≡



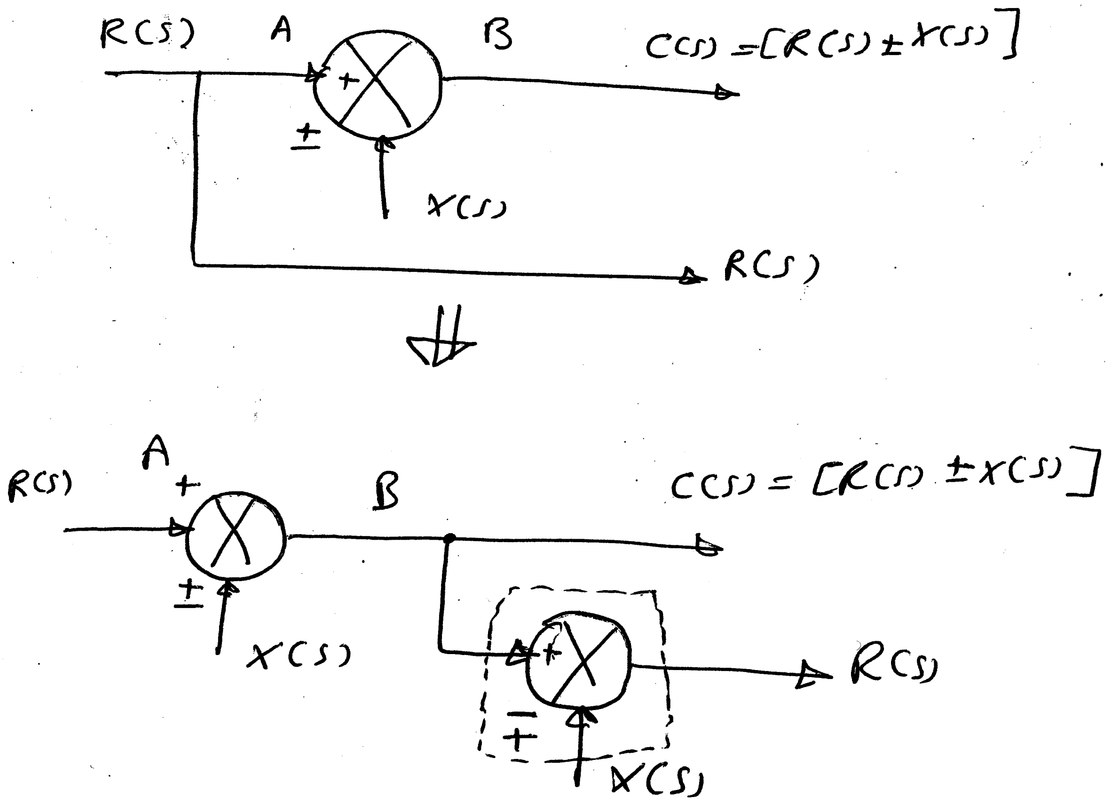
$$C(s) = [R(s)G(s) \pm X(s)G(s)]$$

(4)

(10)

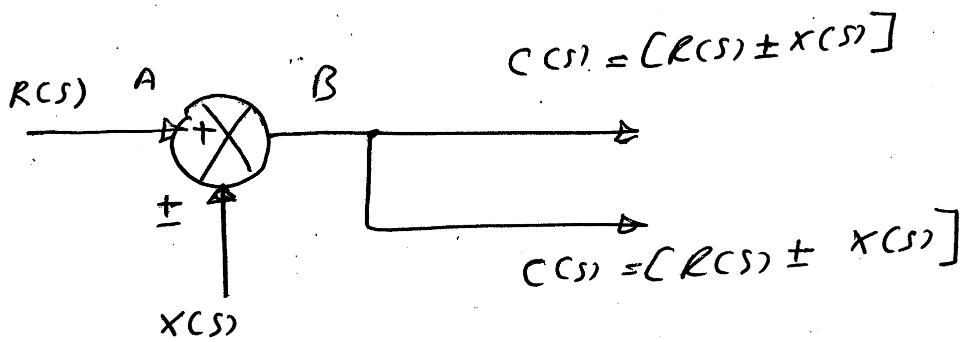

R(s) A
G(s)
R(s) G(s) B
C(s) = [R(s) G(s) ± X(s)]
+
±
X(s)
R(s)
+ A
R(s) ± X(s) 1/G(s)
B
C(s) = [R(s) + X(s) 1/G(s)] G(s)
G(s)
= [R(s) G(s) ± X(s)]
±
1/G(s)
X(s)

(11)

