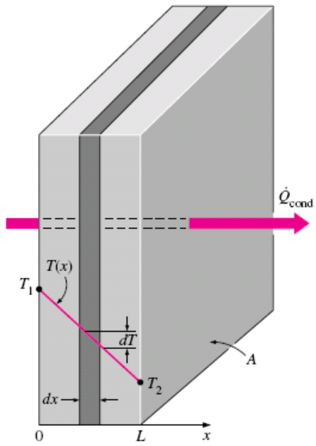
Chapter Three

Steady state Heat Conduction in Plane Walls

1) Considerable temperature difference between the inner and the outer surfaces of the wall (significant temperature gradient in the *x* direction).

2) The wall surface is nearly *isothermal*.



Assuming heat transfer is the only energy interaction and there is no heat generation, the *energy balance* can be expressed as

(Rate of heat transfer into the wall) - (Rate of heat transfer out of the wall) = (Rate of change of the energy of the wall)

$$\dot{Q}_{in} - \dot{Q}_{out} = \overset{=0}{\frac{dE_{wall}}{dt}} = 0$$

Then Fourier's law of heat conduction for the wall can be expressed as

$$\dot{Q}_{cond,wall} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

Integrating the above equation and rearranging yields

$$\dot{Q}_{cond,wall} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

Conduction Resistance

The above equation for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{cond,wall} = \frac{T_1 - T_2}{R_{wall}} \qquad \text{(W)}$$

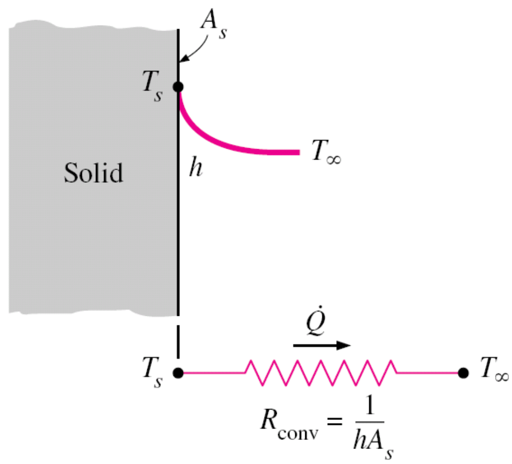
Where R_{wall} is the **conduction resistance** expressed as

$$R_{wall} = \frac{L}{kA} \qquad (^\circ\text{C/W})$$

Convection Resistance

Newton's law of cooling for convection heat transfer rate ($\dot{Q}_{conv} = hA_s(T_s - T_\infty)$) can be rearranged as

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \qquad \text{(W)}$$



R_{conv} is the convection resistance

$$R_{conv} = \frac{1}{hA_s} \qquad (^\circ\text{C/W})$$

Radiation Resistance

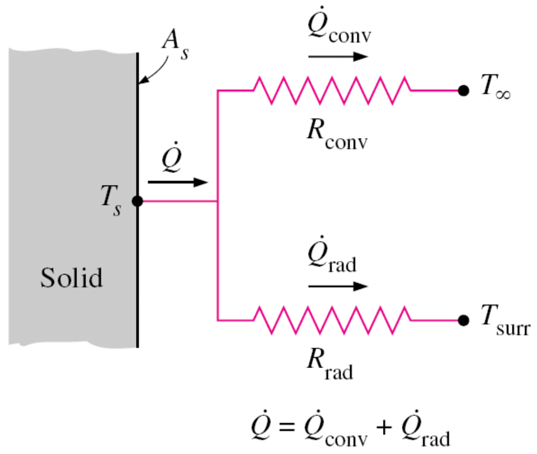
$$\dot{Q}_{rad} = \varepsilon\sigma A_s\left(T_s^4 - T_{surr}^4\right) = h_{rad}A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \text{ (W)}$$

$$R_{rad} = \frac{1}{h_{rad}A_s} \qquad \text{(K/W)}$$

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma\left(T_s^2 + T_{surr}^2\right)\left(T_s + T_{surr}\right) \ (\text{W/m}^2\cdot\text{K})$$

