Chapter One

Introduction

-Methods of heat transfer

Heat transfer: is transit energy between the bodies of materials due to the temperature difference*, and its measured by (W=J/s)

1-Conduction heat transfer

Conduction: is heat transfer (energy) from the higher particles activity to lower activity due to the temperature difference, and this phenomenon replay to random motion, translational motion, rotational motion and internal vibrational motion of particles.

-Fourier's law

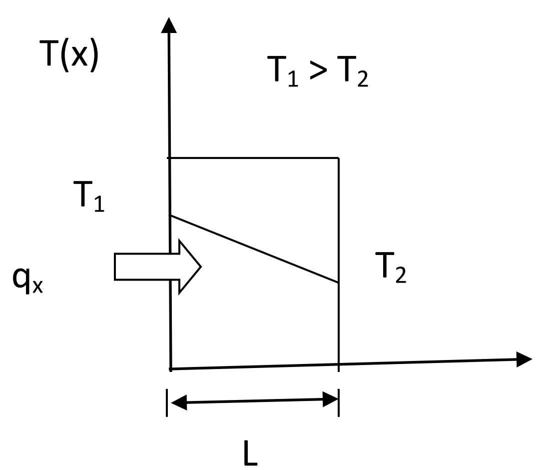
Heat will flow from the higher temperature to the lower temperature

$$q_x \;\alpha\; A\,\frac{dT}{dx}$$

$$q_x = -k\;A\,\frac{dT(x)}{dx} \quad (W) \quad heat\ transfer\ rate$$

Or

$$q''_x = -k\,\frac{dT(x)}{dx} \qquad (W/m^2)\ heat\ flux$$



Where

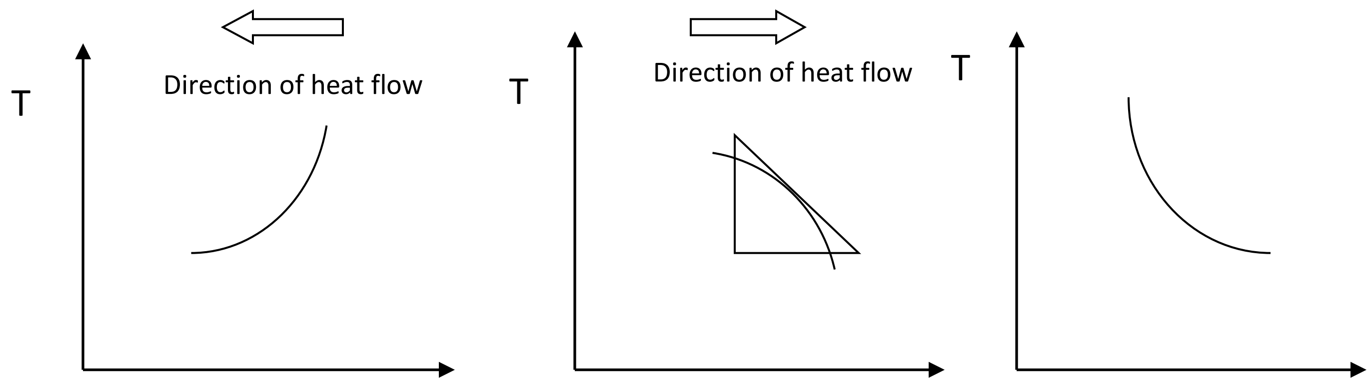
A: the area m^2

T(x): the temperature distribution K or °C

X: the distance m

k: the thermal conductivity W/m.K

the minus sign is a consequence of the second law of thermodynamics, which requires that heat must flow from higher to lower temperature. As illustrated below;



(a)Positive $(\frac{dT}{dx})$ x (b)Negative $(\frac{dT}{dx})$ x (c) General n

For three dimension bodies there are three components for heat transfer

$$q_x = -k\ A_x\ \frac{dT}{dx}, \qquad q_y = -k\ A_y\ \frac{dT}{dx}, \qquad q_z = -k\ A_z\ \frac{dT}{dx}$$

$$q = i\ q_x + j\ q_y + k\ q_z\ ,|q| = \sqrt{q_x^2 + q_y^2 + q_z^2}$$

The temperature gradients can be expressed as

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

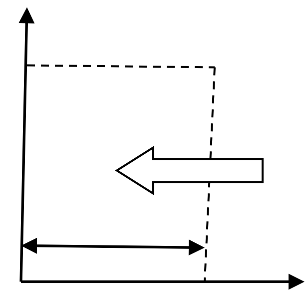
And then the heat flux becomes;

$$q_x'' = -k\ \frac{T_2 - T_1}{L} = k\ \frac{T_1 - T_2}{L} = k\ \frac{\Delta T}{L}$$

Example1: plane wall has thickness 100mm and thermal conductivity k=100W/m.K, and the steady state conditions occurs at T_1=400K, T_2=600K. Calculate the heat flux q''_x and temperature gradient $\frac{dT}{dx}$?

Sol.

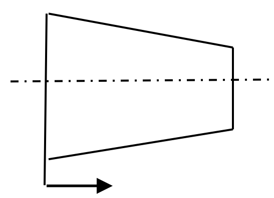
$$\frac{dT}{dx} = \frac{T_2 - T_1}{L} = \frac{600 - 400}{0.1} = 2000 \ K/m$$



$$q''_x = -100 \times 2000 = -20 \ kW/m^2$$

Example2: Find $\frac{dT}{dx}$ and thermal conductivity of the following figure?

$$T(x) = 300(1 - 2x - x^3) \ K$$
$$A(x) = (1 - x) \ m^2$$
$$q = 6000 \ W$$



Sol.

$$q_x = -k \ A \frac{dT(x)}{dx} \quad (W)$$

$$\frac{dT(x)}{dx} = 300(-2 - 3x^2) = -300(2 + 3x^2) \quad K/m$$

$$k(x) = \frac{-6000}{-(1-x)\times 300(2+3x^2)} = \frac{20}{(1-x)(2+3x^2)}$$

For example at x=0 , k=10, A=1 and dT/dx=-600

$$q = -10\times 1\times(-600) = 6000 \quad W$$

Thermal Conductivities

Thermal conductivity is transitional property which its defined as

$$k \equiv \frac{q''_x}{\left|\partial T / \partial x\right|}$$

q''_x : the heat flux per unit area normal to the surface.

