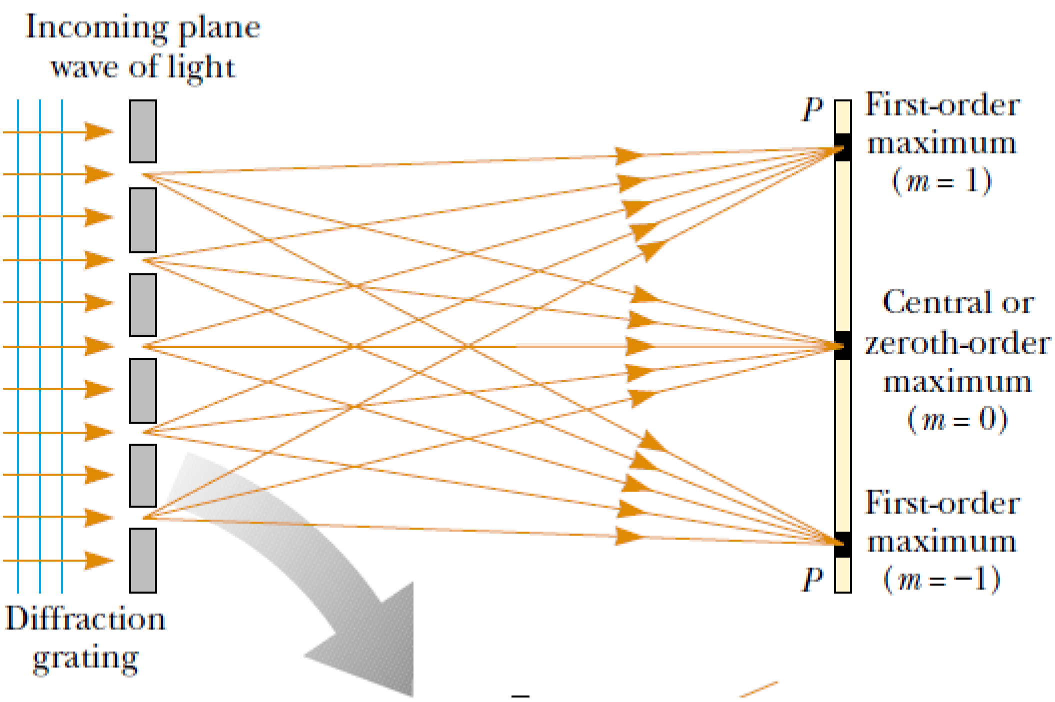
The Diffraction Grating

The diffraction grating, a useful device for analyzing light sources, consists of a large number of equally spaced parallel slits. A *transmission grating* can be made by cutting parallel grooves on a glass plate with a precision ruling machine. The spaces between the grooves are transparent to the light and hence act as separate slits. A *reflection grating* can be made by cutting parallel grooves on the surface of a reflective material. A section of a diffraction grating is illustrated in Figure 38.16. A plane wave is incident from the left, normal to the plane of the grating. The pattern observed on the screen (far to the right of Figure 38.16) is the result of the combined effects of interference and diffraction. Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern. The waves from all slits are in phase as they leave the slits. However, for some arbitrary direction θ measured from the horizontal, the waves must travel different path lengths before reaching the screen. From Figure 38.16, note that the path difference δ between rays from any two adjacent slits is equal to $d \sin\theta$. If this path difference equals one wavelength or some integral multiple of a wavelength, then waves from all slits are in phase at the screen and a bright fringe is observed. Therefore, the condition for *maxima* in the interference pattern at the angle θ_{bright} is

$$d \sin\theta_{\text{bright}} = m\lambda \qquad m = 0, \pm 1, \pm 2, \pm 3, \ldots \qquad (38.10)$$

Condition for interference maxima for a grating



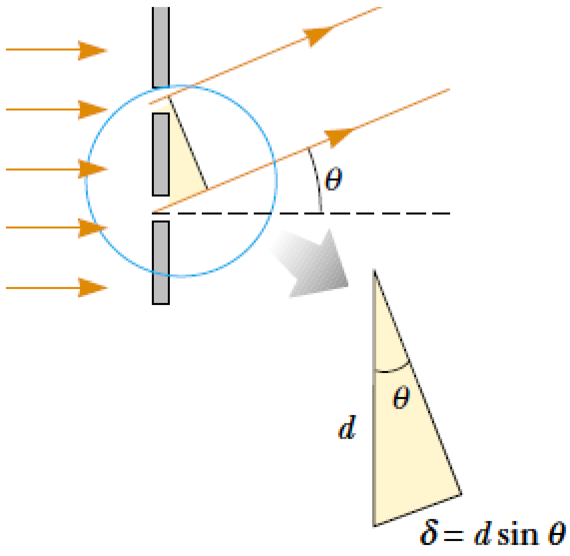


Figure 38.16 Side view of a diffraction grating. The slit separation is d, and the path difference between adjacent slits is $d\sin\theta$.

