Chapter Eight

Complex Stresses

8.1 Stresses on oblique planes

Consider a bar under direct load F giving rise to stress σ_y vertically, as shown in Fig. 8.1.

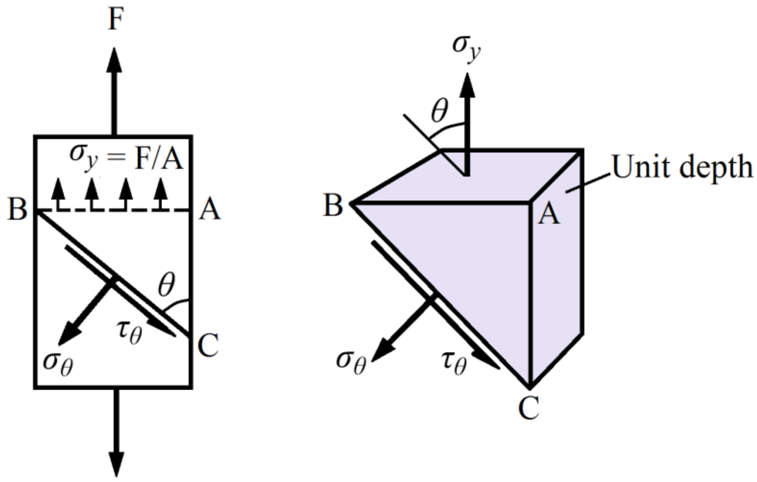


Fig. 8.1 Bar subjected to direct stress, showing stresses acting on any inclined plane.

Let the block be of unit depth, then considering the equilibrium of forces on the triangular portion ABC:

resolving forces perpendicular to BC,

$$\sigma_\theta \times BC \times 1 = \sigma_y \times AB \times 1 \times \sin\theta$$

But $\quad AB = BC \sin\theta$

$$\therefore \qquad \boldsymbol{\sigma_\theta = \sigma_y \sin^2\theta} \qquad (8.1)$$

Now resolving forces parallel to BC,

$$\tau_\theta \times BC \times 1 = \sigma_y \times AB \times 1 \times \cos\theta$$

Again $\quad AB = BC \sin\theta$

$$\therefore \qquad \tau_\theta = \sigma_y \sin\theta \cos\theta$$

$$\boldsymbol{= \frac{1}{2}\sigma_y \sin 2\theta} \qquad (8.2)$$

The direct stress σ_θ has a maximum value of σ_y, when θ = 90° whilst the shear stress τ_θ has a maximum value of $\frac{1}{2}\sigma_y$, when θ = 45°.

8.2 Material subjected to pure shear

Consider the shear stresses is applied on an element to the sides *AB* and *DC*. ***Complementary shear stresses*** of equal value but of opposite effect are then set up on sides *AD* and *BC* in order to prevent rotation of the element. Since the applied and complementary shears are of equal value on the *x* and *y* planes, they are both given the symbol τ_{xy} .

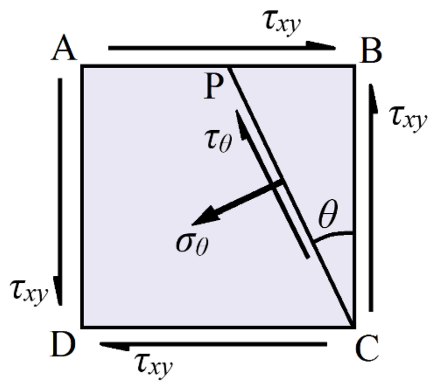


Fig. 8.2 Stresses on an element subjected to pure shear.

Consider the equilibrium of portion *PBC*.

Resolving normal to *PC assuming unit depth,*

$$\sigma_\theta \times PC = \tau_{xy} \times BC\ \sin\theta + \tau_{xy} \times PB\ \cos\theta$$

$$= \tau_{xy} \times PC\ \cos\theta\ \sin\theta + \tau_{xy} \times PC\ \sin\theta\ \cos\theta$$

$$\therefore \qquad \boldsymbol{\sigma_\theta = \tau_{xy} \sin 2\theta} \qquad (8.3)$$

The maximum value of σ_θ is τ_{xy} when $\theta = 45°$.

Similarly, resolving forces parallel to *PC*,

$$\tau_\theta \times PC = \tau_{xy} \times PB\ \sin\theta - \tau_{xy} \times BC\ \cos\theta$$

$$= \tau_{xy} \times PC\ \sin^2\theta - \tau_{xy} \times PC\ \cos^2\theta$$

$$\therefore \qquad \boldsymbol{\tau_\theta = -\tau_{xy} \cos 2\theta} \qquad (8.4)$$

The negative sign means that the sense of τ_θ is opposite to that assumed in Fig. 8.2.

The maximum value of τ_θ, is τ_{xy} when $\theta = 0°$ or $90°$ and it has a value of zero when $\theta = 45°$, i.e. on the planes of maximum direct stress.

The system of pure shear stresses produces an equivalent direct stress system as shown in Fig. 8.3, one set compressive and one tensile, each at 45° to the original shear directions, and equal in magnitude to the applied shear.

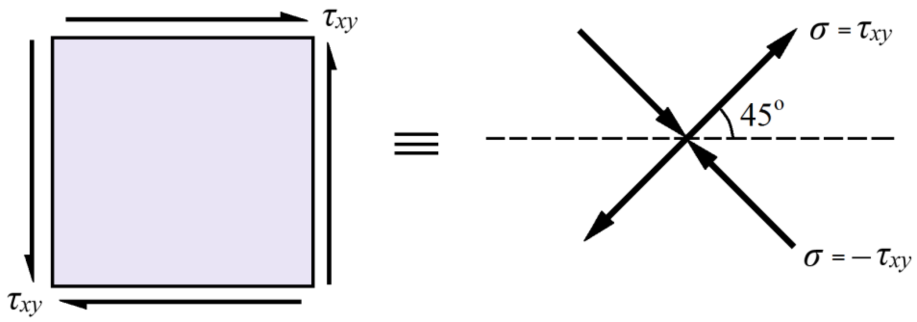


Fig. 8.3 Direct stresses due to shear.

8.3 Material subjected to two mutually perpendicular direct stresses

Consider the rectangular element of unit depth shown in Fig. 8.4 subjected to a system of two direct stresses, both tensile, at right angles, σ_x and σ_y.

For equilibrium of the portion *ABC*, resolving perpendicular to *AC*,

$$\sigma_\theta \times AC \times 1 = \sigma_x \times BC \times 1 \times \cos\theta + \sigma_y \times AB \times 1 \times \sin\theta$$

$$= \sigma_x \times AC\cos^2\theta + \sigma_y \times AC\sin^2\theta$$

$$\therefore \qquad \sigma_\theta = \frac{1}{2}\sigma_x\,(1+\cos 2\theta) + \frac{1}{2}\sigma_y\,(1-\cos 2\theta)$$

$$\boldsymbol{\sigma_\theta = \frac{1}{2}\left(\sigma_x + \sigma_y\right) + \frac{1}{2}\left(\sigma_x - \sigma_y\right)\cos 2\theta} \qquad (8.5)$$

