

Materials Selection in Design

- The Role of Materials Selection in Design
- Exploring relationships - Materials Property Charts
- The Materials Selection Process –Design Models
- Selecting materials –Materials Indices
- Case Studies of Materials Selection using CES

Demystifying Material Indices

Material properties --

the "Physicists" view of materials, e.g.

Cost,	C_m
Density,	ρ
Modulus,	E
Strength,	σ_y
Endurance limit,	σ_e
Thermal conductivity,	λ
T- expansion coefficient,	α

Material indices --

the "Engineers" view of materials

Objective: minimise mass

Function	Stiffness	Strength
Tension (tie)	ρ/E	ρ/σ_y
Bending (beam)	$\rho/E^{1/2}$	$\rho/\sigma_y^{2/3}$
Bending (panel)	$\rho/E^{1/3}$	$\rho/\sigma_y^{1/2}$

Minimise these!

Using Materials Indices with Materials Selection Charts

Index $M = \frac{\rho}{E^{1/2}}$

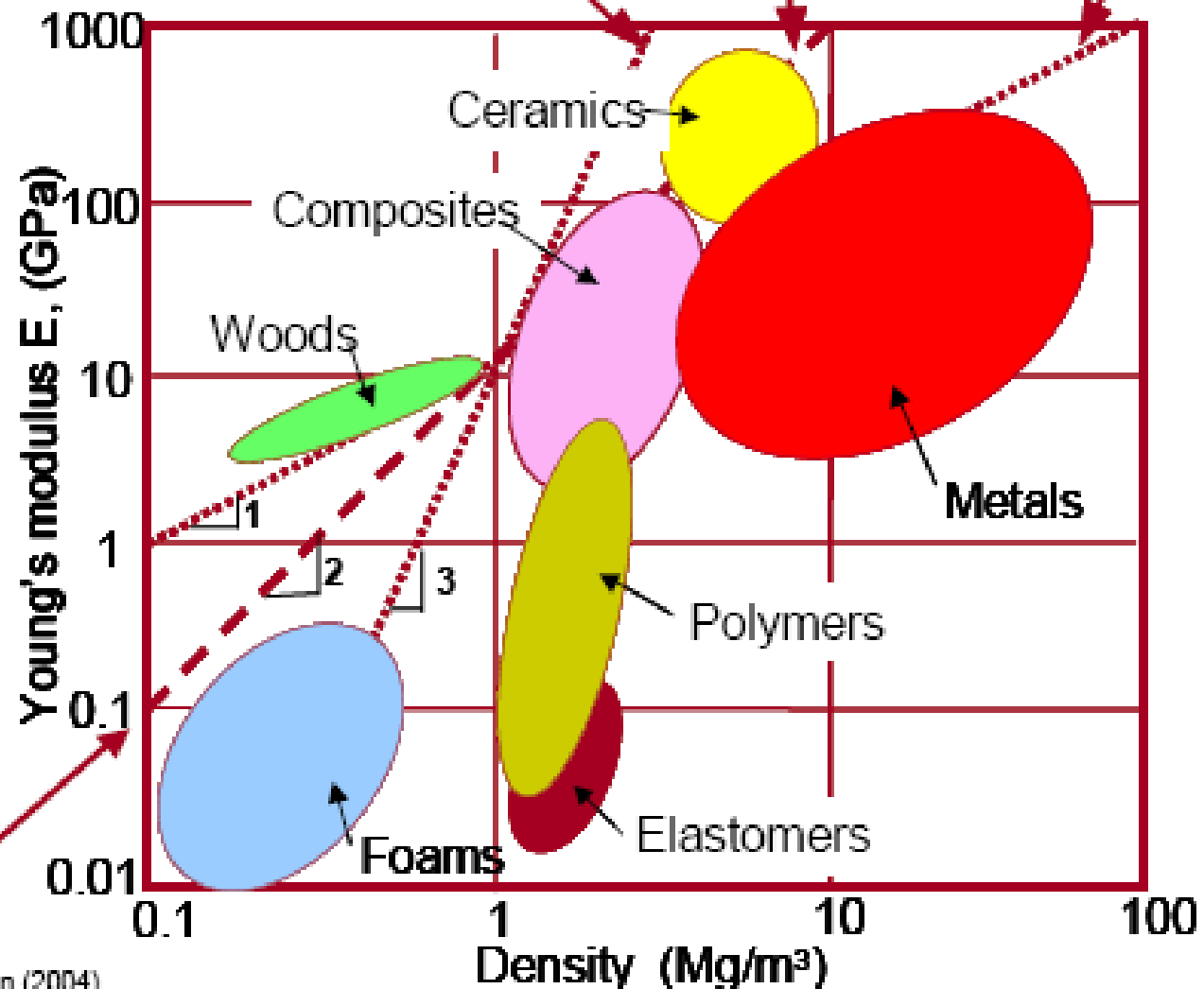
$\Rightarrow E = \rho^2 / M^2$

$\text{Log}(E) = 2\text{Log}(\rho) - 2\text{Log}(M)$

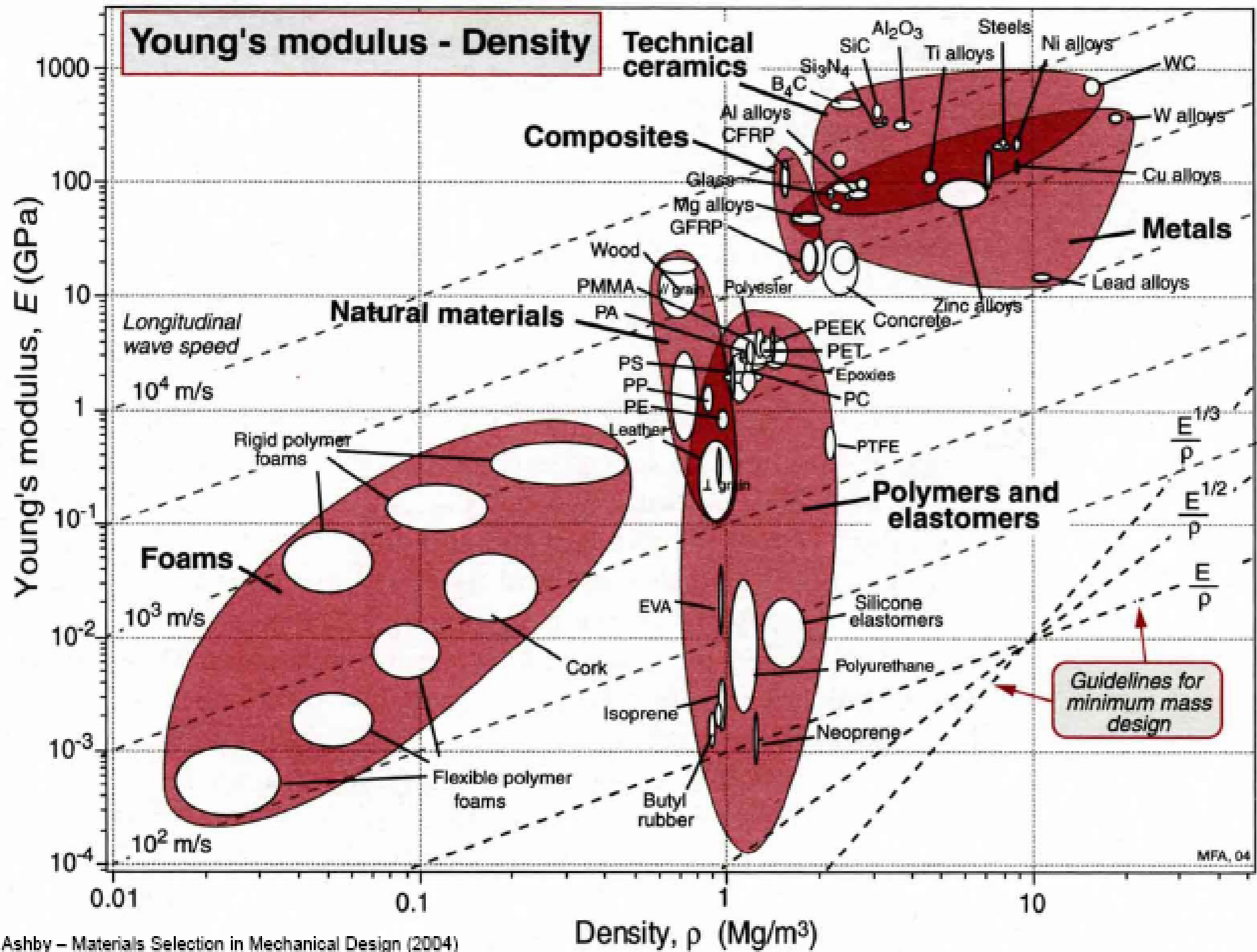
$\frac{\rho}{E^{1/3}} = C$

$\frac{\rho}{E^{1/2}} = C$

$\frac{\rho}{E} = C$

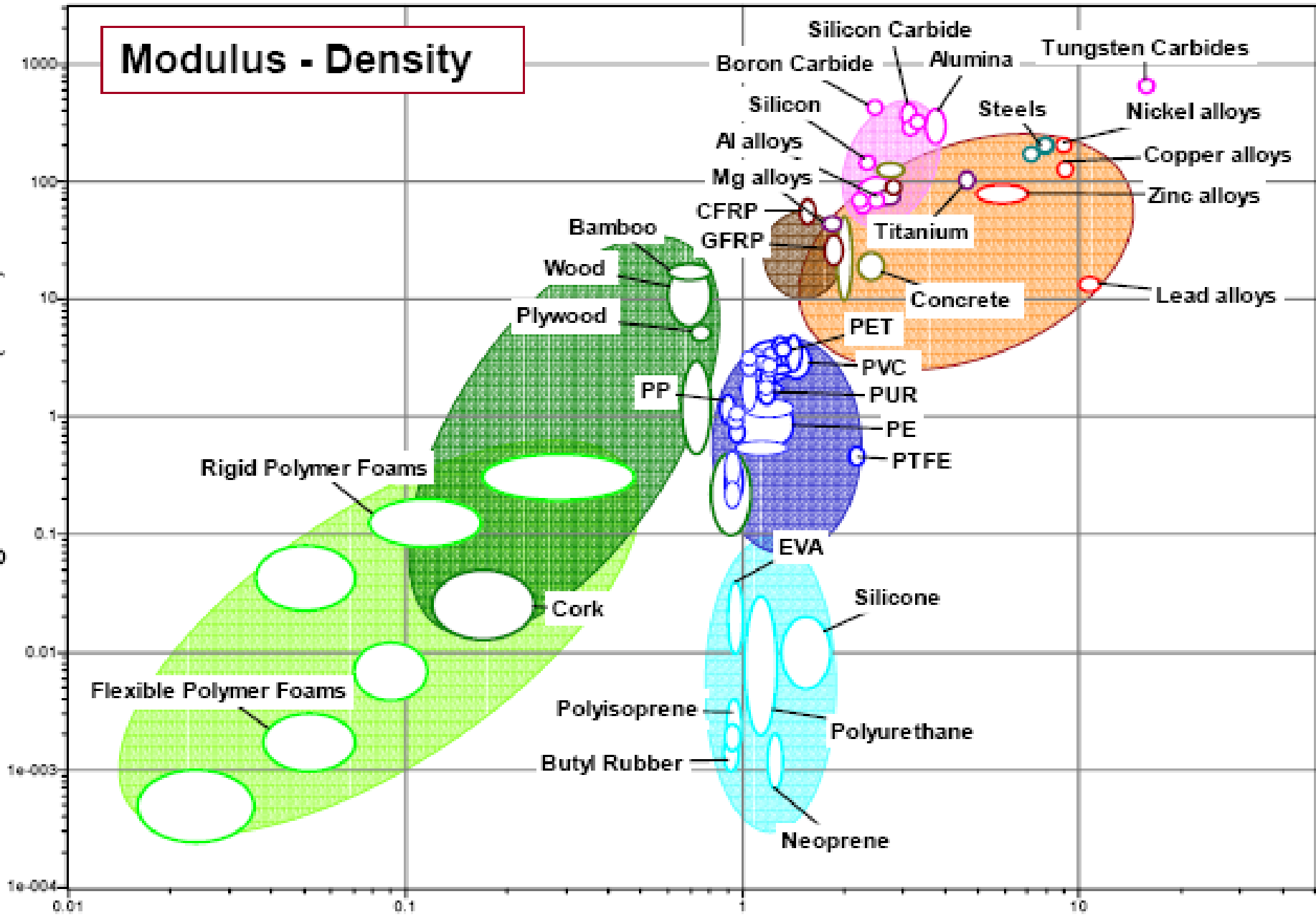


Young's modulus - Density



Modulus - Density

Young's modulus (GPa)



