Important solved problems by Ref. Incropera – sixth edition

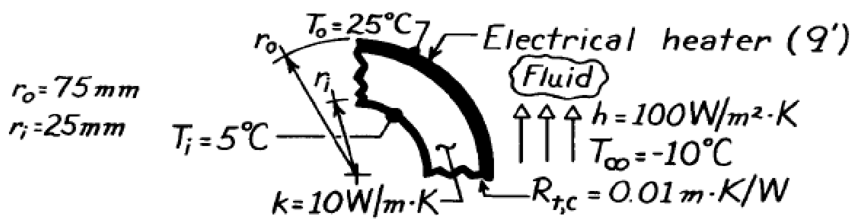
3.37 A thin electrical heater is wrapped around the outer surface of a long cylindrical tube whose inner surface is maintained at a temperature of 5°C. The tube wall has inner and outer radii of 25 and 75 mm, respectively, and a thermal conductivity of 10 W/m · K. The thermal contact resistance between the heater and the outer surface of the tube (per unit length of the tube) is $R'_{t,c} = 0.01$ m · K/W. The outer surface of the heater is exposed to a fluid with $T_\infty = -10$°C and a convection coefficient of $h = 100$ W/m^2 · K. Determine the heater power per unit length of tube required to maintain the heater at $T_o = 25$°C.

PROBLEM 3.37

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface and is exposed to a fluid of prescribed h and T_∞. Thermal contact resistance between heater and tube wall and wall inner surface temperature.

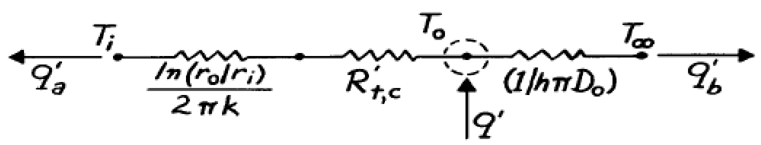
FIND: Heater power per unit length required to maintain a heater temperature of 25°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater.

ANALYSIS: The thermal circuit has the form



Applying an energy balance to a control surface about the heater,

$$q' = q'_a + q'_b$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/h\pi D_o)}$$

$$q' = \frac{(25\text{-}5)^\circ C}{\frac{\ln(75\text{mm}/25\text{mm})}{2\pi \times 10 \text{ W/m}\cdot\text{K}} + 0.01\frac{\text{m}\cdot\text{K}}{\text{W}}} + \frac{[25-(-10)]^\circ C}{\left[1/\left(100 \text{ W/m}^2\cdot\text{K} \times \pi \times 0.15\text{m}\right)\right]}$$

$$q' = (728 + 1649) \text{ W/m}$$

$$q' = 2377 \text{ W/m.}$$ <

COMMENTS: The conduction, contact and convection resistances are 0.0175, 0.01 and 0.021 m ·K/W, respectively,

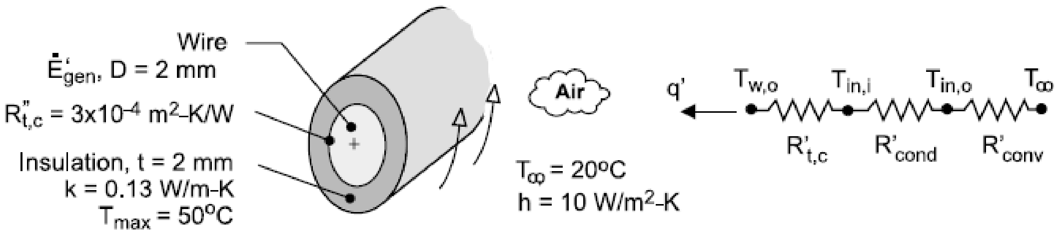
3.43 A 2-mm-diameter electrical wire is insulated by a 2-mm-thick rubberized sheath (k = 0.13 W/m · K), and the wire/sheath interface is characterized by a thermal contact resistance of $R''_{t,c} = 3 \times 10^{-4} \text{m}^2 \cdot \text{K/W}$. The convection heat transfer coefficient at the outer surface of the sheath is 10 W/m^2 · K, and the temperature of the ambient air is 20°C. If the temperature of the insulation may not exceed 50°C, what is the maximum allowable electrical power that may be dissipated per unit length of the conductor? What is the critical radius of the insulation?