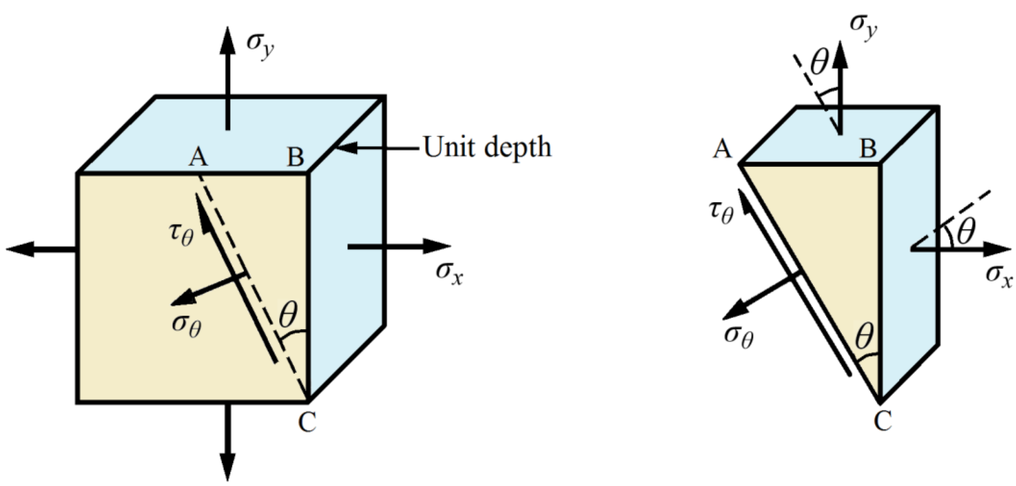


Fig. 8.4 Element subjected to two mutually perpendicular direct stresses.

Resolving parallel to AC:

$$\tau_\theta \times AC \times 1 = \sigma_x \times BC \times 1 \times \sin\theta - \sigma_y \times AB \times 1 \times \cos\theta$$

$$\tau_\theta = \sigma_x \cos\theta \sin\theta - \sigma_y \cos\theta \sin\theta$$

$$\therefore \qquad \boldsymbol{\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta} \qquad (8.6)$$

The maximum direct stress will equal σ_x or σ_y, when $\theta = 0$ or $90°$. The maximum shear stress ***in the plane of the applied stresses*** occurs when $\theta = 45°$,

$$\therefore \qquad \boldsymbol{\tau_{max} = \frac{1}{2}(\sigma_x - \sigma_y)} \qquad (8.7)$$

8.4 Material subjected to combined direct and shear stresses

Consider the complex stress system shown in Fig. 8.5 acting on an element of material. The stresses σ_x and σ_y may be compressive or tensile and may be the result of direct forces or bending. The shear stresses may be as shown or completely reversed and occur as a result of either shear forces or torsion.

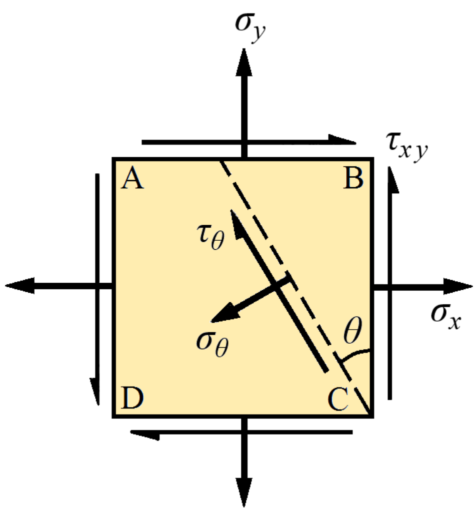


Fig. 8.5 Two-dimensional complex stress system.

$$\boldsymbol{\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta} \qquad (8.8)$$

and
$$\boldsymbol{\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta} \qquad (8.9)$$

The ***maximum and minimum stresses*** in the material can be determined as follows:

For σ_θ to be a maximum or minimum $\quad \dfrac{d\sigma_\theta}{d\theta} = 0$

The maximum and minimum direct stresses are given by equation

$$\boldsymbol{\sigma_1} \quad \text{or} \quad \boldsymbol{\sigma_2} = \frac{1}{2}\left(\sigma_x + \sigma_y\right) \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\,\tau^2_{x\,y}} \qquad (8.10)$$

These are termed the ***principal stresses*** of the system.

The complex stress system of Fig. 8.5 can be reduced to the equivalent system of principal stresses shown in Fig. 8.6.

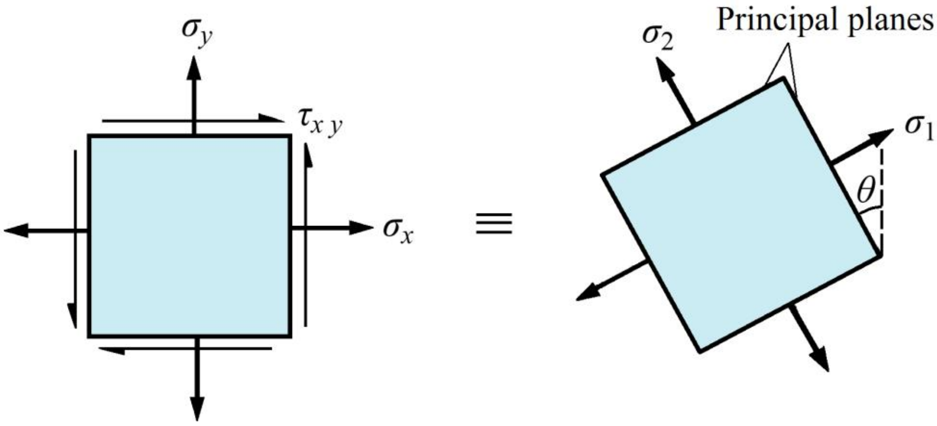


Fig. 8.6 Principal planes and stresses.

From eq. (8.7) the maximum shear stress present in the system is given by

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2) \qquad (8.11)$$

$$= \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\,\tau^2_{x\,y}} \qquad (8.12)$$

and this occurs ***on planes at 45° to the principal planes.***

8.5 Principal plane inclination in terms of the associated principal stress

The angle of inclination can be expressed as the following

$$\tan 2\theta = \frac{2\tau_{x\,y}}{\left(\sigma_x - \sigma_y\right)} \qquad (8.13)$$

Yields two values of the inclination θ of the two principal planes on which the principal stresses σ_1 and σ_2 act.

Consider the equilibrium of a triangular block of unit depth of material, the principal plane *AC* on which a principal stress σ_p acts, and the shear stress is zero.

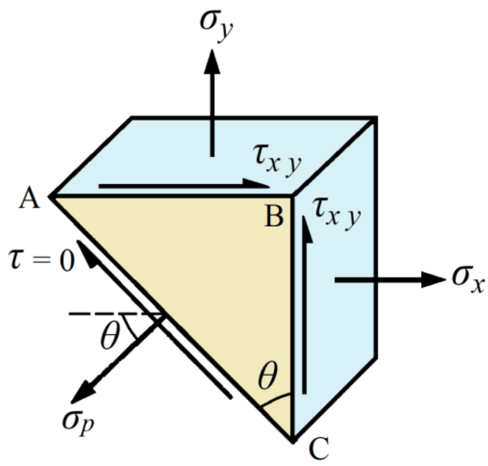


Fig. 8.7.

Resolving forces horizontally,

$$(\sigma_x \times BC \times 1) + (\tau_{xy} \times AB \times 1) = (\sigma_p \times AC \times 1)\ \cos\theta$$

$$\sigma_x + \tau_{xy}\ \tan\theta = \sigma_p$$

$$\boldsymbol{\tan\theta = \frac{\sigma_p - \sigma_x}{\tau_{xy}}} \tag{8.14}$$

The above equation for the inclination of the principal planes ***in terms of the principal stress***.