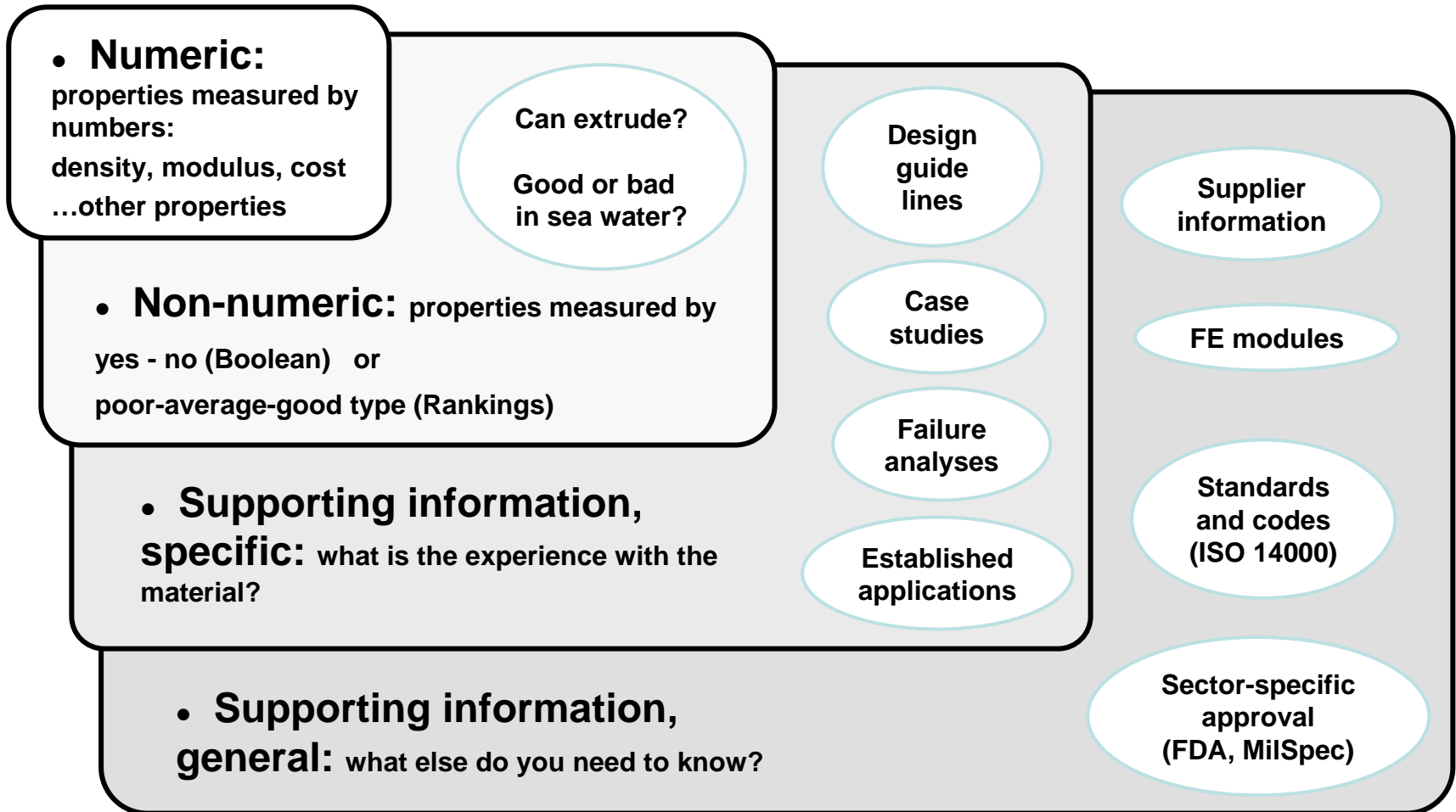
Density (Mg/m³)

Commonly used Materials Indices (MI's)

Function, objective, and constraints	Index
Tie, minimum weight, stiffness prescribed	$\frac{E}{\rho}$
Beam, minimum weight, stiffness prescribed	$\frac{E^{1/2}}{\rho}$
Beam, minimum weight, strength prescribed	$\frac{\sigma_y^{2/3}}{\rho}$
Beam, minimum cost, stiffness prescribed	$\frac{E^{1/2}}{C_m \rho}$
Beam, minimum cost, strength prescribed	$\frac{\sigma_y^{2/3}}{C_m \rho}$
Column, minimum cost, buckling load prescribed	$\frac{E^{1/2}}{C_m \rho}$
Spring, minimum weight for given energy storage	$\frac{\sigma_y^2}{E \rho}$
Thermal Insulation, minimum cost, heat flux prescribed	$\frac{1}{\lambda C_p \rho}$
Electromagnet, maximum field, temperature rise prescribed	$\frac{C_p \rho}{\rho_e}$

ρ = density; E = Young's modulus; σ_y = elastic limit; C_m = cost/kg; λ = thermal conductivity; ρ_e = electrical resistivity; C_p = specific heat.

The nature of material data



“Structured” and “Unstructured” data

Handbooks,
data sheets

Reports, papers,
the Web

Other Materials Selection Charts

- Modulus-Relative Cost
- **Strength-Relative Cost**
- **Modulus-Strength**
- **Specific Modulus-Specific Strength**
- **Fracture Toughness-Modulus**
- **Fracture Toughness-Strength**
- **Loss Coefficient-Modulus**
- Fracture Toughness-Density
- **Conductivity-Diffusivity**
- **Expansion-Conductivity**
- **Expansion-Modulus**
- **Strength-Expansion**
- **Strength Temperature**
- Wear Rate-Hardness
- Environmental Attack Chart

Summary: Material Indices

- A method is necessary for translating design requirements into a prescription for a material
- Modulus-Density charts
 - Reveal a method of using lines of constant

$$\frac{E^{1/n}}{\rho} \quad n = 1, 2, 3$$

to allow selection of materials for minimum weight and deflection-limited design.

- Material Index
 - Combination of material properties which characterize performance in a given application.
- Performance of a material:

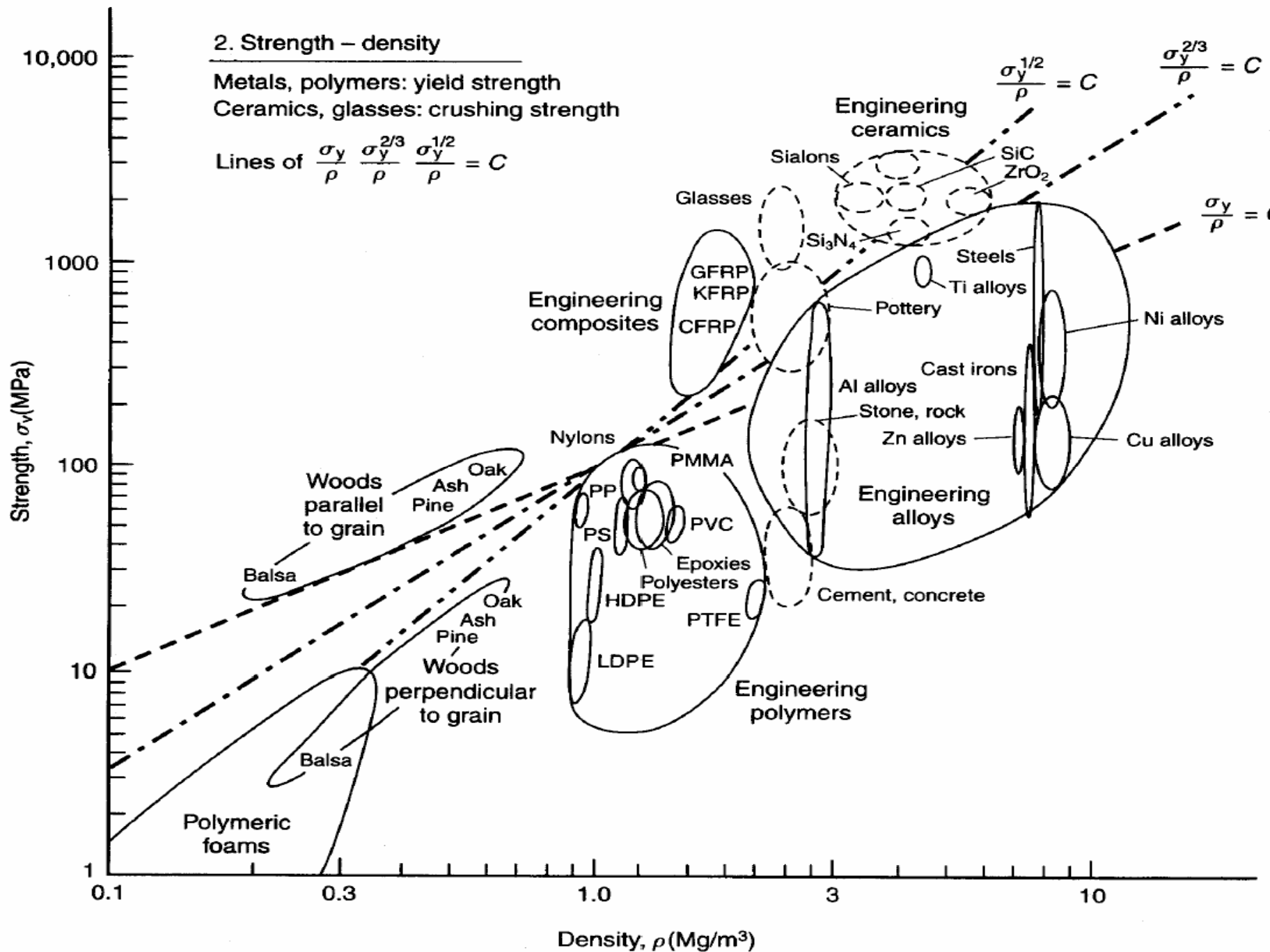
$$p = f \left[\left(\begin{array}{l} \textit{Functional} \\ \textit{Needs, } F \end{array} \right), \left(\begin{array}{l} \textit{Geometric} \\ \textit{Parameters, } G \end{array} \right), \left(\begin{array}{l} \textit{Material} \\ \textit{Characteristics, } M \end{array} \right) \right]$$

