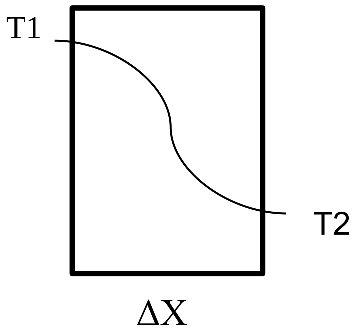
Steady state conduction one diemension

1- Plane wall



From Fourier equation $q = -KA\frac{dT}{dx}$ →

q dx =- k A dT $\qquad q = -KA\frac{(T2-T1)}{(X2-X1)}$ →

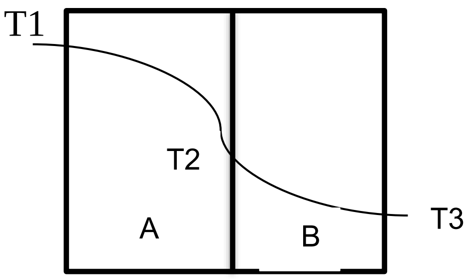
$q = -KA\frac{\Delta T}{\Delta X}$ ----------1

H.W. If k =k°(1+Bt) find q

2- Composite wall (multi layers)

Equation 1 may be rearranged as $q = \frac{(T1-T2)}{\frac{\Delta X}{KA}}$ -----------2

In the figure



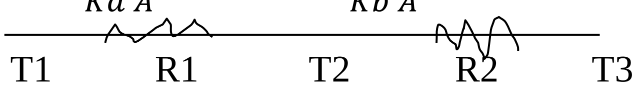
$$q = \frac{(T1-T2)}{\frac{\Delta Xa}{Ka\,A}} = \frac{(T2-T3)}{\frac{\Delta Xb}{Kb\,A}} \longrightarrow$$

$$q = \frac{(Tin-Tout)}{\frac{\Delta Xa}{Ka\,A}+\frac{\Delta Xb}{Kb\,A}} \quad \text{-----------3}$$

$$\text{Heat flow} = \frac{\text{thermal potential differince}}{\text{thermal resistance}}$$

A relation quite like Ohms law in electric –circuit theory, in equation 2 the resistance is Δx/kA, and in equation 3 it is the sum of two terms

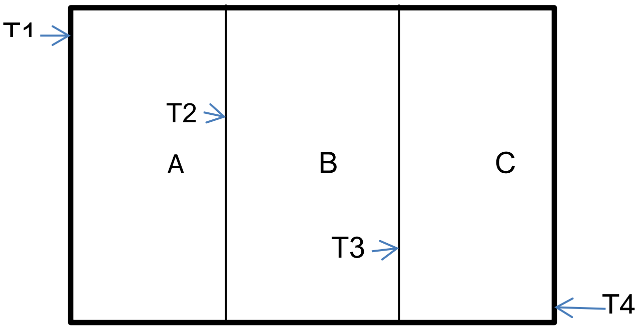
$R1 = \frac{\Delta Xa}{Ka\,A}$, $R2 = \frac{\Delta Xb}{Kb\,A}$, $R_{total} = R1+R2$, $q = \frac{(Tin-Tout)}{R1+R2}$



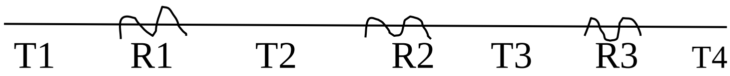
For 3 layers

$$q = \frac{(T1-T2)}{\frac{\Delta Xa}{Ka\,A}} = \frac{(T2-T3)}{\frac{\Delta Xb}{Kb\,A}} = \frac{(T3-T4)}{\frac{\Delta Xc}{Kc\,A}}$$

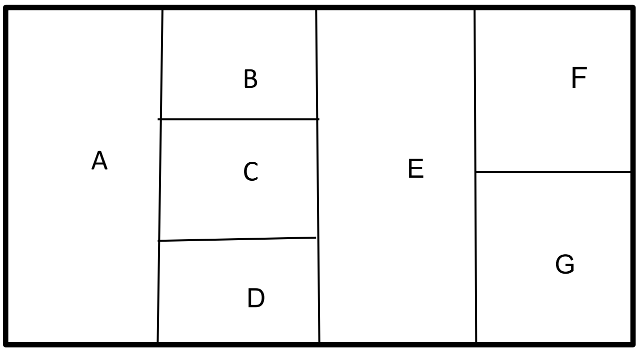
$$q = \frac{(Tin-Tout)}{\frac{\Delta Xa}{Ka\,A}+\frac{\Delta Xb}{Kb\,A}+\frac{\Delta Xc}{Kc\,A}}$$