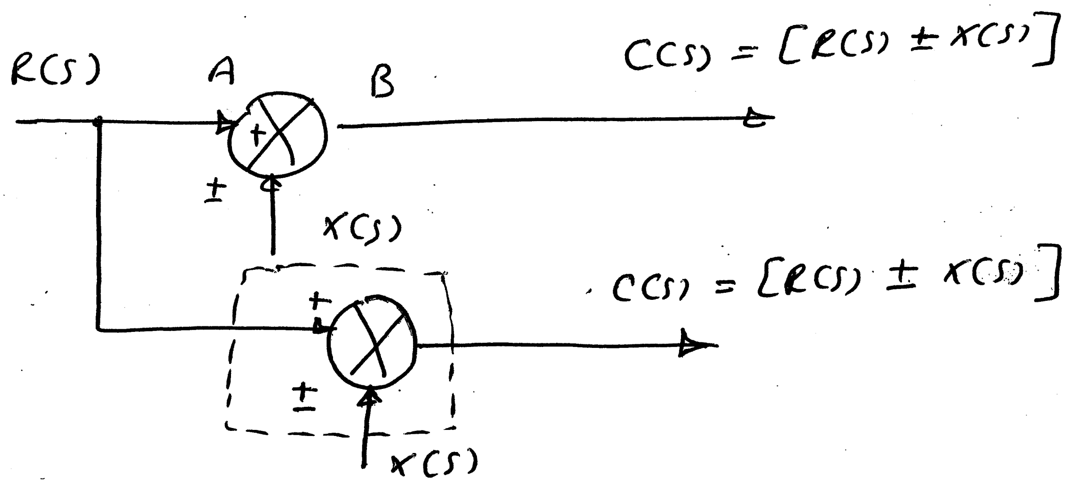

R(s) A
B
C(s) = [R(s) ± X(s)]
+
±
X(s)
R(s)
R(s)
A
+
B
C(s) = [R(s) ± X(s)]
±
X(s)
+
∓
R(s)
X(s)

(5)

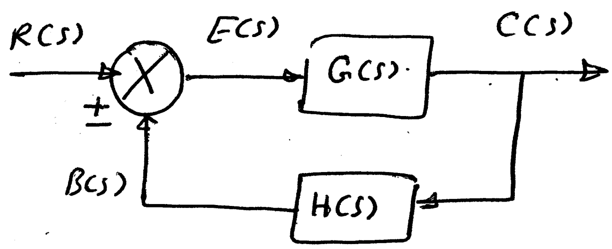
(12)



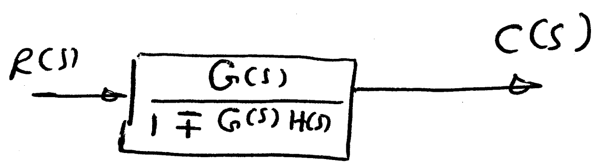
$\Downarrow$



(13)

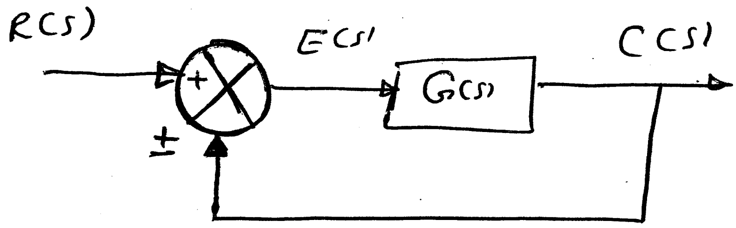


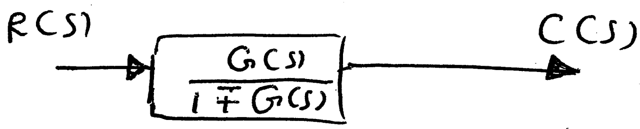
$\Downarrow$



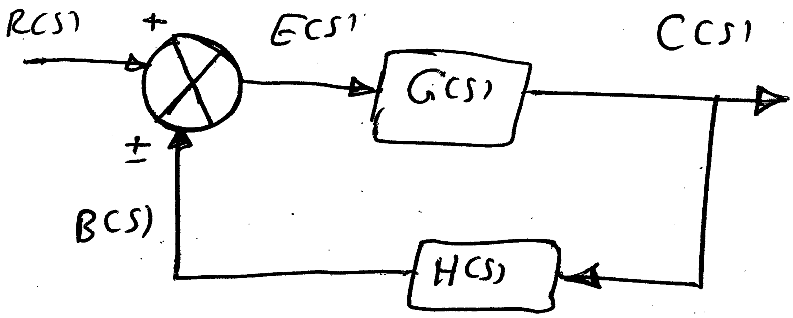
6

(14)



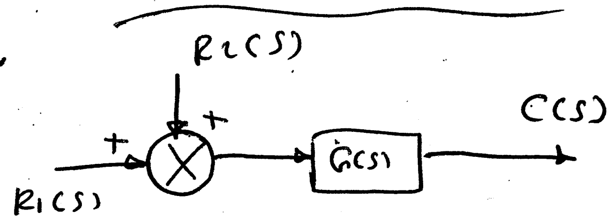


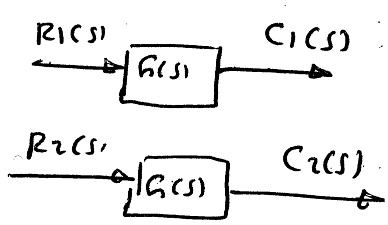
(15)



$$\frac{E(s)}{R(s)} = \frac{1}{1 \mp G(s)\,H(s)}$$

16





$$C_1(s) = R_1(s)\,G(s)$$

$$C_2(s) = R_2(s)\,G(s)$$

$$C(s) = C_1(s) + C_2(s)$$

$$= R_1(s)\,G(s) + R_2(s)\,G(s)$$

7

where :

$R(s)$: Reference input signal

$C(s)$: output signal

$E(s)$: Error Signal

$G(s) = \frac{C(s)}{E(s)}$: Forward path transfer function

$B(s)$: Feedback Signal

$H(s)$: Feedback path transfer function

$G(s)\,H(s)$: open-Loop transfer function

$\frac{C(s)}{R(s)}$: overall (Closed-Loop) transfer function or control ratio

$\frac{E(s)}{R(s)}$: Error ratio

$\frac{B(s)}{R(s)}$: primary feedback ratio.