$$p = f_1(F) f_2(G) f_3(M)$$

2. Strength – density

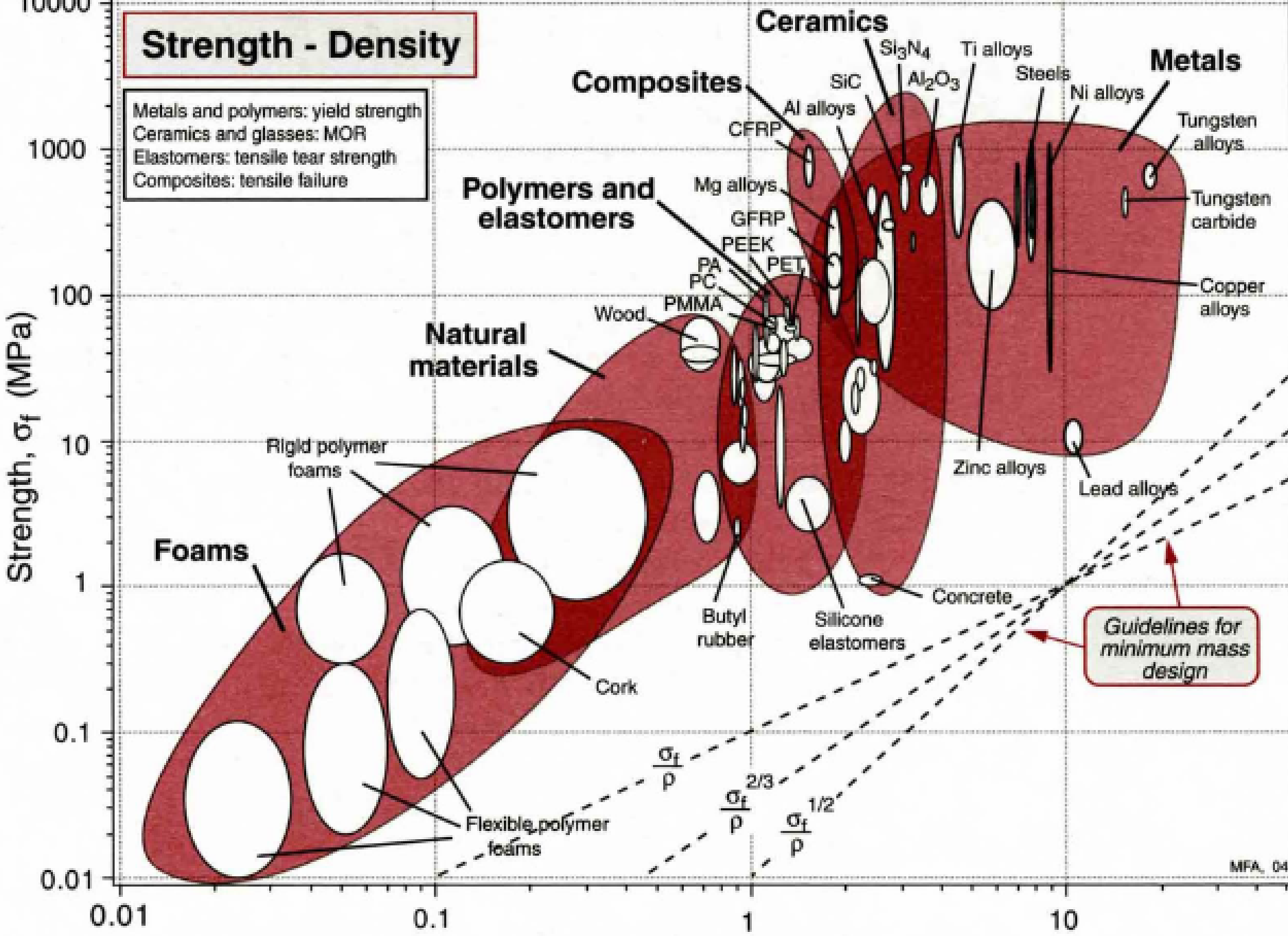
Metals, polymers: yield strength
 Ceramics, glasses: crushing strength

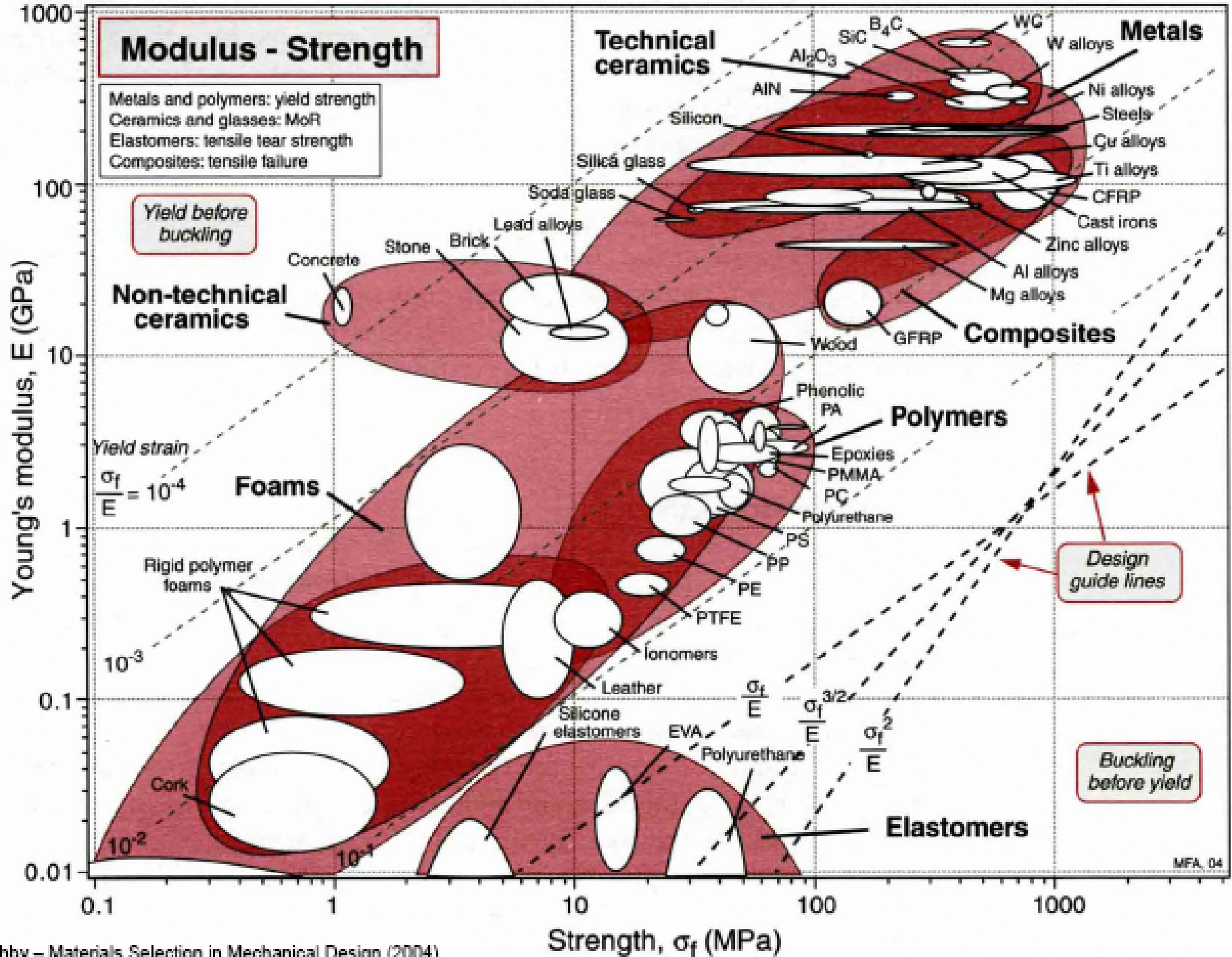
Lines of $\frac{\sigma_y}{\rho}$ $\frac{\sigma_y^{2/3}}{\rho}$ $\frac{\sigma_y^{1/2}}{\rho} = C$



Strength - Density

Metals and polymers: yield strength
 Ceramics and glasses: MOR
 Elastomers: tensile tear strength
 Composites: tensile failure