8.6 Graphical solution - Mohr's stress circle

Consider the complex stress system of Fig. 8.5 as stated below that represents a complete stress system in two dimensions.

In order to find graphically the direct stress σ_θ and shear stress τ_θ on any plane inclined at θ to the plane on which σ_x acts, proceed as follows:

(1) Label the block *ABCD*.

(2) Set up axes for direct stress (as abscissa) and shear stress (as ordinate) (Fig. 8.8).

(3) Plot the stresses acting on two adjacent faces, *AB* and *BC*, using the following sign conventions:

direct stresses: tensile, *positive*; compressive, *negative*;

shear stresses: tending to turn block clockwise, *positive*; tending to turn block counterclockwise, *negative*.

This gives two points on the graph, which may be labelled $\overline{AB}$ and $\overline{BC}$ respectively to denote stresses on these planes.

(4) Join $\overline{AB}$ and $\overline{BC}$.

(5) The point P where this line cuts the σ axis is then the center of Mohr's circle, and the line is the diameter; therefore the circle can be drawn.

Every point on the circumference of the circle then represents a state of stress on some plane through C.

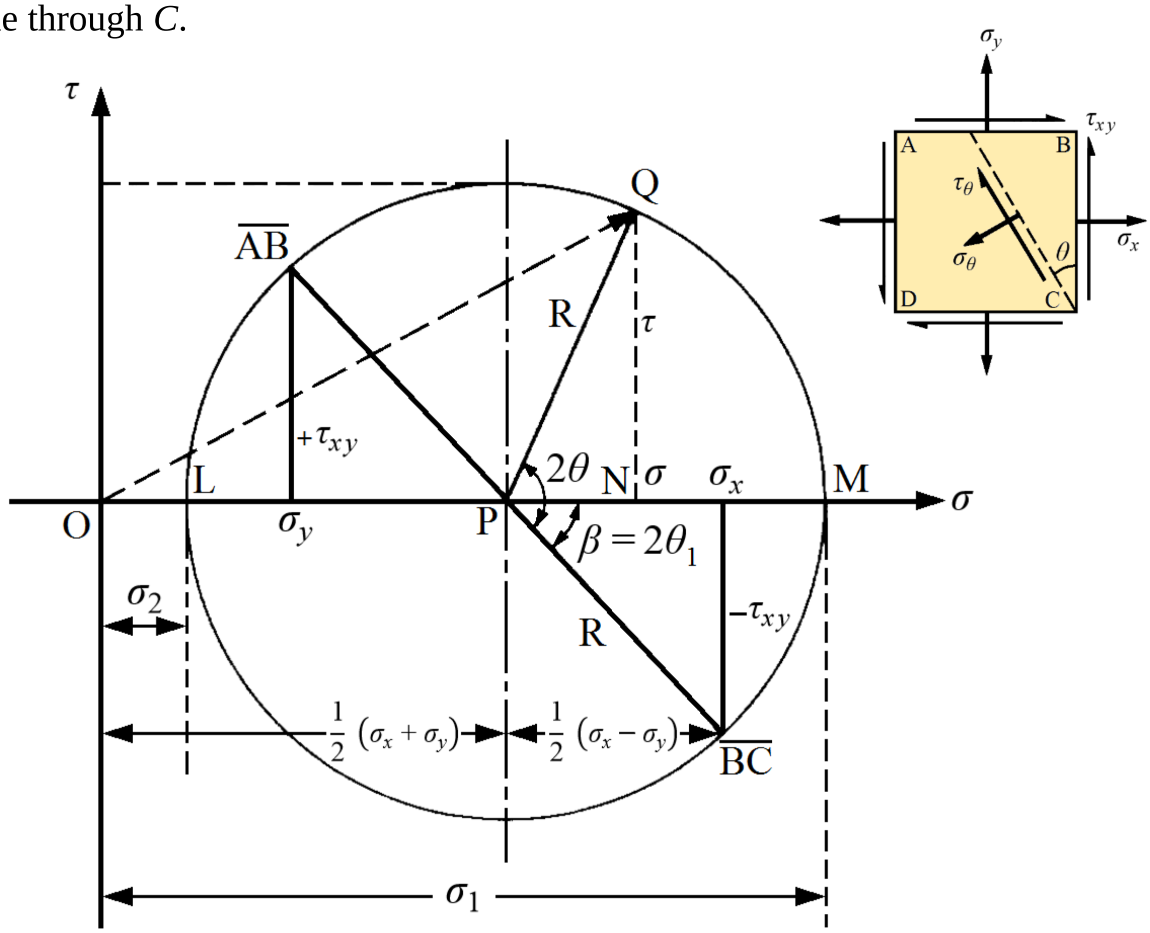


Fig. 8.8 Mohr's stress circle.

Example 8.1

A circular bar 40 mm diameter carries an axial tensile load of 100 kN. What is the value of the shear stress on the planes on which the normal stress has a value of 50 MN/m^2 tensile?

Solution:

Tensile stress $\sigma_y = \dfrac{F}{A} = \dfrac{100 \times 10^3}{\pi \times (0.02)^2} = \mathbf{79.6\ \ MN/m^2}$

Now, the normal stress on an oblique plane is given by eq. (8.1):

$$\sigma_\theta = \sigma_y \sin^2\theta$$

$$50 \times 10^6 = 79.6 \times 10^6 \sin^2\theta$$

$$\boldsymbol{\theta = 52° \ 28'}$$

The shear stress on the oblique plane is given by eq. (8.2):

$$\tau_\theta = \frac{1}{2}\ \sigma_y \sin 2\theta$$

$$= \frac{1}{2} \times 79.6 \times 10^6 \times \sin 104° \ 56'$$

$$= 38.6 \times 10^6$$

The required shear stress is 38.6 MN/m^2.

Example 8.2

Under certain loading conditions, the stresses in the walls of a cylinder are as follows:

(a) 80 MN/m^2 tensile.

(b) 30 MN/m^2 tensile at right angles to (a).

(c) shear stresses of 60 MN/m^2 on the planes on which the stresses (a) and (b) act; the shear couple acting on planes carrying the 30 MN/m^2 stress is clockwise in effect.

Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged?

Solution:

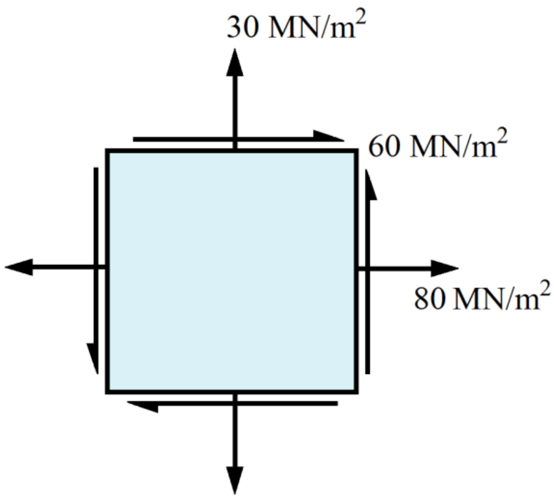


Fig. 8.9.