For many materials, the thermal conductivity can be approximated as a linear function of temperature over limited temperature ranges:

$$k(T) = k_0 \ (1+\beta_k T)$$

β_k : empirical constant

k_0 :the value of conductivity at a reference temperature.

Material	Thermal conductivity W/m.K	Material	Thermal conductivity W/m.K
Silver	410	Lead	35
Copper	385	Water	0.556
Aluminum	202	Air	0.024
Nickel	93	Ice	2.22
Iron	73	Saw dust	0.059
Carbon steel	43	Glass	0.78

Thermal diffusivity

The **thermal diffusivity** represents how fast heat diffuses through a material. Appears in the transient heat conduction analysis. A material that has a high thermal conductivity or a low heat capacity will have a large thermal diffusivity. The larger the thermal diffusivity, the faster the propagation of heat into the medium.

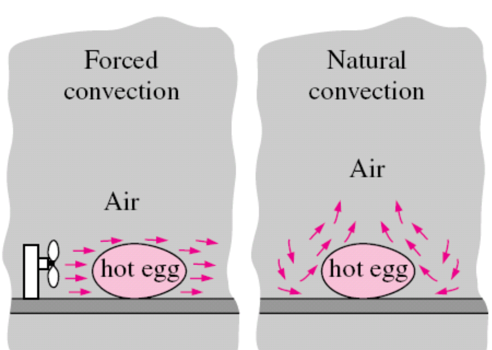
$$\alpha = \frac{\text{Heat conducted}}{\text{Heat stored}} = \frac{k}{\rho c_p} \qquad (\mathrm{m^2/s})$$

Convection

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion.

Convection is commonly classified into three sub-modes:

– Forced convection,
– Natural (or free) convection,
– Change of phase (liquid/vapor, solid/liquid, etc.)



The rate of *convection heat transfer* is expressed by **Newton's law of cooling** as

$$\dot{Q}_{convection} = hA_s(T_s - T_\infty)$$

h is the *convection heat transfer coefficient* in **W/m^2 °C**.

h depends on variables such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity.

Mode	h W/m^2.K
Free convection	5-25
Forced convection	
Gases	25-250
Liquids	50-20000
Convection with phase change (condensation and boiling)	2500-100000

Radiation

Radiation is the energy emitted by matter in the form of *electromagnetic waves* (or *photons*) as a result of the changes in the electronic configurations of the atoms or molecules. Heat transfer by radiation does not require the presence of an *intervening medium.* In heat transfer studies we are interested in *thermal radiation (*radiation emitted by bodies because of their temperature).

Radiation – Emission

The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s (in K or R) is given by the Stefan–Boltzmann law

$$\dot{Q}_{emit,\max} = \sigma A_s T_s^4 \quad \text{(W)}$$

σ=5.670X10^{8} W/m^{2}·K^{4} is the *Stefan–Boltzmann constant.*

The idealized surface that emits radiation at this maximum rate is called a **blackbody.** The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as :

$$\dot{Q}_{emit,\max} = \varepsilon \sigma A_s T_s^4 \quad \text{(W)}$$

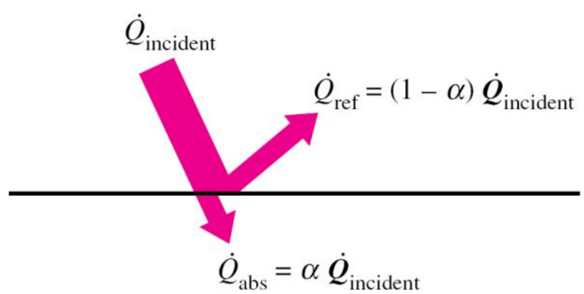
$0 \le \varepsilon \le 1$ ε is the **emissivity** of the surface.

Radiation - Absorption

The fraction of the radiation energy incident on a surface that is absorbed by the surface is termed the **absorptivity** α. Both ε and α of a surface depend on the

temperature and the wavelength of the radiation.

$$0 \le \alpha \le 1$$



Example

A 10 cm diameter copper ball is to be heated from 100^{o}C to an average temperature of 150^{o}C in 30 minutes . Taking the average density and specific heat of copper in this temperature range to be ρ=8950 kg/m^3 and C_p=0.395 kJ/kg^{o}C ,respectively . Determine (a) the total amount of heat transfer to the copper ball

(b) the average rate rate of heat transfer to the ball and (c) the average heat flux .

Ans : a) 92.6 kJ b) 51.4 W c) 1636 W/m^2

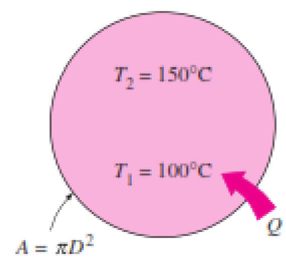
Properties The average density and specific heat of copper are given to be ρ = 8950 kg/m^3 and C_p = 0.395 kJ/kg · °C.