We can use this expression to calculate the wavelength if we know the grating spacing d and the angle θ_{bright}. If the incident radiation contains several wavelengths, the mth-order maximum for each wavelength occurs at a specific angle. All wavelengths are seen at $\theta = 0$, corresponding to $m = 0$, the zeroth-order maximum. The first-order maximum ($m = 1$) is observed at an angle that satisfies the relationship $\sin\theta_{bright} = \lambda/d$; the second-order maximum ($m = 2$) is observed at a larger angle θ_{bright} , and so on.

The intensity distribution for a diffraction grating obtained with the use of a monochromatic source is shown in Figure 38.17. Note the sharpness of the principal maxima and the broadness of the dark areas. This is in contrast to the broad bright fringes characteristic of the two-slit interference pattern (see Fig. 37.7). You should also review Figure 37.14, which shows that the width of the intensity maxima they decreases as the number of slits increases. Because the principal maxima are so sharp, are much brighter than two-slit interference maxima.

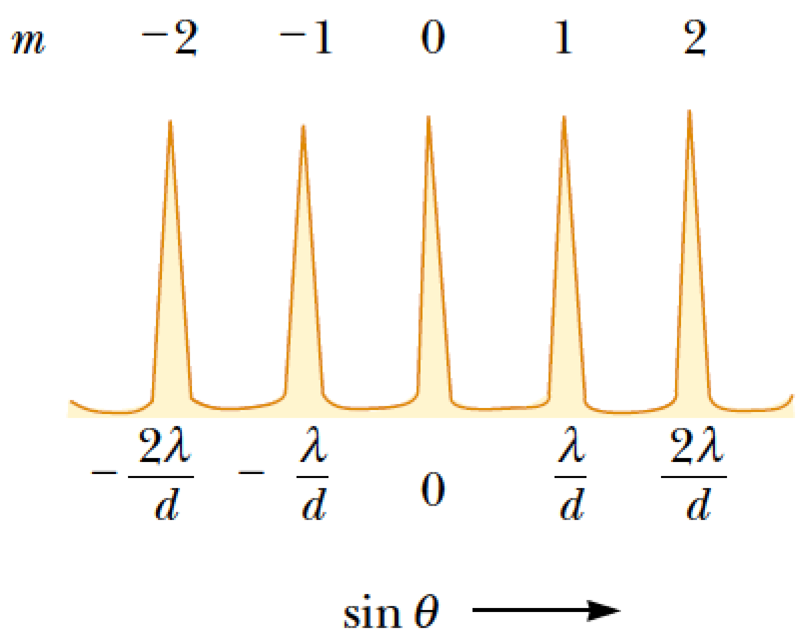


Figure 38.17 Intensity versus $\sin\theta$ for a diffraction grating. The zeroth-, first-, and second-order maxima are shown.

A schematic drawing of a simple apparatus used to measure angles in a diffraction pattern is shown in Figure 38.18. This apparatus is a *diffraction grating spectrometer*. The light to be analyzed passes through a slit, and a collimated beam of light is incident on the grating. The diffracted light leaves the grating at angles that satisfy Equation 38.10, and a telescope is used to view the image of the slit. The wavelength can be determined by measuring the precise angles at which the images of the slit appear for the various orders.