The principal stresses are given by the formula

$$\sigma_1 \text{ and } \sigma_2 = \frac{1}{2}\left(\sigma_x + \sigma_y\right) \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\,\tau_{xy}^2}$$

$$= \frac{1}{2}(80 + 30) \pm \frac{1}{2}\sqrt{(80 - 30)^2 + (4 \times 60^2)}$$

$$= 55 \pm 5\sqrt{(25 + 144)}$$

$$= 55 \pm 65$$

$\therefore$ $\boldsymbol{\sigma_1 = 120}$ **MN/m^2**

and $\boldsymbol{\sigma_2 = -10}$ **MN/m^2** **(compressive)**

The planes on which these stresses act can be determined from eqn. (8.14),

$$\tan\theta_1 = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

$\therefore$ $$\tan\theta_1 = \frac{120 - 80}{60} = 0.6667$$

$\therefore$ $$\theta_1 = 33° \ 41'$$

Also $$\tan\theta_2 = \frac{-10 - 80}{60} = 1.50$$

$\therefore$ $$\theta_2 = \mathbf{-56° \ 19' \quad or \quad 123° \ 41'}$$

The resulting angles are at 90° to each other as expected.

If the loading is changed so that the 80 MN/m^2 stress becomes compressive:

$$\sigma_1 = \frac{1}{2}(-80+30) + \frac{1}{2}\sqrt{(-80-30)^2 + (4\times 60^2)}$$

$$= -25 + 5\sqrt{(121 + 144)}$$

$$= -25 + 81.5 = \mathbf{56.5 \ MN/m^2}$$

and $$\sigma_2 = -25 - 81.5 = \mathbf{-106.5 \ MN/m^2}$$

The $$\tan\theta_1 = \frac{56.5 - (-80)}{60} = 2.28$$

$\therefore$ $$\theta_1 = \mathbf{66° \ 19'}$$

and $$\theta_2 = 66° \ 19' + 90 = \mathbf{156° \ 19'}$$

Mohr's circle solutions

In the first part of the question the stress system and associated Mohr's circle are as drawn in Fig. 8.10.

By measurement:

$$\sigma_1 = \mathbf{120 \ MN/m^2 \ tensile}$$

$$\sigma_2 = \mathbf{10 \ MN/m^2 \ compressive}$$

and $$\theta_1 = \mathbf{34° \ counterclockwise \ from} \ \boldsymbol{BC}$$

$$\theta_2 = \mathbf{124° \ counterclockwise \ from} \ \boldsymbol{BC}$$

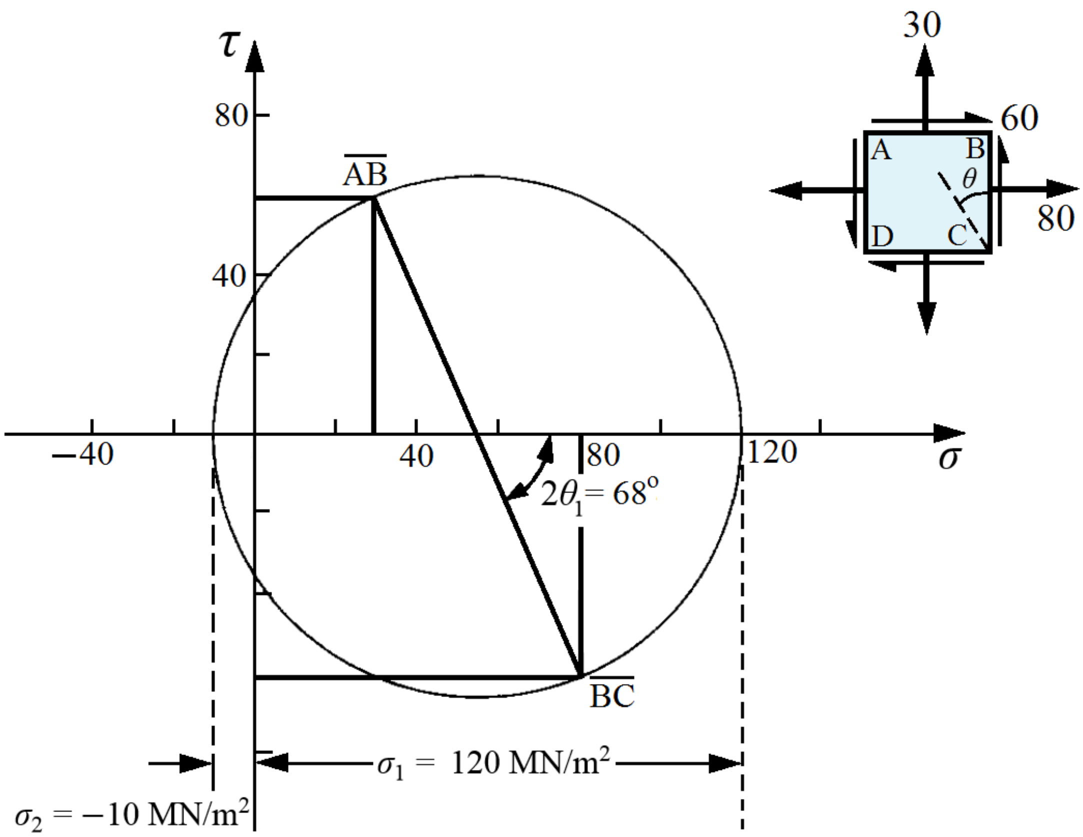
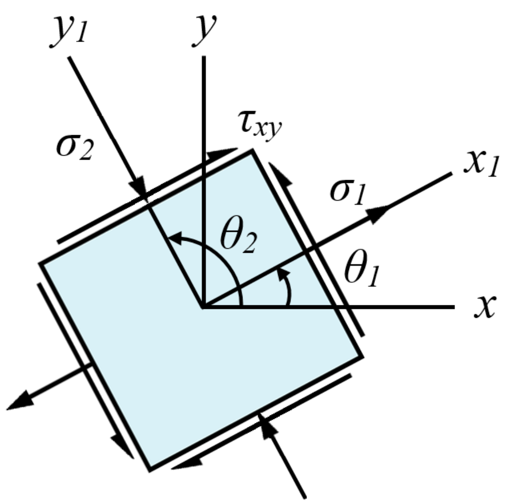


Fig. 8.10.



When the 80 MN/m^2 stress is reversed, the stress system is shown in Fig. 8.11, giving Mohr's circle as drawn.

The required values are then:

$$\sigma_1 = \textbf{56.5 MN/m}^{\mathbf{2}} \ \textbf{tensile}$$

$$\sigma_2 = \textbf{106.5 MN/m}^{\mathbf{2}} \ \textbf{compressive}$$

$$\theta_1 = \textbf{66° 15}' \ \textbf{counterclockwise to } \boldsymbol{BC}$$

and $$\theta_2 = \textbf{156° 15}' \ \textbf{counterclockwise to } \boldsymbol{BC}$$

