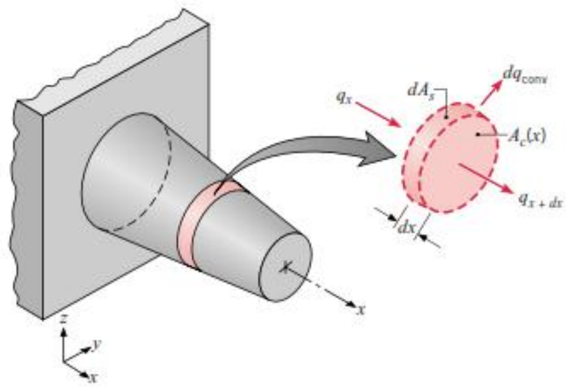
Heat Transfer from Extended Surfaces

Two ways to increase the rate of heat transfer:

- increasing the heat transfer coefficient,
- increase the surface area →fins



$$q_x = q_{x+dx} + dq_{\text{conv}}$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}\,dx$$

$$q_{x+dx} = -kA_c\frac{dT}{dx} - k\frac{d}{dx}\left(A_c\frac{dT}{dx}\right)dx$$

The convection heat transfer rate may be expressed as

$$dq_{\text{conv}} = h\,dA_s(T - T_\infty)$$

$$\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

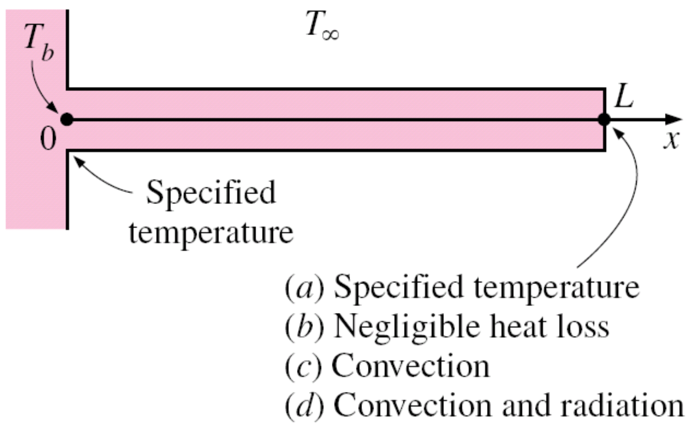
$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad \theta = T - T_\infty \quad \text{and} \quad m^2 \equiv \frac{hP}{kA_c}$$

The general solution of this equation is :

$$\theta(x) = C_1e^{mx} + C_2e^{-mx}$$

Boundary Conditions

Several boundary conditions are typically employed as shown in the Figure



Boundary condition at fin base: $$\theta(0) = \theta_b = T_b - T_\infty$$

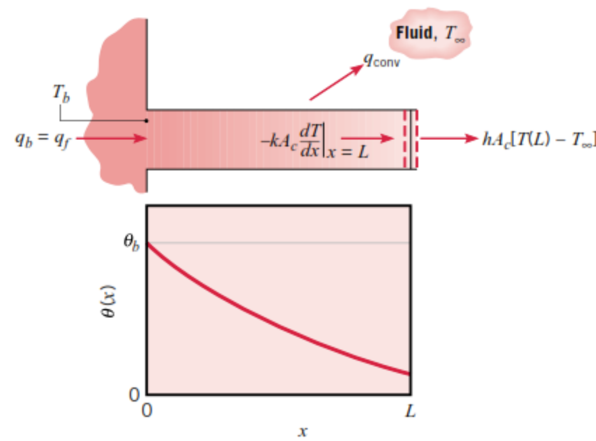
1 - Convection heat transfer at the fin tip :

$$hA_c[T(L) - T_\infty] = -kA_c \frac{dT}{dx}\bigg|_{x=L}$$

$$h\theta(L) = -k\frac{d\theta}{dx}\bigg|_{x=L}$$

$$\theta_b = C_1 + C_2$$

$$h(C_1e^{mL} + C_2e^{-mL}) = km(C_2e^{-mL} - C_1e^{mL})$$



Solving for C_1 and C_2, it may be shown, after some manipulation, that

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

$$q_f = q_b = -kA_c\frac{dT}{dx}\bigg|_{x=0} = -kA_c\frac{d\theta}{dx}\bigg|_{x=0}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

2- *Insulated fin tip ($Q_{fin\ tip}$ = 0)*

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$C_1 e^{mL} - C_2 e^{-mL} = 0$$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \tanh mL$$

3- *Specific temperature end fin ($\Theta_{(L)} = \Theta_L$)*

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\cosh mL - \theta_L/\theta_b}{\sinh mL}$$

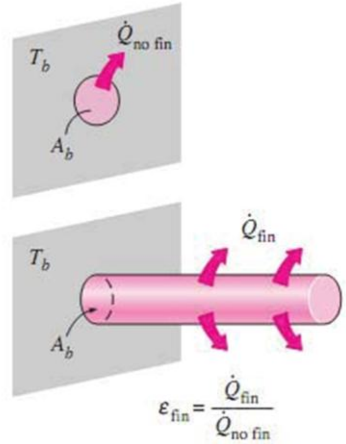
4- Infinity Fin ($T_{tip} = T_\infty$)

$$\frac{\theta}{\theta_b} = e^{-mx}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b$$

Fin Effectiveness

The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case.