Analysis (*a*) The amount of heat transferred to the copper ball is simply the change in its internal energy, and is determined from

$$\text{Energy transfer to the system} = \text{Energy increase of the system}$$
$$Q = \Delta U = mC_{\text{ave}}(T_2 - T_1)$$

where

$$m = \rho V = \frac{\pi}{6}\rho D^3 = \frac{\pi}{6}(8950\ \text{kg/m}^3)(0.1\ \text{m})^3 = 4.69\ \text{kg}$$



Substituting,

$$Q = (4.69\ \text{kg})(0.395\ \text{kJ/kg}\cdot{}^\circ\text{C})(150 - 100)^\circ\text{C} = \mathbf{92.6\ kJ}$$

Therefore, 92.6 kJ of heat needs to be transferred to the copper ball to heat it from 100°C to 150°C.

(*b*) The rate of heat transfer normally changes during a process with time. However, we can determine the *average* rate of heat transfer by dividing the total amount of heat transfer by the time interval. Therefore,

$$\dot{Q}_{\text{ave}} = \frac{Q}{\Delta t} = \frac{92.6\ \text{kJ}}{1800\ \text{s}} = 0.0514\ \text{kJ/s} = \mathbf{51.4\ W}$$

(*c*) Heat flux is defined as the heat transfer per unit time per unit area, or the rate of heat transfer per unit area. Therefore, the average heat flux in this case is

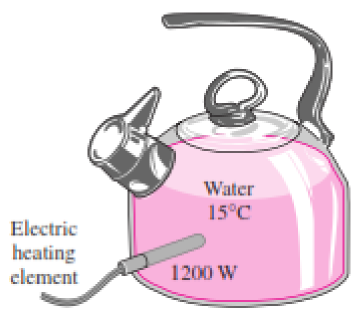
$$\dot{q}_{\text{ave}} = \frac{\dot{Q}_{\text{ave}}}{A} = \frac{\dot{Q}_{\text{ave}}}{\pi D^2} = \frac{51.4\ \text{W}}{\pi(0.1\ \text{m})^2} = \mathbf{1636\ W/m^2}$$

Example

1.2 kg of liquid water initially at 15°C is to be heat to 95°C in a teapot equipped with a 1200 W electric heating element inside . The teapot is 0.5 kg and has an average specific heat of 0.7 kJ/kg °C .Taking the specific heat of water to be 4.18 kJ/kg ° C and disregarding any heat loss from the teapot, determine how long it will take for the water to be heated . Ans : 6 min

Properties The average specific heats are given to be 0.7 kJ/kg · °C for the teapot and 4.18 kJ/kg · °C for water.

$$E_{in} = (mC\Delta T)_{water} + (mC\Delta T)_{teapot}$$
$$= (1.2\ kg)(4.18\ kJ/kg\cdot{}^\circ C)(95-15)^\circ C + (0.5\ kg)(0.7\ kJ/kg\cdot{}^\circ C)(95-15)^\circ C$$
$$= 429.3\ kJ$$
$$\Delta t = \frac{\text{Total energy transferred}}{\text{Rate of energy transfer}} = \frac{E_{in}}{\dot{E}_{transfer}} = \frac{429.3\ kJ}{1.2\ kJ/s} = 358\ s = 6.0\ min$$

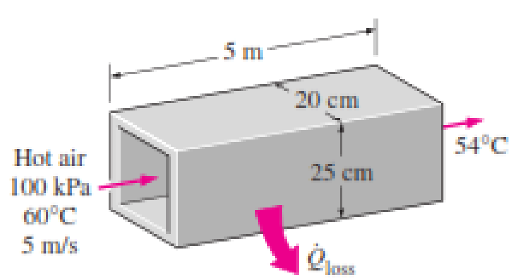


Example

5 m long section of an air heating system of a house passess through an unheated space in the basement . The cross section of the rectangular duct of the heating system is 20 cm × 25 cm . Hot air enters the duct at 100 kPa and 60°C at an average velocity of 5 m/s . The temperature of the air in the duct drops to 54°C as a result of heat loss to the cool space in the basement . Determine the rate of heat loss from the air in the duct to the basement under steady condition .

Properties The constant pressure specific heat of air at the average temperature of (54 + 60)/2 = 57°C is 1.007 kJ/kg · °C (Table A-15).

$$\dot{m} = \rho \mathcal{V} A_c = (1.046\ kg/m^3)(5\ m/s)(0.05\ m^2) = 0.2615\ kg/s$$
$$\dot{Q}_{loss} = \dot{m} C_p (T_{in} - T_{out})$$
$$= (0.2615\ kg/s)(1.007\ kJ/kg\cdot{}^\circ C)(60-54)^\circ C$$
$$= 1.580\ kJ/s$$

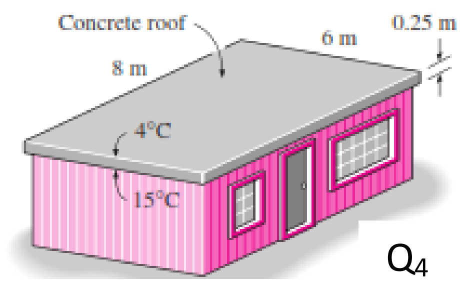


$$\text{Cost of heat loss} = \frac{(\text{Rate of heat loss})(\text{Unit cost of energy input})}{\text{Furnace efficiency}}$$
$$= \frac{(5688 \text{ kJ/h})(\$0.60/\text{therm})}{0.80}\left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right)$$
$$= \$0.040/\text{h}$$

Example

The roof of an electrically heated home is 6 m long 8 m wide and 0.25 m thick , and is made of a flat layer of concrete whose thermal conductivity is k=0.8 W/m ^{o}C . The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15 ^{o}C and 4^{o}C respectively for a period of 10 hours . Determine (a) the rate of heat loss through the roof that night and (b) the cost of the heat loss to the home owner if the cost of electricity is $ 0.08 /kWh .