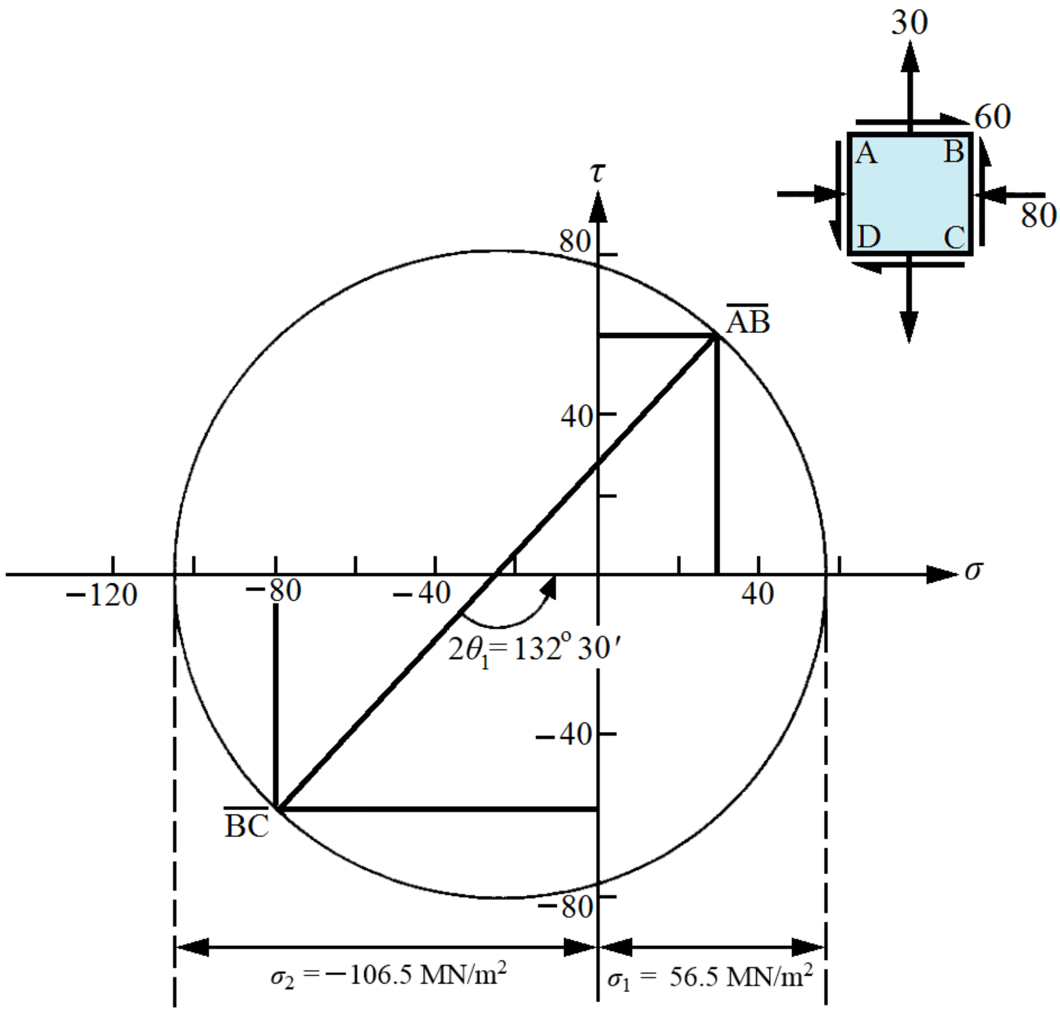
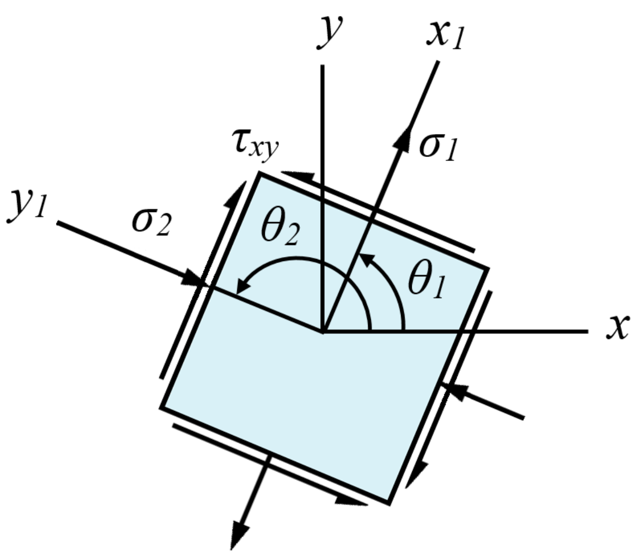

30
60
80
A
B
C
D
τ
80
40
−40
−80
$\overline{AB}$
$\overline{BC}$
σ
−120
−80
−40
40
$2\theta_1 = 132^\circ 30'$
$\sigma_2 = -106.5$ MN/m²
$\sigma_1 = 56.5$ MN/m²

Fig. 8.11.


y
x_1
τ_{xy}
σ_1
y_1
σ_2
θ_2
θ_1
x

Example 8.3

A material is subjected to two mutually perpendicular direct stresses of 80 MN/m^2 tensile and 50 MN/m^2 compressive, together with a shear stress of 30 MN/m^2. The shear couple acting on planes carrying the 80 MN/m^2 stress is clockwise in effect. Calculate:

(a) the magnitude and nature of the principal stresses.

(b) the magnitude of the maximum shear stresses in the plane of the given stress system.

(c) the direction of the planes on which these stresses act.

Confirm your answer by means of a Mohr's stress circle diagram, and from the diagram determine the magnitude of the normal stress on a plane inclined at 20° counterclockwise to the plane on which the 50 MN/m^2 stress acts.

Solution:

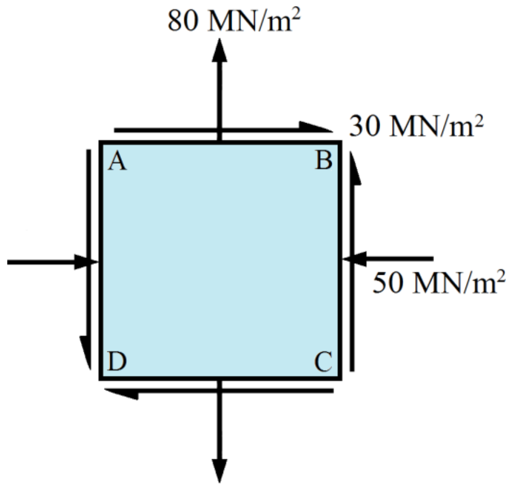


Fig. 8.12.

(a) To find the principal stresses:

$$\sigma_1 \text{ and } \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2}$$

$$= \frac{1}{2}(-50 + 80) \pm \frac{1}{2}\sqrt{(-50 - 80)^2 + (4 \times 900)}$$

$$= 5(3 \pm \sqrt{169 + 36}) = 5(3 \pm 14.31)$$

$\therefore$ $\boldsymbol{\sigma_1 = 86.55}$ **MN/m^2** and $\boldsymbol{\sigma_2 = -56.55}$ **MN/m^2**

The principal stresses are

86.55 MN/m^2 tensile and 56.55 MN/m^2 compressive.

(b) To find the maximum shear stress:

$$\tau_{max} = \frac{(\sigma_1 - \sigma_2)}{2} = \frac{86.55 - (-56.55)}{2} = \frac{143.1}{2} = 71.6 \text{ MN/m}^2$$

Maximum shear stress = **71.6 MN/m²**

(c) To find the directions of the principal planes:

$$\tan\theta_1 = \frac{\sigma_p - \sigma_x}{\tau_{xy}}$$

$$\therefore \quad \tan\theta_1 = \frac{86.55 - (-50)}{30} = 4.552$$

$$\therefore \quad \theta_1 = \mathbf{77° \, 36'}$$

$$\therefore \quad \theta_2 = 77° \, 36' + 90° = \mathbf{167° \, 36'}$$

The principal planes are inclined at 77° 36′ to the plane on which the 50 MN/m² stress acts. The maximum shear planes are at 45° to the principal planes.

Mohr's circle solution

The stress system shown in Fig. 8.12 gives the Mohr's circle in Fig. 8.13.