PROBLEM 3.43

KNOWN: Diameter of electrical wire. Thickness and thermal conductivity of rubberized sheath. Contact resistance between sheath and wire. Convection coefficient and ambient air temperature. Maximum allowable sheath temperature.

FIND: Maximum allowable power dissipation per unit length of wire. Critical radius of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible radiation exchange with surroundings.

ANALYSIS: The maximum insulation temperature corresponds to its inner surface and is independent of the contact resistance. From the thermal circuit, we may write

$$\dot{E}'_g = q' = \frac{T_{in,i} - T_\infty}{R'_{cond} + R'_{conv}} = \frac{T_{in,i} - T_\infty}{\left[\ell n\left(r_{in,o}/r_{in,i}\right)/2\pi k\right] + \left(1/2\pi r_{in,o} h\right)}$$

where $r_{in,i} = D/2 = 0.001\text{m}$, $r_{in,o} = r_{in,i} + t = 0.003\text{m}$, and $T_{in,i} = T_{max} = 50°\text{C}$ yields the maximum allowable power dissipation. Hence,

$$\dot{E}'_{g,max} = \frac{(50-20)°\text{C}}{\dfrac{\ell n 3}{2\pi \times 0.13\,\text{W/m}\cdot\text{K}} + \dfrac{1}{2\pi\left(0.003\text{m}\right)10\,\text{W/m}^2\cdot\text{K}}} = \frac{30°\text{C}}{\left(1.35 + 5.31\right)\text{m}\cdot\text{K/W}} = 4.51\,\text{W/m} \quad <$$

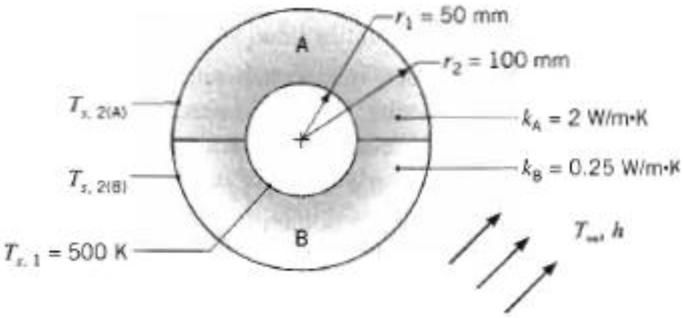
The critical insulation radius is also unaffected by the contact resistance and is given by

$$r_{cr} = \frac{k}{h} = \frac{0.13\,\text{W/m}\cdot\text{K}}{10\,\text{W/m}^2\cdot\text{K}} = 0.013\text{m} = 13\,\text{mm} \quad <$$

Hence, $r_{in,o} < r_{cr}$ and $\dot{E}'_{g,max}$ could be increased by increasing $r_{in,o}$ up to a value of 13 mm (t = 12 mm).

COMMENTS: The contact resistance affects the temperature of the wire, and for $q' = \dot{E}'_{g,max} = 4.51\,\text{W/m}$, the outer surface temperature of the wire is $T_{w,o} = T_{in,i} + q' R'_{t,c} = 50°\text{C} + (4.51\,\text{W/m})\left(3\times10^{-4}\text{m}^2\cdot\text{K/W}\right)/\pi\left(0.002\text{m}\right) = 50.2°\text{C}$. Hence, the temperature change across the contact resistance is negligible.

3.52 Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B.



The interface between the two materials may be assumed to have an infinite contact resistance, and the entire outer surface is exposed to air for which T_∞ = 300 K and h = 25 W/m^2 · K.

(a) Sketch the thermal circuit of the system. Label (using the above symbols) all pertinent nodes and resistances.

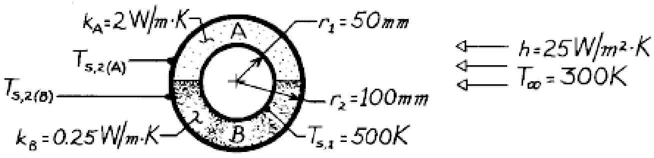
(b) For the prescribed conditions, what is the total heat loss from the pipe? What are the outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$?

