Fin efficiency

$$\eta = \frac{q\ actual}{q\ max}$$

Applied this equation on case 1

$$\eta = \frac{(hpkA)^{0.5}\,(T^{\circ}-T\infty)}{hpL\,(T^{\circ}-T\infty)} = \frac{1}{m\,L}$$

For case 2

$$\eta = \frac{(hpkA)^{0.5}(T^{\circ}-T\infty)\tanh(m\,L)}{hpL\,(T^{\circ}-T\infty)} = \frac{\tanh(m\,L)}{m\,L}$$

The value of η may be found from figure 1 or figure 2

Figure 1 Efficiencies of straight rectangular and triangular fins.

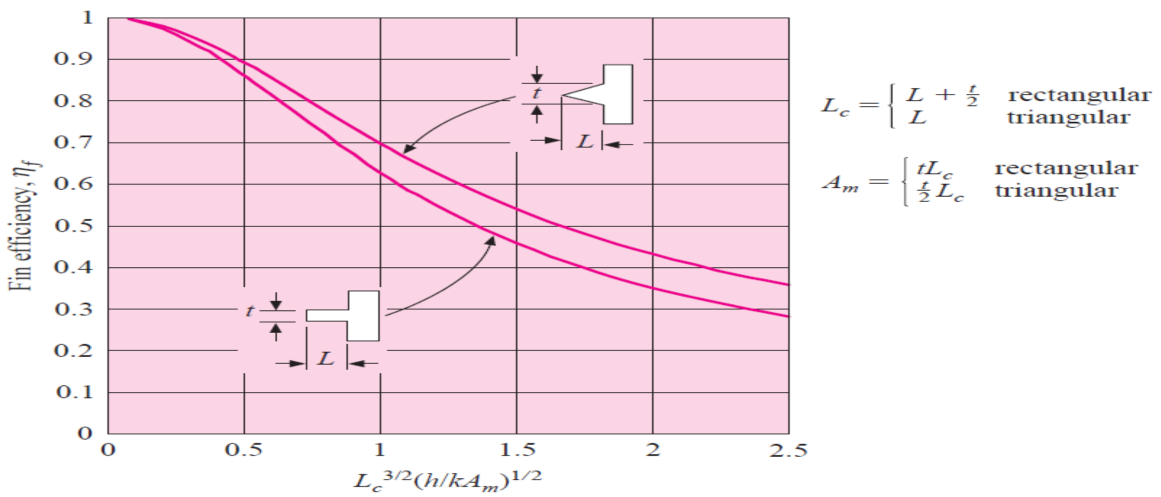
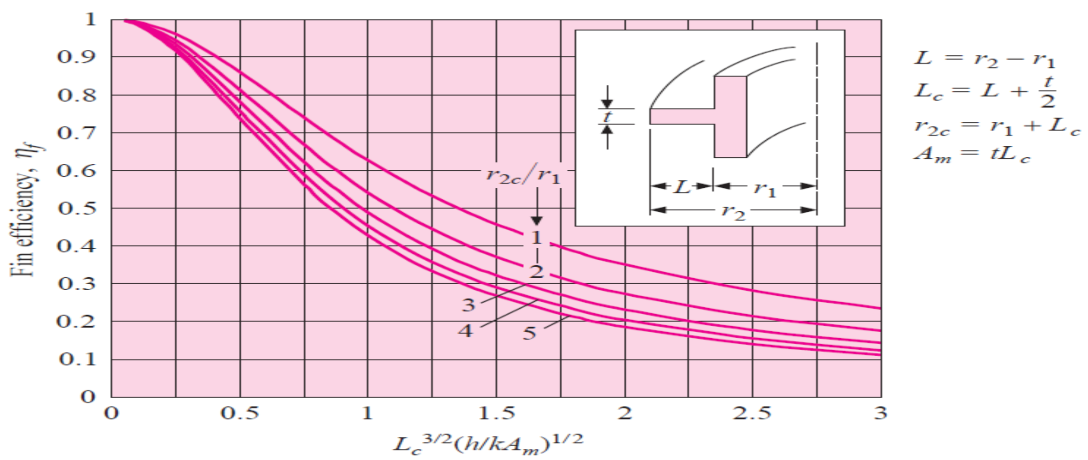


Figure 2 Efficiencies of circumferential fins of rectangular profile.



q max for circumferential fins = $2\pi\, h(r_{2c}{}^2 - r_1{}^2)\,(T° - T\infty)$

q max for rectangular fins= hp L θ°

Fin performance

In some cases a valid method of evaluating fin performance is to compare the heat transfer with the fin to that which would be obtained without the fin. The ratio of these quantities

$$\frac{\text{q with fin}}{\text{q without fin}} = \frac{\eta f \;\; Af \;\; h \;\; \theta°}{\text{h Ab } \theta°}$$

where *Af* is the total surface area of the fin and *Ab* is the base area.