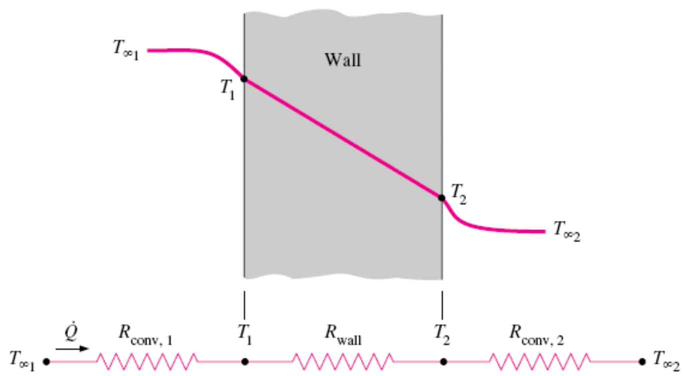
Radiation and Convection Resistance

$$h_{combined} = h_{conv} + h_{rad}$$



Thermal Resistance Network

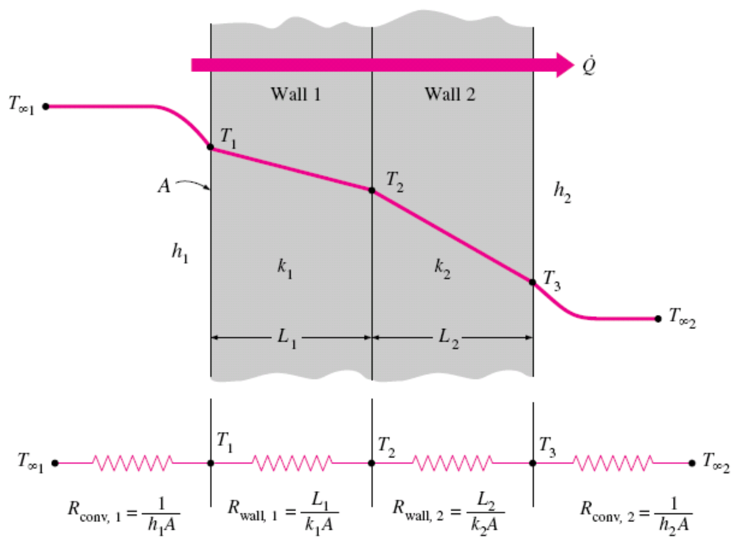
$$\dot{Q} = h_1 A\left(T_{\infty,1} - T_1\right) =$$
$$kA\frac{T_1 - T_2}{L} = h_2 A\left(T_2 - T_{\infty,2}\right)$$



$$R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \quad (^\circ\mathrm{C/W})$$

Multilayer Plane Walls

$$R_{total} = R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2}$$
$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$



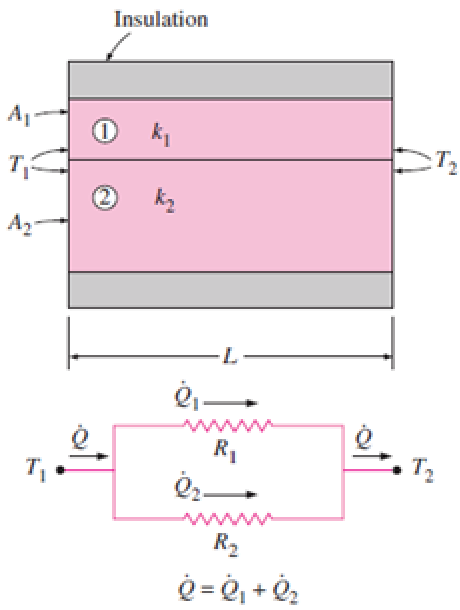
Generalized Thermal Resistance Networks

The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Utilizing electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}} \quad where \quad \frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$



