Lectures of Nuclear Physics

(Phy.414)

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Fourth Grade

Second Lecture

Mirror nuclei

Two nuclides are called mirror nuclei if they have equal mass numbers A, and if the number of protons Z in one of them is equal to the number of neutrons N in the other.

boron-11 and carbon-11

$$ {}^{11}_{5}B - {}^{11}_{6}C $$

,

carbon-13 and nitrogen-13

$$ {}^{13}_{6}C_7 - {}^{13}_{7}N_6 $$

Angular Moments in the Nucleus

In classical mechanics, the angular momentum ***L*** of a particle moving with a linear momentum **p** at a location **r** around a reference point is:

$$ \mathbf{L} = \mathbf{r} \times \mathbf{p} $$

Orbital angular momentum

Since protons and neutrons move in an average field and so cause orbital angular momentum to build up, the *expectation value of the angular momentum* of a nucleon is evaluated, and for simplicity by calculating the magnitude of the average value of $\langle L^2 \rangle$, instead of $\langle L \rangle$.

Since the nucleus is an isolated system (no external torque acting on a system), then its angular momentum is conserved, and represented by ***orbital quantum number ℓ*** with a value:

$$\langle L^2 \rangle = \hbar^2 \ell(\ell + 1)$$

or,

$$|\mathrm{L}| = \hbar\sqrt{\ell(\ell + 1}$$

with ***orbital quantum number*** restricted to the values **ℓ = 0, 1, 2, 3.**

In addition,

$$< L^2 >=< L_x^2 + L_y^2 + L_z^2 >$$

is a function of position, as shown in Fig. (#).

Analogy to the atomic physics, the nuclear physics use the same spectroscopic notation as shown below:

ℓ value	0	1	2	3	4	5	6
symbol	s	p	d	f	g	h	i

It is the very act of measuring one component of L that makes the others indeterminate; we usually choose the z- component to be determined to get:

$$< L_z > = \hbar m_\ell$$

Where

$m_\ell = 0, \pm 1, \pm 2 \ldots, \pm \ell$.

is the ***magnetic quantum number***. The value of *m* can range from **-ℓ *to* +ℓ, *inclusive of zero.*** Notice that:

$$|< L_z >| < |\mathrm{L}| = \hbar\sqrt{\ell(\ell + 1)}$$

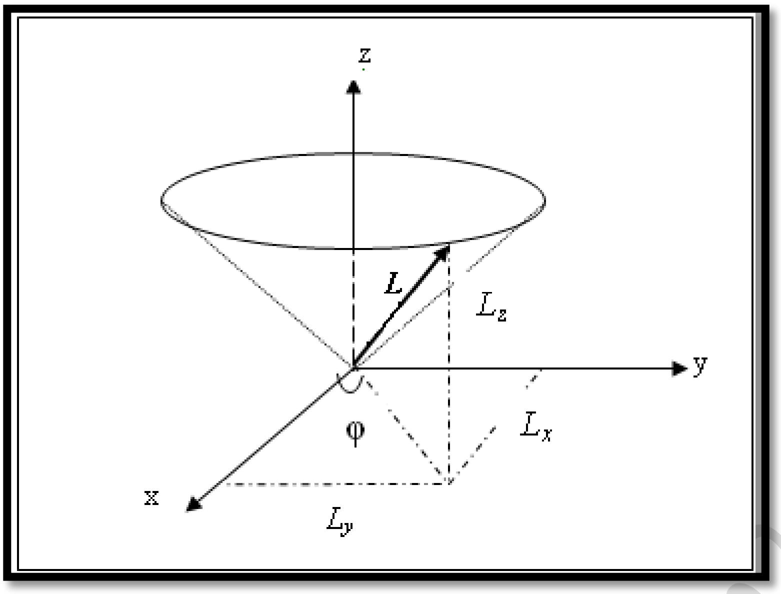

Figure (#) The vector ***L*** rotates about the z-axis, so that *Lz* stays constant, but *Lx* and *Ly* are variable.

