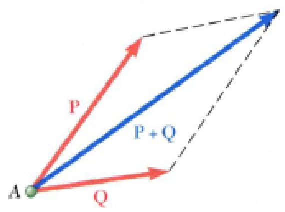
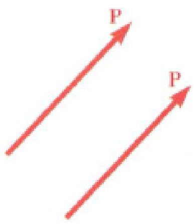
chapter two

Statics of Particles

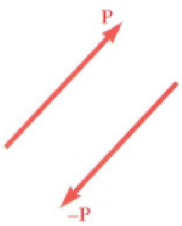
Vectors



- *Vector*: parameters possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.
- *Scalar*: parameters possessing magnitude but not direction. Examples: mass, volume, temperature

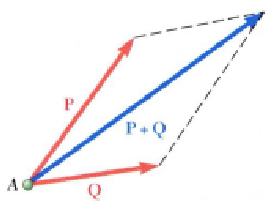


- Vector classifications:
  - *Fixed* or *bound* vectors have well defined points of application that cannot be changed without affecting an analysis.
  - *Free* vectors may be freely moved in space without changing their effect on an analysis.
  - *Sliding* vectors may be applied anywhere along their line of action without affecting an analysis.

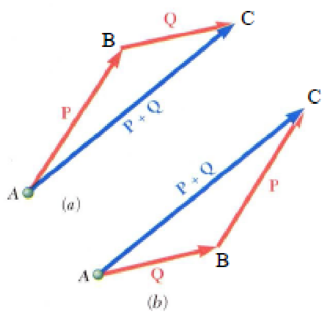


- *Equal* vectors have the same magnitude and direction.
- *Negative* vector of a given vector has the same magnitude and the opposite direction.

Addition of Vectors



- Trapezoid rule for vector addition
- Triangle rule for vector addition



- Law of cosines,

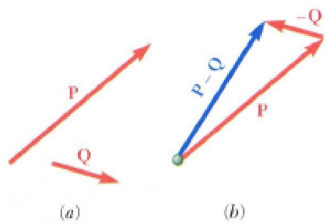
$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

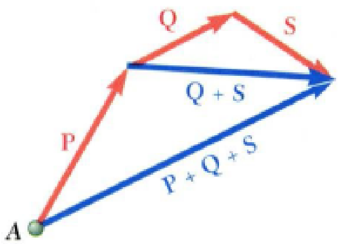
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

- Vector addition is commutative,

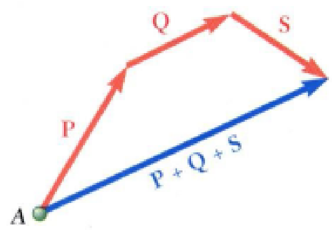
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$



- Vector subtraction

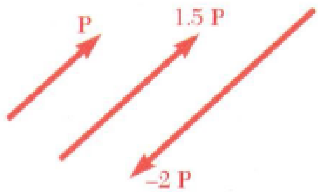


- Addition of three or more vectors through repeated application of the triangle rule



- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$



- Multiplication of a vector by a scalar

Static of particles

Case one: Study of one force

If one force acted in one direction

Example: Find the resultant of force Fx

Solution:

F=Fx+Fy = Fx+0 =Fx

Example: Find the resultant of force Fy

Solution:

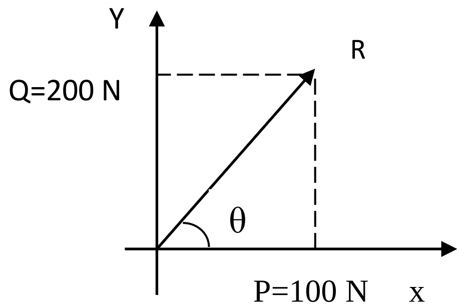
F=Fx+Fy = 0+Fy =Fy

If the force F has an angle with the x axis

Fx=F cosϴ

Fy=F sin ϴ

Forces in a plane: Results of two forces



$$R = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1}\frac{Q}{P}$$

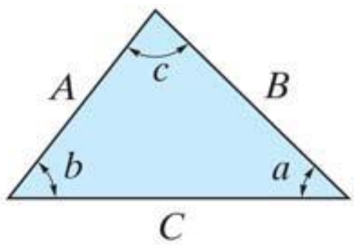
$$\cos\theta = \frac{P}{R} \qquad P = R\cos\theta$$

$$\sin\theta = \frac{Q}{R} \qquad Q = R\sin\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{Q}{R}\frac{R}{P} = \frac{Q}{P}$$

Results of two forces

1- The law of sines:- *the magnitude of the resultant forces cab be determined from the law of cosines, and its direction is determined from the law of sines*