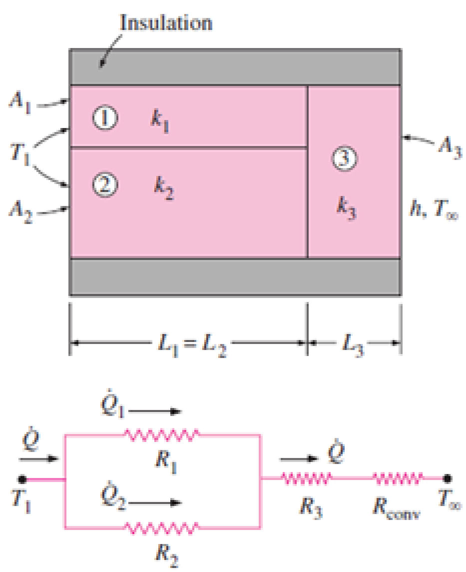
Now consider the combined series-parallel arrangement shown, the total rate of heat transfer through this composite system can again be expressed as,

where
$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$

and
$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$

$$R_1 = \frac{L_1}{k_1 A_1}, \qquad R_2 = \frac{L_2}{k_2 A_2}, \qquad R_3 = \frac{L_3}{k_3 A_3}, \qquad R_{\text{conv}} = \frac{1}{hA_3}$$

once the individual thermal resistances are evaluated, the total resistance and the total rate of heat transfer can easily be determined from the relations above.



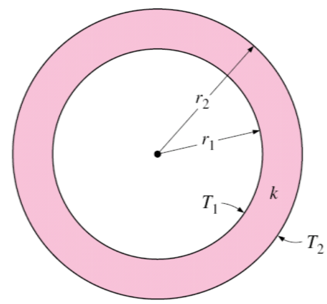
Sphere

Sphere systems may also be treated as on dimensional when the temperature is a function of radius only

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o}$$

Heat Conduction in Cylinders

$$\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr} \qquad \text{(W)}$$



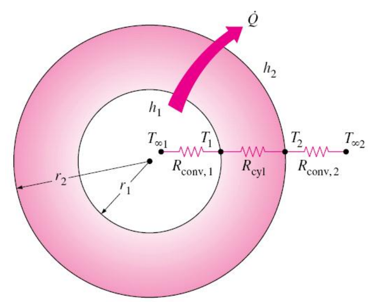
$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{cond,cyl}}{A}dr = -\int_{T=T_1}^{T_2} kdT$$

$$\dot{Q}_{cond,cyl} = 2\pi Lk\,\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

Thermal Resistance with Convection

$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{R_{cyl}}$$

$$\dot{Q} = \frac{T_{\infty,1} - T_{\infty,2}}{R_{total}}$$



$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$

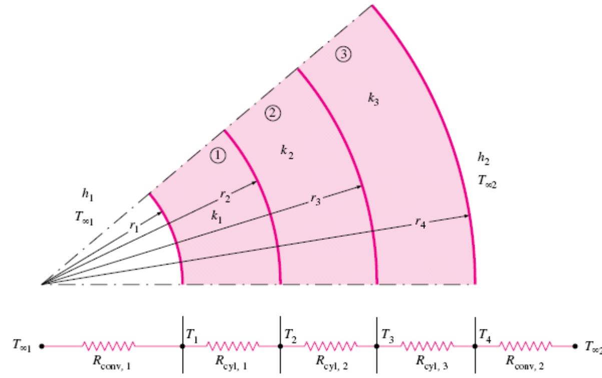
$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2} =$$

$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk} + \frac{1}{(2\pi r_2 L)h_2}$$

Multilayered Cylinders

$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,3} + R_{cyl,3} + R_{conv,2} =$$

$$= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi Lk_1} + \frac{\ln(r_3/r_2)}{2\pi Lk_2} + \frac{\ln(r_4/r_3)}{2\pi Lk_3} + \frac{1}{(2\pi r_2 L)h_2}$$



1
2
3
h_1
$T_{\infty 1}$
r_1
r_2
r_3
r_4
k_1
k_2
k_3
h_2
$T_{\infty 2}$
$T_{\infty 1}$
T_1
T_2
T_3
T_4
$T_{\infty 2}$
$R_{\text{conv, 1}}$
$R_{\text{cyl, 1}}$
$R_{\text{cyl, 2}}$
$R_{\text{cyl, 3}}$
$R_{\text{conv, 2}}$