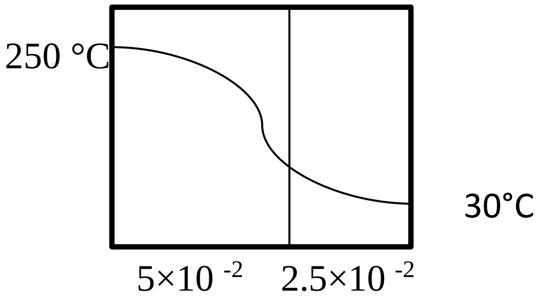
$$q = \frac{(Tin - Tout)}{R1 + R2 + R3}$$



H.W. Draw the electric analog for the figure



Example: One side of copper block 5cm thick (k = 386 W/m.°C) is maintained at 250 °C.The other side is covered with layer of Fiberglas 2.5 cm thick(k=0.038 W/m.°C)at 35 °C.The total heat through the copper – fiberglass is 52 kw .What is the area of slab.



Solution:

$$\frac{q}{A} = = \frac{(Tin - Tout)}{\frac{\Delta Xa}{Ka} + \frac{\Delta Xb}{Kb}}$$

$$R1 = \frac{\Delta Xa}{Ka} = \frac{0.05}{386} = 1.295 \times 10^{-4}$$

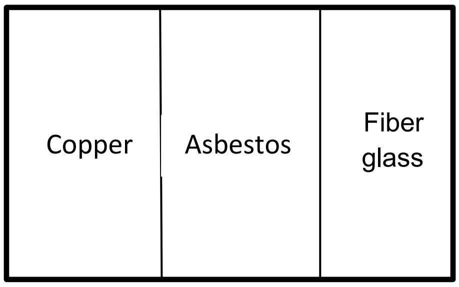
$$R2 = \frac{\Delta Xb}{Kb} = \frac{0.025}{0.038} = 0.65789$$

$$\frac{q}{A} = \frac{(Tin - Tout)}{R1 + R2} = \frac{(250 - 35)}{1.295 \times 10\ \hat{}-4 + 0.65789} \longrightarrow$$

$$\frac{52000}{A} = \frac{215}{0.6580} \longrightarrow A = 159.144\ m^2$$

Example: A composite wall is formed of 2.5 cm copper (k = 386 W/m.°C), 3.2 mm layer of asbestos (k = 0.16 W/m.°C)and 5 cm layer of fiberglass (k=0.038 W/m.°C). The wall is subjected to an overall temperature difference of 560 °C. Calculate the heat flow per unit area through the composite structure.

$$\frac{q}{A} = \frac{(Tin - Tout)}{\frac{\Delta Xa}{Ka} + \frac{\Delta Xb}{Kb} + \frac{\Delta Xc}{KA}}$$



$$\frac{q}{A} = \frac{(Tin - Tout)}{R1 + R2 + R3}$$

$$R1 = \frac{\Delta Xa}{Ka} = \frac{2.5 \times 10^{-2}}{386} = 6.476 \times 10^{-5}$$

$$R2 = \frac{\Delta Xb}{Kb} = \frac{3.2 \times 10^{-3}}{0.16} = 0.02$$

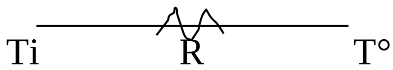
$$R3 = \frac{\Delta Xc}{Kc} = \frac{5 \times 10^{-2}}{0.038} = 1.3158$$

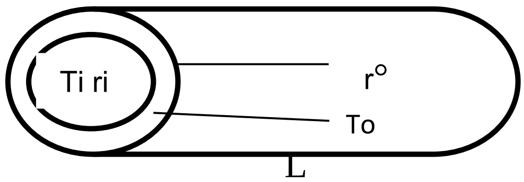
$$\frac{q}{A} = \frac{560}{6.476 \times 10^{-5} + 0.02 + 1.3158} = \frac{560}{1.3358} = 419.21 W/m^2$$