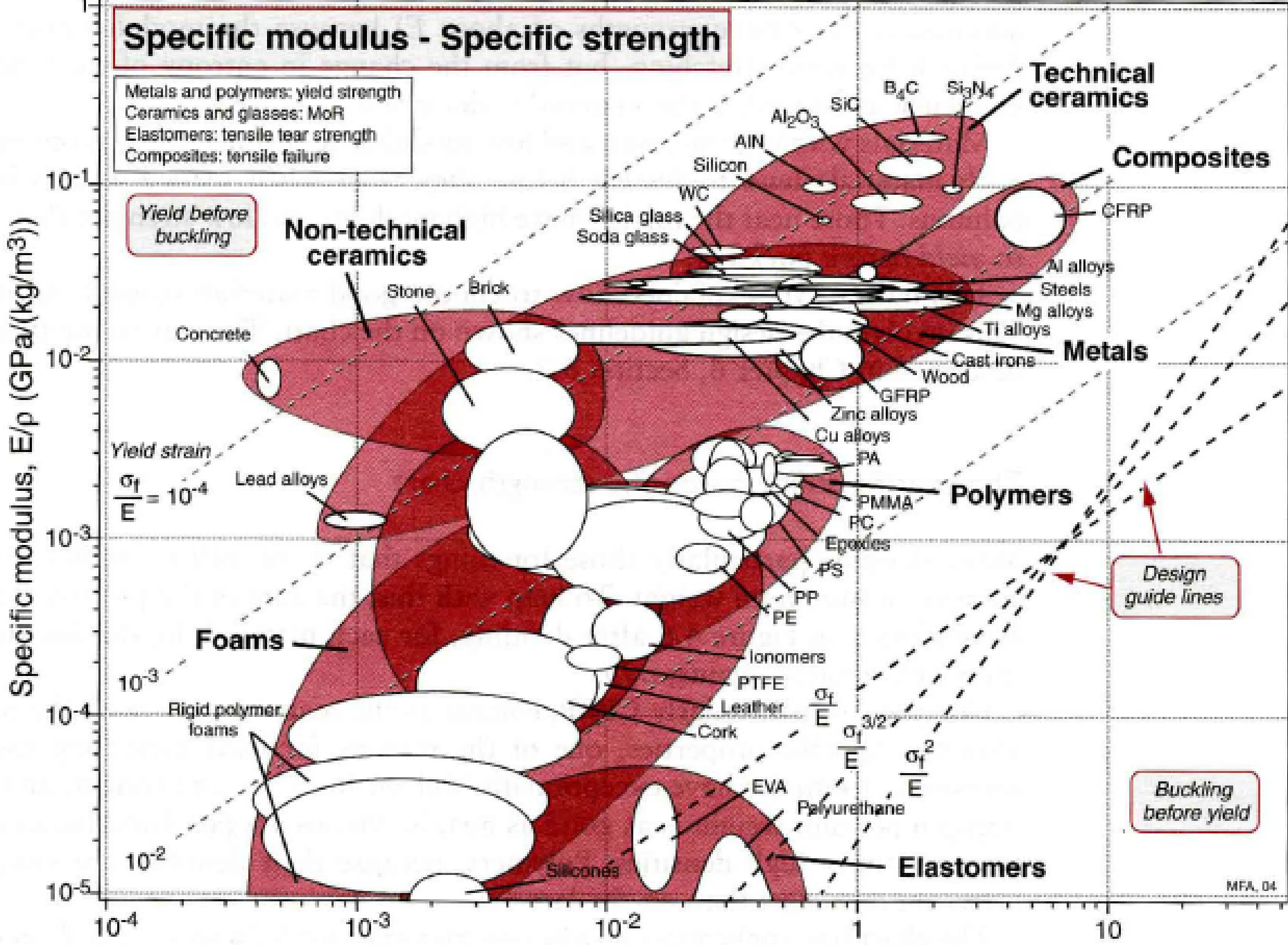




Specific modulus - Specific strength

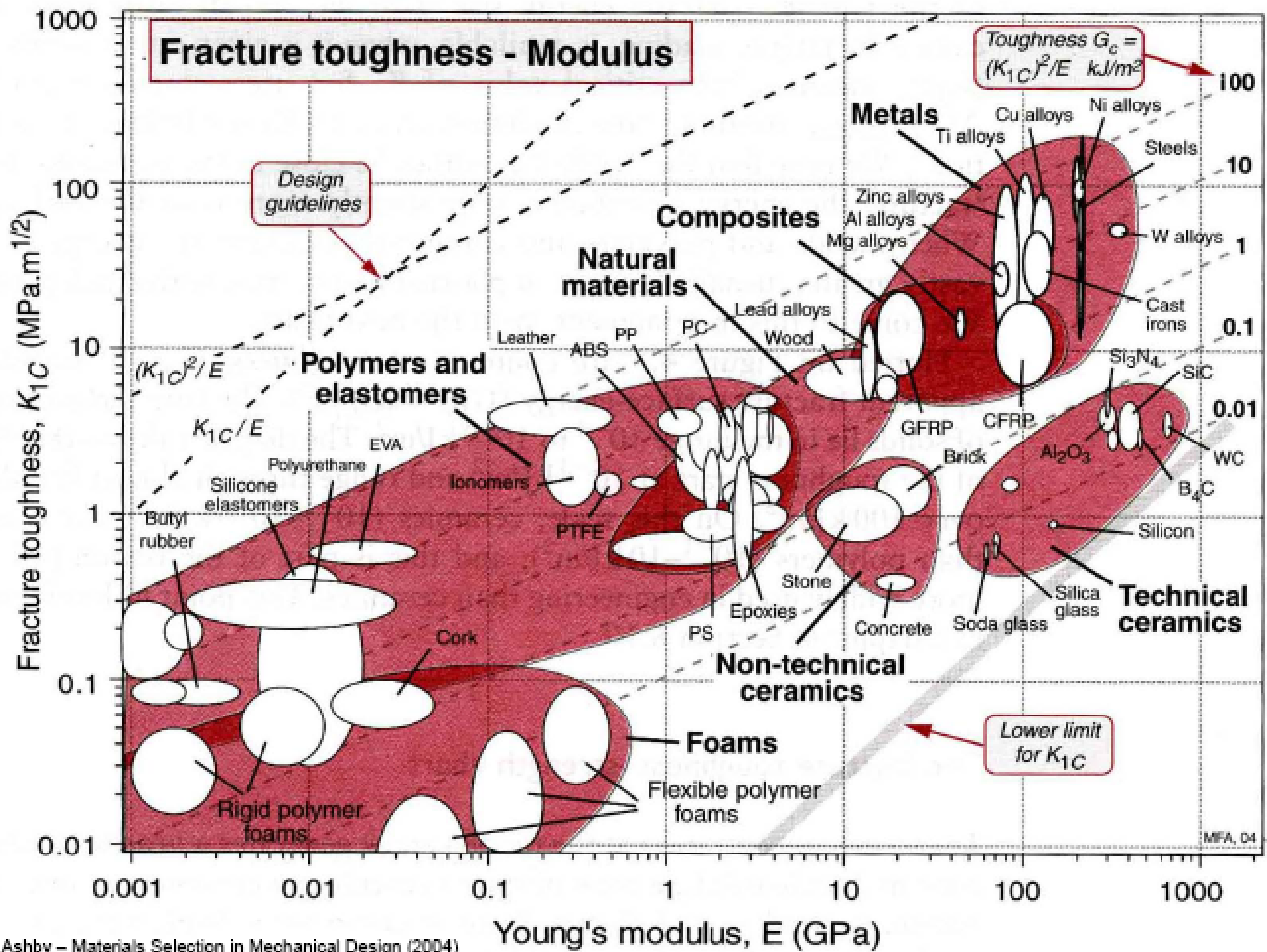
Metals and polymers: yield strength
 Ceramics and glasses: MoR
 Elastomers: tensile tear strength
 Composites: tensile failure