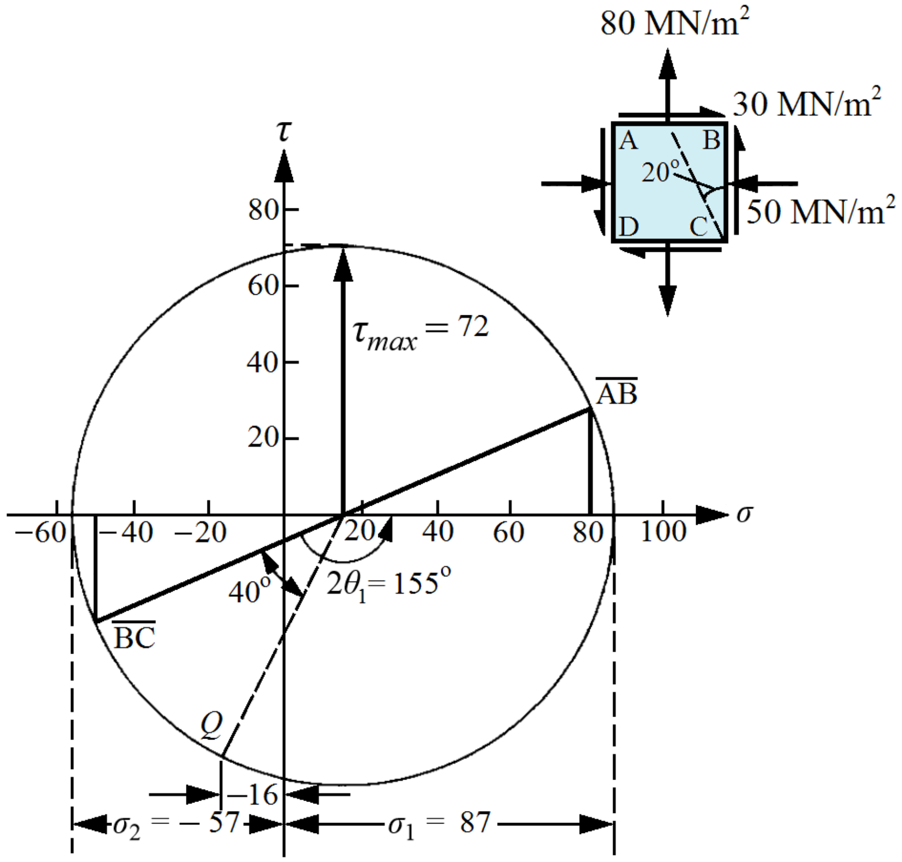


Fig. 8.13.

By measurement

$$\sigma_1 = \mathbf{87\ \ MN/m^2\ \ tensile}$$

$$\sigma_2 = \mathbf{57\ \ MN/m^2\ \ compressive}$$

$$\tau_{max} = \mathbf{72\ \ MN/m^2}$$

and
$$\theta_1 = \frac{155°}{2} = \mathbf{77°\ \ 30'}$$

The direct or normal stress on a plane inclined at 20° counterclockwise to *BC* is obtained by measuring from $\overline{BC}$ on the Mohr's circle through 2 × 20° = 40° in the same direction. This gives $\boldsymbol{\sigma}$ **= 16 MN/m² compressive**

Example 8.4

At a given section, a shaft is subjected to a bending stress of 20 MN/m² and a shear stress of 40 MN/m². Determine:

(a) the principal stresses.

(b) the directions of the principal planes.

(c) the maximum shear stress and the planes on which this act.

(d) the tensile stress which, acting alone, would produce the same maximum shear stress.

(e) the shear stress which, acting alone, would produce the same maximum tensile principal stress.

Solution:

(a) The bending stress is a direct stress and can be treated as acting on the *x*-axis, so that σ_x = 20 MN/m²; since no other direct stresses are given, σ_y = 0.

Principal stress
$$\sigma_1 = \frac{1}{2}\left(\sigma_x + \sigma_y\right) \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\,\tau_{xy}^2}$$

$$= \frac{1}{2} \times 20 + \frac{1}{2}\sqrt{20^2 + (4 \times 40^2)}$$

$$= 10 + 5\sqrt{68} = 10 + (5 \times 8.246)$$

∴
$$\sigma_1 = \mathbf{51.23\ \ MN/m^2}$$

and
$$\sigma_2 = 10 - 41.23 \qquad \therefore \quad \sigma_2 = \mathbf{-\,31.23\ \ MN/m^2}$$

(b) Then $$\tan\theta_1 = \frac{\sigma_p - \sigma_x}{\tau_{xy}} = \frac{51.23 - 20}{40} = \frac{31.23}{40} = 0.7808$$

$\therefore$ $$\theta_1 = \mathbf{37^\circ\ 59'}$$

$$\tan\theta_2 = \frac{-31.23 - 20}{40} = \frac{-51.23}{40} = -1.2808$$

$\therefore$ $$\theta_2 = \mathbf{-52^\circ\ 1'} \quad \text{or} \quad \mathbf{127^\circ\ 59'}$$

both angles being measured counterclockwise from the plane on which the 20 MN/m^2 stress acts.

(c) Maximum shear stress

$$\tau_{max} = \frac{(\sigma_1 - \sigma_2)}{2} = \frac{51.23 - (-31.23)}{2} = \frac{82.46}{2} = \mathbf{41.23\ MN/m^2}$$

This acts on planes at 45° to the principal planes,

i.e. **at 82° 59′ or −7° 1′**

(d) the tensile stress

Maximum shear stress $$\tau_{max} = \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\,\tau_{xy}^2}$$

Thus, if a tensile stress is to act alone to give the same maximum shear stress ($\sigma_y = 0$ and $\tau_{xy} = 0$):

$$\text{maximum shear stress} = \frac{1}{2}\sqrt{(\sigma_x)^2} = \frac{1}{2}\sigma_x$$

$$41.23 = \frac{1}{2}\sigma_x$$

$$\boldsymbol{\sigma_x = 82.46\ \mathbf{MN/m^2}}$$

The required tensile stress is 82.46 MN/m^2.

(e) the shear stress

Principal stress $$\sigma_1 = \frac{1}{2}\left(\sigma_x + \sigma_y\right) + \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\,\tau_{xy}^2}$$

Thus, if a shear stress is to act alone to give the same principal stress ($\sigma_x = \sigma_y = 0$):

$$\sigma_1 = \frac{1}{2}\sqrt{(4\,\tau_{xy}^2)} = \tau_{xy}$$

$\therefore$ $\tau_{xy} = 51.23$ The required shear stress is **51.23 MN/m²**.

Mohr's circle solutions

(a), (b), (c) The stress system and corresponding Mohr's circle as shown in Fig. 8.14.

