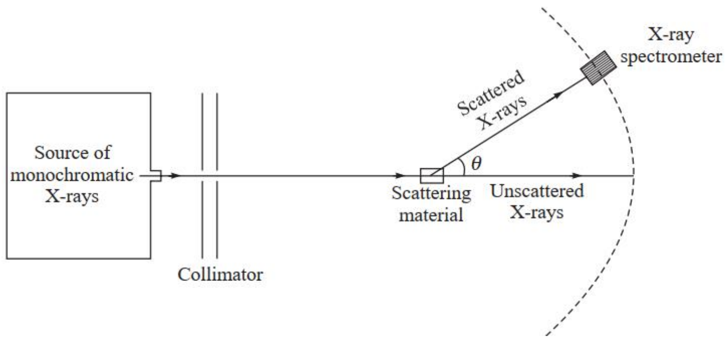
1.3 THE COMPTON EFFECT (1923)

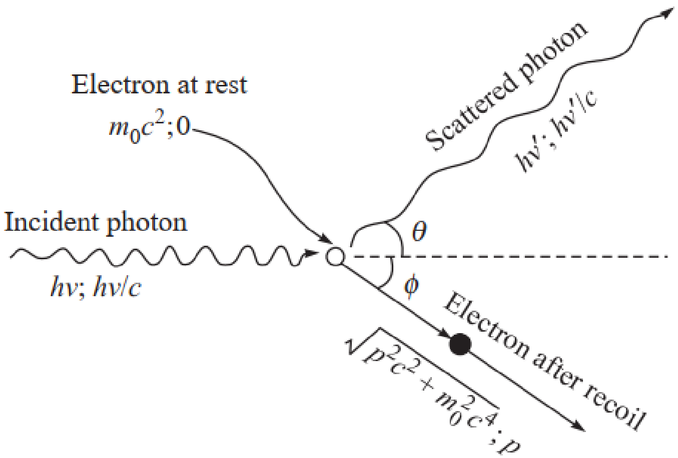
When a monochromatic beam of X-rays is scattered by an element of low atomic weight (for example carbon), it is observed that the scattered X-rays, at all angles, have maximum intensities at two wavelengths, one at the original wavelength and the other at a slightly longer wavelength. The wavelength shift is independent of the wavelength of the incident beam and the scattering material; it depends only on the scattering angle. *This phenomenon is called the Compton effect.*



the original wavelength (λ_o) and the other due to the modified wavelength (λ). The wavelength shift ($\Delta\lambda$) increases with the increase of scattering angle θ.

Compton's Explanation

Compton was able to explain this phenomenon using the quantum theory of radiation, developed by Planck and Einstein. He considered the incident X-rays as a stream of photons, each of energy $h\nu$ and momentum $h\nu/c$. The scattering process is treated as an elastic collision between a photon and a "free" electron, which is initially at rest. In the collision, a part of the photon energy is transferred to the electron which recoils. Therefore, the scattered photon has a smaller energy and hence a lower frequency (higher wavelength).



Let ν' be the frequency of the scattered photon, m_0 be the rest mass of the electron and p be the recoil momentum of the electron. According to the theory of relativity, the energy of the electron at rest is m_0c^2 and that after recoil is $(p^2c^2 + m_0^2c^4)^{1/2}$. From the law of conservation of energy,

$$h\nu' + (p^2c^2 + m_0^2c^4)^{1/2} = h\nu + m_0c^2$$

or $$p^2c^2 + m_0^2c^4 = [h(\nu - \nu') + m_0c^2]^2$$

or $$p^2c^2 + m_0^2c^4 = h^2(\nu - \nu')^2 + 2h(\nu - \nu')m_0c^2 + m_0^2c^4$$

or $$\frac{p^2c^2}{h^2} = (\nu - \nu')^2 + \frac{2m_0c^2}{h}(\nu - \nu') \qquad \ldots(1.10)$$

