$$R = \frac{\lambda}{\Delta\lambda} = \frac{589.30\text{ nm}}{589.59\text{ nm} - 589.00\text{ nm}} = \frac{589.30}{0.59} = \boxed{999}$$

(B) To resolve these lines in the second-order spectrum, how many slits of the grating must be illuminated?

Solution From Equation 38.12 and the result to part (A), we find that

$$N = \frac{R}{m} = \frac{999}{2} = \boxed{500\text{ slits}}$$

Diffraction of X-Rays by Crystals

In principle, the wavelength of any electromagnetic wave can be determined if a grating of the proper spacing (on the order of ✦) is available. X-rays, discovered by Wilhelm Roentgen (1845–1923) in 1895, are electromagnetic waves of very short wavelength (on the order of 0.1 nm). However, the atomic spacing in a solid is known to be about 0.1 nm. In 1913, Max von Laue (1879–1960) suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for x-rays. Subsequent experiments confirmed this prediction. The diffraction patterns from crystals are complex because of the three-dimensional nature of crystal structure. Nevertheless, x-ray diffraction has proved to be an invaluable technique for elucidating these structures and for understanding the structure of matter.

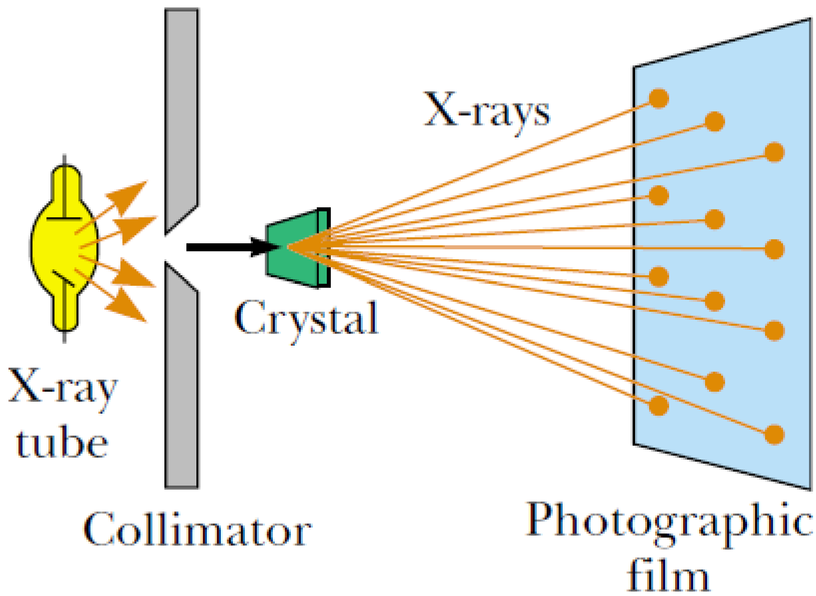
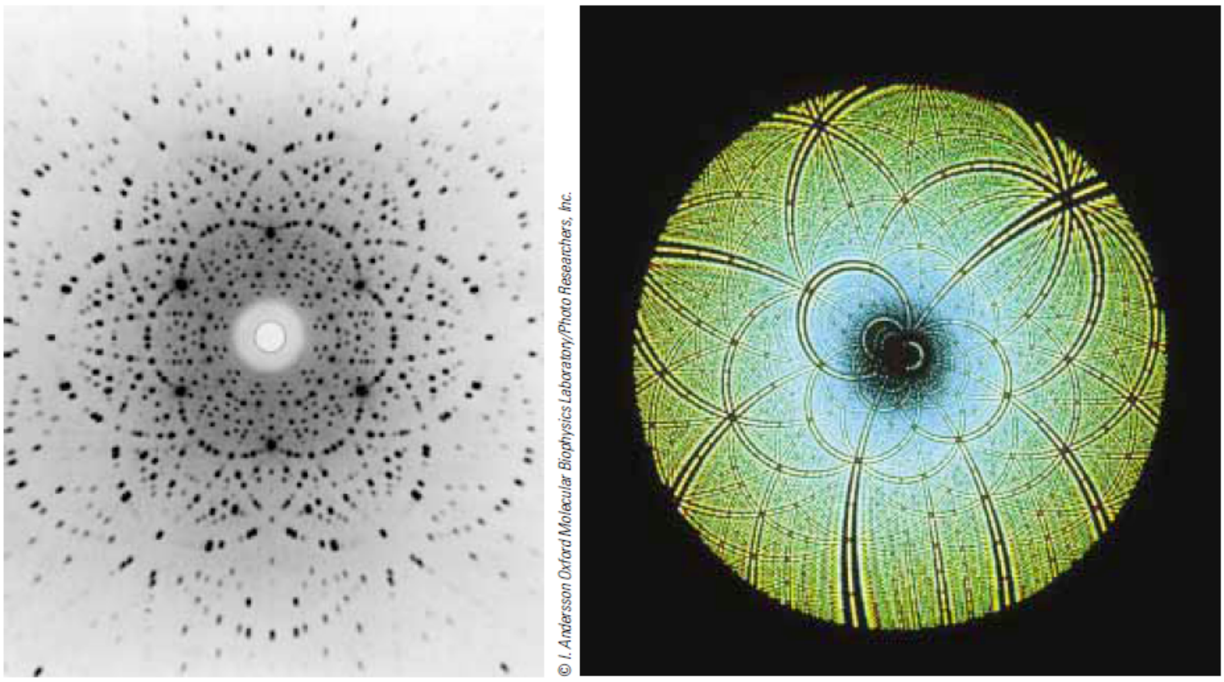