(a) $\sigma_1 \approx$ **51 MN/m² tensile**

$\sigma_2 \approx$ **31 MN/m² compressive**

(b) $\theta_1 = \frac{76°}{2} = \mathbf{38°}$

$\theta_2 = 38° + 90° = \mathbf{128°}$

(c) $\tau_{max} \approx$ **41 MN/m²**

Angle of maximum shear plane

$$\theta = \frac{166°}{2} = \mathbf{83°}$$

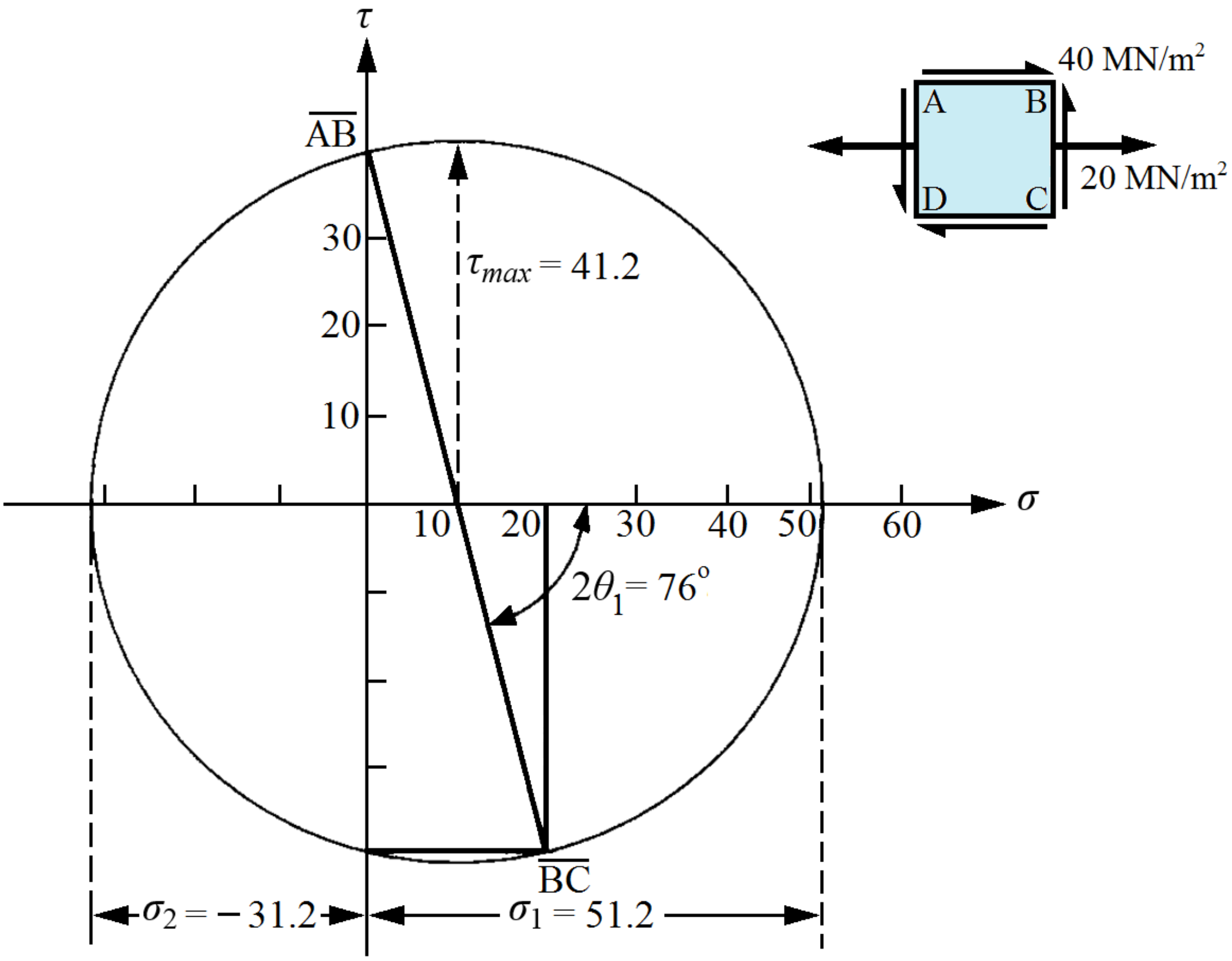


Fig. 8.14.

(d) If a tensile stress σ_x is to act alone to give the same maximum shear stress, then $\sigma_y = 0$, $\tau_{xy} = 0$ and $\tau_{\max} = 41$ MN/m^2. The Mohr's circle has a radius of **41 MN/m^2** and passes through the origin (Fig. 8.15). Hence, the required tensile stress is **82 MN/m^2**.

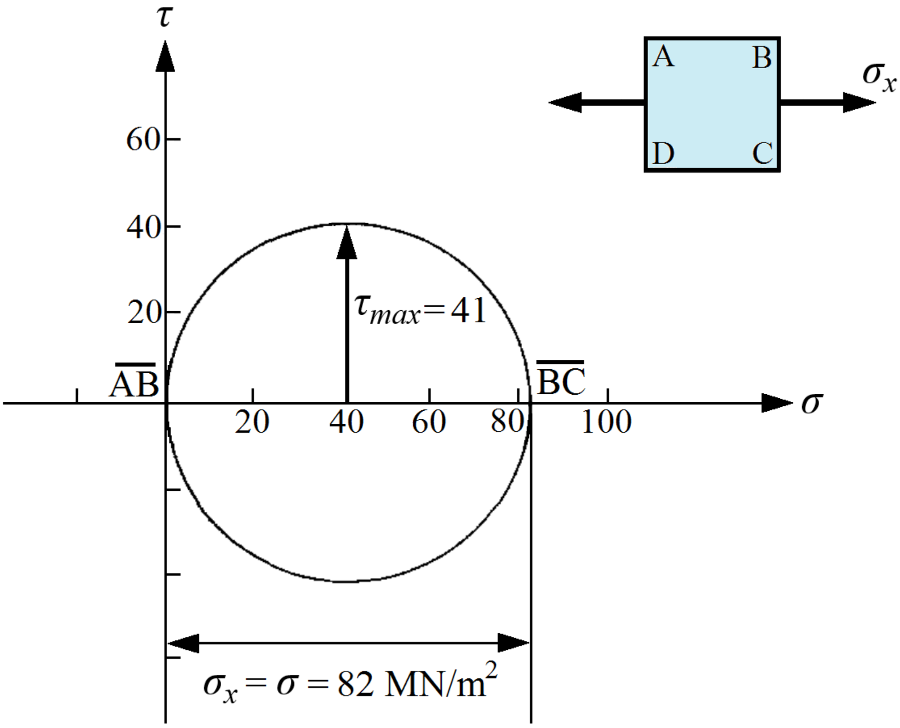


Fig. 8.15.

(e) If a shear stress is to act alone to produce the same principal stress, $\sigma_x = 0$, $\sigma_y = 0$ and $\sigma_1 = 51$ MN/m^2. The Mohr's circle has its center at the origin and passes through $\sigma = 51$ MN/m^2 (Fig. 8.16). Hence, the required shear stress is **51 MN/m^2**.

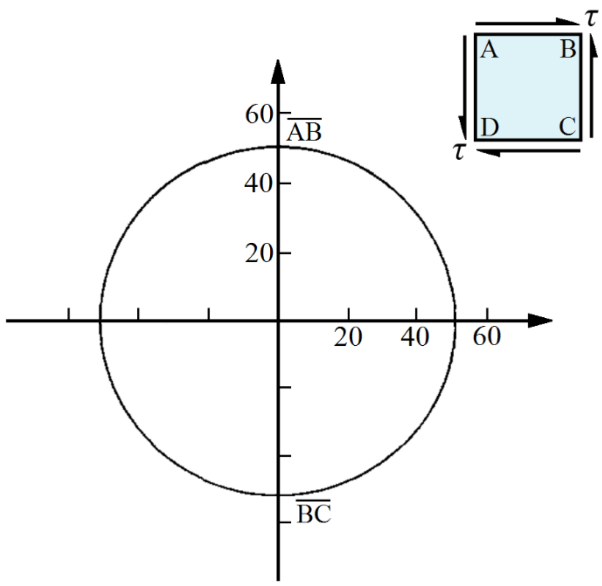


Fig. 8.16.

