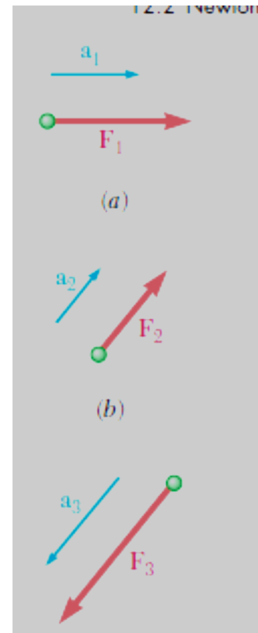
Kinetics of Particles "Newton's Second Law"

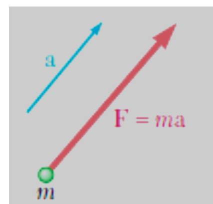
2-1 NEWTON'S SECOND LAW OF MOTION

If the resultant force acting on a particle is not zero, the particle will have acceleration proportional to the magnitude of the resultant and in the direction of this resultant force



$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \cdots = \text{constant}$$

$$\mathbf{F} = m\mathbf{a}$$

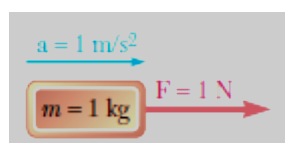


$$\Sigma\mathbf{F} = m\mathbf{a}$$

where F represents the sum, or resultant, of all the forces acting on the particle.

2-2 SYSTEMS OF UNITS

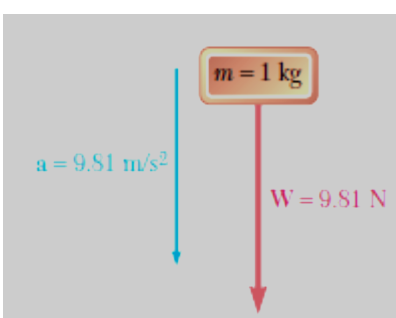
A-International System of Units (SI Units)



$$1\ \text{N} = (1\ \text{kg})(1\ \text{m/s}^2) = 1\ \text{kg} \cdot \text{m/s}^2$$

$$W = (1\ \text{kg})(9.81\ \text{m/s}^2) = 9.81\ \text{N}$$

$$1\ \text{km} = 1000\ \text{m} \qquad 1\ \text{mm} = 0.001\ \text{m}$$
$$1\ \text{Mg} = 1000\ \text{kg} \qquad 1\ \text{g} = 0.001\ \text{kg}$$
$$1\ \text{kN} = 1000\ \text{N}$$



B- U.S. Customary Units

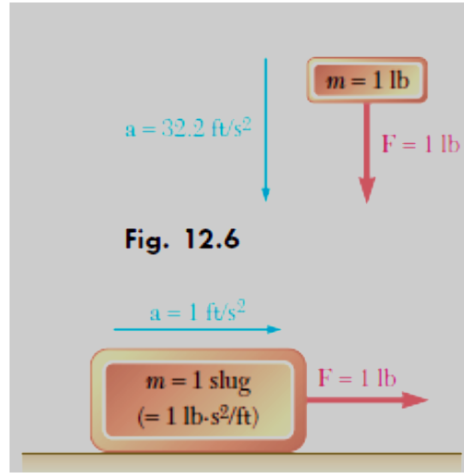


Fig. 12.6

$$F = ma \qquad 1\ \text{lb} = (1\ \text{slug})(1\ \text{ft/s}^2)$$

$$1\ \text{slug} = \frac{1\ \text{lb}}{1\ \text{ft/s}^2} = 1\ \text{lb}\cdot\text{s}^2/\text{ft}$$

C- Conversion from One System of Units to Another

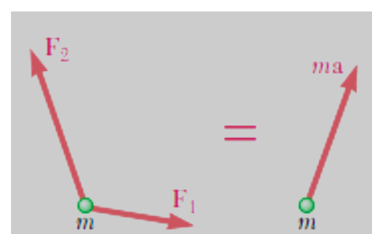
Length:	$1\ \text{ft} = 0.3048\ \text{m}$
Force:	$1\ \text{lb} = 4.448\ \text{N}$
Mass:	$1\ \text{slug} = 1\ \text{lb}\cdot\text{s}^2/\text{ft} = 14.59\ \text{kg}$

$$1\ \text{pound-mass} = 0.4536\ \text{kg}$$

2-3 EQUATIONS OF MOTION

A- **Rectangular Components**

Resolving each force **F** and the acceleration **a** into rectangular components, we write