Figure 38.24 Schematic diagram of the technique used to observe the diffraction of x-rays by a crystal. The array of spots formed on the film is called a Laue pattern.

Figure 38.24 is one experimental arrangement for observing x-ray diffraction from a crystal. A collimated beam of monochromatic x-rays is incident on a crystal. The diffracted beams are very intense in certain directions, corresponding to constructive interference from waves reflected from layers of atoms in the crystal. The diffracted beams, which can be detected by a photographic film, form an array of spots known as a *Laue pattern*, as in Figure 38.25a. One can deduce the crystalline structure by analyzing the positions and intensities of the various spots in the pattern. Fig. 38.25b shows a Laue pattern from a crystalline enzyme, using a wide range of wavelengths so that a swirling pattern results.



(a) (b)

Figure 38.25 (a) A Laue pattern of a single crystal of the mineral beryl (beryllium aluminum silicate). Each dot represents a point of constructive interference. (b) A Laue npattern of the enzyme Rubisco, produced with a wide-band x-ray spectrum. This enzyme is present in plants and takes part in the process of photosynthesis. The Laue pattern is used to determine the crystal structure of Rubisco.

The arrangement of atoms in a crystal of sodium chloride (NaCl) is shown in Figure 38.26. Each unit cell (the geometric solid that repeats throughout the crystal) is a cube having an edge length *a*. A careful examination of the NaCl structure shows that the ions lie in discrete planes (the shaded areas in Fig. 38.26). Now suppose that aAn incident x-ray beam makes an angle ! with one of the planes, as in Figure 38.27. The beam can be reflected from both the upper plane and the lower one. However,

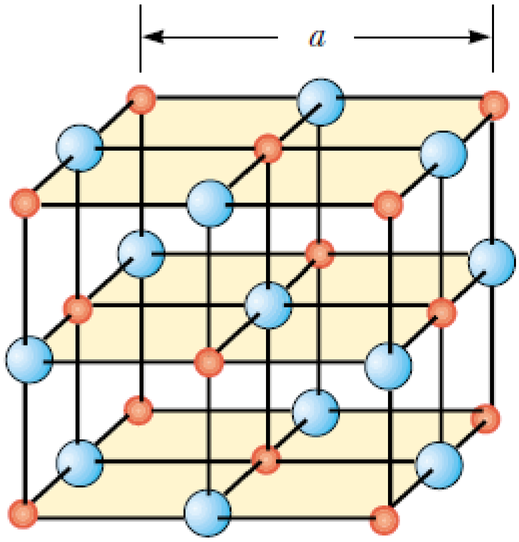


Figure 38.26 Crystalline structure of sodium chloride (NaCl). The blue spheres represent Cl^- ions, and the red spheres nrepresent Na, ions. The length of the cube edge is a = 0.562 737 nm.