Intrinsic angular momentum

As in the electronic state in atom, it is required to introduce a new quantum number called intrinsic quantum number or simply ***spin*** denoted by (**s**) with a value **s = ½** for the nucleon. The spin can be treated as an angular momentum. Thus:

$$\langle s^2 \rangle = \hbar^2 s\,(s+1)$$

or

$$|s| = \hbar\sqrt{s(s+1}$$

and,

$$\langle s_z \rangle = \hbar m_s$$

Where

$m_s = \pm\frac{1}{2}$ **is the *spin quantum number*.**

It is often useful to imagine the spin as a vector **s** with possible z components of value:

$$< s_z > = \pm\tfrac{1}{2}\hbar$$

Total angular quantum momentum

The bounded nucleons inside the nucleus, like electrons of the atom, move in a central potential with orbital angular momentum ***L*** and spin **s** will have a total angular momentum.

$$j = L + S$$

The total angular momentum ***j*** has the same behavior of ***L*** and **s,** so that:

$$<j^2> = \hbar^2 j\,(j + 1)$$

or

$$|\mathrm{j}| = \hbar\sqrt{j(j+1}$$

Where, ***j*** *is the total angular momentum quantum number*,

then it can be written in term of the vector sum of the orbital angular momentum and spin momentum as shown in Fig. (), with:

$$<j_z> = <\ell_z + s_z> = \hbar m_j$$

where,

$\boldsymbol{m_j} = -j, -j+1, -j+2 \ldots, j-2, j-1, j \; (mj \neq 0)$

$= m\ell + ms = m\ell \pm \frac{1}{2}$

is the *total angular quantum number*.

Since m_ℓ is always integer, m_j must be half-integer:

mj = ±1/2, ±3/2, ±5/2,……, then j also is a half integer.

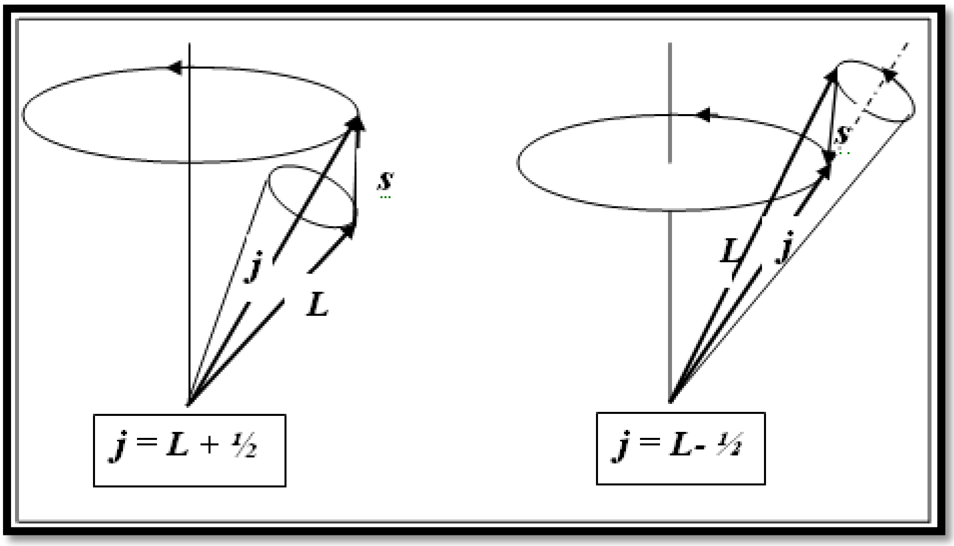

Fig. (),The coupling of orbital angular momentum ***L*** to spin angular momentum ***s*** giving total angular momentum ***j***.

Nuclear Magnetic Moments

Both, the protons (positive charge e) and neutrons (no charge) are moving inside the nucleus. Consequently, there are charge, mass and current densities. As a result, magnetic dipole and electric quadrupole moment produced.

Associated with the nuclear spin is a magnetic moment **μ**, which can take on any value because it *is not quantized*. This lead to additional information on the nature of nuclear forces and help in selecting an appropriate nuclear model.

Just like for electrons of the atom, the nuclear magnetic moment of the orbital motion of a proton is expressed in terms of a *nuclear magneton* μ_N.

$$\mu_N = e\hbar/2m_p$$

where, m_p is the mass of the proton.

For the intrinsic spin of the nucleus, introduce a nuclear **g** factor, called gyromagnatic ratio **g**, to relate the magnetic moment μ of a nucleus to its spin angular momentum **I**. The nuclear **g factor** is defined as the ratio of the nuclear magnetic moment, expressed in terms of nuclear magneton, to the spin angular momentum, expressed in units of **ħ**:

$g = \mu/I\ \mu_N$

and,

$\mu/g = I\ \mu_N = I\ e\hbar/2m_p$

where,

$\mu_N = \mu_B\ /1840$

$= 0.505\times10^{-23}$ **ergs/gauss**

$= 3.15245\times10^{-14}$ **MeV/T**

Here, ***μ_B is the Bohr magneton = 5.78838×10-11 MeV/T***.