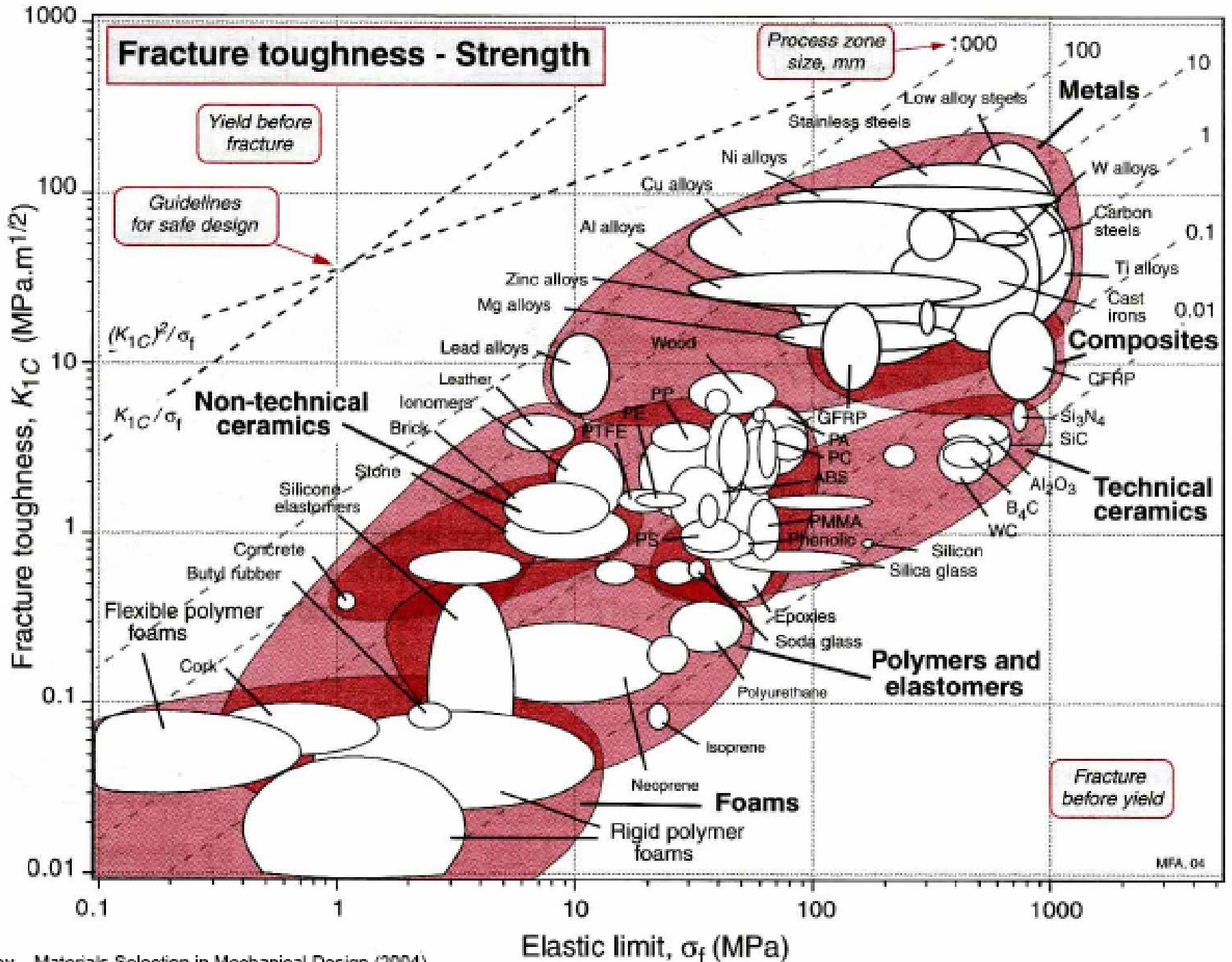
Yield before buckling



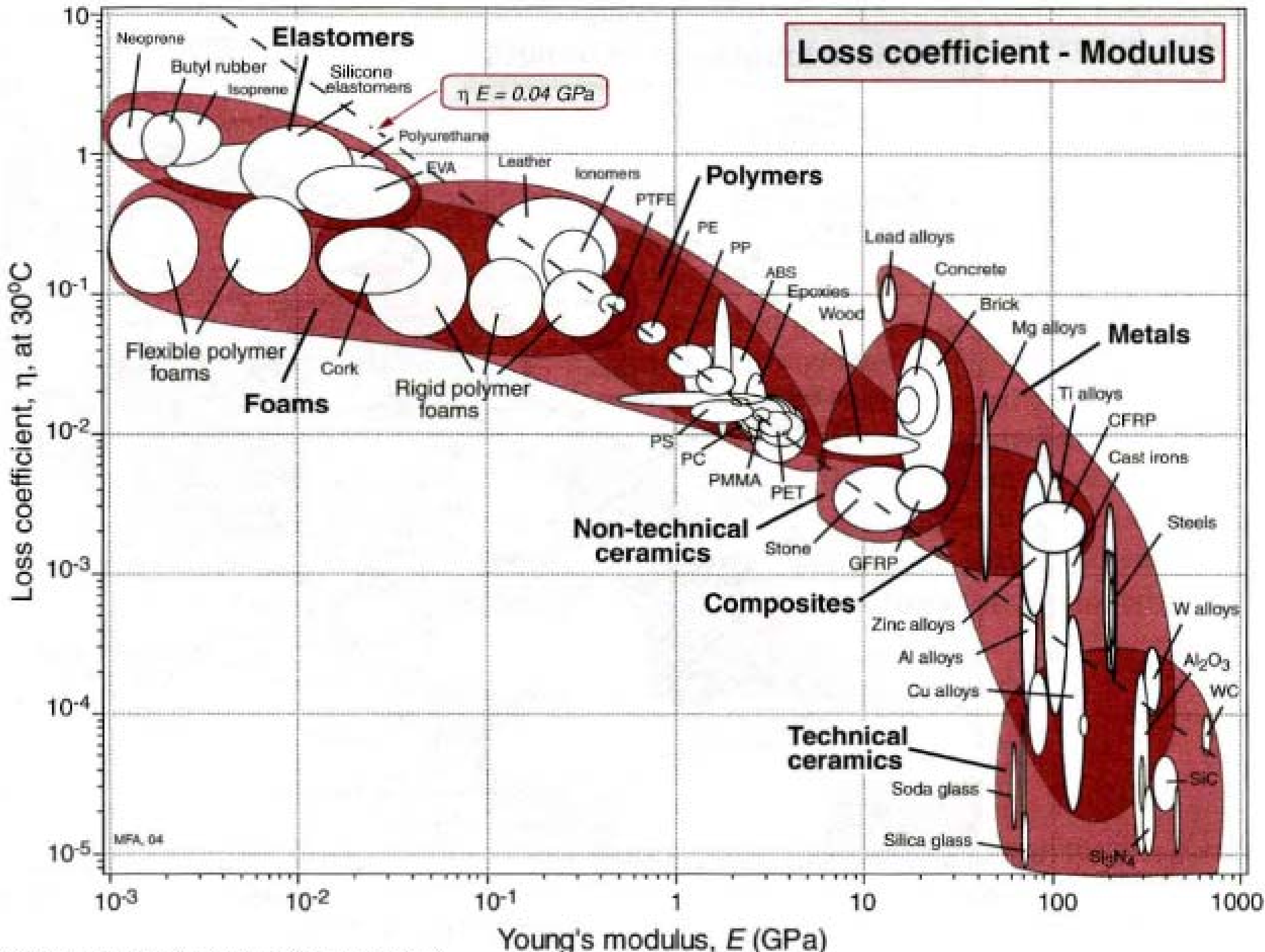
Design guide lines

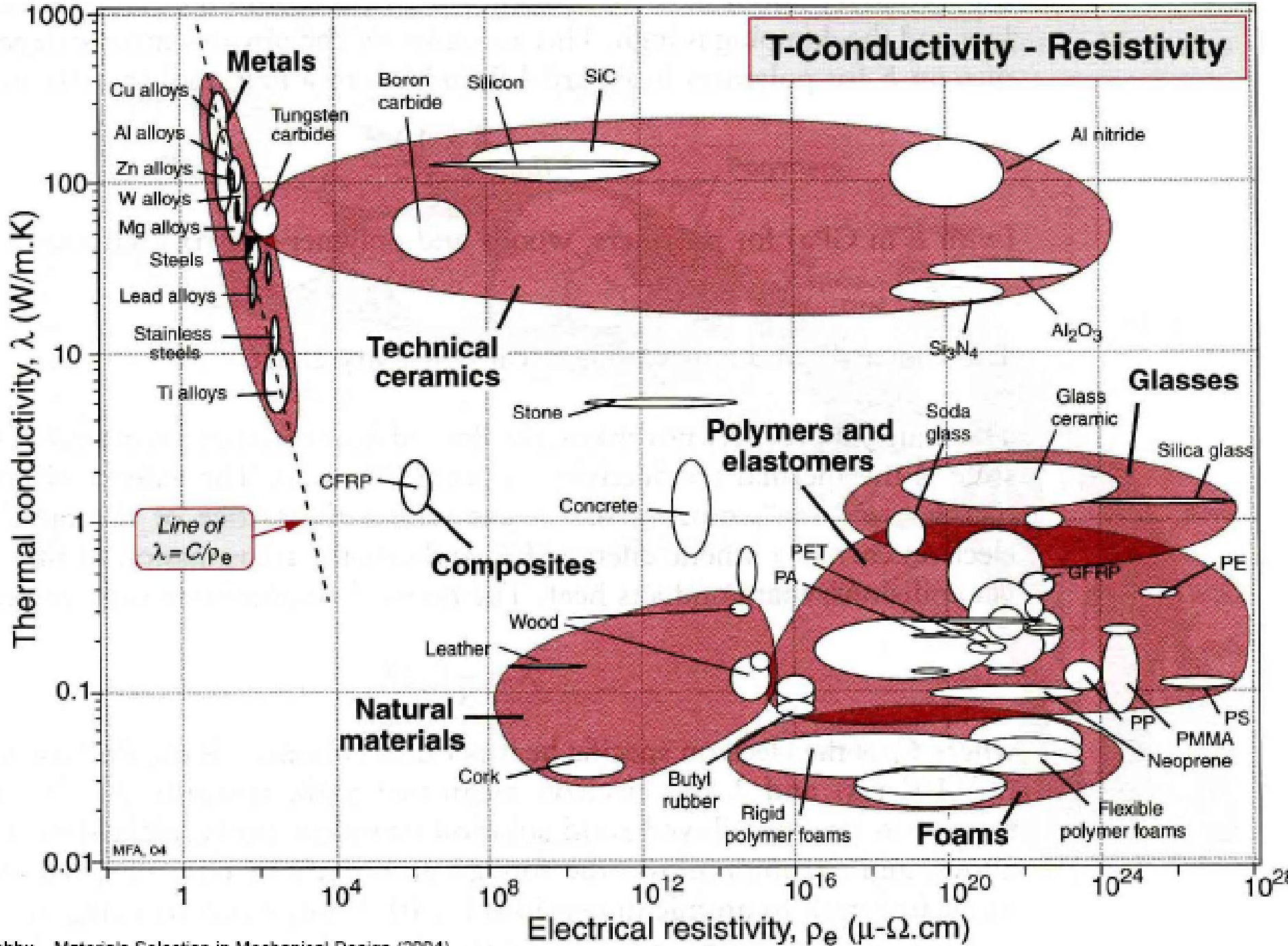
Buckling before yield

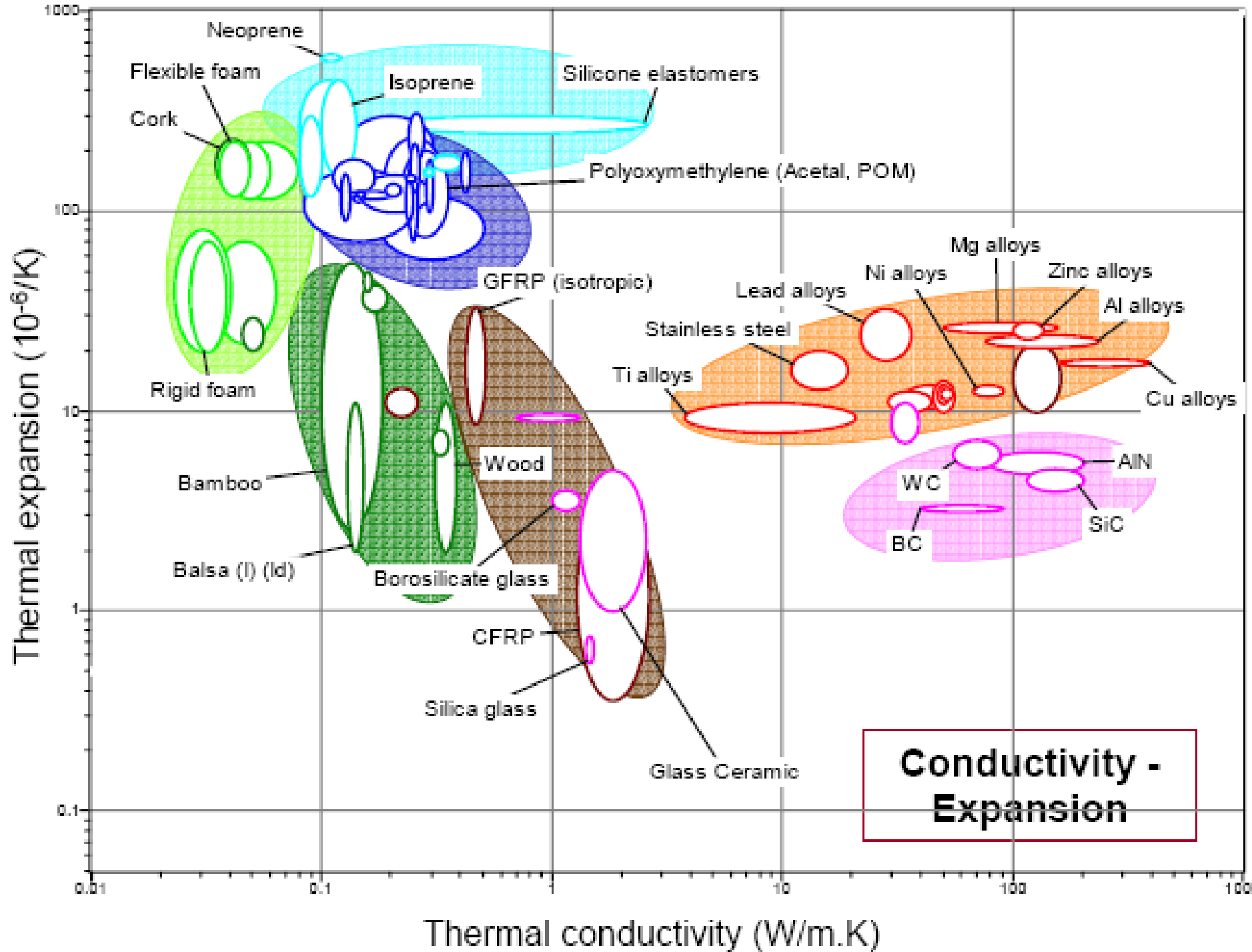




Loss coefficient - Modulus





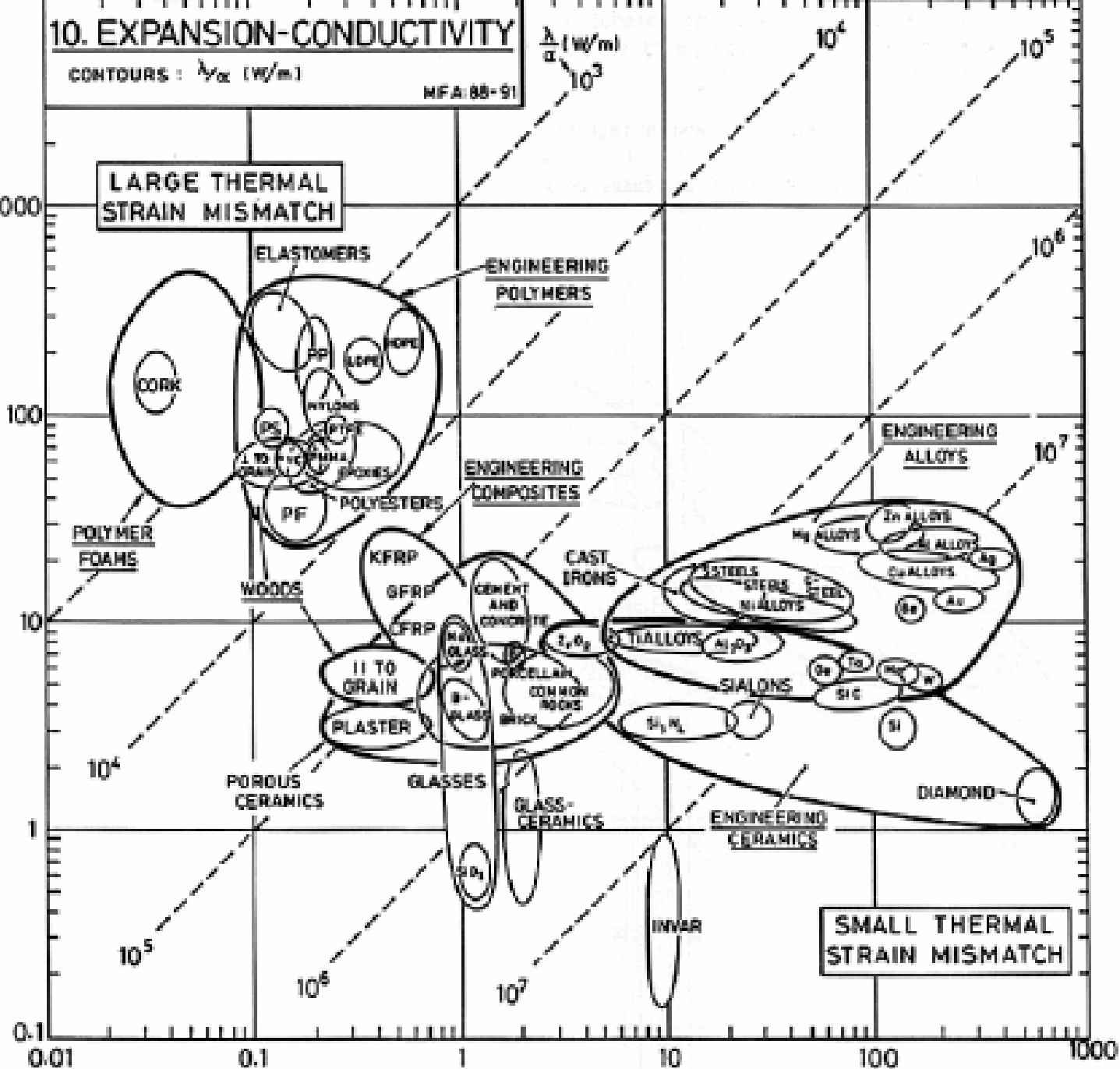


10. EXPANSION-CONDUCTIVITY

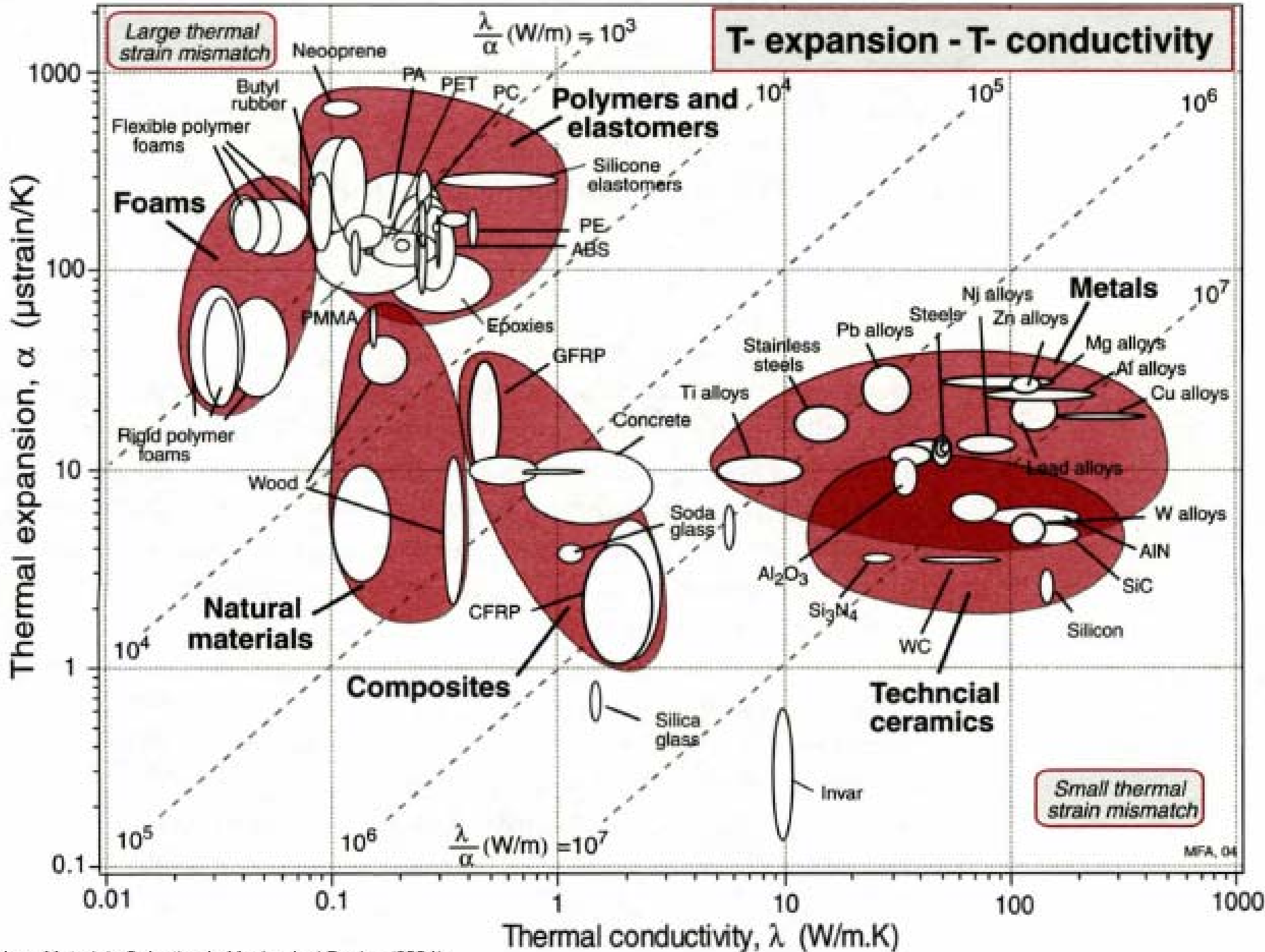
CONTOURS: λ/α (W/m)