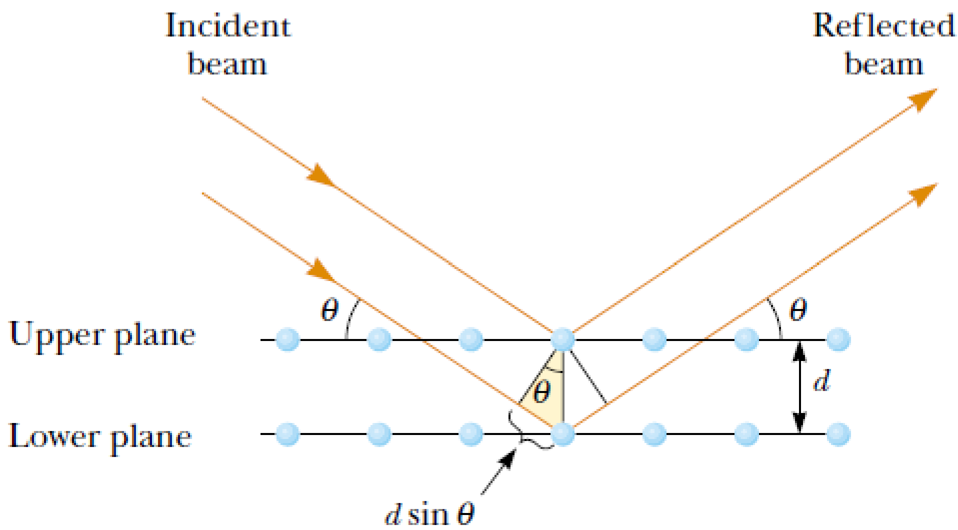


Figure 38.27 A two-dimensional description of the reflection of an x-ray beam from two parallel crystalline planes separated by a distance d. The beam reflected from the lower plane travels farther than the one reflected from the upper plane by a distance $2d \sin \theta$.

the beam reflected from the lower plane travels farther than the beam reflected from the upper plane. The effective path difference is $2d \sin \theta$. The two beams reinforce each other (constructive interference) when this path difference equals some integer multiple of λ. The same is true for reflection from the entire family of parallel planes. Hence, the condition for *constructive* interference (maxima in the reflected beam) is

$$2d \sin \theta = m\lambda \qquad m = 1, 2, 3, \ldots \qquad (38.13)$$

Bragg's law

This condition is known as Bragg's law, after W. L. Bragg (1890–1971), who first derived the relationship. If the wavelength and diffraction angle are measured, Equation 38.13 can be used to calculate the spacing between atomic planes.

Diffraction Patterns from Narrow Slits

Example 1. Helium–neon laser light (λ=632.8 nm) is sent through a 0.3mm-wide single slit. What is the width of the central maximum on a screen 1 m from the slit?

Solution:

$$\sin\theta = \frac{\lambda}{a} = \frac{6.328\times10^{-7}}{3\ \times10^{-4}} = 2.11\times10^{-3}$$

$$\frac{y}{1\ \text{m}} = \tan\theta \approx \sin\theta = \theta\ \ (\text{for small }\theta)$$

$$2y = \boxed{4.22\ \text{mm}}$$

Example 2. A screen is placed 50 cm from a single slit, which is illuminated with 690 nm light. If the distance between the first and third minima in the diffraction pattern is 3 mm, what is the width of the slit?

