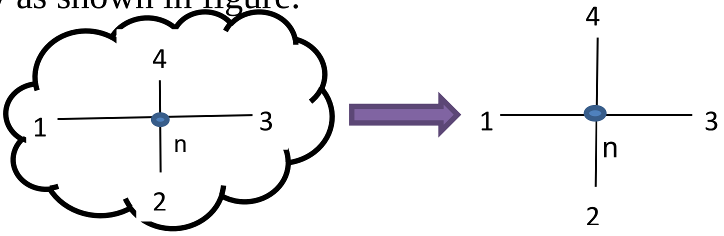
Numerical analysis for the steady state conduction two dimensions

Consider a two dimensions body as shown in figure:

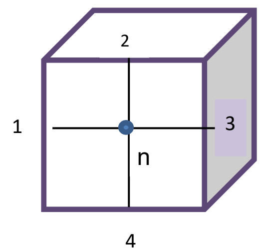


For internal point

Node point n , $\Delta x = \Delta y$

When q is taken as a positive for

Heat flow in to in

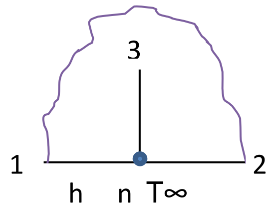


$$KL\,\Delta y\,\frac{T1-Tn}{\Delta x} + KL\,\Delta x\,\frac{T2-Tn}{\Delta y} + KL\,\Delta y\,\frac{T3-Tn}{\Delta x} + KL\Delta\,x\,\frac{T4-Tn}{\Delta y} = 0$$

$\Delta x = \Delta y$ and divide equation to KL gives

$$T_1+T_2+T_3+T_4 - 4Tn = 0$$

For external point:



Consider the body (node n)subjected convection heat transfer

$$KL\,\frac{\Delta y}{2}\,\frac{T1-Tn}{\Delta x} + KL\,\frac{\Delta y}{2}\,\frac{T2-Tn}{\Delta x} + KL\,\Delta x\,\frac{T3-Tn}{\Delta y} + hL\Delta x(T\infty - Tn) = 0$$

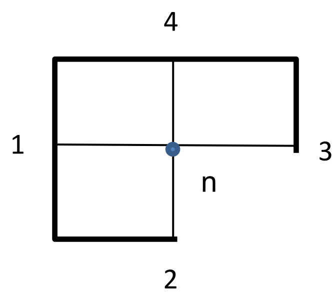
$\Delta x = \Delta y$ and divide to KL

$$\frac{1}{2}(T_1-Tn)+\frac{1}{2}(T_2-Tn)+(T_3-Tn)+\left(h\frac{\Delta x}{k}T\infty - h\frac{\Delta x}{k}Tn\right)=0$$

$$\frac{1}{2}(T_1+T_2+2T_3)+ h\frac{\Delta x}{k}T\infty - \left(h\frac{\Delta x}{k}+2\right)Tn=0$$

For enternal corner:

$$KL\,\Delta y\,\frac{T1-Tn}{\Delta x} + KL\,\frac{\Delta x}{2}\,\frac{T2-Tn}{\Delta y} + KL\,\frac{\Delta y}{2}\,\frac{T3-Tn}{\Delta x} +$$
$$KL\,\Delta x\,\frac{T4-Tn}{\Delta y} + hL\Delta x(T\infty - Tn) = 0$$



$\Delta x = \Delta y$ and divide to KL

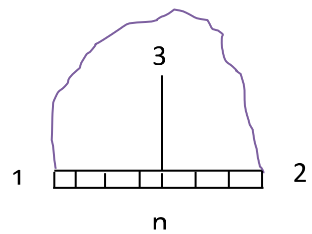
$$\frac{1}{2}(T_2+T_3)+T_1+T_4+ h\frac{\Delta x}{k}T\infty - \left(h\frac{\Delta x}{k}+3\right)Tn=0$$

<u>For insulated:</u>

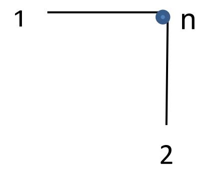
$$KL\,\frac{\Delta y}{2}\,\frac{T1-Tn}{\Delta x}+KL\,\frac{\Delta y}{2}\,\frac{T2-Tn}{\Delta x}+KL\;\Delta x\,\frac{T3-Tn}{\Delta y}=0$$

$\Delta x = \Delta y \quad , \div KL$

$\frac{1}{2}(T_1+T_2)+T_3-2Tn=0$

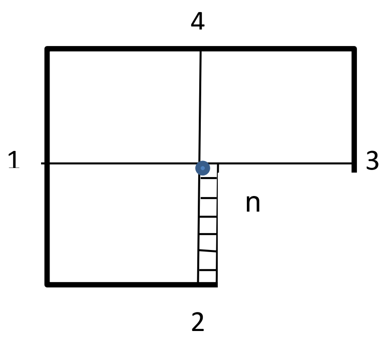


<u>For internal corner:</u>



Answer: $\frac{1}{2}(T_1+T_2)+h\frac{\Delta x}{k}T\infty-(h\frac{\Delta x}{k}+1)Tn=0$

<u>H.W.</u>



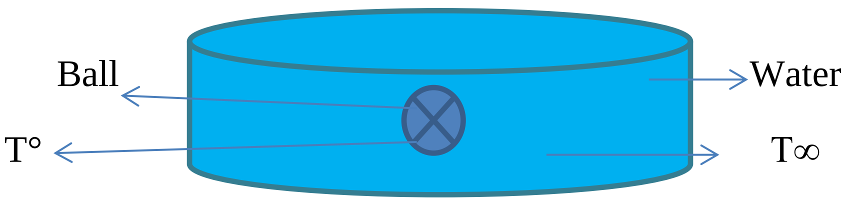
Unsteady state heat transfer

Its large number of heating and cooling process with time that may be applied in the industrial applications.