PROBLEM 3.52

KNOWN: Inner surface temperature of insulation blanket comprised of two semi-cylindrical shells of different materials. Ambient air conditions.

FIND: (a) Equivalent thermal circuit, (b) Total heat loss and material outer surface temperatures.

SCHEMATIC:



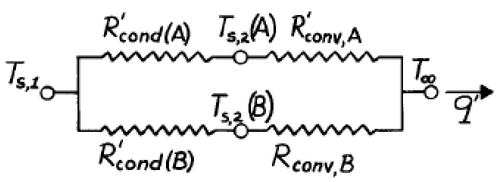
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction, (3) Infinite contact resistance between materials, (4) Constant properties.

ANALYSIS: (a) The thermal circuit is,

$$R'_{conv,A} = R'_{conv,B} = 1/\pi r_2 h$$

$$R'_{cond(A)} = \frac{\ln(r_2/r_1)}{\pi k_A}$$

$$R'_{cond(B)} = \frac{\ln(r_2/r_1)}{\pi k_B}$$



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The conduction resistances follow from Section 3.3.1 and Eq. 3.28. Each resistance is larger by a factor of 2 than the result of Eq. 3.28 due to the reduced area.

(b) Evaluating the thermal resistances and the heat rate $(q' = q'_A + q'_B)$,

$$R'_{conv} = \left(\pi \times 0.1\text{m} \times 25\ \text{W/m}^2 \cdot \text{K}\right)^{-1} = 0.1273\ \text{m} \cdot \text{K/W}$$

$$R'_{cond(A)} = \frac{\ln(0.1\text{m}/0.05\text{m})}{\pi \times 2\ \text{W/m}\cdot\text{K}} = 0.1103\ \text{m}\cdot\text{K/W} \quad R'_{cond(B)} = 8\ R'_{cond(A)} = 0.8825\ \text{m}\cdot\text{K/W}$$

$$q' = \frac{T_{s,1} - T_\infty}{R'_{cond(A)} + R'_{conv}} + \frac{T_{s,1} - T_\infty}{R'_{cond(B)} + R'_{conv}}$$

$$q' = \frac{(500-300)\text{K}}{(0.1103+0.1273)\text{m}\cdot\text{K/W}} + \frac{(500-300)\text{K}}{(0.8825+0.1273)\text{m}\cdot\text{K/W}} = (842+198)\ \text{W/m} = 1040\ \text{W/m}. \qquad <$$

Hence, the temperatures are

$$T_{s,2(A)} = T_{s,1} - q'_A R'_{cond(A)} = 500\text{K} - 842\frac{\text{W}}{\text{m}} \times 0.1103\frac{\text{m}\cdot\text{K}}{\text{W}} = 407\text{K} \qquad <$$

$$T_{s,2(B)} = T_{s,1} - q'_B R'_{cond(B)} = 500\text{K} - 198\frac{\text{W}}{\text{m}} \times 0.8825\frac{\text{m}\cdot\text{K}}{\text{W}} = 325\text{K}. \qquad <$$

COMMENTS: The total heat loss can also be computed from $q' = \left(T_{s,1} - T_\infty\right)/R_{equiv}$,

where $R_{equiv} = \left[\left(R'_{cond(A)} + R'_{conv,A}\right)^{-1} + \left(R'_{cond(B)} + R'_{conv,B}\right)^{-1}\right]^{-1} = 0.1923\ \text{m}\cdot\text{K/W}.$

Hence $q' = (500-300)\text{K}/0.1923\ \text{m}\cdot\text{K/W} = 1040\ \text{W/m}.$

3.60 A spherical vessel used as a reactor for producing pharmaceuticals has a 10-mm-thick stainless steel wall (k = 17 W/m · K) and an inner diameter of 1 m. The exterior surface of the vessel is exposed to ambient air (T_∞ = 25°C) for which a convection coefficient of 6 W/m^2 · K may be assumed.

(a) During steady-state operation, an inner surface temperature of 50°C is maintained by energy generated within the reactor. What is the heat loss from the vessel?