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8 \text{ W/m}\cdot{}^{\circ}\text{C})(48 \text{ m}^2)\frac{(15-4)^{\circ}\text{C}}{0.25 \text{ m}} = 1690 \text{ W} = 1.69 \text{ kW}$$
$$Q = \dot{Q}\,\Delta t = (1.69 \text{ kW})(10 \text{ h}) = 16.9 \text{ kWh}$$
$$\text{Cost} = (\text{Amount of energy})(\text{Unit cost of energy})$$
$$= (16.9 \text{ kWh})(\$0.08/\text{kWh}) = \$1.35$$



Q4

Example

2m long 0.3 diameter electrical wire extends across a room at 15^{o}C . Heat is generated in the wire as a result of resistance heat ing , and the surface temperature of the wire is measured to be 152^{o}C in steady operation Also the voltage drop and electric current through the wire are measured to be60 V and 1.5 A respectively . Disregarding any heat transfer by radiation

,determine the convection heat transfer Coefficient for heat transfer between the outer surface of the wire and the air in the room

$$\dot{Q} = \dot{E}_{generated} = VI = (60 \text{ V})(1.5 \text{ A}) = 90 \text{ W}$$

$$A_s = \pi DL = \pi(0.003 \text{ m})(2 \text{ m}) = 0.01885 \text{ m}^2$$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

$$h = \frac{\dot{Q}_{conv}}{A_s(T_s - T_\infty)} = \frac{90 \text{ W}}{(0.01885 \text{ m}^2)(152 - 15)°\text{C}} = 34.9 \text{ W/m}^2 \cdot °\text{C}$$

Example

Consider a person standing in a breezy room at 20°C . Determine the total rate of heat transfer from this person if the exposed area and the average outer surface temperature of the person are 1.6 m^2 and 29°C respectively and the convection heat transfer coefficient is 6 W/m^2 °C .

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (6 \text{ W/m}^2 \cdot °\text{C})(1.6 \text{ m}^2)(29 - 20)°\text{C} = 86.4 \text{ W}$$

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = (0.95)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.6 \text{ m}^2) \times [(29 + 273)^4 - (20 + 273)^4] \text{ K}^4 = 81.7 \text{ W}$$

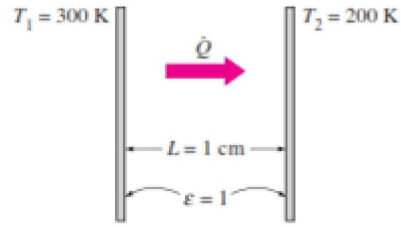
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = (86.4 + 81.7) \text{ W} = 168.1 \text{ W}$$

Example

Consider steady heat transfer between two large parallel plates at constant temperature of T_1=300 K and T_2=200 K that are L =1m apart .Assuming the surfaces to be black (ε=1) , determine the rate of heat transfer between the plates when filled with atmospheric air . Take k_{air} = 0.026 W/m . C°

Properties The thermal conductivity at the average temperature of 250 K is k = 0.0219 W/m · °C for air (Table A-11), 0.026 W/m · °C for urethane insulation (Table A-6), and 0.00002 W/m · °C for the superinsulation.

$$\dot{Q}_{cond} = kA\frac{T_1 - T_2}{L} = (0.0219 \text{ W/m}\cdot{}^\circ\text{C})(1 \text{ m}^2)\frac{(300 - 200)^\circ\text{C}}{0.01 \text{ m}} = 219 \text{ W}$$



and

$$\dot{Q}_{rad} = \varepsilon\sigma A(T_1^4 - T_2^4)$$
$$= (1)(5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)(1 \text{ m}^2)[(300 \text{ K})^4 - (200 \text{ K})^4] = 368 \text{ W}$$

Therefore,

$$\dot{Q}_{total} = \dot{Q}_{cond} + \dot{Q}_{rad} = 219 + 368 = \mathbf{587\ W}$$

the voids in the insulating material. The rate of heat transfer through the urethane insulation is

$$\dot{Q}_{total} = \dot{Q}_{cond} = kA\frac{T_1 - T_2}{L} = (0.026 \text{ W/m}\cdot{}^\circ\text{C})(1 \text{ m}^2)\frac{(300 - 200)^\circ\text{C}}{0.01 \text{ m}} = \mathbf{260\ W}$$

Chapter Two

Introduction to Conduction heat transfer

-Heat diffusion equation in wall plane

rate of heat conduction into control volume+ rate of heat generation inside control volume=rate of heat conduction out of control volume +rate of energy storage inside control volume.

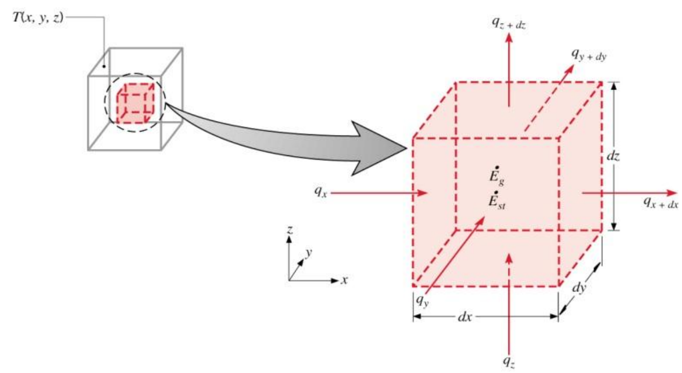
$$q_x + q_y + q_z + \dot{q}\, dx\, dy\, dz = q_{x+dx} + q_{y+dy} + q_{z+dz} + \frac{\partial}{\partial t}(\rho\ dV\ c\ T)$$

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

- Cartesian Coordinates:



t: time sec

ρ:density Kg/m^3

c:specific heat J/kg.K

q:the rate of heat generation per unit volume inside the control volume; W/m^3.