MFA 88-91

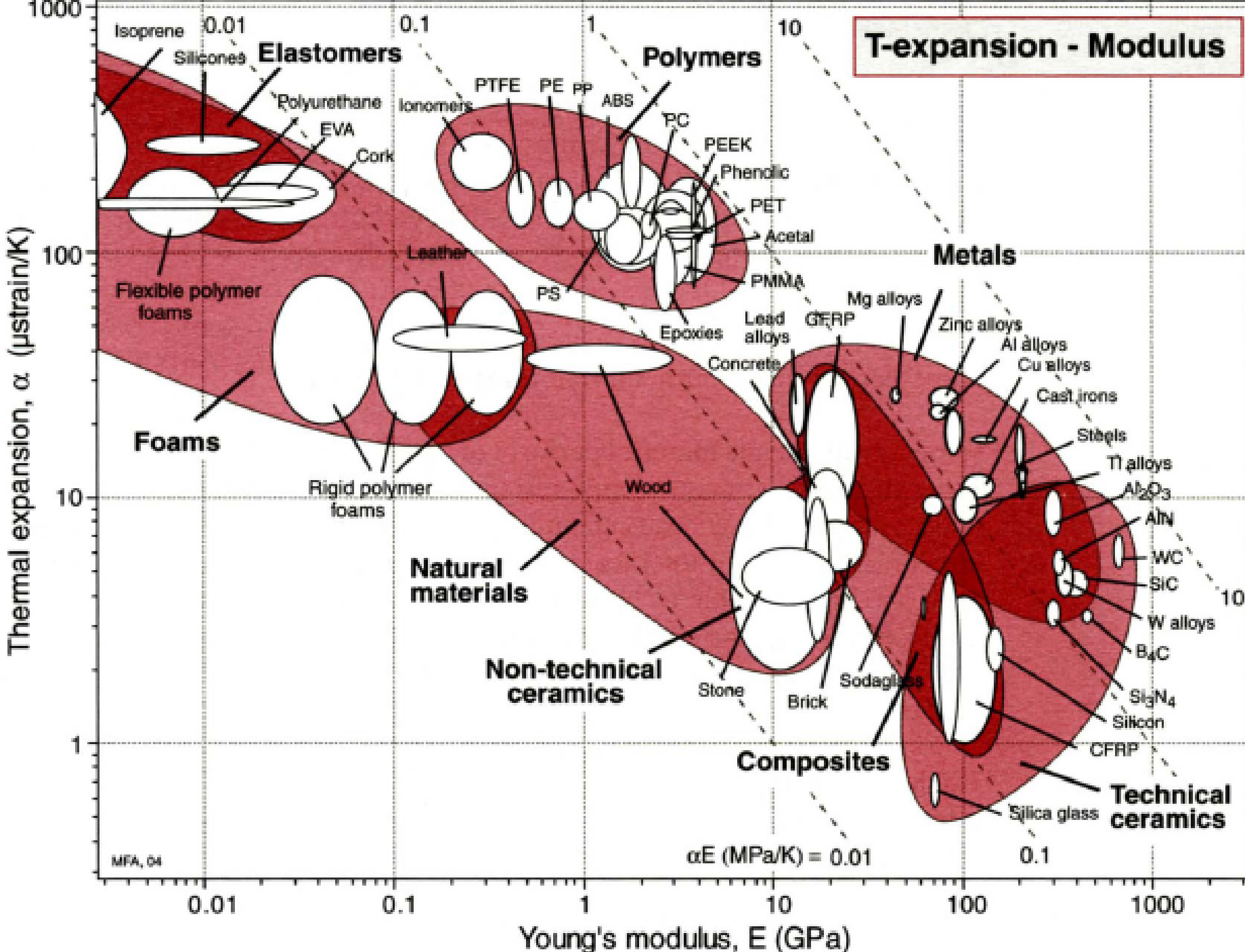
LINEAR EXPANSION COEFFICIENT α ($10^{-6}/K$)

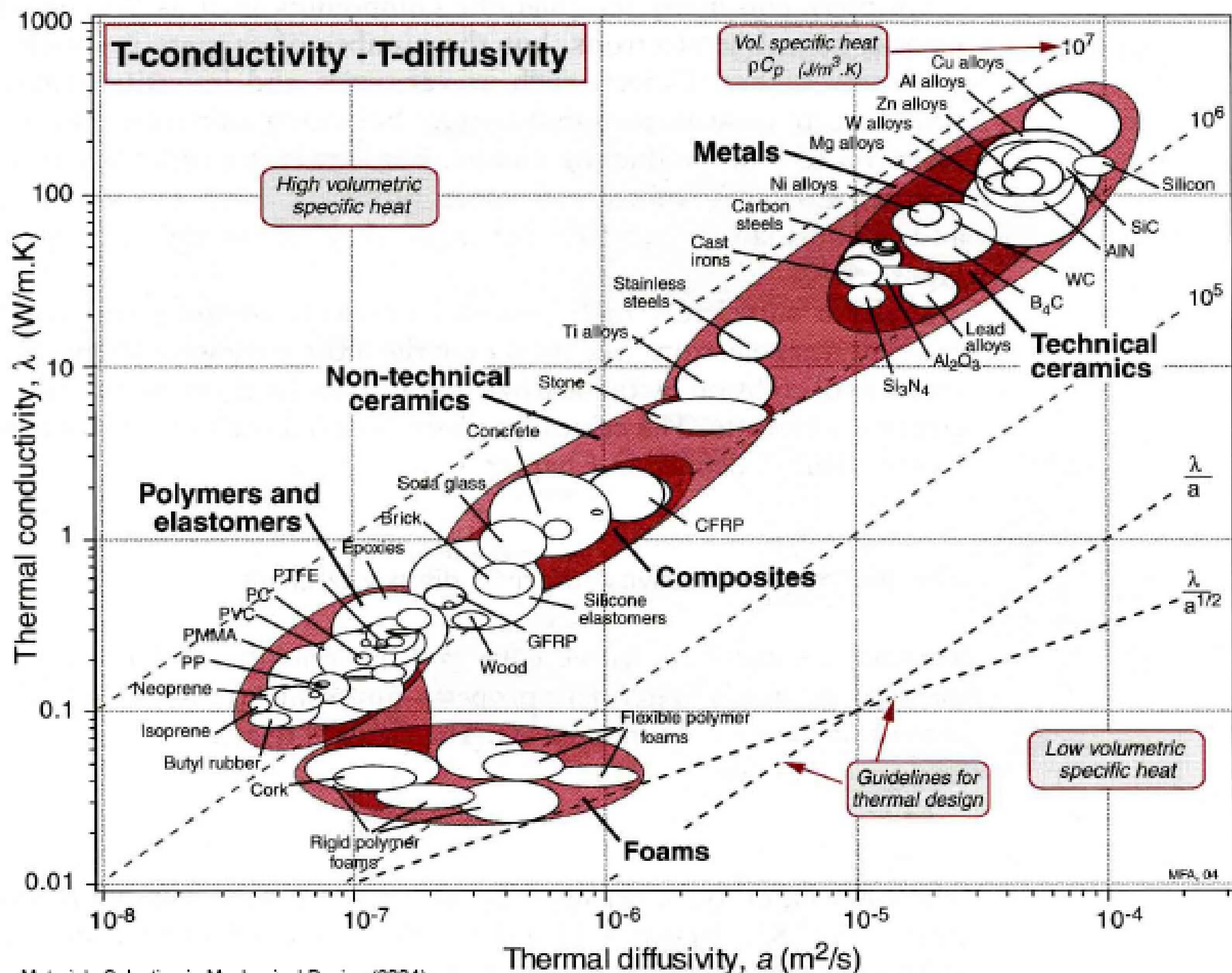


THERMAL CONDUCTIVITY λ W/(mK)