<u>**Radial system :**</u>

1- Cylinder:

Consider along cylinder of inside radius (ri), outside radius (r°) and length (L) as shown below:





Fourier law is used by inserting the proper area relation

$$q_r = - k A_r \frac{dT}{dr} \qquad , A_r = 2 \pi r L \longrightarrow$$

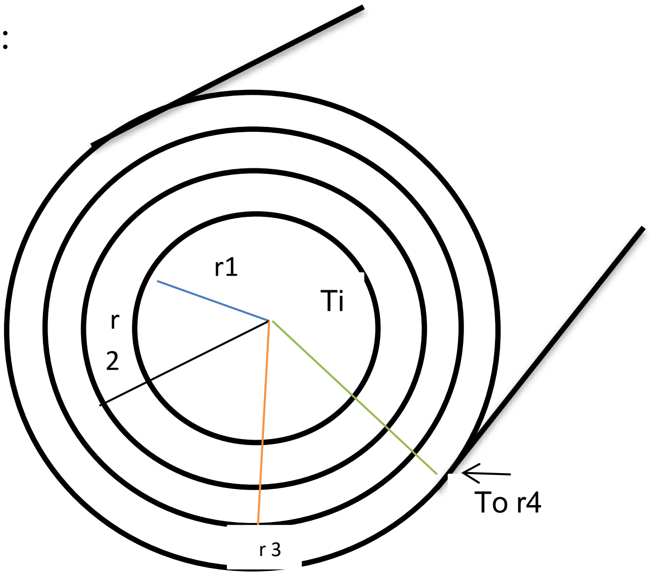
$$q_r = -2\pi k r L \frac{dT}{dr} \qquad q_r \frac{dr}{r} = -2\pi k L dt \longrightarrow$$

$$q_r = \frac{2\pi KL(Ti - To)}{\ln(\frac{ro}{ri})}$$

$$q_r = \frac{(Ti - To)}{\frac{\ln(\frac{ro}{ri})}{2\pi K L}} \quad , \quad qr = \frac{(Ti - To)}{R1} \quad , \quad R1 = \frac{\ln(\frac{ro}{ri})}{2\pi K L}$$

The thermal resistance concept may be used for multiple layer cylindrical walls just as it was used for plane wall.

For three layers system as shown in figure:



T1 R1 T2 R2 T3 R3 T4

$$R1 = \frac{\ln\left(\frac{r2}{r1}\right)}{2\pi\, K1\, L}$$

$$R2 = \frac{\ln\left(\frac{r3}{r2}\right)}{2\pi\, K2\, L}$$

$$R3 = \frac{\ln\left(\frac{r4}{r3}\right)}{2\pi\, K3\, L}$$

$$q_r = \frac{(Ti - To)}{\frac{\ln\left(\frac{r2}{r1}\right)}{2\pi\, K1\, L} + \frac{\ln\left(\frac{r3}{r2}\right)}{2\pi\, K2\, L} + \frac{\ln\left(\frac{r4}{r3}\right)}{2\pi\, K3\, L}} \qquad qr = \frac{(Ti - To)}{R1 + R2 + R3}$$

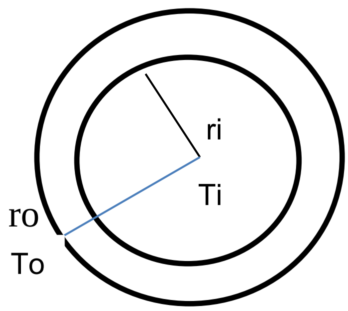
Or $$q_r = \frac{2\pi L (Ti - To)}{\frac{\ln\left(\frac{r2}{r1}\right)}{K1} + \frac{\ln\left(\frac{r3}{r2}\right)}{K2} + \frac{\ln\left(\frac{r4}{r3}\right)}{K3}}$$

2- Spheres:

Spherical system may also be treated as one dimensional when the temperature is a function of radius only .The heat flow is then:

$$q_r = -k A_r \frac{dT}{dr} \quad , A_r = 4\pi r^2$$

$$q_r = -k(4\pi r^2)\frac{dT}{dr} \quad , \quad q_r \frac{dr}{r^{\wedge}2} = -4\pi k\, dt$$



$$q_r = \frac{(Ti - To)}{\frac{\left(\frac{1}{r1}\right) - \left(\frac{1}{r2}\right)}{4\pi k}} \quad , \quad qr = \frac{(Ti - To)}{R1}$$