Problems

8.1 An axial tensile load of 10 kN is applied to a 12 mm diameter bar. Determine the maximum shearing stress in the bar and the planes on which it acts. Find also the value of the normal stresses on these planes. [44.1 MN/m^2 at 45° and 135°; ± 44.2 MN/m^2]

8.2 A compressive member of a structure is of 25 mm square cross-section and carries a load of 50 kN. Determine, from first principles, the normal, tangential and resultant stresses on a plane inclined at 60° to the axis of the bar. [60, 34.6, 69.3 MN/m^2]

8.3 A rectangular block of material is subjected to a shear stress of 30 MN/m^2 together with its associated complementary shear stress. Determine the magnitude of the stresses on a plane inclined at 30° to the directions of the applied stresses, which may be taken as horizontal. [26, 15 MN/m^2]

8.4 A material is subjected to two mutually perpendicular stresses, one 60 MN/m^2 compressive and the other 45 MN/m^2 tensile. Determine the direct, shear and resultant stresses on a plane inclined at 60° to the plane on which the 45 MN/m^2 stress acts. [18.75, 45.5, 49.2 MN/m^2]

8.5 The material of Problem 8.4 is subjected to an additional shearing stress of 10 MN/m^2. Determine the principal stresses acting on the material and the maximum shear stress. [46, − 61, 53.5 MN/m^2]

8.6 At a certain section in a material under stress, direct stresses of 45 MN/m^2 tensile and 75 MN/m^2 tensile act on perpendicular planes together with a shear stress τ acting on these planes. If the maximum stress in the material is limited to 150 MN/m^2 tensile, determine the value of τ.

[88.7 MN/m^2]

8.7 At a point in a material under stress, there is a compressive stress of 200 MN/m^2 and a shear stress of 300 MN/m^2 acting on the same plane. Determine the principal stresses and the directions of the planes on which they act. [216 MN/m^2 at 54.2° to 200 MN/m^2 plane; − 416 MN/m^2 at 144.2°]

8.8 At a certain point in a material, the following stresses act: a tensile stress of 150 MN/m^2, a compressive stress of 105 MN/m^2 at right angles to the tensile stress and a shear stress clockwise in effect of 30 MN/m^2. Calculate the principal stresses and the directions of the principal planes.

[153.5, − 108.5 MN/m^2; at 6.7° and 96.7° counterclockwise to 150 MN/m^2 plane]

8.9 The stresses across two mutually perpendicular planes at a point in an elastic body are 120 MN/m^2 tensile with 45 MN/m^2 clockwise shear, and 30 MN/m^2 tensile with 45 MN/m^2 counterclockwise shear. Find (i) the principal stresses, (ii) the maximum shear stress, and (iii) the normal and tangential stresses on a plane measured at 20° counterclockwise to the plane on which the 30 MN/m^2 stress acts. Draw sketches showing the positions of the stresses found above and the planes on which they act relative to the original stresses. [138.6, 11.4, 63.6, 69.5, − 63.4 MN/m^2]

8.10 At a point in a strained material, the stresses acting on planes at right angles to each other are 200 MN/m^2 tensile and 80 MN/m^2 compressive, together with associated shear stresses, which may be assumed clockwise in effect on the 80 MN/m^2 planes. If the principal stress is limited to 320 MN/m^2 tensile, calculate: **(a)** the magnitude of the shear stresses.

(b) the directions of the principal planes.

(c) the other principal stress.

(d) the maximum shear stress.

[219 MN/m^2, 28.7 and 118.7° counterclockwise to 200 MN/m^2 plane; − 200 MN/m^2; 260 MN/m^2]

8.11 A solid shaft of 125 mm diameter transmits 0.5 MW at 300 rev/min. It is also subjected to a bending moment of 9 kN m and to a tensile end load. If the maximum principal stress is limited to 75 MN/m^2, determine the permissible end thrust. Determine the position of the plane on which the principal stress acts, and draw a diagram showing the position of the plane relative to the torque and the plane of the bending moment. [61.4 kN; 61° to shaft axis]

8.12 At a certain point in a piece of material there are two planes at right angles to one another on which there are shearing stresses of 150 MN/m^2 together with normal stresses of 300 MN/m^2 tensile on one plane and 150 MN/m^2 tensile on the other plane. If the shear stress on the 150 MN/m^2 planes is taken as clockwise in effect, determine for the given point:

(a) the magnitudes of the principal stresses.

(b) the inclinations of the principal planes.

(c) the maximum shear stress and the inclinations of the planes on which it acts.

(d) the maximum strain if E = 208 GN/m^2 and Poisson's ratio = 0.29. غير مطلوب

[392.7, 57.3 MN/m^2; 31.7°, 121.7°; 167.7 MN/m^2, 76.7°, 166.7°; 1810 $\mu\varepsilon$]

8.13 A 250 mm diameter solid shaft drives a screw propeller with an output of 7 MW. When the forward speed of the vessel is 35 km/h, the speed of revolution of the propeller is 240 rev/min. Find the maximum stress resulting from the torque and the axial compressive stress resulting from the thrust in the shaft; hence find for a point on the surface of the shaft: (a) the principal stresses, and (b) the directions of the principal planes relative to the shaft axis. Make a diagram to show clearly the direction of the principal planes and stresses relative to the shaft axis.

[90.8, 14.7, 98.4, − 83.7 MN/m^2; 47° and 137°]

8.16 A shaft l00 mm diameter is subjected to a twisting moment of 7 kN m, together with a bending moment of 2 kN m. Find, at the surface of the shaft, (a) the principal stresses, (b) the maximum shear stress. [47.3, − 26.9 MN/m^2; 37.1 MN/m^2]

8.17 A material is subjected to a horizontal tensile stress of 90 MN/m^2 and a vertical tensile stress of 120 MN/m^2, together with shear stresses of 75 MN/m^2, those on the 120 MN/m^2 planes being counterclockwise in effect. Determine:

(a) the principal stresses.

(b) the maximum shear stress.

(c) the shear stress which, acting alone, would produce the same principal stress.

(d) the tensile stress which, acting alone, would produce the same maximum shear stress.

[181.5, 28.5 MN/m^2; 76.5 MN/m^2; 181.5 MN/m^2; 153 MN/m^2]

8.18 Two planes *AB* and *BC* in an elastic material under load are inclined at 45° to each other. The loading on the material is such that the stresses on these planes are as follows:

1. on *AB*, 150 MN/m^2 direct stress and 120 MN/m^2 shear.
2. on *BC*, 80 MN/m^2 shear and a direct stress σ.

Determine the value of the unknown stress σ on *BC* and hence determine the principal stresses which exist in the material. [190, 214, − 74 MN/m^2]

8.19 A beam of I-section, 500 mm deep and 200 mm wide, has flanges 25 mm thick and web 12 mm thick. It carries a concentrated load of 300 kN at the center of a simply supported span of 3 m. Calculate the principal stresses set up in the beam at the point where the web meets the flange.

[83.4, − 6.15 MN/m^2]

8.20 At a certain point on the outside of a shaft, which is subjected to a torque, and a bending moment the shear stresses are 100 MN/m^2 and the longitudinal direct stress is 60 MN/m^2 tensile. Find, by calculation from first principles or by graphical construction, which must be justified:

(a) the maximum and minimum principal stresses.

(b) the maximum shear stress.

(c) the inclination of the principal stresses to the original stresses.

Summarize the answers clearly on a diagram, showing their relative positions to the original stresses.

[134.4, − 74.4 MN/m^2; 104.4 MN/m^2; 35.5°]