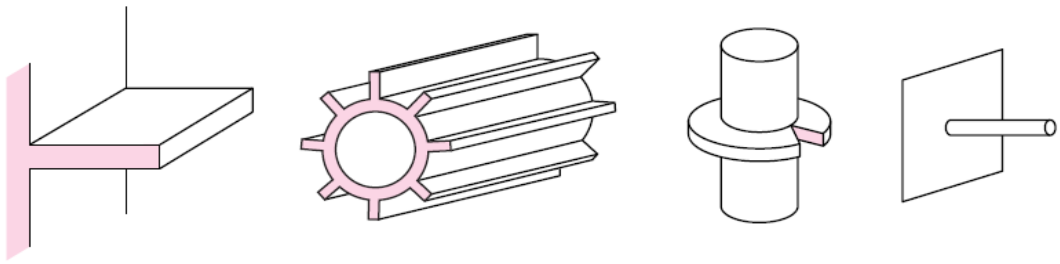
Fins

Extended bodies or fins are used to increase the rate of heat transfer by increasing the effective surface area or convective heat transfer. Fins are used in a large number of applications such as heat exchanger, cooling reactor core, electrical transformer and motors, automobile applications, rectifier, etc

Various types of simple-shaped fins, namely, rectangular, square, annular, cylindrical and tapered, have been used with different geometrical combinations.

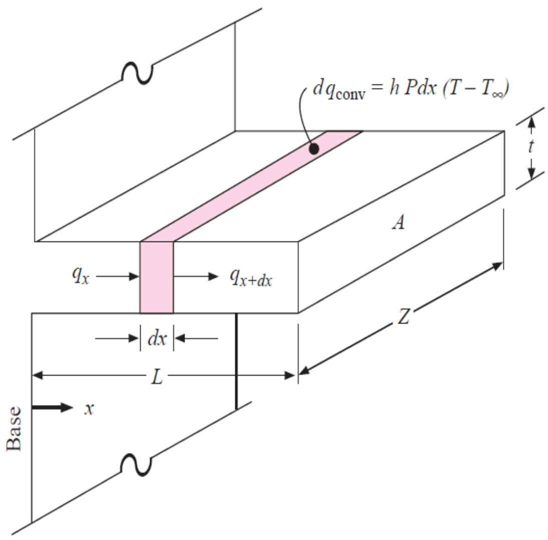
Different types of finned surfaces. (*a*) Straight fin of rectangular profile on plane wall, (*b*) straight fin of rectangular profile on circular tube, (*c*) cylindrical tube with radial fin of rectangular profile, (*d*) cylindrical-spine or circular-rod fin.



For rectangular fin having a temperature T° at the base surrounded by a fluid temperature T∞
Energy in left face = energy out right face + energy lost by convection.

Assuming steady state one dimensional without heat generation

The 3 energies terms are respectively:



$$\text{Energy in left face} = q_x = -kA\frac{dT}{dx}$$

$$\text{Energy out right face} = q_{x+dx} = -kA\left.\frac{dT}{dx}\right]_{x+dx}$$

$$= -kA\left(\frac{dT}{dx} + \frac{d^2T}{dx^2}\,dx\right)$$

Energy by convection = h A (T-T∞) = h p dx (T-T∞)

P is the perimeter = 2 (z + t) , A = z * t

When we combine the quantities, the energy balance yields

Note: for cylinder

A = = $\frac{\pi}{4}$ d²

P = π d

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0 \quad \text{------------------------1}$$

Let θ = (T-T∞) , m = $\sqrt{hp/kA}$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \quad \text{-----------------2}$$

which has a general solution

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \quad \text{---------3}$$

Take 2 boundary condition

T= T° at x = 0

Let θ° = (T°-T∞) Applied B.C.1 on equation 3

T°-T∞= $C_1 e^{-mx} + C_2 e^{mx}$ ⟶

T°-T∞= $C_1 + C_2$

θ°= $C_1 + C_2$ --------------4

The other boundary condition depends on the physical situation. Several cases may be considered:

CASE 1 The fin is very long, and the temperature at the end of the fin is essentially that of the surrounding fluid.

CASE 2 The end of the fin is insulated so that *dT/dx* = 0 at *x* = *L*.

CASE 3 The fin is of finite length and loses heat by convection from its end.

