We can obtain an expression relating the polarizing angle to the index of refraction of the reflecting substance by using Figure 38.32b. From this figure, we see that $\theta_p + 90° + \theta_2 = 180°$; thus $\theta_2 = 90° - \theta_p$. Using Snell's law of refraction and taking $n_1 = 1$ for air and $n_2 = n$, we have

$$\boxed{n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2}}$$

Because $\sin\theta_2 = \sin(90° - \theta_p) = \cos\theta_p$, we can write this expression for n as $n = \sin\theta_p/\cos\theta_p$, which means that

$$\boxed{n = \tan\theta_p \qquad (38.15)}$$

Brewster's law

This expression is called Brewster's law, and the polarizing angle θ_p is sometimes called Brewster's angle, after its discoverer, David Brewster (1781–1868). Because n varies with wavelength for a given substance, Brewster's angle is also a function of wavelength.

We can understand polarization by reflection by imagining that the electric field in the incident light sets electrons at the surface of the material in Fig. 38.32b into oscillation. The component directions of oscillation are (1) parallel to the arrows shown on the refracted beam of light and (2) perpendicular to the page. The oscillating electrons act as antennas radiating light with a polarization parallel to the direction of oscillation. For the oscillations in direction (1), there is no radiation in the perpendicular direction, which is along the reflected ray. For oscillations in direction (2), the electrons radiate light with a polarization perpendicular to the page. Thus, the light reflected from the surface at this angle is completely polarized parallel to the surface.

Polarization by reflection is a common phenomenon. Sunlight reflected from water, glass, and snow is partially polarized. If the surface is horizontal, the electricfield vector of the reflected light has a strong horizontal component.

انه ليس من الصعوبة أن نفهم السبب الفيزيائي لماذا لا ينعكس الضوء الذي يهتز في مستوى السقوط عندما تكون زاوية السقوط هي زاوية بروستر . فالضوء الساقط يجعل الالكترونات في ذرات الوسط في حالة اهتزاز وان الإشعاع من هذه الذرات المتهيجة يولد الضوء المنعكس . والآن ا ذا لوحظ الضوء المنعكس بزاوية 90 مع الضوء المنكسر فالاهتزازات العمودية على مستوى السقوط يمكن مشاهدتها فقط بينما تلك التي تكون في مستوى السقوط ليس لها مركبة مستعرضة بالاتجاه 90 ولهذا فلا تشع في هذا الاتجاه.

Plane-polarizing

It is possible to represent two electromagnetic waves which have same frequency, one x direction and anther in y direction by following equations

$$E_x(z,t) = i' E_{ox} \cos(kz - wt) \qquad (1)$$

$$E_y(z,t) = j' E_{oy} \cos(kz-wt+\varepsilon) \qquad (2)$$

$$E(z,t) = E_x(z,t) + E_y(z,t) \qquad (3)$$

If the phase different equal to zero or integer number multiply by ($\pm 2\pi$) . In this case you can say that the two waves are in phase and the equation (3) can be write

$$E(z,t) = (i' E_{ox} + j' E_{oy}) \cos(kz-wt) \qquad (4)$$

From equation (4) the wave have constant amplitude equal to ($i' E_{ox} + j' E_{oy}$) so the wave plane-polarized**.** If the phase different equal to integer number multiply by ($\pm\pi$) in this case you can say that these waves have phase different 180 and the equation (3) can be write

$$E(z,t) = (i' E_{ox} - j' E_{oy}) \cos(kz-wt) \qquad (5)$$

This wave is plane-polarized because have constant amplitude but the plane vibration has rotation 180 from plane vibration.
If the Ø represent the angle which made by the plane xz

$$\tan Ø = \pm E_{oy}/E_{ox} \qquad (6)$$

Circular polarizing

If we returned to equation (1) and (2) which represent two waves motion in same direction with same frequency. If the phase different equal to $-\pi/2 + 2m\pi$ where