Applying the law of conservation of momentum along and perpendicular to the direction of the incident photon,

$$p\cos\phi + \frac{h\nu'}{c}\cos\theta = \frac{h\nu}{c}$$

and $$p\sin\phi = \frac{h\nu'}{c}\sin\theta$$

Rearranging these equations,

$$\frac{pc}{h}\cos\phi = \nu - \nu'\cos\theta \qquad \ldots(1.11)$$

and $$\frac{pc}{h}\sin\phi = \nu'\sin\theta \qquad \ldots(1.12)$$

Squaring and adding (2.11) and (2.12),

$$\frac{p^2c^2}{h^2} = (\nu - \nu'\cos\theta)^2 + \nu'^2\sin^2\theta$$
$$= \nu^2 - 2\nu\nu'\cos\theta + \nu'^2$$
$$= (\nu - \nu')^2 + 2\nu\nu' - 2\nu\nu'\cos\theta$$
$$= (\nu - \nu')^2 + 2\nu\nu'(1 - \cos\theta) \qquad \ldots(1.13)$$

Comparing (2.10) and (2.13), we get

$$\frac{2m_0c^2}{h}(\nu - \nu') = 2\nu\nu'(1 - \cos\theta)$$

or
$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

or
$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\theta) \qquad \ldots(1.14)$$

If λ and λ' are the wavelengths of the incident and scattered photons, then

$$\nu' = \frac{c}{\lambda'} \quad \text{and} \quad \nu = \frac{c}{\lambda}$$

Therefore, (2.14) can be expressed as

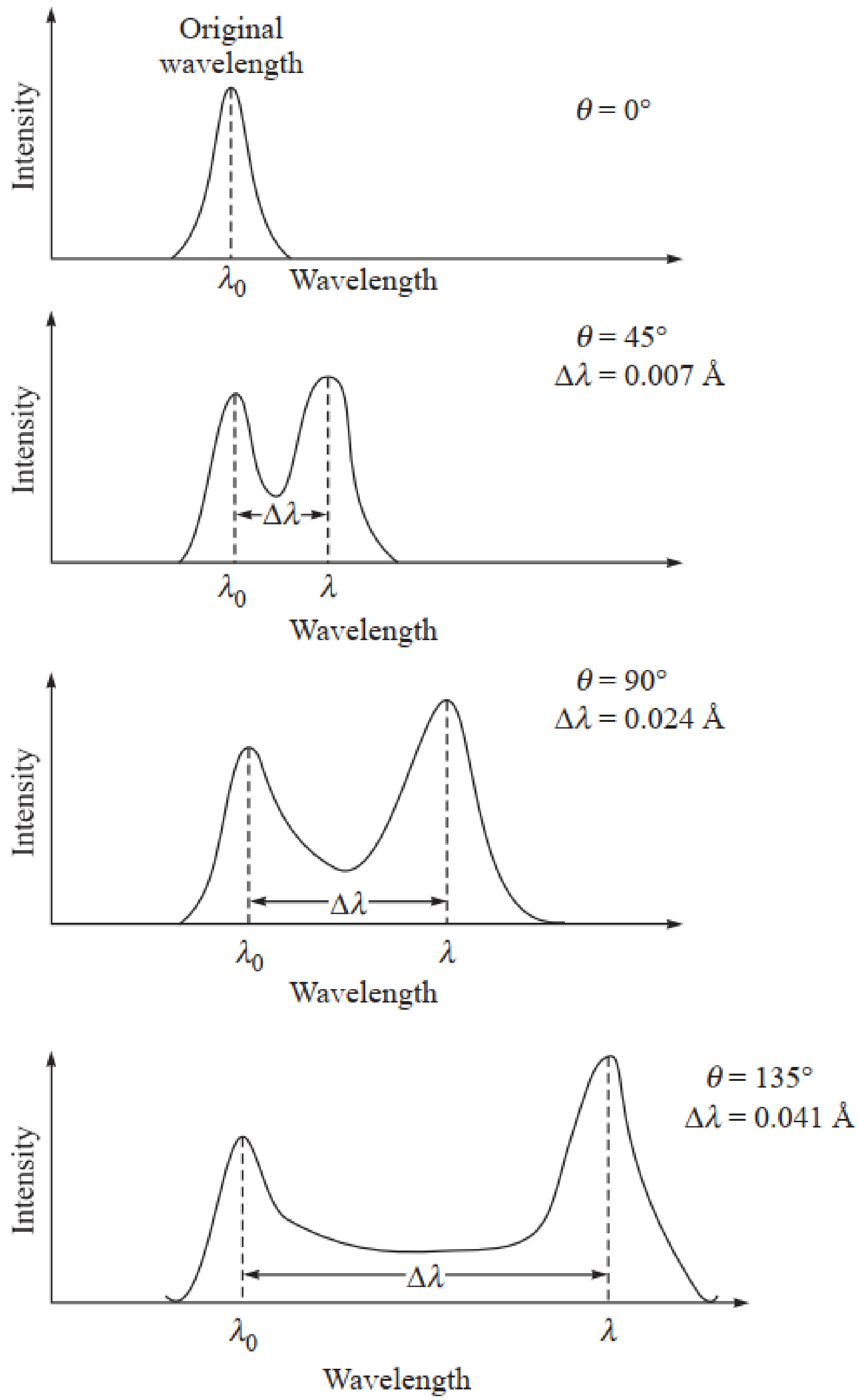
$$\boxed{\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0c}(1 - \cos\theta)} \qquad \ldots(1.15)$$

