model of an arch structure under load conditions observed between perpendicular polarizers. Such patterns are useful in the optimal design of architectural components. Figure 38.37 illustrates how sunlight becomes polarized when it is scattered. The phenomenon is similar to that creating completely polarized light upon reflection from a surface at Brewster's angle. An unpolarized beam of sunlight traveling in the horizontal direction (parallel to the ground) strikes a molecule of one of the gases that make up air, setting the electrons of the molecule into vibration. These vibrating charges act like the vibrating charges in an antenna. The horizontal component of the electric field vector in the incident wave results in a horizontal component of the vibration of the charges, and the vertical component of the vector results in a vertical component of vibration. If the observer in Figure 38.37 is looking straight up (perpendicular to the original direction of propagation of the light), the vertical oscillations of the charges send no radiation toward the observer. Thus, the observer sees light that is completely polarized in the horizontal direction, as indicated by the brown arrows. If the observer looks in other directions, the light is partially polarized in the horizontal direction.

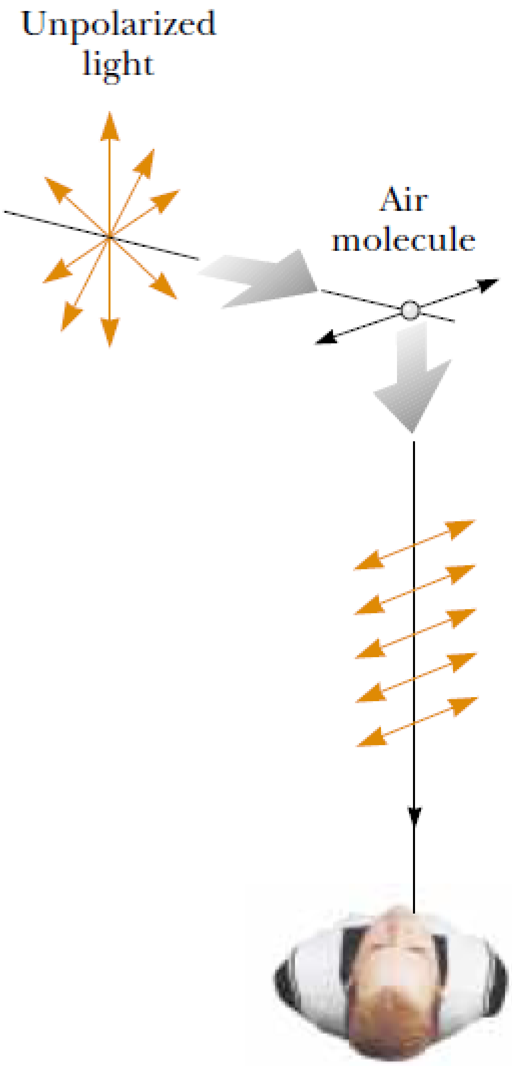


Figure 38.37 The scattering of unpolarized sunlight by air molecules. The scattered light traveling perpendicular to the incident light is plane-polarized because the vertical vibrations of the charges in the air molecule send no light in this direction.

Some phenomena involving the scattering of light in the atmosphere can be understood as follows. When light of various wavelengths % is incident on gas molecules of diameter *d*, where ***d*** **<<✦,** the relative intensity of the scattered light varies as 1/✦⁴. The condition ***d*** **<<✦** is satisfied for scattering from oxygen (O_2) and nitrogen (N_2) molecules in the atmosphere, whose diameters are about 0.2 nm. Hence, short wavelengths (blue light) are scattered more efficiently than long wavelengths (red light). Therefore, when sunlight is scattered by gas molecules in the air, the short-wavelength radiation (blue) is scattered more intensely than the long-wavelength radiation (red).

When you look up into the sky in a direction that is not toward the Sun, you see the scattered light, which is predominantly blue; hence, you see a blue sky. If you look toward the west at sunset (or toward the east at sunrise), you are looking in a direction toward the Sun and are seeing light that has passed through a large distance of air. Most of the blue light has been scattered by the air between you and the Sun. The light that survives this trip through the air to you has had much of its blue component scattered and is thus heavily weighted toward the red end of the spectrum; as a result, you see the red and orange colors of sunset.

Example.1: Plane-polarized light is incident on a single polarizing disk with the direction of $\mathbf{E}_0$ parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3, (b) 5, (c) 10?