$m = 0, \pm 1, \pm 2, \ldots\ldots$

and the amplitude is equal i.e. $E_{ox} = E_{oy} = E_o$. Under these conditions we can write the equations (1) and (2) as following

$$E_x(z,t) = i' E_o \cos(kz-wt) \qquad (7)$$

$$E_y(z,t) = j' E_o \sin(kz-wt) \qquad (8)$$

$$E_R(z,t) = E_o [i' \cos(kz-wt) + j' \sin(kz-wt)] \qquad (9)$$

Wave amplitude is E_o.
If the phase different equal to $\pi/2 + 2m\pi$ where
$m = 0, \pm 1, \pm 2, \ldots\ldots$
Under these conditions we can write the equations (1) and (2) as following

$$E_x(z,t) = i' E_o \cos (kz-wt)$$

$$E_y(z,t) = -j' E_o \sin (kz-wt)$$

$$E_L(z,t) = E_o [i' \cos (kz-wt) - j' \sin (kz-wt)] \qquad (10)$$

$$E (z,t)= E_R(z,t) + E_L(z,t)$$

$$E (z,t) = 2 E_o i' \cos (kz-wt) \qquad (11)$$

Elliptical polarizing

The most general type of vibration is elliptical, of which linear and circular vibrations are a special case of elliptical. If we returned to equation (1) and (2)

$$E_x(z,t) = E_{ox} \cos (kz-wt)$$

$$E_y(z,t) = E_{oy} \cos (kz-wt+\varepsilon)$$

From eq. (1) we obtained

$$\sin (kz-wt)= [1- (E_x/E_{ox})^2]^{1/2} \qquad (12)$$

From eq. (2) we obtained

$$E_y/E_{oy} = \cos (kz-wt) \cos \varepsilon - \sin(kz-wt) \sin \varepsilon \qquad (13)$$

Sub. in eq. (13) the values of cos (kz-wt) , sin(kz-wt) from (1) , (12) we obtained

$$E_y/E_{oy}= (E_x/E_{ox}) \cos \varepsilon - [1- (E_x/E_{ox})^2]^{1/2} \sin \varepsilon$$

$$E_y/E_{oy}- (E_x/E_{ox}) \cos \varepsilon = - [1- (E_x/E_{ox})^2]^{1/2} \sin \varepsilon$$

$$(E_y/E_{oy})^2 - (E_x/E_{ox})^2 - 2 (E_x/E_{ox}) (E_y/E_{oy}) \cos \varepsilon = \sin^2 \varepsilon \qquad (14)$$

$$\tan \alpha = [(2 E_{ox}E_{oy})/(E_{ox}{}^2 - E_{oy}{}^2)] \cos \varepsilon \qquad (15)$$

If $\alpha = 0$ this meaning

$$\varepsilon = \pm\pi/2, \pm (3/2) \pi, \pm (5/2) \pi, \ldots\ldots. \qquad (16)$$

If we sub. eq. (16) in eq. (14) we obtained

Polarization by Double Refraction

Solids can be classified on the basis of internal structure. Those in which the atoms are arranged in a specific order are called *crystalline;* the NaCl structure of Figure 38.26 is just one example of a crystalline solid. Those solids in which the atoms are distributed randomly are called *amorphous.* When light travels through an amorphous material, such as glass, it travels with a speed that is the same in all directions. That is, glass has a single index of refraction. In certain crystalline materials, however, such as calcite and quartz, the speed of light is not the same in all directions. Such materials arecharacterized by two indices of refraction. Hence, they are often referred to as doublerefracting or birefringent materials.

Upon entering a calcite crystal, unpolarized light splits into two plane polarized rays that travel with different velocities, corresponding to two angles of refraction, as shown in Figure 38.33. The two rays are polarized in two mutually perpendicular directions, as indicated by the dots and arrows. One ray, called the ordinary (O) ray, is characterized by an index of refraction n_O that is the same in all directions. This means that if one could place a point source of light inside the crystal, as in Figure 38.34, the ordinary waves would spread out from the source as spheres.