Lumped heat capacity system

For lumped heat capacity system the temperature is uniformly distributed throughout the solid body and the surface convection resistance.

If a hot or cool still ball is immersed in a cool or hot water the following of heat balance may be applied.



(Heat out of objects during time dt) = (decrease of internal thermal energy of objects during dt)

$$h\,A\,(T-T_\infty) = -\,\rho\,Cp\,v\,\frac{dT}{dt} \quad \text{--------------------} \; 1$$

when:

h= Heat transfer coefficient w/ m^2.°C

A= Surface area of the ball m^2

T= Temperature of the ball °C

T° = Initial temperature °C

T_∞ = Environmental temperature °C

ρ= Density of the ball kg/m^3

Cp = Specific heat J/kg.°C

t = Time sec

Divide equation 1 on the ρ Cp v $\longrightarrow$

$$\frac{dT}{dt} = -\frac{h\,A}{\rho\,Cp\,v}\,(T-T\infty)$$

Re arrangement

$$\frac{dT}{(T-T\infty)} = -\frac{h\,A}{\rho\,Cp\,v}\,dt$$

The initial condition is T = T° at t =0

$$\int_{T^\circ}^{T}\frac{dT}{(T-T\infty)} = -\frac{h\,A}{\rho\,Cp\,v}\int_0^t dt \quad \text{gives}$$

$$\frac{(T-T\infty)}{(T^\circ-T\infty)} = \exp\ -\left(\frac{h\,A}{\rho\,Cp\,v}\right)t \quad \text{-------------------2}$$

Equation 2 can be applied , the following condition must be tested:

Bi. no. < 0.1

$$\text{Bi. no.} = \frac{h(\frac{v}{A})}{K}$$

When Bi is the Biot number .

Note : the words (suddenly, lumped, immersed ,time ..etc)refers to the unsteady state heat transfer.

Example:

Stainless steel rod 6.4 mm in diameter is initially at 25 °C and suddenly immersed in the liquid T= 150 °C with heat transfer coefficient h= 120 W/m^2 °C . Calculate the time for the rod temperature to reach 120 °C if ρ= 7817 kg/m^3 , Cp = 460 J/kg. °C , k=19 w/m· °C

Solution

$$\text{Bi. no.} = \frac{h(\frac{v}{A})}{K}$$

$$\frac{v}{A} = \left(\frac{\left(\frac{\pi}{4}\right)\, d*d*L}{\pi*d*L}\right) = \frac{d}{4} = \frac{0.0064}{4} = 0.0016 \text{ m}$$

$$Bi = \frac{120*0.0016}{19} = 0.01 < 0.1 \longrightarrow \text{Lumped heat}$$

$$\frac{A}{v} = \frac{4}{d} = 625 \ m^{-1}$$

$$\frac{(T-T\infty)}{(T°-T\infty)} = \exp -\left(\frac{h\,A}{\rho\,Cp\,v}\right) t$$

$$\frac{(120-150)}{(25-150)} = \exp -\left(\frac{120*625}{7817*460}\right) t$$

$$\frac{-30}{-125} = \exp -0.02086\,t \Longrightarrow 0.24 = \exp -0.02086\,t$$

$$\ln 0.24 = 0.02086\,t \Longrightarrow t = 68.4 \text{ sec}$$

Example:

A still ball (Cp = 0.46 kj/kg . °C , K = 35 W /m. °C) 5 cm in diameter and initially at a uniform a temperature of 450 °C is suddenly placed in a controlled environment convection heat transfer 10 w/m^2 .°C. Calculate the time required for the ball to attain a temperature of 150 °C. ρ= 7817 kg/m^3 . The environment temperature is 100 °C.

Solution

Check Bi. No.

$$Bi.\ no. = \frac{h(\frac{V}{A})}{K}$$

$$Bi.\ no. = \frac{10*(\frac{4}{3})\,\pi\,(0.025*0.025*0.025)}{4\,\pi\,(0.025*0.025)*35} = 0.0023 < 0.1$$

$$\Longrightarrow \text{Lumped heat}$$

$$\frac{h\,A}{\rho\,Cp\,v} = \frac{10*4\,\pi\,(0.025*0.025)}{7800*460*\left(\frac{4}{3}\right)\pi*(0.025*0.025*0.025)} = 3.344\times 10^{-4}$$

$$\frac{(T-T\infty)}{(T°-T\infty)} = \exp -\left(\frac{h\,A}{\rho\,Cp\,v}\right) t$$

$$\frac{(150-100)}{(450-100)} = \exp -(3.344\times 10^{-4})\,t$$

$\frac{50}{350}$ = exp - (3.344× 10^{-4}) t

t = 5819 sec

H.w:

Aluminum plate having a thickness of 4cm and initial temperature is 200 °C. The plate is subjected to a convection with h= 500 w/m^2.°C and T= 25 °C. Using lumped heat method , calculate the temperature at time of 24.2 sec.

Take the density = 2707 kg/m^3 , Cp =896 j/kg. °C , k= 204 w/m. °C