Solution:

$$\Delta y = 3\ \times10^{-3}\ \text{nm} \qquad \frac{y}{L} \approx \sin\theta = \frac{m\lambda}{a}$$

$$\Delta m = 3-1 = 2 \quad \text{and} \qquad a = \frac{\Delta m\lambda L}{\Delta y}$$

$$a = \frac{2\left(690\times10^{-9}\ \text{m}\right)(0.5\ \text{m})}{\left(3\ \times10^{-3}\ \text{m}\right)} = \boxed{2.3\ \times10^{-4}\ \text{m}}$$

Example 3. Light of wavelength 587.5 nm illuminates a single slit 0.750 mm in width.

At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 0.850 mm from the center of the principal maximum?

Solution:

$$\sin\theta = \frac{y}{L} = \frac{m\lambda}{a}$$

Therefore, for first minimum, $m = 1$ and

$$L = \frac{ay}{m\lambda} = \frac{(7.50\times10^{-4}\ \text{m})(8.50\times10^{-4}\ \text{m})}{(1)(587.5\times10^{-9}\ \text{m})} = \boxed{1.09\ \text{m}}$$

Resolution of Single-Slit and Circular Apertures

Example 4. The pupil of a cat's eye narrows to a vertical slit of width 0.5 mm in daylight. What is the angular resolution for horizontally separated mice? Assume that the average wavelength of the light is 500 nm.

Solution:

$$\sin\theta = \frac{\lambda}{a} =$$

$$\boxed{1.00\times10^{-3}\ \text{rad}}$$

Example 5. Find the radius a star image forms on the retina of the eye if the aperture diameter (the pupil) at night is 0.7 cm and the length of the eye is 3 cm. Assume the representative wavelength of starlight in the eye is 500 nm.

Solution:

y = radius of star-image L = length of eye

λ = 500 nm D = pupil diameter

θ = half angle

$$\theta_{\text{min}} = \frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$y = \frac{(1.22)(5\times10^{-7})(0.03)}{7\times10^{-3}} = \boxed{2.61\ \mu\text{m}}$$

The Diffraction Grating

Example 5. White light is spread out into its spectral components by a diffraction grating. If the grating has 2 000 grooves per centimeter, at what angle does red light of wavelength 640 nm appear in first order?

Solution:

$$d = \frac{1 \text{ cm}}{2\,000} = \frac{1 \times 10^{-2} \text{ m}}{2\,000} = 5 \ \mu\text{m}$$

$$\sin\theta = \frac{m\lambda}{d} = \frac{1(640 \times 10^{-9} \text{ m})}{5 \times 10^{-6} \text{ m}} = 0.128 \qquad \theta = \boxed{7.35°}$$

Example 6. A helium–neon laser (λ = 632.8 nm) is used to calibrate a diffraction grating. If the first-order maximum occurs at 20.5°, what is the spacing between adjacent grooves in the grating?

Solution:

$$\sin\theta = 0.350: \qquad d = \frac{\lambda}{\sin\theta} = \frac{632.8 \text{ nm}}{0.350} = 1.81 \times 10^{3} \text{ nm}$$

$$\text{Line spacing} = \boxed{1.81 \ \mu\text{m}}$$

Example 7. Three discrete spectral lines occur at angles of 10.09°, 13.71°, and 14.77° in the first-order spectrum of a grating spectrometer. If the grating has 3 660 slits/cm, what are the wavelengths of the light?

Solution:

$$d = \frac{1}{3\,660 \text{ lines/cm}} = 2.732 \times 10^{-4} \text{ cm} = 2.732 \times 10^{-6} \text{ m} = 2\,732 \text{ nm}$$

$$\lambda = \frac{d\sin\theta}{m}: \qquad \text{At } \theta = 10.09° \qquad \lambda = \boxed{478.7 \text{ nm}}$$

$$\text{At } \theta = 13.71°, \qquad \lambda = \boxed{647.6 \text{ nm}}$$

$$\text{At } \theta = 14.77°, \qquad \lambda = \boxed{696.6 \text{ nm}}$$