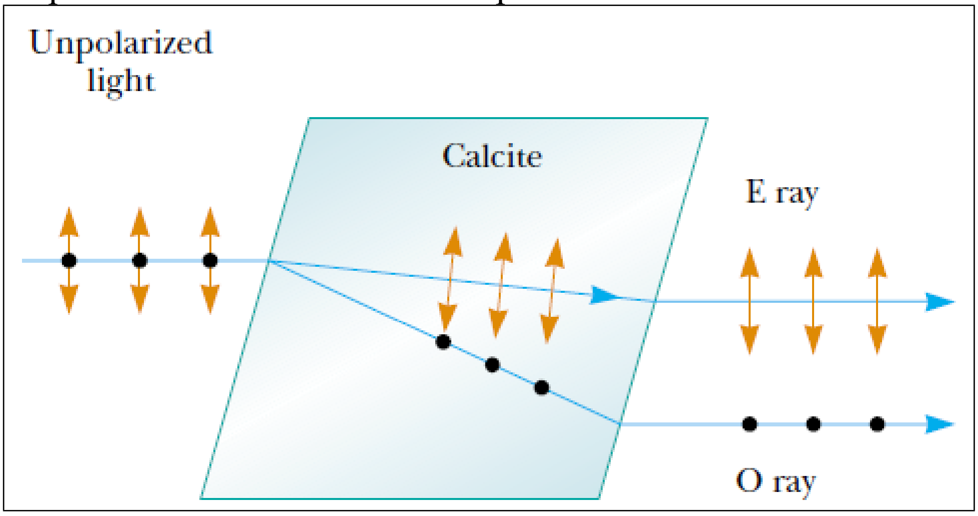

Figure 38.33 Unpolarized light incident on a calcite crystal splits into an ordinary **(O)** ray and an extraordinary **(E)** ray. These two rays are polarized in mutually perpendicular directions.

The second plane-polarized ray, called the extraordinary **(E)** ray, travels with different speeds in different directions and hence is characterized by an index of refraction n_E that varies with the direction of propagation. Consider again the point source within a birefringent material, as in Figure 38.34. The source sends out an extraordinary wave having wave fronts that are elliptical in cross section. Note from Figure 38.34 that there is one direction, called the optic axis, along which the ordinary and extraordinary rays have the same speed, corresponding to the direction for which $n_O = n_E$. The difference in speed for the two rays is a maximum in the direction perpendicular to the optic axis. For example, in calcite, $n_O = 1.658$ at a wavelength of 589.3 nm, and n_E varies from 1.658 along the optic axis to 1.486 perpendicular to the optic axis.

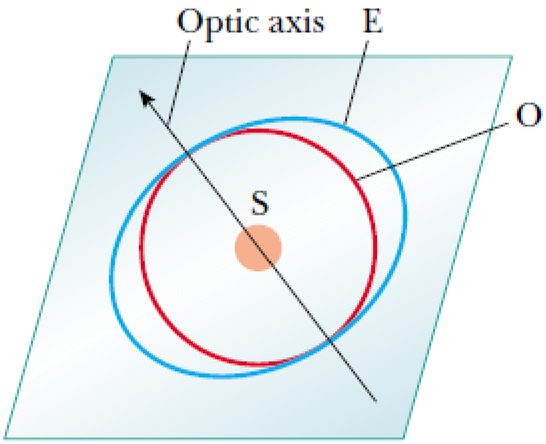

Figure 38.34 A point source **S** inside a double-refracting crystal produces a spherical wave front corresponding to the ordinary ray and an elliptical wave front corresponding to the extraordinary ray. The two waves propagate with the same velocity along the optic axis.

Polarization by Scattering

When light is incident on any material, the electrons in the material can absorb and reradiate part of the light. Such absorption and reradiation of light by electrons in the gas molecules that make up air is what causes sunlight reaching an observer on the Earth to be partially polarized. You can observe this effect- called scattering-by looking directly up at the sky through a pair of sunglasses whose lenses are made of polarizing material. Less light passes through at certain orientations of the lenses than at others.


Peter Aprahamian/Science Photo Library

(a) (b)

Figure 38.36 (a) Strain distribution in a plastic model of a hip replacement used in a medical research laboratory. The pattern is produced when the plastic model is viewed between a polarizer and analyzer oriented perpendicular to each other. (b) A plastic