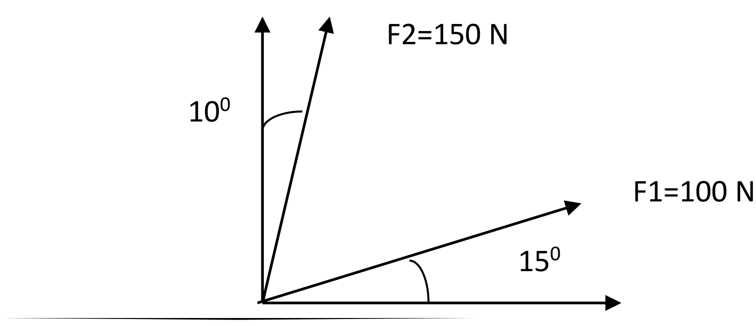
Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Example: determine the magnitude of the resultant force and its direction shown in fig. below



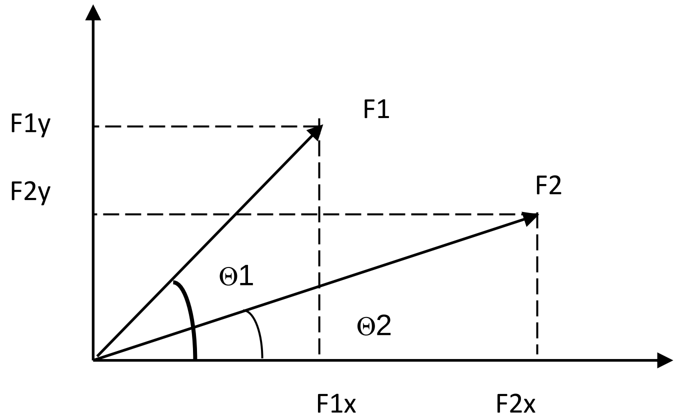
$R=[100^2+150^2-2*100*150\cos 115]^{0.5}$

R=213 N

$$\frac{F2}{\sin\theta}=\frac{R}{\sin 115} \Rightarrow \frac{150}{\sin\theta}=\frac{213}{\sin 115}$$

$$\sin\theta=\frac{150}{213}\sin 115 \qquad \theta=39.8^0 \qquad \alpha=\theta+15=54.8^0$$

3-Rectangular components



$$R_x=\Sigma F_x \qquad R_y=\Sigma F_y$$

$$R=\sqrt{R_x^2+R_y^2} \qquad \theta=\tan^{-1}\frac{R_y}{R_x}$$

F1=F1xi+F1yj

F2=F2xi+F2yj

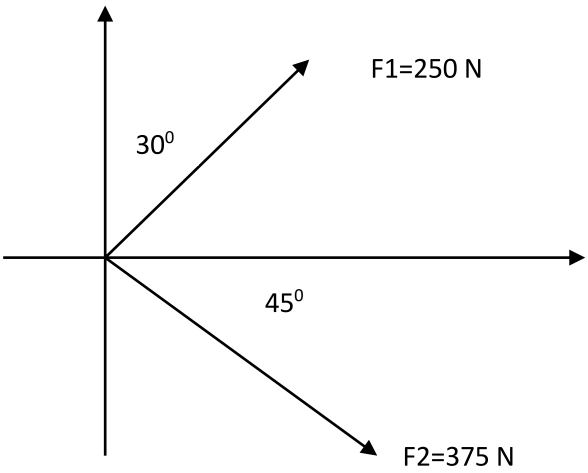
F1y=F1 sinθ1

F1x=F1 cos θ1

F2y=F2 sin θ2

F2x= F2 cos θ2

Example: determine the magnitude of the resultant force and its direction



$$R_x=\Sigma F_x$$

Rx=F1cos 60+F2 cos 45

=250 cos60+375 cos45

=390.17 N

Ry=F1 sin60- F2 sin45

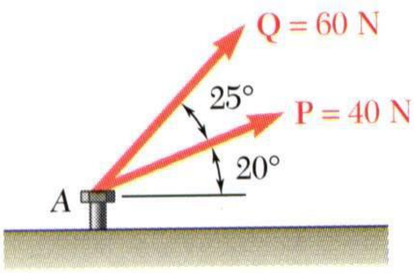
=250 sin60-375 sin45

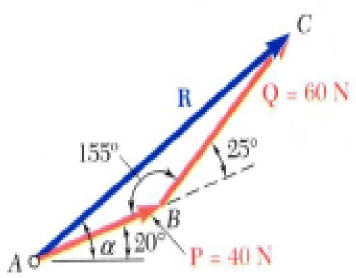
=-48.66 N

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

R=393 N θ=-7.1

***Example*:** The two forces act on a bolt at A. Determine their resultant.





- Trigonometric solution - Apply the triangle rule.