When a nucleus of magnetic moment μ is in a constant magnetic field B, it will presses about the direction of B with a frequency f given by Larmor's theorem.

$f = \mu B/I\hbar$

The magnetic moment μ of a nucleus can thus be found by measuring Larmor's frequency f.

of very great importance in nuclear physics are the magnetic moments of the proton μp, neutron μn, and deuteron μd, their measured values are:

μp = 2.792847 μN

μn = -1.913043 μN

μd = 0.857438 μN

It is worthwhile to mention the fact that, since the nuclear magnetic moments are only of the order of magnitude of the nuclear magneton (<< Bohr magneton) is, therefore, another strong argument against the existence of electrons inside the nucleus.

Mass Defect

Careful measurements have shown that the mass of a particular atom or isotope is always slightly less than the number of nucleons (sum of the individual neutrons and protons) of which the atom consists. The difference between the atomic mass of the atom and the total number of nucleons (A) in the nucleus is called the ***mass defect or mass excess*** **(Δm)**. The mass defect can be expressed in terms of atomic mass units and/or in terms of energy as:,

$$\Delta m = M(Z,N) - A \ \text{u} \quad (1)$$

$$\Delta m = \{M(Z,N) - A\} 931.5 \ MeV \quad (2)$$

where: Δm = mass defect (u or MeV)

$M(Z, N)$ = mass of nuclide ${}^{A}_{Z}X$ (u)

A = mass number

Q1/

Calculate the mass defect for lithium-7. The mass of ^{7}**Li** is 7.016003 u.

Solution:

Apply Eq. 1

$\Delta m = 7.016003 - 7 \ \text{u}$

Applying Eq. 2

$\Delta m = (7.016003 - 7)931.5\,\boldsymbol{MeV}$

$= 14.9067945$ MeV

Binding energy

Binding energy is defined as the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles (nucleons). **It can also be understood** as the amount of energy that would be released if the nucleus was formed from the separate particles. The binding energy of nuclei is always a positive number, since all nuclei require net energy to separate them into individual protons and neutrons.

Since **1 u** is equivalent to **931.5 MeV** of energy, the binding energy can be calculated by the mass difference between the nucleus and the sum of those of the free nucleons,

$$BE(Z,A) = \{Zm_H + Nm_n - M(Z,N)\}931.5\ MeV$$

$m_p =$ mass of proton (1.0072764 u)

$m_n =$ mass of neutron (1.008665 u)

$m_e =$ mass of electron (0.000548597 u)

$m_H = m_p + m_e =$ mass of hydrogen atom = (1.007825 u)

According to Einstein's special theory of relativity, mass is a measure of the total energy of a system (**$E=mc^2$**$= \Delta mC^2$). Thus, the total energy of a nucleus is less than the sum of the energies of its constituent nucleons.

