University of Basra-College of Engineering
Petroleum Engineering Department

Subject: **Reservoir Simulation** Class: 4^{th} Year

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Syllabus

- solution of system of difference equations tridiagonal algorithms, use of irregular Gridding, transmissibility,
- the finite difference from of the flow equation in terms of transmissibility,
- Averaging of rock and fluid properties, solution of radial from of the flow equation, two dimensional flow,
- setting up the finite difference from, ordering schemes, standard row ordering, standard column ordering,
- resulting matrix structure,
- Introduction to multi-phase flow through porous media.

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Tridiagonal matrix algorithm

In numerical linear algebra, the **tridiagonal matrix algorithm**, also known as the **Thomas algorithm** (named after Llewellyn Thomas), is a simplified form of Gaussian elimination that can be used to solve tridiagonal systems of equations.

$$A = \begin{bmatrix} d_1 & u_1 & 0 & \cdots & 0 & 0 \\ l_2 & d_2 & u_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & d_{n-1} & u_{n-1} \\ 0 & 0 & \cdots & \cdots & l_n & d_n \end{bmatrix}$$

or

$$\begin{bmatrix} a_1 & c_1 & & \\ b_2 & a_2 & c_2 & \\ & b_3 & a_3 & c_3 \\ & & b_4 & a_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}$$

For Example

Let A is a Matrix ; X, B are Vectors and
A * X=B, then:

$$\begin{pmatrix} -2.25 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2.25 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2.25 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2.25 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2.25 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2.25 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2.25 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 100 \end{pmatrix}$$

This is a tri-diagonal Matrix.

Solution of The tri-diagonal Matrix

Example:-

$$2x_1 + x_2 = 1$$
$$2x_1 + 3x_2 + x_3 = 2$$
$$x_2 + 4x_3 + 2x_4 = 3$$
$$x_3 + 3x_4 = 4$$

For:

A X =B

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Since:

$$\begin{bmatrix} a_1 & c_1 & & \\ b_2 & a_2 & c_2 & \\ & b_3 & a_3 & c_3 \\ & & b_4 & a_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix}$$

Let;

$$\propto_k = a_k - c_{k-1}\,\beta_k$$

$$\beta_k = \frac{b_k}{\alpha_{k-1}}$$

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$a_1 = 2$	$a_2 = 3$	$a_3 = 4$	$a_4 = 3$
$b_2 = 2$	$b_3 = 1$	$b_4 = 1$	
$c_1 = 1$	$c_2 = 1$	$c_3 = 2$	
$d_1 = 1$	$d_2 = 2$	$d_3 = 3$	$d_4 = 4$

Then:

$$\propto_1 = a_1 = 2$$

$$\beta_2 = \frac{b_2}{\alpha_1} = \frac{2}{2} = 1$$

$$\propto_2 = a_2 - C_1\,\beta_2 = 3 - 1 * 1 = 2$$

$$\beta_3 = \frac{b_3}{\alpha_2} = \frac{1}{2}$$

$$\propto_3 = a_3 - C_2\,\beta_3 = 4 - 1 * \frac{1}{2} = \frac{7}{2}.$$

$$\beta_4 = \frac{b_4}{\alpha_3} = \frac{1}{3.5} = \frac{2}{7}$$

$$\propto_4 = a_4 - C_3\,\beta_4 = 3 - 2 * \frac{2}{7} = \frac{17}{7}.$$

$$\boxed{V_k = d_k - \beta_k\,V_{k-1}}$$

$$V_1 = d_1 - \beta_1\,V_{1-1} = d_1 = 1$$

$$V_2 = d_2 - \beta_2\,V_1 = 2 - 1 * 1 = 1$$

$$V_3 = d_3 - \beta_3\,V_2 = 3 - \frac{1}{2} * 1 = \frac{5}{2}$$

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$$V_4 = d_4 - \beta_4 V_3 = 4 - \frac{2}{7} * \frac{5}{2} = \frac{23}{7}$$

Now we will use the following formula to find the value of (x)

$$\boxed{x_k = \frac{V_k - c_k\, x_{k+1}}{\propto_k}}$$

$$\rightarrow x_4 = \frac{V_4 - \cancel{C_4\, x_{4+1}}}{\propto_4} = \frac{23}{1}$$

$$\rightarrow x_3 = \frac{V_3 - C_3\, x_4}{\propto_3} = -\frac{1}{7}$$

$$\rightarrow x_2 = \frac{V_2 - C_2\, x_3}{\propto_2} = \frac{9}{17}$$

$$\rightarrow x_1 = \frac{V_1 - C_1\, x_2}{\propto_1} = \frac{4}{17}$$

HW: By using Matlab, write files for the following;

1- How to create Tridiagonal matrix

2- The steps to solve your own example

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