$$-\frac{\partial q_x}{\partial x} dx - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz + \dot{q} dV = \rho\ dV\ c\ \frac{\partial T}{\partial t}$$

$$\frac{\partial q_x}{\partial x} dx = \frac{\partial}{\partial x}\left(-k\ dy\ dz \frac{\partial T}{\partial x} \right) dx$$

$$\frac{\partial q_y}{\partial y}\,dy = \frac{\partial}{\partial y}\left(-k\ dx\ dz\,\frac{\partial T}{\partial y}\right)dy$$

$$\frac{\partial q_z}{\partial z}\,dz = \frac{\partial}{\partial z}\left(-k\ dx\ dy\,\frac{\partial T}{\partial z}\right)dz$$

Where ρ ,c & dV constant with time.

$$\frac{\partial}{\partial x}\left(k\ \frac{\partial T}{\partial x}\right)dV + \frac{\partial}{\partial y}\left(k\ \frac{\partial T}{\partial y}\right)dV + \frac{\partial}{\partial z}\left(k\ \frac{\partial T}{\partial z}\right)dV + \dot{q}\ dV = \rho\, c\, dV\,\frac{\partial T}{\partial t}$$

$$\frac{\partial}{\partial x}\left(k\ \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\ \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\ \frac{\partial T}{\partial z}\right) + \dot{q} = \rho\, c\,\frac{\partial T}{\partial t}$$

for steady state $\frac{\partial T}{\partial t} = 0$ for unsteady state

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{\dot{q}}{k} = 0$$

Without heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$\nabla^2 T = 0$ Laplace equation

$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ Laplace operator

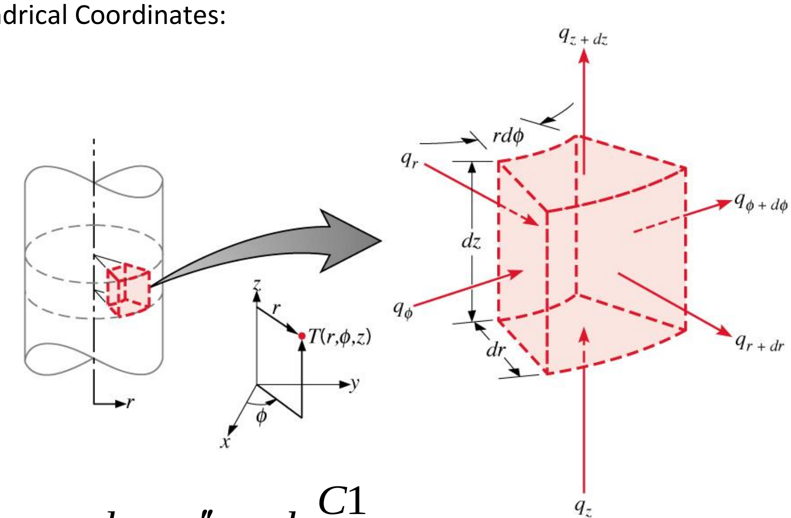
$\frac{d^2 T}{dx^2} = 0$ one dimension

Cylindirical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- Cylindrical Coordinates:

For T(r), steady state and q=0



$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

$$\frac{dT}{dr}=\frac{C1}{r} \quad \Rightarrow \quad T=C1\ln r+C2 \quad and \quad q''=-k\frac{C1}{r}$$

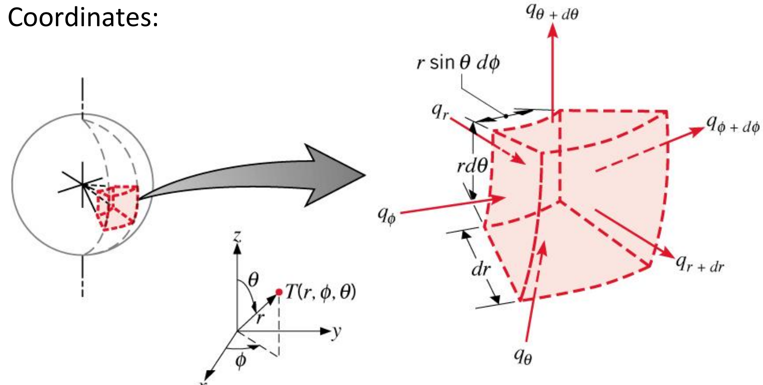
$$q=q''\ A(r)=-\frac{k\ C1}{r}\ 2\pi rL=cons\tan t$$

Spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- Spherical Coordinates:

For T(r) and q=0



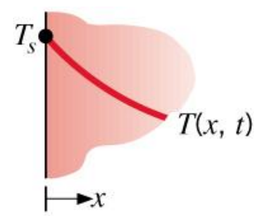
$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$

$$\frac{dT}{dr} = \frac{C1}{r^2} \quad \Rightarrow \quad T = -\frac{C1}{r} + C2 \quad and \quad q'' = -k\frac{C1}{r^2}$$

<u>Specified Boundary Condition</u>

1- Constant surface temperature.