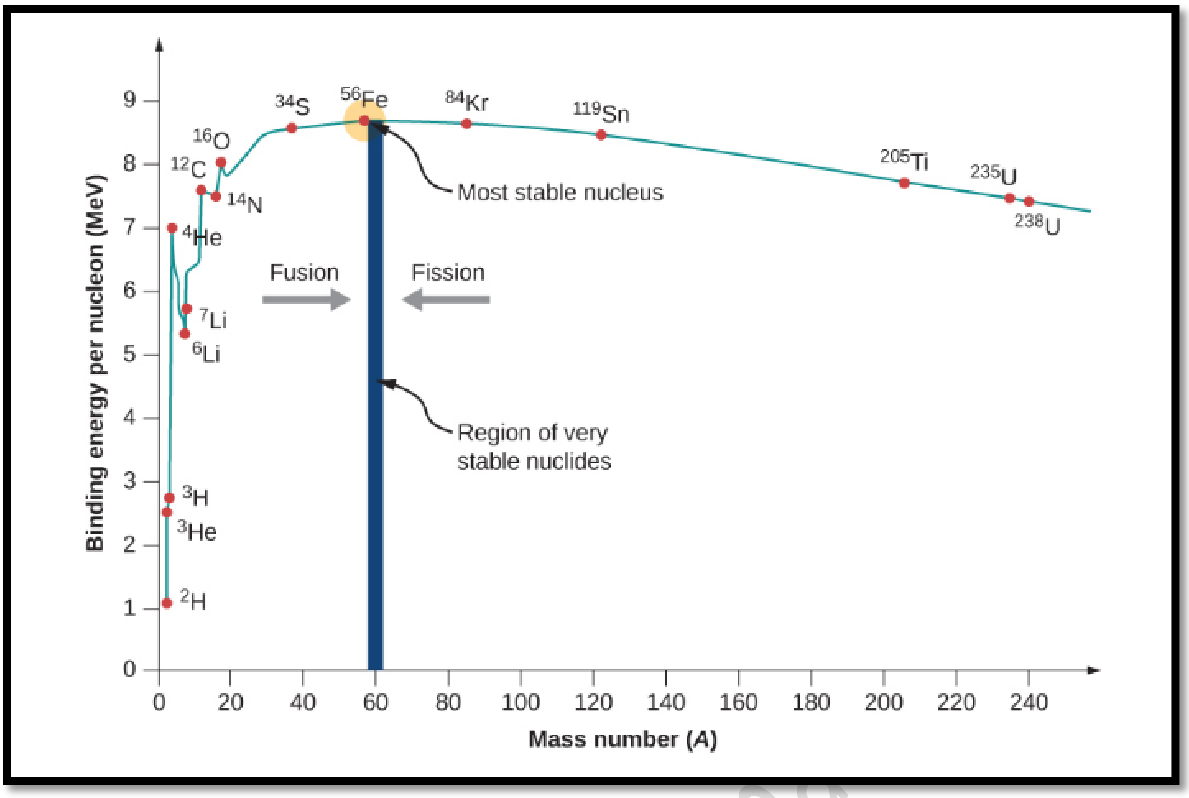

Fig.Nuclear binding energy curve *This graph shows the nuclear binding energy (in MeV) per nucleon as a function of the number of nucleons in the nucleus. Notice that iron-56 has the most binding energy per nucleon, making it the most stable nucleus*

from the above figure, we notice the following.

1- The relationship curve is almost constant except for the light cores
2- The average binding energy for each nucleus is 8Mev .
3- The maximum value that the curve reaches at the mass number 60, where the nucleus is tight, especially the iron nucleus.
4- The nuclei with mass number less than 60, the nuclear energy can be emitted through nuclear fusion
5- The nuclei with mass number great than 60, the nuclear energy can be emitted through nuclear fission

The binding energy is the energy required to break a nucleus into its constituent protons and neutrons. A system of separated nucleons has a greater mass than a system of bound nucleons.

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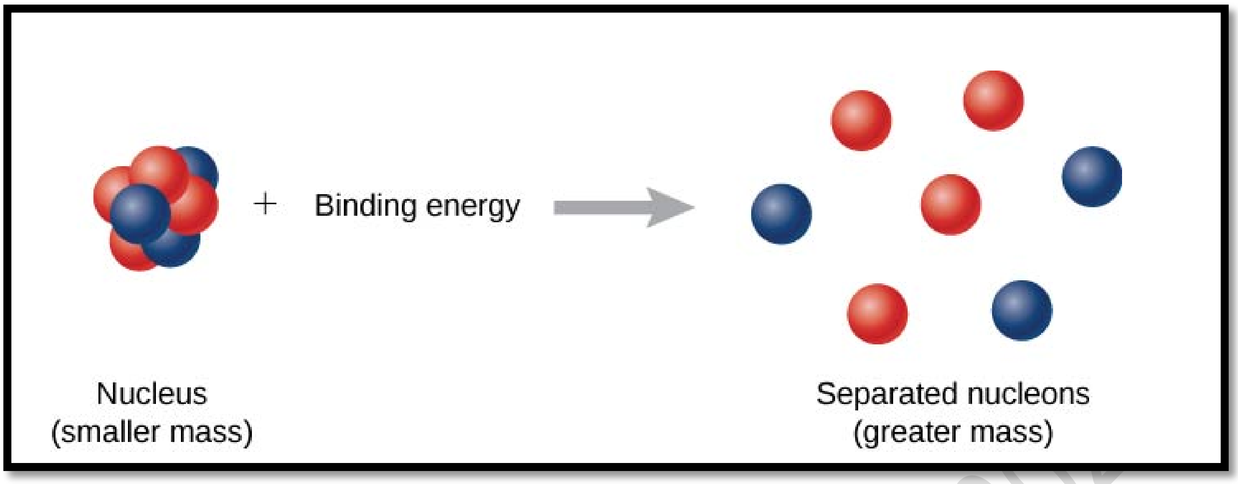