or, equivalently,

$$\boxed{\Delta\lambda = \frac{2h}{m_0c}\sin^2\frac{\theta}{2}} \qquad \ldots(1.16)$$

The above equation shows that the ***Compton shift*** in wavelength is independent of the wavelength (or energy) of the incident photon and depends only on the angle of scattering.

The quantity (h/m_oc) is called the *Compton wavelength* of the electron. Its value is 0.0242 Å.



Variation of the shifted line with scattering angle. The peak at λ_0 is due to the incident beam.

Conclusion: Compton's work established the existence of photons as real particles having momentum as well as energy.

PROBLEM: X-rays of wavelength 2.0 Å are scattered from a carbon block. The scattered photons are observed at right angles to the direction of the incident beam. Calculate (a) the wavelength of the scattered photon, (b) the energy of the recoil electron.

Solution: (a) If λ and λ' are the wavelengths of the incident and the scattered photons, respectively, and θ is the scattering angle, then

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$= \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1 - \cos 90°)$$

$$= 2.4 \times 10^{-12} \text{ m}$$

$$= 0.024 \text{ Å}$$

Therefore, $$\lambda' = \lambda + \Delta\lambda = \boxed{2.024 \text{ Å}}$$

(b) Neglecting the binding energy of the electron, its recoil energy is given by

$$E = h(\nu - \nu')$$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$$

$$= \frac{hc(\lambda' - \lambda)}{\lambda\lambda'}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}\times0.024\times10^{-10}}{2.0\times10^{-10}\times2.024\times10^{-10}}$$

$$= \boxed{1.17\times10^{-17} \text{ J}}$$

Kinetic Energy of the Recoil Electron

The kinetic energy of the recoil electron is

$$E = h\nu - h\nu'$$

From equation of Compton shift

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

or $$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0 c^2}(1 - \cos\theta)$$

We get $$\frac{\nu}{\nu'} = 1 + \alpha(1 - \cos\theta) \quad \text{where } \alpha = h\nu/m_0 c^2.$$

$$\nu' = \frac{\nu}{1 + \alpha(1 - \cos\theta)} \qquad \ldots(1.17)$$

Therefore,

$$E = h\nu\left[1 - \frac{1}{1 + \alpha(1 - \cos\theta)}\right]$$

or $$\boxed{E = h\nu\,\frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)}} \qquad \ldots(1.18)$$

H.W.

1- In a Compton scattering experiment, the incident radiation has wavelength 2 Å while the wavelength of the radiation scattered through 180° is 2.048 Å. Calculate (a) the wavelength of the radiation scattered at an angle of 60° to the direction of incidence, and (b) the energy of the recoil electron which scatters the radiation through 60°.

2- A photon of energy 0.9 MeV is scattered through 120° by a free electron. Calculate the energy of the scattered photon.