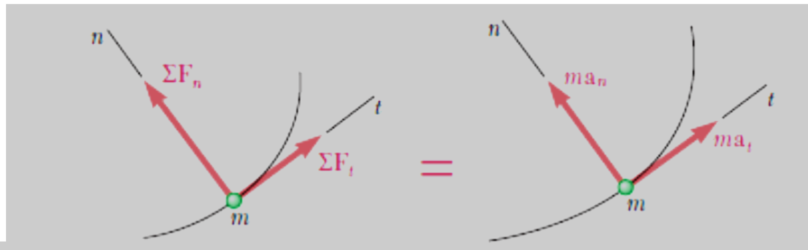
$$\Sigma(F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}) = m(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

from which it follows that

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z$$

$$\Sigma F_x = m\ddot{x} \qquad \Sigma F_y = m\ddot{y} \qquad \Sigma F_z = m\ddot{z}$$

B- **Tangential and Normal Components**

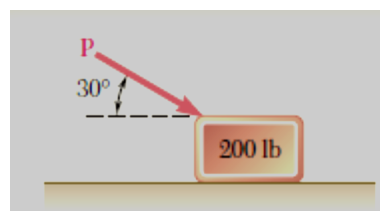


$$\Sigma F_t = ma_t \qquad \Sigma F_n = ma_n$$

Substituting for a_t and a_n from Eqs. (11.40), we have

$$\Sigma F_t = m\frac{dv}{dt} \qquad \Sigma F_n = m\frac{v^2}{\rho}$$

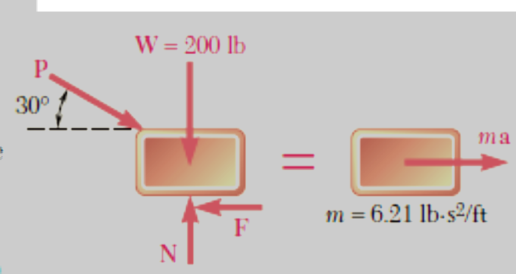
Example -1: A 200-lb block rests on a horizontal plane. Find the magnitude of the force **P** required to give the block an acceleration of 10 ft/s2 to the right. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.



The mass of the block is

$$m = \frac{W}{g} = \frac{200 \text{ lb}}{32.2 \text{ ft/s}^2} = 6.21 \text{ lb} \cdot \text{s}^2/\text{ft}$$

We note that $F = \mu_k N = 0.25N$ and that $a = 10$ ft/s^2. Expressing that the forces acting on the block are equivalent to the vector $m\mathbf{a}$, we write



$$\xrightarrow{+} \Sigma F_x = ma: \quad P\cos 30° - 0.25N = (6.21 \text{ lb} \cdot \text{s}^2/\text{ft})(10 \text{ ft/s}^2)$$

$$P\cos 30° - 0.25N = 62.1 \text{ lb} \qquad (1)$$

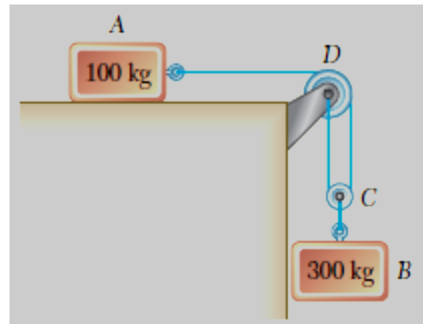
$$+\uparrow \Sigma F_y = 0: \quad N - P\sin 30° - 200 \text{ lb} = 0 \qquad (2)$$

Solving (2) for N and substituting the result into (1), we obtain

$$N = P\sin 30° + 200 \text{ lb}$$

$$P\cos 30° - 0.25(P\sin 30° + 200 \text{ lb}) = 62.1 \text{ lb} \qquad P = 151 \text{ lb}$$

Example -2: The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in each cord.



$$x_B = \tfrac{1}{2}x_A$$

ect to t, we have

$$a_B = \tfrac{1}{2}a_A$$

Block A. Denoting by T_1 the tension in cord ACD, we write

$\overset{+}{\rightarrow}\Sigma F_x = m_A a_A$: $\qquad T_1 = 100a_A$

Block B. Observing that the weight of block B is

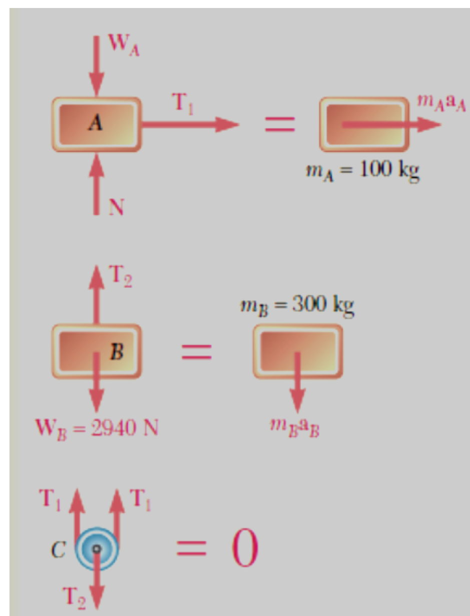
$$W_B = m_B g = (300\text{ kg})(9.81\text{ m/s}^2) = 2940\text{ N}$$