$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{hA_b\left(T_b - T_\infty\right)} =$$

$$\varepsilon_{fin} \geq 2$$

For the infinite approximate

$$\varepsilon_f = \left(\frac{kP}{hA_c}\right)^{1/2}$$

Where $A_{c,b}$ is the cross section area at the base , in general the use of fin may rarely be justified unless $\varepsilon_f \geq 2$

Fin efficiency

The maximum rate at which a fin dissipate energy is the rate that would exist if the entire fin surface were at the base temperature

$$\eta_f \equiv \frac{q_f}{q_{max}} = \frac{q_f}{hA_f\theta_b}$$

Where A_f is the surface area of the fin .

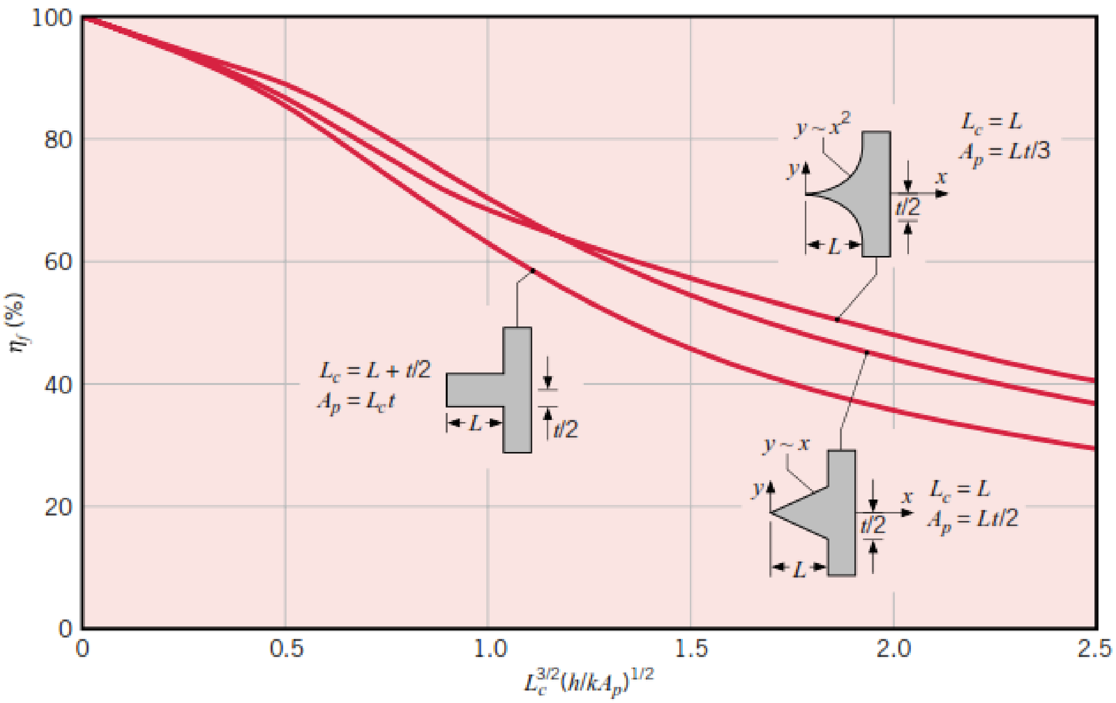
For a straight fin of uniform cross section and an adiabatic tip .