Af = Area of fin Ab = Area of base

The heat ratio become in case 2

$$\frac{\text{q with fin}}{\text{q without fin}} = \frac{\tanh ml}{\left(\frac{\text{hA}}{\text{kp}}\right)^{0.5}}$$

The ratio of heat called fin effectivness

Example:

Aluminum fins 1.5 cm wide and 1.0 mm thick are placed on a 2.5cm diameter tube to dissipate the heat.The tube surface temperature is170◦ C, and the ambient-fluid temperature is 25◦C. Calculate the heat loss per fin for h = 130 W/m^2 · ◦ C. Assume k = 200 W/m · ◦ C for aluminum.

Solution

Lc = L + t/2 = 1.5 + 0.05 = 1.55 cm
r1 = 2.5/2 = 1.25 cm
r2c = r1 + Lc = 1.25 + 1.55 = 2.80 cm
*r*2*c* /*r*1 = 2.80/1.25 = 2.24
Am = t*Lc = (0.001)(1.55)(10^{-2}) = 1.55 × 10^{-5} m^2

$$Lc^{3/2}\left(\frac{h}{k\,Am}\right)^{1/2} = (0.0155)^{3/2}\left(\frac{130}{200 * 1.55\times 10-5}\right)^{1/2} = 0.369$$

From Figure 2, ηf = 82 percent

q max = $2\pi(r_{2c}{}^2 - r_1{}^2)(h)\,(T° - T\infty)$

$= 2\pi(2.8^2 - 1.25^2)(10^{-4})(130)(170 - 25) = 74.35$ W

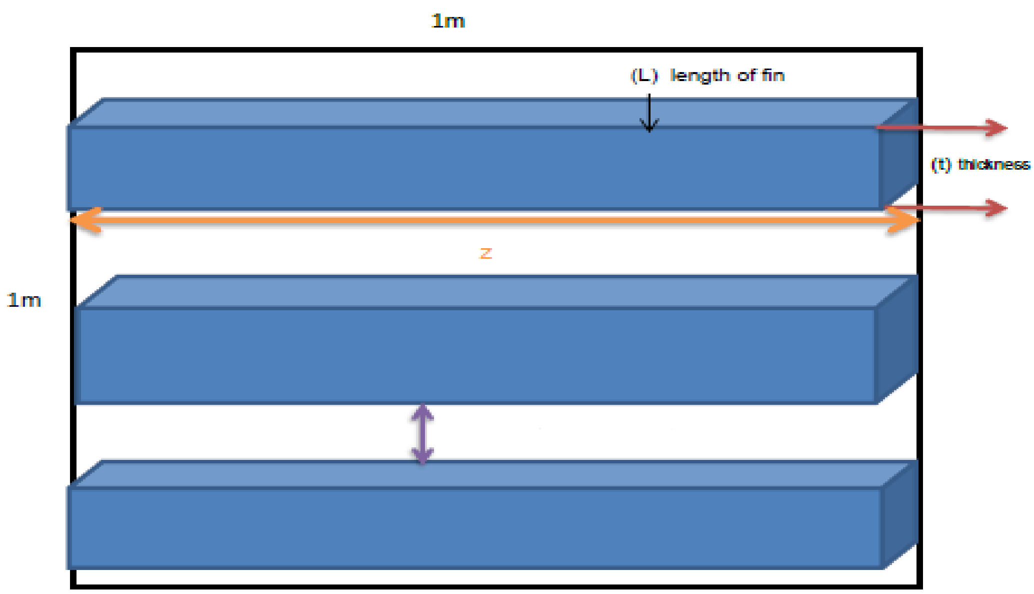
$\eta = \frac{q\ actual}{q\ max}$ qactual = (0.82)*(74.35) = 60.97 W

Example:

Calculate the heat loss from 250 rectangular thin fins of uniform cross section of thickness 0.5 mm and height 50 mm on (1*1)m surface ,separated by a distance of 4 mm .The heat transfer coefficient is 30 w/m^2 °C and the thermal conductivity is 200 W/m °C.

Solution:

q with fins = no. of fin * q fin + q between fins



q fin = k A m θ° tanh (mLc)

Lc = L + t/2 = 5+ 0.025 = 5.025 cm

$m = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2*30}{200*0.5*10^{-3}}} = 24.5$

m Lc = 24.5 *5.025 *10^{-2} = 1.23

q fin = 200 *24.5 *(1*0.5 *10^{-3}) * θ°* tanh(1.23)= 2.06 θ°

q between fin = h $A_{\text{between fins}}$ θ°

$A_{\text{between fins}}$ = 1(1-(0.5 *10 $^{-3}$ *1 *250))= 0.875m^2

q between fins = 30 * 0.875* θ°= 26 .25 θ°

q with fins=250 * 2.06 θ°+26.25 θ° = 541.25 θ°

*Calculate the percentage of increase of heat transfer rate due to addition of number of rectangular fins ,if the heat transfer coefficient is 40w/m^2 °C before addition the fins

q with fins = 541.25 θ°

q without fins = h Ab θ° = 40 *(1*1)* θ° = 40 θ°

$$\frac{\text{q with fins}}{\text{q without fins}} = \frac{541.25\ \theta°}{40\ \theta°} = 13.53$$