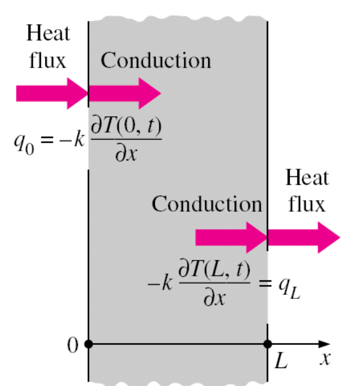
$$T(o,t) = T_s$$



2- Specified Heat Flux Boundary Condition

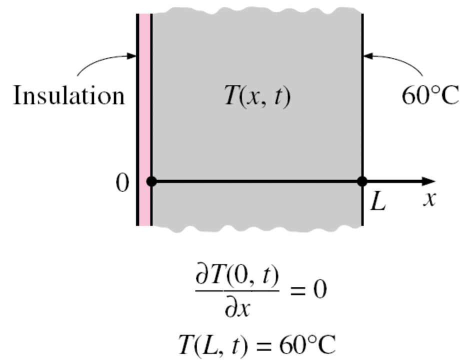
The heat flux in the positive *x*-direction anywhere in the medium, including the boundaries, can be expressed by *Fourier's law* of heat conduction as

$$\dot{q} = -k\frac{dT}{dx} =$$ (**Heat flux in the positive x-direction**)



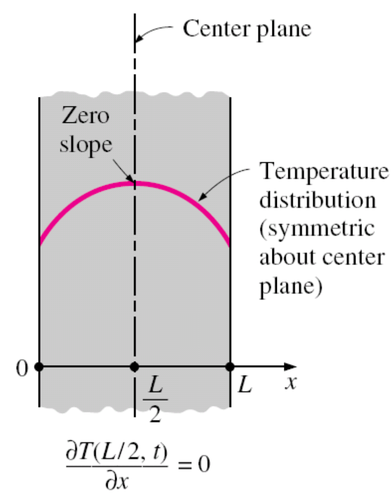
The sign of the specified heat flux is determined by inspection: *positive* if the heat flux is in the positive direction of the coordinate axis, and *negative* if it is in the opposite direction.

3-Adiabatic boundary conditions



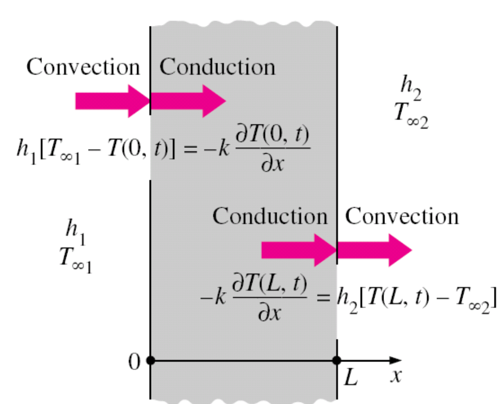
$$k\frac{\partial T(0,t)}{\partial x} = 0 \qquad \text{or} \qquad \frac{\partial T(0,t)}{\partial x} = 0$$

4- Thermal symmetry



$$\frac{\partial T\left(L/2, t\right)}{\partial x} = 0$$

5-Convection Boundary Condition



$$-k\frac{\partial T(L,t)}{\partial x} = h_2\left[T(L,t) - T_{\infty 2}\right]$$

$$-k\frac{\partial T(0,t)}{\partial x} = h_1\left[T_{\infty 1} - T(0,t)\right]$$

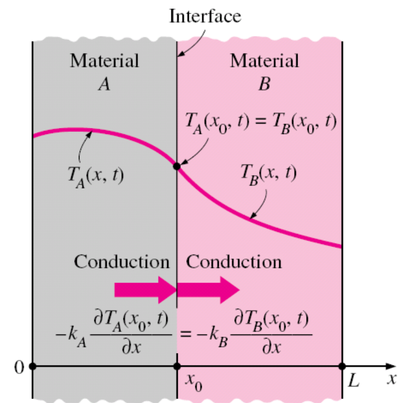
At the interface the requirements are:

(1) two bodies in contact must have the *same temperature* at the area of contact,

(2) an interface (which is a surface) cannot store any energy, and thus the *heat flux* on the two sides of an interface *must be the same.*

$$T_A(x_0, t) = T_B(x_0, t)$$

$$-k_A \frac{\partial T_A(x_0,t)}{\partial x} = -k_B \frac{\partial T_B(x_0,t)}{\partial x}$$



Variable Thermal Conductivity for One-Dimensional Cases

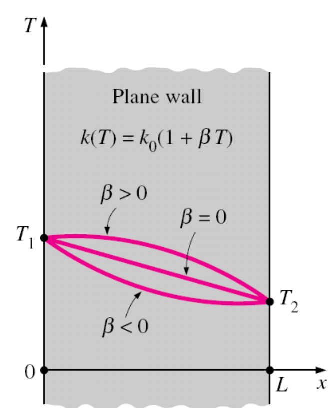
$$Thermal\ resistance = \frac{thermal\ potential\ differance}{heat\ flow}$$

$$R_{t,cond} = \frac{\Delta T}{q} = \frac{T_1 - T_2}{q} = \frac{L}{kA} \equiv R = \frac{V}{I}$$

For variable k

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0, \quad k(t) = k_o(1 + \beta\ T)$$

$$q = -k(t)A\frac{dT}{dx}$$



$$q = -k_o(1+\beta\ T)A\frac{dT}{dx}$$

$$\frac{q}{A}\int_0^L dx = -k_o\int_{T1}^{T2}(1+\beta\ T)dT$$

$$\frac{qL}{A} = k_o(T_1 - T_2)\left[1+\frac{\beta}{2}(T_1+T_2)\right]$$

$$k_{av} = k_o\left[1+\frac{\beta}{2}(T_{s1}+T_{s2})\right]$$

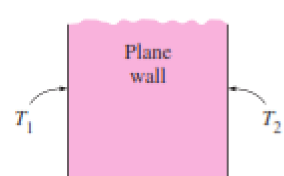
$$q = k_{av}A\frac{T_{s1}-T_{s2}}{L}, \quad R = \frac{LA}{k_{av}}$$

Example : The resistance wire of a 1200-W iron is 80 cm long and has a diameter of *0.3 cm. Determine the rate of heat* generation in the wire per unit volume, in W/cm^3 , and the heat flux on the outer surface of the wire as a result of this heat generation.