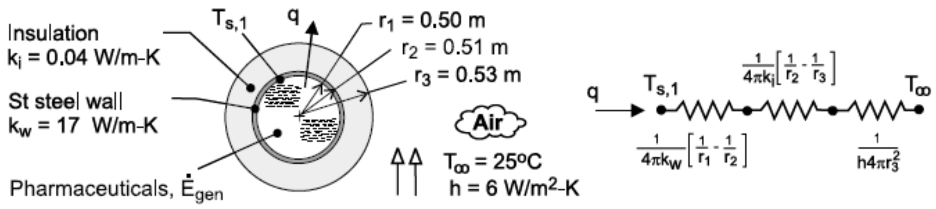
(b) If a 20-mm-thick layer of fiberglass insulation (k = 0.040 W/m · K) is applied to the exterior of the vessel and the rate of thermal energy generation is unchanged, what is the inner surface temperature of the vessel?

PROBLEM 3.60

KNOWN: Inner diameter, wall thickness and thermal conductivity of spherical vessel containing heat generating medium. Inner surface temperature without insulation. Thickness and thermal conductivity of insulation. Ambient air temperature and convection coefficient.

FIND: (a) Thermal energy generated within vessel, (b) Inner surface temperature of vessel with insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

ANALYSIS: (a) From an energy balance performed at an instant for a control surface about the pharmaceuticals, $\dot{E}_g = q$, in which case, without the insulation

$$\dot{E}_g = q = \frac{T_{s,1} - T_\infty}{\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi r_2^2 h}} = \frac{(50-25)°C}{\frac{1}{4\pi\,(17\,W/m\cdot K)}\left(\frac{1}{0.50m} - \frac{1}{0.51m}\right) + \frac{1}{4\pi\,(0.51m)^2\,6\,W/m^2\cdot K}}$$

$$\dot{E}_g = q = \frac{25°C}{\left(1.84\times10^{-4} + 5.10\times10^{-2}\right)K/W} = 489\,W \qquad <$$

(b) With the insulation,

$$T_{s,1} = T_\infty + q\left[\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{4\pi r_3^2 h}\right]$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + \frac{1}{4\pi\,(0.04)}\left(\frac{1}{0.51} - \frac{1}{0.53}\right) + \frac{1}{4\pi\,(0.53)^2\,6}\right]\frac{K}{W}$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + 0.147 + 0.047\right]\frac{K}{W} = 120°C \qquad <$$

COMMENTS: The thermal resistance associated with the vessel wall is negligible, and without the insulation the dominant resistance is due to convection. The thermal resistance of the insulation is approximately three times that due to convection.

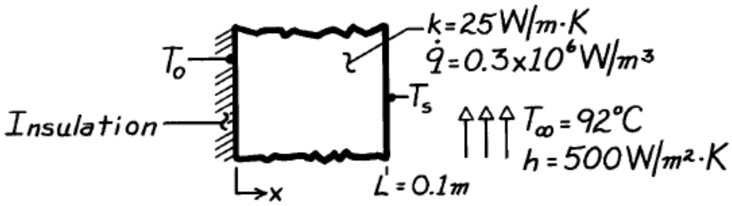
3.72 A plane wall of thickness 0.1 m and thermal conductivity 25 W/m · K having uniform volumetric heat generation of 0.3 MW/m³ is insulated on one side, while the other side is exposed to a fluid at 92°C. The convection heat transfer coefficient between the wall and the fluid is 500 W/m² · K. Determine the maximum temperature in the wall.

PROBLEM 3.72

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: From Eq. 3.42, the temperature at the inner surface is given by Eq. 3.43 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2/2k + T_s.$$

The outer surface temperature follows from Eq. 3.46,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92^\circ C + 0.3\times10^6 \frac{W}{m^3} \times 0.1m/500 W/m^2\cdot K = 92^\circ C + 60^\circ C = 152^\circ C.$$

It follows that

$$T_o = 0.3\times10^6 W/m^3 \times (0.1m)^2/2\times25 W/m\cdot K + 152^\circ C$$

$$T_o = 60^\circ C + 152^\circ C = 212^\circ C.$$ <

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 500 W/m^2\cdot K(152-92)^\circ C = 30 kW/m^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

$$q''_{conv} = \dot{q}L = 0.3\times10^6 W/m^3 \times 0.1m = 30 kW/m^2.$$