Or $$q_r = \frac{4\pi k (Ti - To)}{\left(\frac{1}{r1}\right) - \left(\frac{1}{r2}\right)}$$

For Two layers spheres

$$qr = \frac{4\pi(Ti-To)}{\frac{1}{k1}\{\left(\frac{1}{r1}\right)-\left(\frac{1}{r2}\right)\}+\frac{1}{k2}\{\left(\frac{1}{r2}\right)-\left(\frac{1}{r3}\right)\}}$$

Example: A hot pipe k= 40 W/m.°C having an inside surface temperature of 300 °C has an inside diameter 10 cm a wall thickness 6 mm is covered with 5 cm layer of insulation having k = 0.2 W/.m.°C followed by 3 cm layer of insulation having k= 0.4W/m.°C . The outside temperature of insulation is 40 °C. Calculate the heat lost per meter of length.

Solution

r1= 5 cm = 5× 10^{-2} m r2 = 5+0.6 = 5.6 cm = 5.6× 10^{-2} m

r3 =5+0.6+5= 10.6 cm = 10.6× 10^{-2} m

r4=5+0.6+5+3=13.6 cm =13.6× 10^{-2} m

$$\frac{q}{L} = \frac{(Ti-To)}{R1+R2+R3}$$

$$R1= \frac{\ln\left(\frac{r2}{r1}\right)}{2\pi\, K1} = = \frac{\ln\left(\frac{5.6\times 10^{-2}}{5\times 10^{-2}}\right)}{2\pi*40} = 4.5\times 10^{-4}$$

$$R2= \frac{\ln\left(\frac{r3}{r2}\right)}{2\pi\, K2} = = \frac{\ln\left(\frac{10.6\times 10^{-2}}{5.6\times 10^{-2}}\right)}{2\pi*0.2} = 0.508$$

$$R3= \frac{\ln\left(\frac{r4}{r3}\right)}{2\pi\, K3} = = \frac{\ln\left(\frac{13.6\times 10^{-2}}{10.6\times 10^{-2}}\right)}{2\pi*0.4} = 0.0992$$

$$\frac{q}{L} = \frac{300-40}{4.5\times 10^{-4}+0.508+0.0992} = 427.87 \text{ w/m}$$

Example: A hollow sphere is constructed of stainless steel k= 15 W/m.C with an inner diameter of 4 cm and outer diameter of 8 cm the inside and outer temperature is 100 and 50 C respectively .Calculate the heat rate

Solution

r1= 0.02 m r2= 0.04 m

$$q_r = \frac{4\pi k(Ti-To)}{\left(\frac{1}{r1}\right)-\left(\frac{1}{r2}\right)}$$

$q_r = \dfrac{4\pi * 15(100-50)}{\left(\frac{1}{0.02}\right) - \left(\frac{1}{0.04}\right)} = 376.8$ W

Example: 1.6 cm diameter of stainless steel pipe k= 15 w/m.°C of 0.2 cm thick is insulated with thick plastic cover k= 0.15 w/m.°C .What thickness of insulation must be added to ensure that 146w/m. The difference temperature is 80 °C.

Solution

R1= 0.8 cm

r2= 0.8+0.2 =1 cm

r3= 0.8 +0.2+x= 1+x cm

$\frac{q}{L} = \dfrac{(Ti-To)}{R1+R2} \longrightarrow 146 = \dfrac{80}{R\ total} \longrightarrow$ R total =0.548

R total = R1+R2= 0.548

$R1 = \dfrac{\ln\left(\frac{r2}{r1}\right)}{2\pi K1} = \dfrac{\ln\left(\frac{1}{0.8}\right)}{2\pi * 15} = 2.368\times10^{-3}$

R2 = R total – R1

$R2 = 0.548 - 2.368\times10^{-3} = 0.546$

$R2 = \dfrac{\ln\left(\frac{r3}{r2}\right)}{2\pi K2} \longrightarrow 0.546 = \dfrac{\ln\left(\frac{r3}{1}\right)}{2\pi * 0.15}$

ln r3 = 0.514

r3=1. 672 cm

r3 = 0.8 +0.2 +thickness

1. 672= 1+ thickness

Thickness = 0. 672 cm