$$\dot{g} = \frac{\dot{G}}{V_{wire}} = \frac{\dot{G}}{(\pi D^2/4)L} = \frac{1200\text{ W}}{[\pi(0.3\text{ cm})^2/4](80\text{ cm})} = 212\text{ W/cm}^3$$

$$\dot{q} = \frac{\dot{G}}{A_{wire}} = \frac{\dot{G}}{\pi DL} = \frac{1200\text{ W}}{\pi(0.3\text{ cm})(80\text{ cm})} = 15.9\text{ W/cm}^2$$

<u>Example</u> : Consider a large plane wall of thickness *L = 0.2 m* , thermal conductivity *k = 1.2 W/m · °C, and surface area A* = 15 m^2. The left side of the wall is maintained at a constant temperature of *T1 = 120 °C while the right side loses heat by convection* to the surrounding air at T_2 = *50 °C. Determine* (*a*) *the* variation of temperature in the wall and the value



of temperature at x=0.1 m and (*c*) *the rate of heat transfer* through the wall.

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 120°C$$

$$T(L) = T_2 = 50°C$$

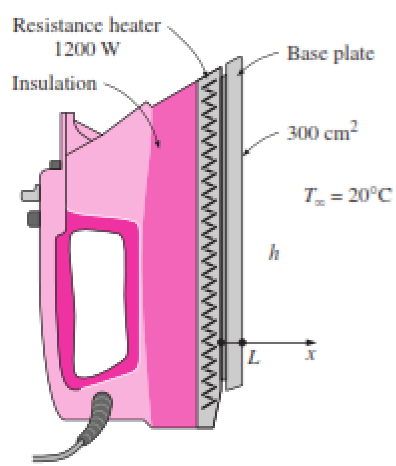
$$T(x) = C_1x + C_2$$

$$T(x) = \frac{T_2 - T_1}{L}x + T_1$$

$$T(0.1\text{ m}) = \frac{(50 - 120)°C}{0.2\text{ m}}(0.1\text{ m}) + 120°C = 85°C$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (1.2\text{ W/m}\cdot°C)(15\text{ m}^2)\frac{(120 - 50)°C}{0.2\text{ m}} = 6300\text{ W}$$

<u>Example</u> : Consider the base plate of an 1200 W household iron with a thickness of *L= 0.5 cm, base area of A=300 cm2, and* thermal conductivity of *k = 15 W/m · °C. The inner surface of* the base plate is subjected to uniform heat flux generated by the resistance heaters inside. When steady operating conditions are reached, the outer surface losses heat to the surrounding at T_∞ = 20 °C by convection . Taking h= 80 W/m^2 °C and disregarding the heat loss by radiation (*a*) *obtain a relation for the* variation of temperature in the base plate by solving the differential equation, and (*b*) *evaluate the inner and the outer surface temperature.*



$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{base}} = \frac{1200\text{ W}}{0.03\text{ m}^2} = 40{,}000\text{ W/m}^2$$

$$\frac{d^2T}{dx^2} = 0$$

$$-k\frac{dT(0)}{dx} = \dot{q}_0 = 40{,}000\text{ W/m}^2$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

$$-k\frac{dT(0)}{dx} = \dot q_0 \quad \rightarrow \quad -kC_1 = \dot q_0 \quad \rightarrow \quad C_1 = -\frac{\dot q_0}{k}$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty] \quad \rightarrow \quad -kC_1 = h[(C_1 L + C_2) - T_\infty]$$

$$C_2 = T_\infty + \frac{\dot q_0}{h} + \frac{\dot q_0}{k}L$$

$$T(x) = T_\infty + \dot q_0\left(\frac{L - x}{k} + \frac{1}{h}\right)$$

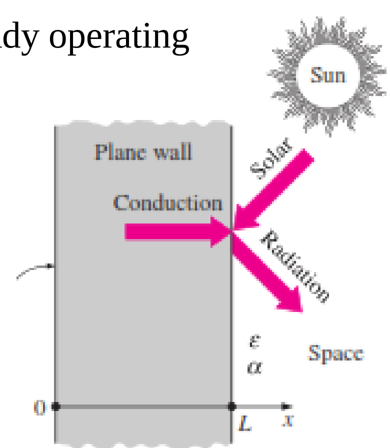
$$T(0) = T_\infty + \dot q_0\left(\frac{L}{k} + \frac{1}{h}\right)$$

$$= 20°C + (40{,}000\ W/m^2)\left(\frac{0.005\ m}{15\ W/m \cdot °C} + \frac{1}{80\ W/m^2 \cdot °C}\right) = 533°C$$

and

$$T(L) = T_\infty + \dot q_0\left(0 + \frac{1}{h}\right) = 20°C + \frac{40{,}000\ W/m^2}{80\ W/m^2 \cdot °C} = 520°C$$

<u>Example</u> : Consider a large plan wall of thickness L=0.06 m and thermal conductivity k=1.2 W/m °C in space . The wall is covered with white porcelain tiles that have an emissivity of $\varepsilon = 0.85$ and a solar absorptivity of $\alpha = 0.26$. The inner surface of the wall is maintained at T_1=300 K at all times , while the outer surface is exposed to solar radiation that is incident at a rate of $q_{solar} = 800\ w/m^2$. The outer surface is also losing heat by radiation to deep space at 0 K . Determine the temperature of the outer surface of the wall and the rate of heat transfer through the wall when steady operating



conditions are reached . What would you response be if no solar radiation was incident on the surface .