Solution

$$I = I_{max}\cos^2\theta \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{I}{I_{max}}}$$

$$\text{(a)} \qquad \frac{I}{I_{max}} = \frac{1}{3} \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{1}{3}} = \boxed{54.7°}$$

$$\text{(b)} \qquad \frac{I}{I_{max}} = \frac{1}{5} \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{1}{5}} = \boxed{63.4°}$$

$$\text{(c)} \qquad \frac{I}{I_{max}} = \frac{1}{10} \qquad \Rightarrow \qquad \theta = \cos^{-1}\sqrt{\frac{1}{10}} = \boxed{71.6°}$$

Example.2: The angle of incidence of a light beam onto a reflecting surface is continuously variable. The reflected ray is found to be completely polarized when the angle of incidence is 48°. What is the index of refraction of the reflecting material?

Solution:

$$\text{By Brewster's law,} \qquad n = \tan\theta_p = \tan(48°) = \boxed{1.11}\,.$$

Example.3: The critical angle for total internal reflection for sapphire surrounded by air is 34.4°. Calculate the polarizing angle for sapphire.

Solution:

$$\sin\theta_c = \frac{1}{n} \qquad \text{or} \qquad n = \frac{1}{\sin\theta_c} = \frac{1}{\sin 34.4°} = 1.77\,.$$

$$\text{Also, } \tan\theta_p = n\,. \qquad \text{Thus,} \qquad \theta_p = \tan^{-1}(n) = \tan^{-1}(1.77) = \boxed{60.5°}\,.$$

Example.4: For a particular transparent medium surrounded by air, show that the critical angle for total internal reflection and the polarizing angle are related by $\cot\theta_p = \sin\theta_c$.

Solution:

$$\sin\theta_c = \frac{1}{n} \text{ and } \tan\theta_p = n$$

$$\text{Thus, } \sin\theta_c = \frac{1}{\tan\theta_p} \text{ or } \boxed{\cot\theta_p = \sin\theta_c}\,.$$

Example.5: How far above the horizon is the Moon when its image reflected in calm water is completely polarized? ($n_{water} = 1.33$)

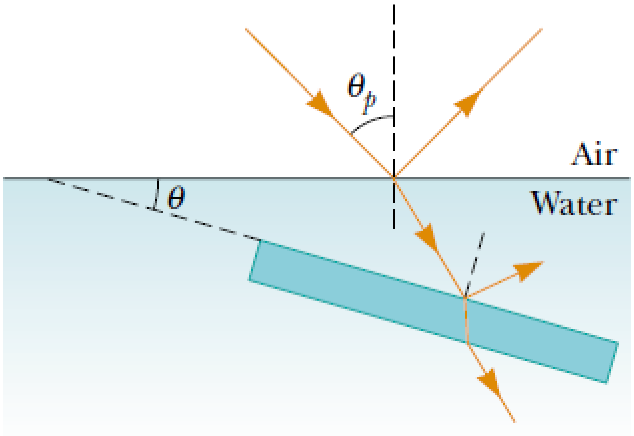
Solution:

Complete polarization occurs at Brewster's angle

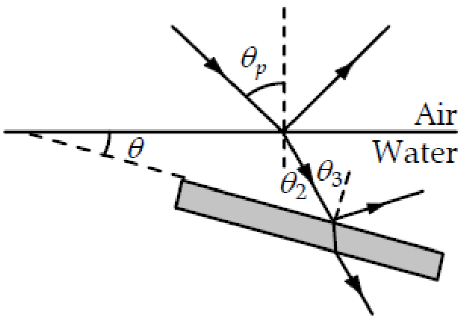
$$\tan\theta_p = 1.33 \qquad\qquad \theta_p = 53.1°\,.$$

Thus, the Moon is $\boxed{36.9°}$ above the horizon.

Example.6 : Light strikes a water surface at the polarizing angle. The part of the beam refracted into the water strikes a submerged glass slab (index of refraction, 1.50), as shown in Figure below . The light reflected from the upper surface of the slab is completely polarized. Find the angle between the water surface and the glass slab.



Solution:



For the air-to-water interface,

$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1} \qquad \theta_p = 53.1°$$

and $$(1)\sin\theta_p = (1.33)\sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9° .$$

For the water-to-glass interface, $\tan\theta_p = \tan\theta_3 = \frac{n_{\text{glass}}}{n_{\text{water}}} = \frac{1.5}{1.33}$ so

$\theta_3 = 48.4°$.

The angle between surfaces is $\theta = \theta_3 - \theta_2 = \boxed{11.5°}$.