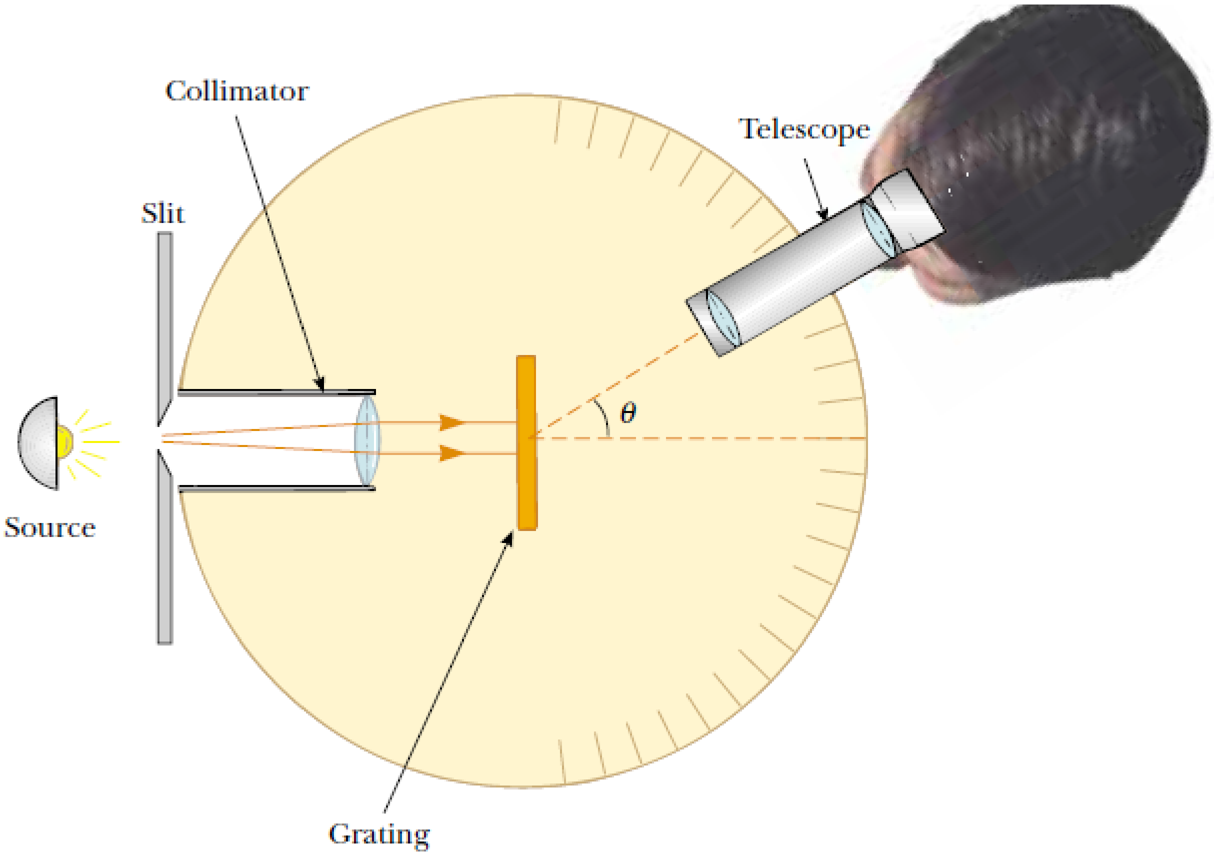


Figure 38.18 Diagram of a diffraction grating spectrometer. The collimated beam incident on the grating is spread into its various wavelength components with constructive interference for a particular wavelength occurring at the angles θ_{bright} that satisfy the equation $d \sin\theta_{\text{bright}} = m\lambda$, where $m = 0, 1, 2, \ldots$.

Example 38.7 The Orders of a Diffraction Grating

Monochromatic light from a helium–neon laser ($\lambda = 632.8$ nm) is incident normally on a diffraction grating containing 6 000 grooves per centimeter. Find the angles at which the first- and second-order maxima are observed.

Solution First, we must calculate the slit separation, which is equal to the inverse of the number of grooves per centimeter:

$$d = \frac{1}{6\,000}\ \text{cm} = 1.667 \times 10^{-4}\ \text{cm} = 1\,667\ \text{nm}$$

For the first-order maximum ($m = 1$), we obtain

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{632.8\ \text{nm}}{1\,667\ \text{nm}} = 0.379\,6$$

$$\theta_1 = \boxed{22.31°}$$

For the second-order maximum ($m = 2$), we find

$$\sin \theta_2 = \frac{2\lambda}{d} = \frac{2(632.8 \text{ nm})}{1\,667 \text{ nm}} = 0.759\,2$$

$$\theta_2 = \boxed{49.39°}$$

Effect of Increasing The Number of Slits

When the intensity pattern due to one, two, three, and more slits of the same width is photographed, a series of pictures like those shown in Fig. 17*A(a)* to (f) is obtained.The arrangement of light source, slit, lenses, and recording plate used in taking these pictures was similar to that described in previous chapters, and the light used was the blue line from a mercury arc. These patterns therefore are produced by *Fraunhofer diffraction.* In fact, it was because of Fraunhofer's original investigations ofthe diffraction of parallel light by gratings in 1819 that his name became associated with this type of diffraction. Fraunhofer's first gratings were made by winding fine wires around two parallel screws. Those used in preparing Fig. 17A were made by cutting narrow transparent lines in the gelatin emulsion on a photographic plate. The most striking modification in the pattern as the number of slits is increased consists of a narrowing of the interference maxima. For two slits these are diffuse, having an intensity which was shown in the last chapter to vary essentially as the square of the cosine. With more slits the sharpness of these *principal maxima* increases rapidly, and in pattern *(f)* of the figure, with 20 slits, they have become narrow lines. Another change, of less importance, which can be seen in patterns *(c), (d),* and *(e)* is the appearance of weak *secondary maxima* between the principal maxima, their number increasing with the number of slits. For three slits only one secondary maximum is present, its intensity being 11.1 percent of the principal maximum. Figure 17B shows an intensity curve for this case, plotted according to the theoretical equation (l7b) given in the next section. Here the individual slits were assumed very narrow. Actually the intensities of all maxima are governed by the pattern of a single slit of width equal to that of anyone of the slits used. The width of the intensity envelopes would be identical in the various patterns of Fig. 17A if the slits had been of the same width in all cases. In fact there were slight differences in the slit widths used for some of the patterns.