and denoting by T_2 the tension in cord BC, we write

$+\downarrow\Sigma F_y = m_B a_B$: $\qquad 2940 - T_2 = 300a_B$

or, substituting for a_B from (1),

$$2940 - T_2 = 300(\tfrac{1}{2}a_A)$$
$$T_2 = 2940 - 150a_A$$



Pulley C. Since m_C is assumed to be zero, we have

$$+\downarrow\Sigma F_y = m_C a_C = 0: \qquad T_2 - 2T_1 = 0 \qquad (4)$$

Substituting for T_1 and T_2 from (2) and (3), respectively, into (4) we write

$$2940 - 150a_A - 2(100a_A) = 0$$
$$2940 - 350a_A = 0 \qquad a_A = 8.40 \text{ m/s}^2 \blacktriangleleft$$

Substituting the value obtained for a_A into (1) and (2), we have

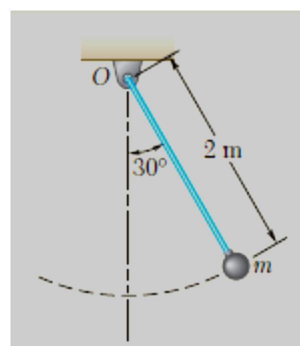
$$a_B = \tfrac{1}{2}a_A = \tfrac{1}{2}(8.40 \text{ m/s}^2) \qquad a_B = 4.20 \text{ m/s}^2 \blacktriangleleft$$
$$T_1 = 100a_A = (100 \text{ kg})(8.40 \text{ m/s}^2) \qquad T_1 = 840 \text{ N} \blacktriangleleft$$

Recalling (4), we write

$$T_2 = 2T_1 \qquad T_2 = 2(840 \text{ N}) \qquad T_2 = 1680 \text{ N} \blacktriangleleft$$

We note that the value obtained for T_2 is *not* equal to the weight of block B.

Example -3: The bob of a 2-m pendulum describes an arc of circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, find the velocity and the acceleration of the bob in that position.



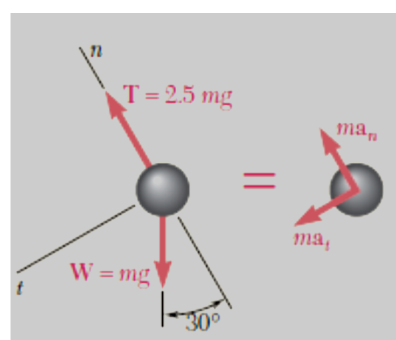
The weight of the bob is $W = mg$; the tension in the cord is thus 2.5 mg. Recalling that $\mathbf{a}_n$ is directed toward O and assuming $\mathbf{a}_t$ as shown, we apply Newton's second law and obtain

$$+\swarrow\Sigma F_t = ma_t: \qquad mg \sin 30^\circ = ma_t$$
$$a_t = g \sin 30^\circ = +4.90 \text{ m/s}^2 \qquad \mathbf{a}_t = 4.90 \text{ m/s}^2 \swarrow \blacktriangleleft$$

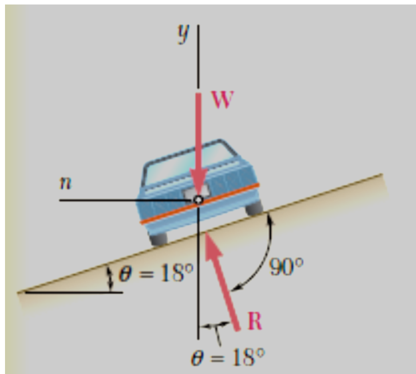
$$+\nwarrow\Sigma F_n = ma_n: \qquad 2.5\,mg - mg\cos 30^\circ = ma_n$$
$$a_n = 1.634\,g = +16.03 \text{ m/s}^2 \qquad \mathbf{a}_n = 16.03 \text{ m/s}^2 \nwarrow \blacktriangleleft$$

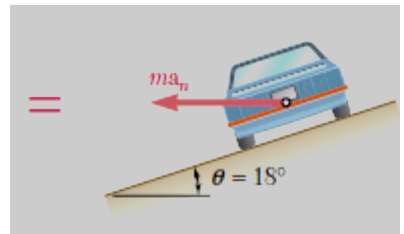
Since $a_n = v^2/\rho$, we have $v^2 = \rho a_n = (2 \text{ m})(16.03 \text{ m/s}^2)$

$$v = \pm 5.66 \text{ m/s} \qquad \mathbf{v} = 5.66 \text{ m/s} \swarrow\!\!\nearrow \text{ (up or down)} \blacktriangleleft$$