From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$\boxed{R = 97.73\text{N}}$$

From the Law of Sines,

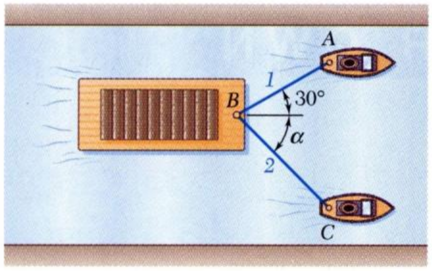
$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B\frac{Q}{R}$$
$$= \sin 155°\frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Example : A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

a) the tension in each of the ropes for α = 45°,

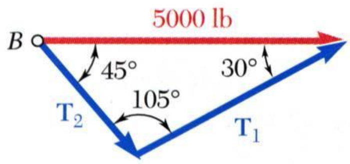
b) the value of α for which the tension in rope 2 is a minimum.

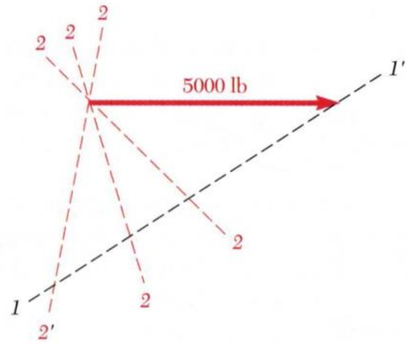


- Trigonometric solution - Triangle Rule with Law of Sines

$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\,\text{lbf}}{\sin 105°}$$

$$\boxed{T_1 = 3660\,\text{lbf} \quad T_2 = 2590\,\text{lbf}}$$



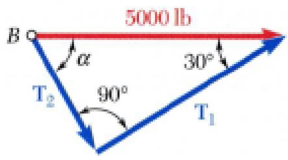


- The minimum tension in rope 2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular.

$$T_2 = (5000\,\text{lbf})\sin 30° \qquad \boxed{T_2 = 2500\,\text{lbf}}$$

$$T_1 = (5000\,\text{lbf})\cos 30° \qquad \boxed{T_1 = 4330\,\text{lbf}}$$

$$\alpha = 90° - 30° \qquad \boxed{\alpha = 60°}$$



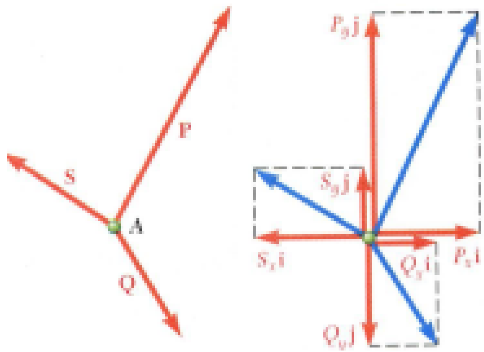
Results of three or more forces

2-Method of projections

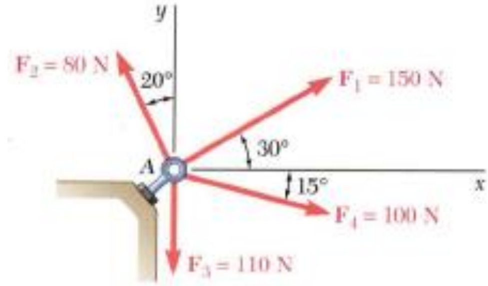
$$R_x = P_x + Q_x + S_x = \sum F_x \qquad R_y = P_y + Q_y + S_y = \sum F_y$$

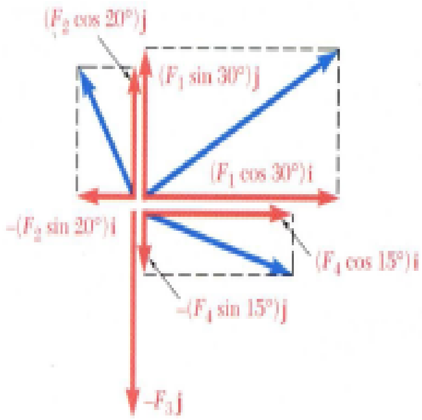
- To find the resultant magnitude and direction,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$



Example : Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

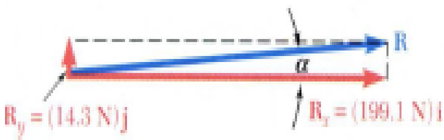




SOLUTION:

- Resolve each force into rectangular components.

force	*mag*	$x-comp$	$y-comp$
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	−27.4	+75.2
$\vec{F}_3$	110	0	−110.0
$\vec{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$



- Determine the components of the resultant by adding the corresponding force components.

- Calculate the magnitude and direction.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\,\mathrm{N}}$$

$$\tan\alpha = \frac{14.3\,\mathrm{N}}{199.1\,\mathrm{N}} \qquad \boxed{\alpha = 4.1^\circ}$$