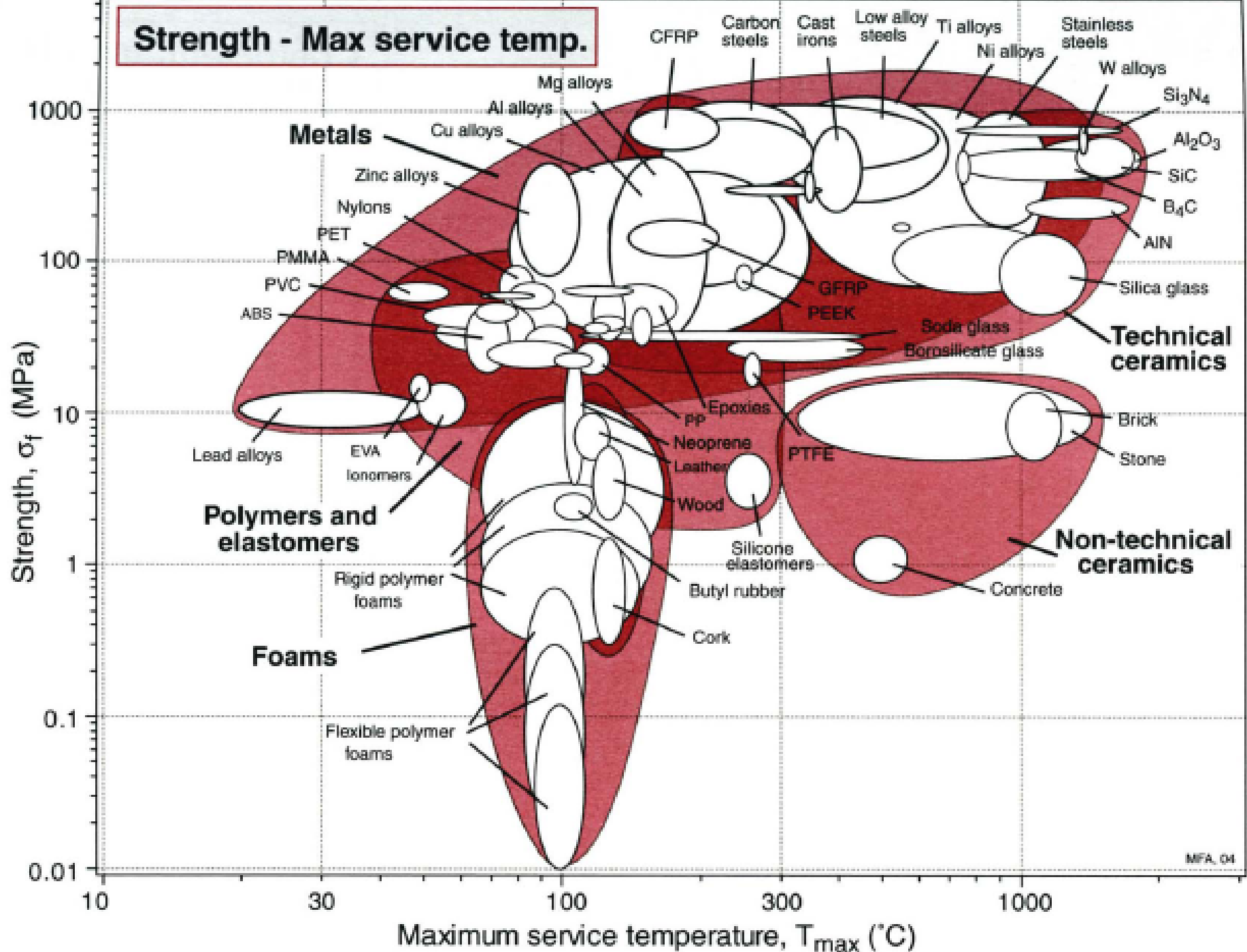


T-expansion - Modulus



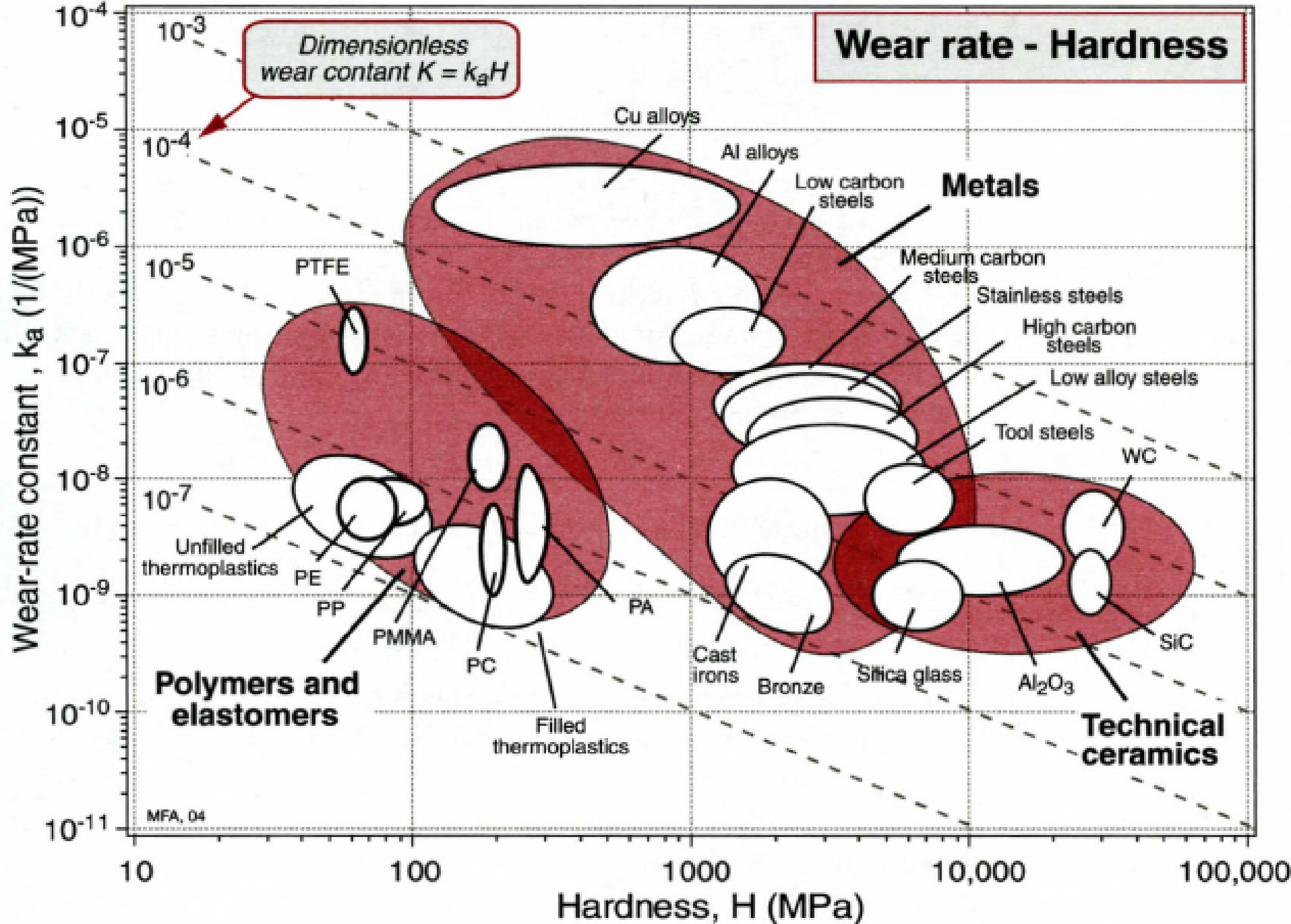


Strength - Max service temp.

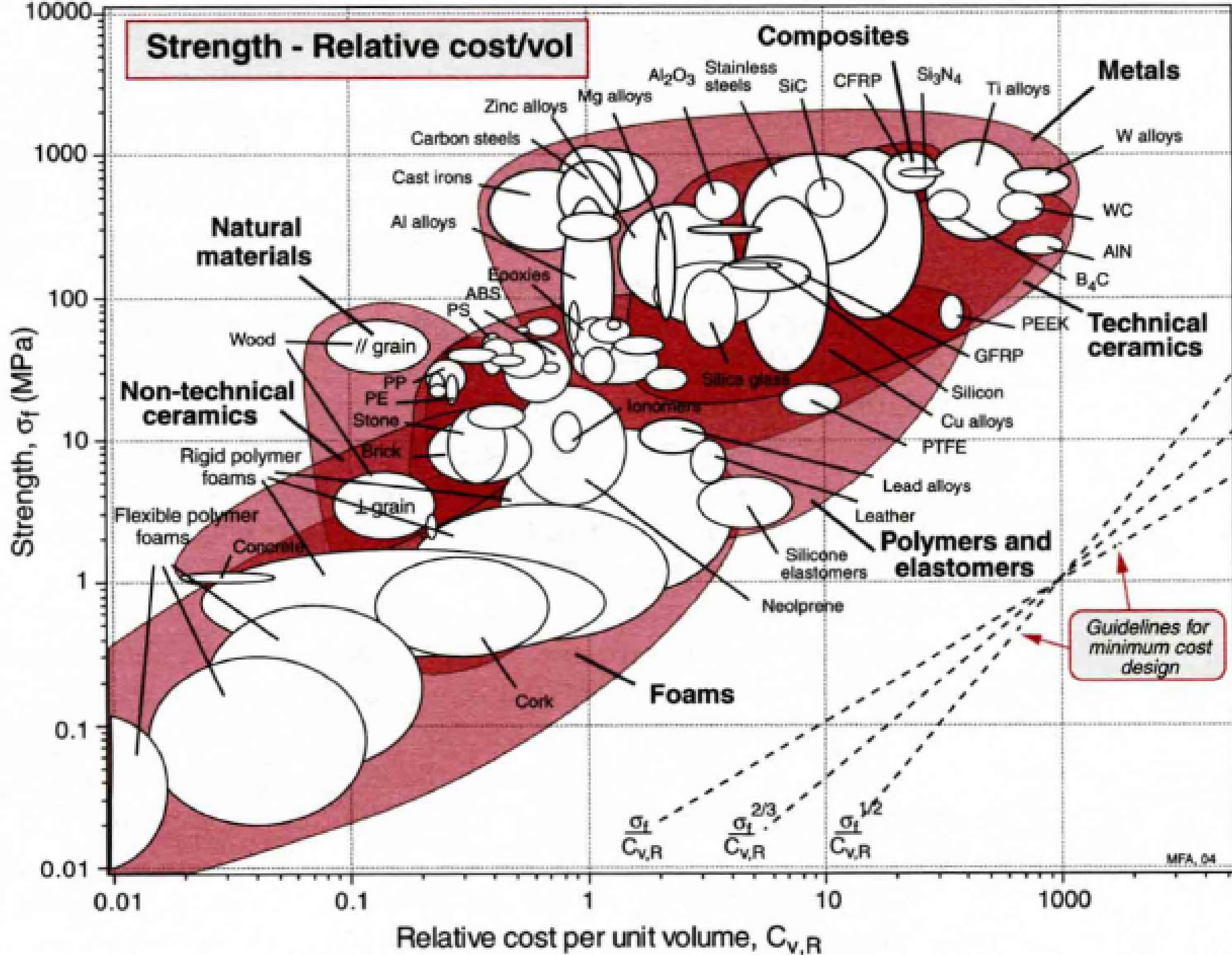


Wear rate - Hardness

Dimensionless wear constant $K = k_a H$

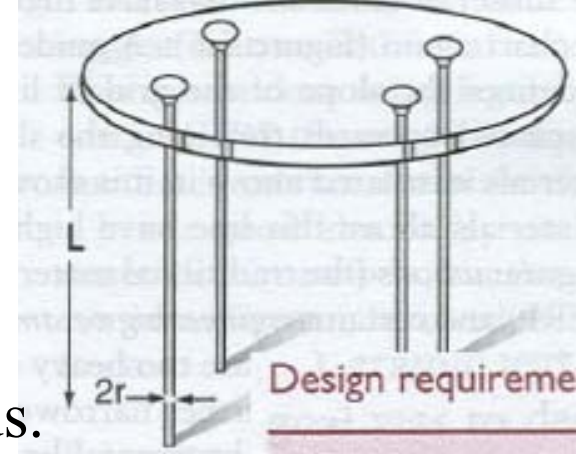


MFA, 04



Case 1: Materials for Table legs

Design a slender, light table legs that will support the applied design load and will not fracture if struck.



Function

Column, supporting compressive loads.

Objective

Minimize mass and maximize slenderness

mass:
$$m = \pi r^2 L \rho \quad (1)$$

Maximum elastic buckling load:

$$F_{crit} = \frac{\pi^2 EI}{L^2} = \frac{\pi^3 E r^4}{4L^2} \quad \text{where} \quad I = \frac{\pi r^4}{4} \quad (2)$$

Constraints

Specified length,
Must not buckle

Must not fracture if struck

Solving for r

Free Variables

Diameter of the legs
Choice of materials

$$m \geq \left(\frac{4F}{\pi} \right)^{1/2} L^2 \left(\frac{\rho}{E^{1/2}} \right) \quad (3)$$

The weight is minimized by selecting materials with the greatest value of the materials index:

$$M_1 = \left(\frac{E^{1/2}}{\rho} \right) \quad (4)$$

Inverting equation (2) gives an equation for the thinnest legs which will not buckle:

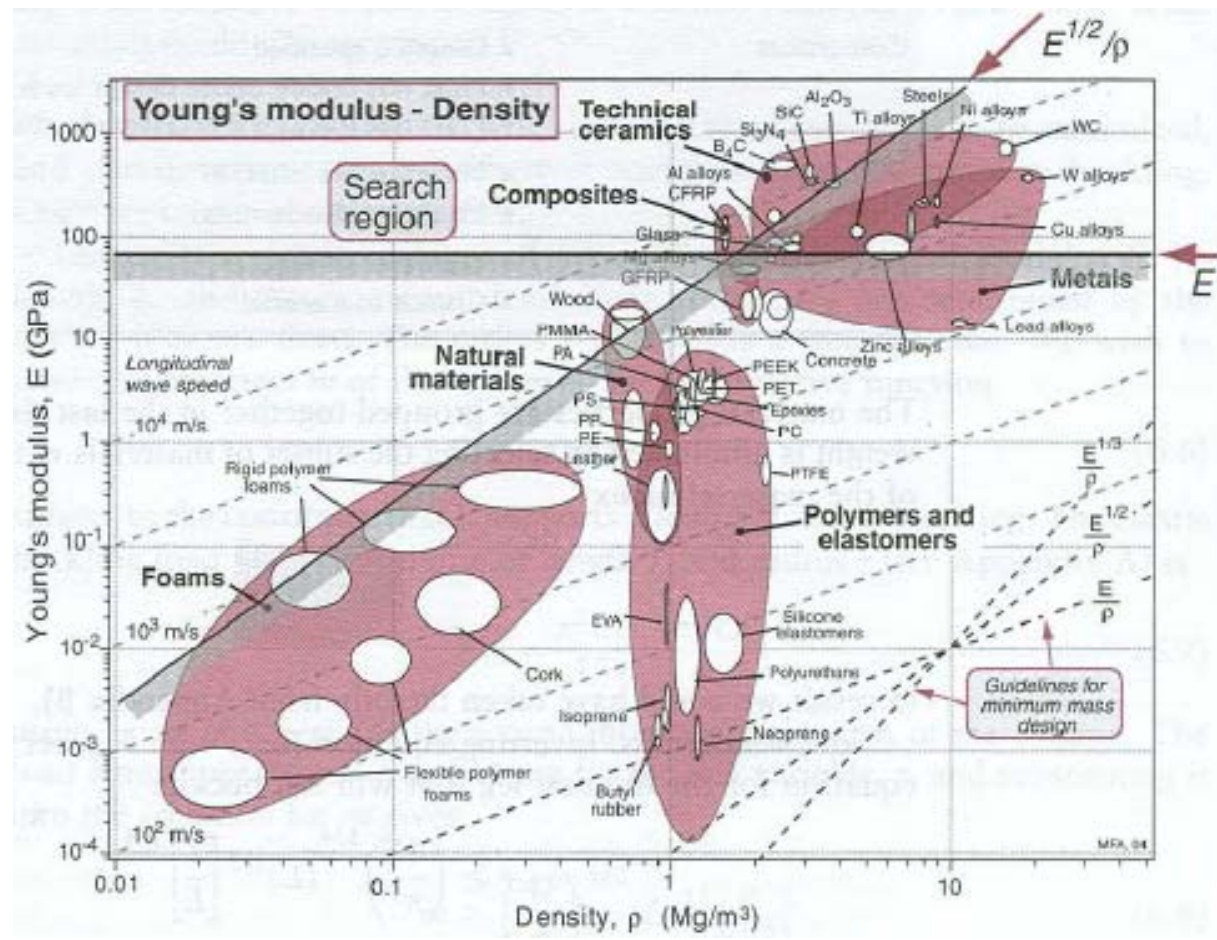
$$r \geq \left(\frac{4F}{\pi^3} \right)^{1/4} L^{1/2} \left(\frac{1}{E} \right)^{1/4} \quad (5)$$

to yield the second materials index (maximize): $M_2 = E$ (6)