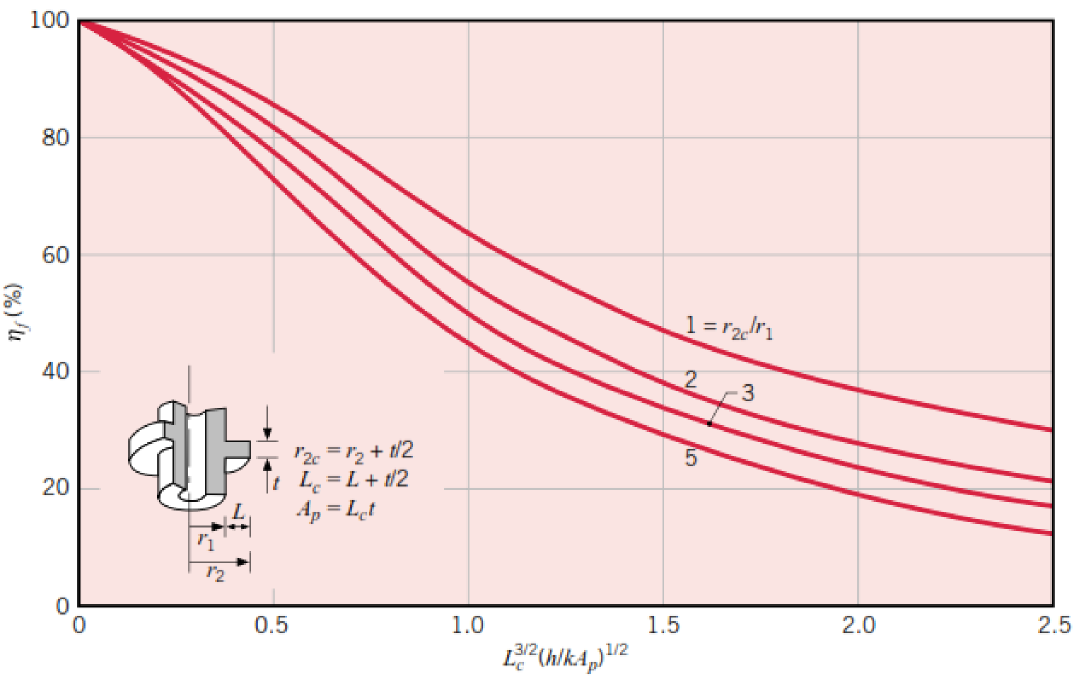
$$\eta_f = \frac{M \tanh mL}{hPL\theta_b} = \frac{\tanh mL}{mL}$$

This results tells as that η_f approaches its maximum and minimum values of 1 and 0 , respectively , as L approaches 0 and ∞

The correction relation for fin efficiency is :

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$





<u>Overall Fin efficiency</u>

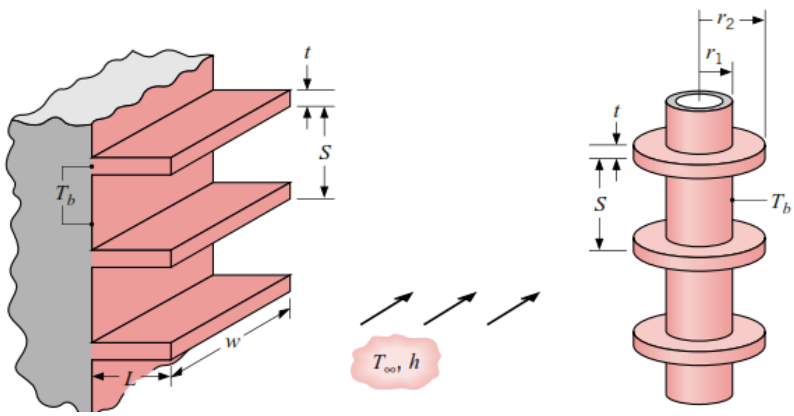
$$\eta_o = \frac{q_t}{q_{\max}} = \frac{q_t}{hA_t\theta_b}$$

$$A_t = NA_f + A_b$$

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b$$

$$q_t = h[N\eta_f A_f + (A_t - NA_f)]\theta_b = hA_t\left[1 - \frac{NA_f}{A_t}(1-\eta_f)\right]\theta_b$$

$$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

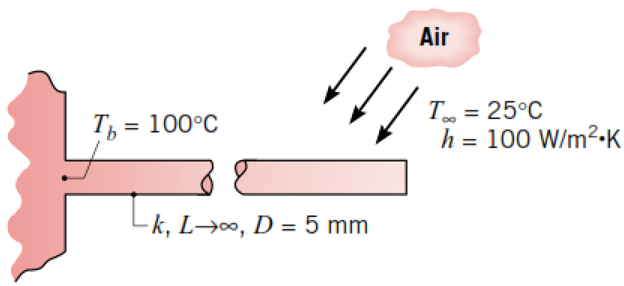


Example :

A very long rod 5 mm in diameter has one end maintained at 100°C. The surface of the rod is exposed to ambient air at 25°C with a convection heat transfer coefficient of 100 W/m^2 · K.

Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?

Take for copper k=398 W/m.k , for Aluminum k=180 w/m.k and for steel k=14 w/m.k



$$q_f = \sqrt{hPkA_c}\,\theta_b$$

Hence for copper,

$$q_f = \left[100\ \text{W/m}^2\cdot\text{K} \times \pi \times 0.005\ \text{m} \times 398\ \text{W/m}\cdot\text{K} \times \frac{\pi}{4}(0.005\ \text{m})^2\right]^{1/2}(100 - 25)^\circ\text{C}$$

$$= 8.3\ \text{W}$$

Similarly, for the aluminum alloy and stainless steel, respectively, the heat rates are q_f = 5.6 W and 1.6 W.