Case	Second B .C	$\frac{\theta}{\theta°}$
1	θ = 0 at x = ∞	e^{-mx}
2	$\frac{d\theta}{dx}$ = 0 at $x = L$	$\frac{\cosh[m(L-x)]}{\cosh(mL)}$
3	-kA$\frac{d\theta}{dx}$ = h p dx θ	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$

Case 1   q= kA m θ° = $\sqrt{kAhp}$ (T° - T∞)

Case 2   q= kA m θ° tanh (mL)

Case 3 q= kA m θ° * $\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$

Notes:

1- For thin fin ,   z >> t ,   p = 2 z

So m = $\sqrt{2h/kt}$

2- For cylindrical rod or pin

P = π d ⟶ m = $\sqrt{2h/kr}$

3- The solution in case 3 may be expressed same from of case 2 when the length is extended by one-half the thickness of the fin. In effect, lengthening of the fin by *t*/2 is assumed to represent the same convection heat transfer as half the fin tip area placed on top and bottom of the fin. A corrected length *Lc* is then used in all the equations that apply for the case of the fin with an insulated tip

For rectangular fin   Lc = L + $\frac{t}{2}$

If a straight cylindrical rod extends from a wall, the corrected fin length is calculated

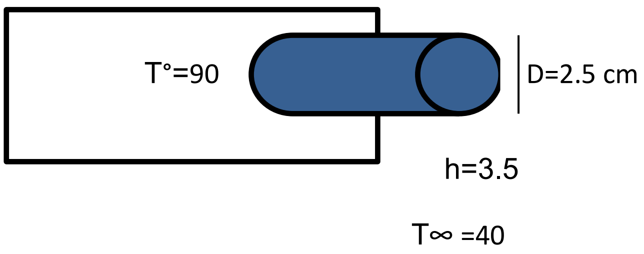
$$L_c = L + \frac{\pi d^2/4}{\pi d} = L + d/4$$

⟶ Lc = L + $\frac{r}{2}$

Example:

A very long copper rod (k= 372 w/m °C) 2.5 cm in diameter has one end at 90 °C ..The rod is exposed to a fluid whose temperature is 40 °C .The heat transfer coefficient is 3.5 w/m^2 °C. Calculate the heat loss from the rod**.**

Solution



$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.025)^2 = 4.91 * 10^{-4}\ m^2$

$P = \pi d = 3.41 * 0.025 = 0.0785$ m

$q = kA\ m\ \theta° = \sqrt{kAhp}\ (T° - T\infty) =$

$q = \sqrt{3.5 * 0.0785 * 372 * 4.91 * 10^{-4}} * (90-40) = 11.2$ W

Example:

An aluminum rectangular thin fin [k = 200 W/m · ° C] [A= 3*10^{-3} m^2]3.0 mm thick and 7.5 cm long. The base is maintained at 300° C, and the ambient temperature is 50° C with h = 10 W/m^2 · ° C. Calculate the heat loss from the fin.

Solution:

Case 3 ⟶ Lc ⟶ Case 2

$Lc = L + \frac{t}{2} = 7.5 + 0.15 = 7.65$ cm

$m = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2*10}{200*0.003}} = 5.774$

$q = kA\ m\ \theta°\ \tanh(mLc)$

$q = (5.774)(200)(3 \times 10^{-3})(300 - 50) \tanh[(5.774)(0.0765)]$

= 359 W

Example :

An aluminum cylindrical rod fin (k=132 Btu/h.ft.°F)having a diameter of 0.375 in and a length of 4 in is attached to a surface having T = 200°F .

The rod is exposed to ambient air 70.°F and heat transfer coefficient is 1.5 Btu / h.ft^2.°F. Determine temperature distribution and heat flux if

a- Neglecting the heat transfer at the end.

b- Accounting the heat transfer at the end.

Solution:

a- Case 2

$$m = \sqrt{\frac{2h}{kr}} = \sqrt{\frac{2*1.5}{132*(\frac{0.1875}{12})}} = 1.206$$

$$\frac{\theta}{\theta^\circ} = \frac{T-T\infty}{T^\circ-T\infty} = \frac{\cosh[m(L-x)]}{\cosh m L}$$

$$\frac{T-70}{200-70} = \frac{\cosh[1.206(\frac{4}{12}-x)]}{\cosh(1.206*\frac{4}{12})}$$

$$T = 70 + 130\,\frac{\cosh[1.206(\frac{4}{12}-x)]}{1.0819}$$

q= kA m θ° tanh (mL)

$= 132 * \frac{\pi}{4} (\frac{0.375}{12})^2 * 1.206 * (200-70) \tanh (1.206*(\frac{4}{12}))$

= 6.055 W

b- H.W