Q2/ Mass Defect and Binding Energy of the Deuteron Calculate the mass defect and the binding energy of the deuteron. The mass of the deuteron is

$m_d = 3.3435 x 10^{-27}$ kg or $1875.6 Mev/c^2$

Solution From (Figure), the mass defect for the deuteron is

$m = m_p + m_n - m_d$

$= 938.27\ Mev/c^2 + 939.57\ Mev/c^2 - 1875.6\ Mev/c^2$

$= 2.24\ Mev/c^2$

The binding energy of the deuteron is then

$E = mc^2$

$E = 2.24\ Mev/c^2 x c^2 = 2.24$ Mev

Q3/:

Calculate the mass defect and binding energy for uranium-235. One uranium-235 atom has a mass of 235.043924 u.

Solution:

Step 1: Calculate the mass defect using Equation (2)

$$\Delta m = \{M(Z,N) - A\} 931.5\ MeV$$
$$= (235.043924 - 235)931.5 = 40.9152\ MeV$$

Step 2: Use the mass defect and Equation (3) to calculate the binding energy

$$BE = \{Zm_H + Nm_n - M(Z,N)\}931.5\ MeV$$
$$= \{[92(1.007826\ u) + (235\text{-}92)1.008665\ u] - 235.043924\}931.5$$
$$BE = 1.91517\ u \times 931.5\ MeV/u = 1784\ MeV$$

The average binding energy for one nucleon is given by the following relationship:

B_{ave} = BE (A,Z) / A

Separation energy

The useful and interesting property of the binding energy is the neutron and proton separation energies. The neutron separation energy **S_n** *is the amount of energy required to remove a neutron from a nucleus,* ${}^{A}_{Z}X$ (sometimes called the binding energy of the last neutron). This is equal to the difference in binding energies between ${}^{A}_{Z}X$ and ${}^{A-1}_{Z}X$

$$S_n = \{M\,({}^{A-1}_{Z}X) - M\,({}^{A}_{Z}X) + m_n\}\,c^2$$

$$S_n = BE\,({}^{A}_{Z}X) - BE\,({}^{A-1}_{Z}X)$$

Similarly one can define proton separation energy *Sp as the energy needed to remove a proton from a nucleus* ${}^{A}_{Z}X$ (also called the binding energy of the last proton) which convert to another nuclide, ${}^{A-1}_{Z-1}X$ and can be calculated as follows.

$$S_p = BE\,({}^{A}_{Z}X) - BE\,({}^{A-1}_{Z-1}Y)$$

$$S_p = \{M\,({}^{A-1}_{Z-1}Y) - M\,({}^{A}_{Z}X) + m\,({}^{1}H)\}\,c^2$$

The Hydrogen mass appears تظهرin this equation instead of proton mass, since the atomic mass is $\mathbf{m\,({}^{1}H) = m_p + m_e}$.,

And, for alpha particles ,

S_α= [M(A-4.Z-2) + M - M(A,Z)]C^2

Particle	Charge	Mass (amu)
Proton	+1	1.00728
Neutron	0	1.00867
Electron	−1	0.000549

- 1 MeV = 1.603 x 10^{-13} Joule.
- Avogadro Number = 6.023 x 10^{23}.

Nuclear force

If only the electrostatic and gravitational forces existed in the nucleus, it would be impossible to have stable nuclei composed of protons and neutrons. The gravitational forces are much too small to hold the nucleons together compared to the electrostatic forces repelling the protons. Since stable atoms of neutrons and protons do exist in nature, there must be other attractive force acting within the nucleus; this force is **called the nuclear force.**

The ***nuclear force*** is a **strong attractive force** that is independent of charge. It acts only between pairs of neutrons, pairs of protons, or a neutron and a proton. The nuclear force has a **very short range**; it acts only over distances approximately equal to the diameter of the nucleus (10^{-15} m), even less. The attractive nuclear force between all nucleons drops off with distance much faster than the repulsive electrostatic force does between protons.

In stable atoms, the attractive and repulsive forces in the nucleus are balanced. If the forces do not balance, the atom cannot be stable, and the nucleus will emit radiation in an attempt to achieve a more stable configuration. Table (1) summarizes the behavior of each force

Table(1) Forces acting in the nucleons

Force	Interaction	Range
Gravitational	very weak attractive force between all nucleons	relatively long
Electrostatic	strong repulsive force between protons	relatively long
Nuclear force	strong attractive force between all nucleons	extremely short