$$\frac{d^2T}{dx^2} = 0$$

with boundary conditions

$$T(0) = T_1 = 300 \text{ K}$$

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma[T(L)^4 - T_{space}^4] - \alpha\dot{q}_{solar}$$

where $T_{space} = 0$. The general solution of the differential equation is again obtained by two successive integrations to be

$$T(x) = C_1x + C_2 \qquad \textbf{(a)}$$

where C_1 and C_2 are arbitrary constants. Applying the first boundary condition yields

$$T(0) = C_1 \times 0 + C_2 \quad \rightarrow \quad C_2 = T_1$$

Noting that $dT/dx = C_1$ and $T(L) = C_1L + C_2 = C_1L + T_1$, the application of the second boundary conditions gives

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma T(L)^4 - \alpha\dot{q}_{solar} \quad \rightarrow \quad -kC_1 = \varepsilon\sigma(C_1L + T_1)^4 - \alpha\dot{q}_{solar}$$

$$-k\frac{dT(L)}{dx} = \varepsilon\sigma T(L)^4 - \alpha\dot{q}_{solar} \quad \rightarrow \quad -kC_1 = \varepsilon\sigma T_L^4 - \alpha\dot{q}_{solar}$$

$$T(x) = \frac{\alpha\dot{q}_{solar} - \varepsilon\sigma T_L^4}{k}x + T_1$$

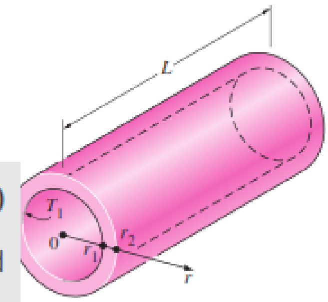
$$T_L = \mathbf{292.7\ K}$$

$$\dot{q} = k\frac{T_0 - T_L}{L} = (1.2 \text{ W/m}\cdot\text{K})\frac{(300 - 292.7)\text{ K}}{0.06 \text{ m}} = \mathbf{146\ W/m^2}$$

<u>Example</u> : Consider a steam pipe of length L =20 m , inner radius r_1=6 cm , outer radius r_2= 8 cm , and thermal conductivity k=20 W/m ^{o}C . The inner and outer surfaces of the pipe are maintained at average temperature of T_1 = 150 ^{o}C and T_2 = 60 ^{o}C respectively . obtain a general relation for the temperature distribution inside the pipe under steady conditions , and determine the rate of heat loss from the steam through the pipe .

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

with boundary conditions



$$T(r) = C_1 \ln r + C_2 \qquad \textbf{(a)}$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in Eq. (*a*) with the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad C_1 \ln r_1 + C_2 = T_1$$
$$T(r_2) = T_2 \quad \rightarrow \quad C_1 \ln r_2 + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = \frac{T_2 - T_1}{\ln(r_2/r_1)} \quad \text{and} \quad C_2 = T_1 - \frac{T_2 - T_1}{\ln(r_2/r_1)} \ln r_1$$

Substituting them into Eq. (*a*) and rearranging, the variation of temperature within the pipe is determined to be

$$T(r) = \left(\frac{\ln(r/r_1)}{\ln(r_2/r_1)}\right)(T_2 - T_1) + T_1 \qquad \textbf{(2-58)}$$

$$\dot{Q}_{\text{cylinder}} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi kLC_1 = 2\pi kL\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

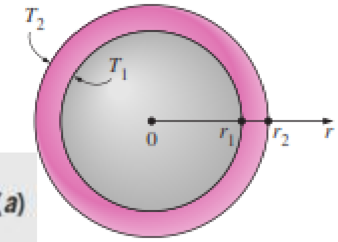
$$\dot{Q} = 2\pi(20\ \text{W/m}\cdot{}^\circ\text{C})(20\ \text{m})\frac{(150 - 60)^\circ\text{C}}{\ln(0.08/0.06)} = \mathbf{786\ kW}$$

<u>Example</u> :Consider spherical container of inner radius r_1=8 cm , outer radius r_2=10 cm and the thermal conductivity k=45 W/m ^{o}C . The inner and outer surfaces of the container are maintained temperature of T_1=200^{o}C and T_2 =80 ^{o}C ,respectively as a result of some chemical reactions occurring inside . Obtain a general relation for the temperature distribution inside the shell under steady conditions and determine the rate of heat loss from the container .

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

with boundary conditions

$$T(r_1) = T_1 = 200^\circ\text{C}$$



$$T(r) = -\frac{C_1}{r} + C_2 \qquad \textbf{(a)}$$

We now apply both boundary conditions by replacing all occurrences of r and $T(r)$ in the relation above by the specified values at the boundaries. We get

$$T(r_1) = T_1 \quad \rightarrow \quad -\frac{C_1}{r_1} + C_2 = T_1$$

$$T(r_2) = T_2 \quad \rightarrow \quad -\frac{C_1}{r_2} + C_2 = T_2$$

which are two equations in two unknowns, C_1 and C_2. Solving them simultaneously gives

$$C_1 = -\frac{r_1 r_2}{r_2 - r_1}(T_1 - T_2) \qquad \text{and} \qquad C_2 = \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

Substituting into Eq. (a), the variation of temperature within the spherical shell is determined to be

$$T(r) = \frac{r_1 r_2}{r(r_2 - r_1)}(T_1 - T_2) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1} \qquad \textbf{(2-60)}$$

The rate of heat loss from the container is simply the total rate of heat conduction through the container wall and is determined from Fourier's law

$$\dot{Q}_{\text{sphere}} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2} = -4\pi k C_1 = 4\pi k r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} \qquad \textbf{(2-61)}$$

The numerical value of the rate of heat conduction through the wall is determined by substituting the given values to be

$$\dot{Q} = 4\pi(45\ \text{W/m}\cdot{}^\circ\text{C})(0.08\ \text{m})(0.10\ \text{m})\frac{(200 - 80)^\circ\text{C}}{(0.10 - 0.08)\ \text{m}} = \mathbf{27{,}140\ W}$$