Set M_1 to be minimum of 5 and M_2 to be greater than 100 (an arbitrary choice –it can be modified later if a wider choice of materials to be screened is desired). Candidate materials include some ceramics, CFRP

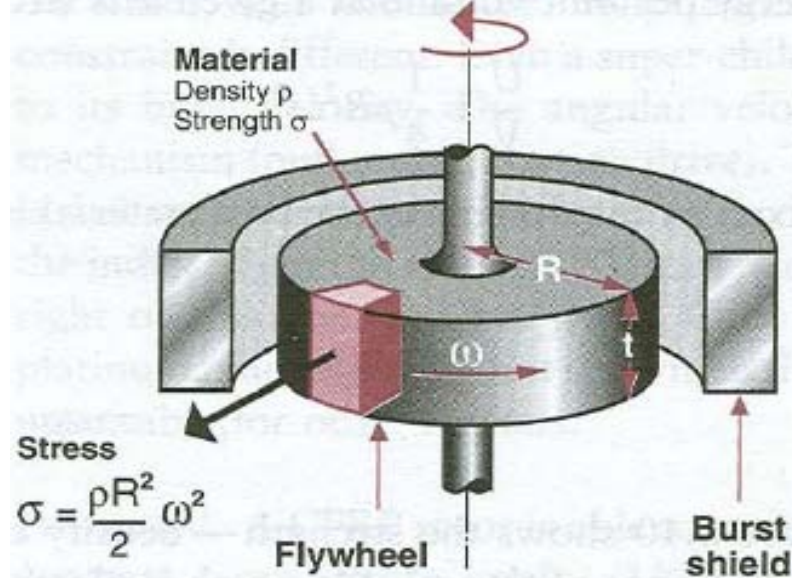
- engineering ceramics are not tough –legs are subjected to abuse and this makes them a bad selection for this application

Selection = CFRP
 must consult designer wrt cost
 -expensive



Case 2: Materials for Flywheels

Flywheels are rotating devices that store rotational energy in applications such as automotive transmissions. An efficient flywheel stores maximum energy per unit volume/mass at a specified angular velocity.



The kinetic energy the device can store is limited by the material strength.

Function

Flywheel for energy storage.

Objective

Maximize kinetic energy per unit mass.

Mass of the disc

$$m = \pi R^2 t \rho$$

Kinetic energy (J is the mass moment of inertia)

$$KE = \frac{1}{2} J \omega^2$$

For a solid round disc J around its rotation axis

$$J = \frac{1}{2} m R^2$$

$$KE = \frac{1}{4} m R^2 \omega^2$$

The quantity to be maximized is the energy per unit mass $\frac{KE}{m} = \frac{1}{4} R^2 \omega^2$

Constraints

The outer radius is fixed.

It must not burst.

It must have adequate toughness (crack tolerance)

Free Variables

Choice of materials

The maximum radial stress (principal stress) is given by the equation:

$$\sigma_{r,Max} = \frac{3+\nu}{8} \rho \omega^2 R^2 \cong \frac{\rho \omega^2 R^2}{2}$$

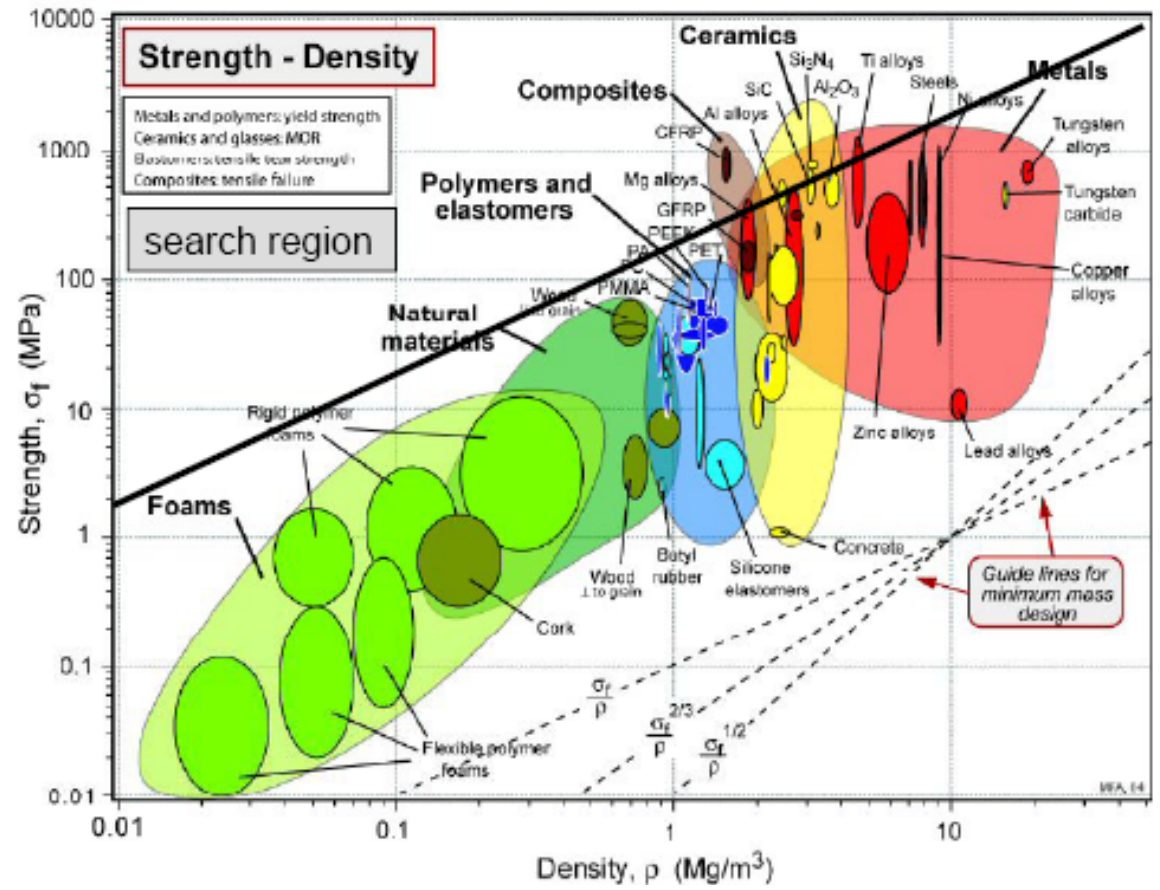
The stress must not exceed the yield stress:

$$\frac{KE}{m} = \frac{1}{2} \left(\frac{\sigma_y}{\rho} \right)$$

Hence, the material index to maximize is:

$$M = \frac{\sigma_y}{\rho}$$

$$M = \frac{\sigma_f}{\rho}$$



Ashby – Materials Selection in Mechanical Design (2004)

The choices are some composites (CFRP), some engineering ceramics and high strength Ti and Al alloys

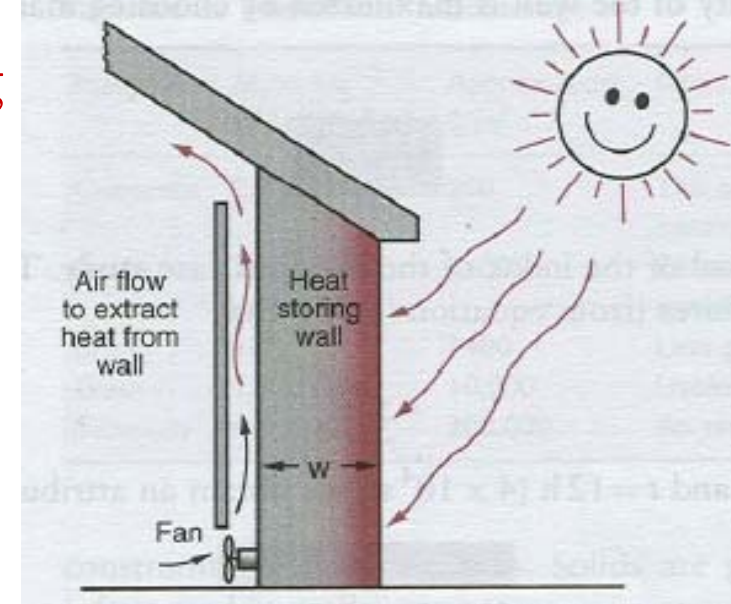
- engineering ceramics eliminated due to lack of toughness
- further selection must be made on the basis of cost and energy storage capacity for specific materials

–e.g. CFRP can store 400kJ/kg

Case 3: Materials for Passive Solar Heating

A simple way of storing solar energy for residential heating is by heating the walls during the day and transferring heat to the interior via forced convection at night.

Need to diffuse heat from the outer to inner surface in 12h. For architectural reasons, the wall thickness (W) cannot exceed 0.5m



Function

Heat storage medium

Objective

Maximize thermal energy storage per unit material cost.

Constraints

Heat diffusion time through wall time (t) $\sim 12\text{h}$

Wall thickness $w < 0.5\text{m}$

Working temperature $T_{Max} \sim 100^\circ\text{C}$

Free Variables

Wall thickness w .

Choice of materials

What material will maximize the thermal energy captured by the wall while retaining the required heat diffusion time of up to 12h?

For a wall of thickness w , the heat (Q) per unit area of wall heated through ΔT is given by:

$$Q = w \rho c_p \Delta T$$

For the heat diffusion distance in time t :

$$w = \sqrt{2Dt}$$

where D is the thermal diffusivity, λ is the thermal conductivity and ρ is the density

$$D = \frac{\lambda}{\rho c_p}$$

$$Q = \sqrt{2t} \rho c_p D^{1/2} \Delta T = \sqrt{2t} \Delta T \left(\frac{\lambda}{\sqrt{D}} \right)$$

The heat capacity of the wall is maximized by choosing a material with a high value of:

$$\frac{\lambda}{\sqrt{D}}$$

The restriction on the wall thickness (w) and diffusion time (t) yield the constraint:

$$D \leq \frac{w^2}{2t} \leq 3 \times 10^{-6} \text{ m}^2/\text{s}$$

Materials for passive solar heat-storage

Ashby – Materials Selection in Mechanical Design (2004)

Material	$M_1 = \lambda/a^{1/2}$ ($\text{W}\cdot\text{s}^{1/2}/\text{m}^2\cdot\text{K}$)	Approx. cost \$/ m^3	Comment
Concrete	2.2×10^3	200	The best choice — good performance at minimum cost
Stone	3.5×10^3	1400	Better performance than concrete because specific heat is greater, but more expensive
Brick	10^3	1400	Less good than concrete
Glass	1.6×10^3	10,000	Useful — part of the wall could be glass
Titanium	4.6×10^3	200,000	An unexpected, but valid, selection. Expensive

Cost must be a significant consideration in this selection because the application is for housing, where cost is always a significant factor. Taking cost into consideration, the most likely choice is concrete, with stone and brick as alternatives.

Literature Resources

M.F. Ashby, “Materials Selection in Mechanical Design, 2ndEd.”

- multiple sources listed in appendices of the book
- materials selection charts in lecture Appendix

Other references :

- ASM Metals Handbook
- Perry’s Chemical Engineering Handbook
- CRC Handbook of Mathematics and Physics
- ASM Handbook of Ceramics and Composites