Example -4: Determine the rated speed of a highway curve of radius ρ = 400 ft banked through an angle φ = 18°. The rated speed of a banked highway curve is the speed at which a car should travel if no lateral friction force is to be exerted on its wheels.





$$+\uparrow \Sigma F_y = 0: \qquad R\cos\theta - W = 0 \qquad R = \frac{W}{\cos\theta} \qquad (1)$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n: \qquad R\sin\theta = \frac{W}{g}a_n \qquad (2)$$

Substituting for R from (1) into (2), and recalling that $a_n = v^2/\rho$,

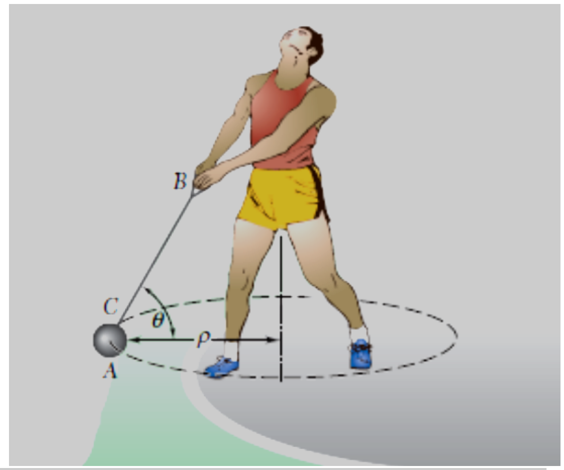
$$\frac{W}{\cos\theta}\sin\theta = \frac{W}{g}\frac{v^2}{\rho} \qquad v^2 = g\rho\tan\theta$$

Substituting $\rho = 400$ ft and $\theta = 18°$ into this equation, we obtain

$$v^2 = (32.2\ \text{ft/s}^2)(400\ \text{ft})\tan 18°$$

$$v = 64.7\ \text{ft/s} \qquad v = 44.1\ \text{mi/h}$$

Example-5: During a hammer thrower's practice swings, the 7.1-kg head A of the hammer revolves at a constant speed v in a horizontal circle as shown. If r = 0.93 m and u = 60°, determine (a) the tension in wire BC, (b) the speed of the hammer's head



SOLUTION

First we note $a_A = a_n = \dfrac{v_A^2}{\rho}$

(*a*) $+\uparrow \Sigma F_y = 0$: $T_{BC}\sin 60° - W_A = 0$

or $$T_{BC} = \frac{7.1\text{ kg} \times 9.81\text{ m/s}^2}{\sin 60°}$$
$$= 80.426\text{ N}$$

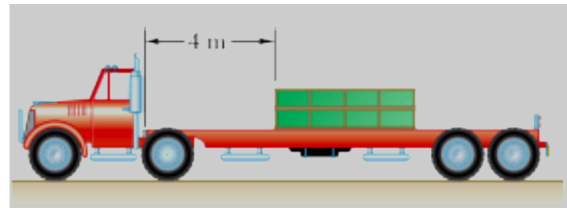
$$T_{BC} = 80.4\text{ N} \blacktriangleleft$$

(*b*) $\xrightarrow{+} \Sigma F_x = m_A a_A$: $T_{BC}\cos 60° = m_A \dfrac{v_A^2}{\rho}$

or $$v_A^2 = \frac{(80.426\text{ N})\cos 60° \times 0.93\text{ m}}{7.1\text{ kg}}$$

or $$v_A = 2.30\text{ m/s} \blacktriangleleft$$

Example-6: The coefficients of friction between the load and the flat-bed trailer shown are μ= 0.30. Knowing that the speed of the rig is 72 km/h, determine the shortest distance in which the rig can be brought to a stop if the load is not to shift.



Deceleration of load is same as deceleration of trailer, which is the maximum allowable deceleration $\mathbf{a}_{max}$.

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \quad N = W$$

$$F_m = \mu_s N = 0.40\, W$$

$$\xrightarrow{+} \Sigma F_x = ma: \quad F_m = ma_{max}$$

$$0.40\ W = \frac{W}{g} a_{max} \qquad a_{max} = 3.924 \text{ m/s}^2$$

$$\mathbf{a}_{max} = 3.92 \text{ m/s}^2 \longrightarrow$$

Uniformly accelerated motion.

$$v^2 = v_0^2 + 2ax \quad \text{with} \quad v = 0 \qquad\qquad v_0 = 72 \text{ km/h} = 20 \text{ m/s}$$

$$a = -a_{max} = 3.924 \text{ m/s}^2$$

$$0 = (20)^2 + 2(-3.924)x \qquad\qquad x = 51.0 \text{ m} \blacktriangleleft$$