

Theory of Chip Formation in Conventional Machining:

The Orthogonal Cutting Model:

- The geometry of most practical CM operations is somewhat complex.
- By assuming that the cutting action is continuous we can develop a continuous model is called "orthogonal cutting model".
- By definition, orthogonal cutting uses a wedge-shaped cutting tool in which the cutting edge is perpendicular to the direction of cutting speed.
- As the cutting tool is forced onto w.p., the chip is formed by shear deformation along a plane called "the shear plane", which is oriented at an angle "φ".
- Along the shear plane, where the bulk of mechanical energy is consumed in machining, the w.p. is plastically deformed.
- The cutting tool in orthogonal cutting has only two elements of geometry: (1) rake angle, (2) clearance or flank or relief angle.
- Figure (4-28) shows orthogonal cutting.



Figure (4-28) orthogonal cutting: (a) 3D process, (b) 2D process

Where:

- φ: shear angle,
- θ : relief angle (clearance angle),

- *t*_o: undeformed chip thickness,
- t_c : deformed chip thickness,
- w: width of chip,
- α : rake angle,
- *l*_s: length of shear plane,

Engineering Analysis of Orthogonal Cutting Model:

1. Chip Thickness Ratio (r_c):

$$r_c = \frac{t_o}{t_c}$$

 r_c : always < 1, because $t_c > t_o$ always.

2. Shear Angle Relationship:



From triangle ABC: $t_o = l_s \sin \phi$

From triangle ACD: $t_c = l_s \cos(\phi - \alpha)$

$$r_{c} = \frac{t_{o}}{t_{c}} = \frac{l_{s} \sin \emptyset}{l_{s} \cos(\emptyset - \alpha)} = \frac{\sin \emptyset}{\cos \emptyset \cos \alpha + \sin \emptyset \sin \alpha}$$
$$\frac{r_{c} (\cos \emptyset \cos \alpha + \sin \emptyset \sin \alpha)}{\sin \emptyset} = 1$$
$$\frac{r_{c} \cos \emptyset \cos \alpha}{\sin \emptyset} + \frac{r_{c} \sin \emptyset \sin \alpha}{\sin \emptyset} = 1$$
$$\frac{r_{c} \cos \alpha}{\sin \emptyset} + r_{c} \sin \alpha = 1$$

$$\therefore \tan \phi = \frac{r_c \cos \alpha}{1 - r_c \sin \alpha}$$
or
$$\tan \phi = \frac{t_0 \cos \alpha}{t_c - t_0 \sin \alpha}$$

3. Shear Strain (γ):

- Shear strain that happens along the shear plane can be determined by checking the following figure (4-29):



Figure (4-29) Shear strain during chip formation: (a) chip formation depicted as a series of parallel plates sliding relative to each other, (b) one of plates isolated to explain the definition of shear strain, and (c) shear strain triangle



- From figure (4-29), each plate experiences a shear strain as in figure 4-29(b). From definition of shear strain and aiding of figure 4-29(c):

$$\gamma = \frac{AC}{BD} = \frac{AD + DC}{BD} = \frac{AD}{BD} + \frac{DC}{BD}$$

From above figure:

$$\tan \phi = \frac{BD}{AD}$$

$$\therefore \frac{AD}{BD} = \frac{1}{\tan \emptyset} = \cot \emptyset , \quad \text{and} \quad DC$$

$$\tan(\emptyset - \alpha) = \frac{DC}{BD}$$

$$\therefore \gamma = \cot \emptyset + \tan(\emptyset - \alpha)$$

Where:

AC: deflection of sheared metal.

BD: orthogonal distance over which deflection occurs.

Example (5):

In a machining operation that approximates orthogonal cutting, the cutting tool has a rake angle = 10° . The chip thickness before the cut $t_{o} = 0.5$ mm and the chip thickness after the cut $t_{c} = 1.125$ mm. Calculate the shear plane angle and the shear strain in the operation.

Solution: $\tan \phi = \frac{t_0 \cos \alpha}{t_c - t_0 \sin \alpha} = \frac{0.5 \cos 10}{1.125 - 0.5 \sin 10} = 0.474$ $\therefore \phi = 25.4^{\circ}$ <u>Answer</u> $\gamma = \cot \phi + \tan(\phi - \alpha) = \cot 25.4 + \tan(25.4 - 10) = 2.381$ <u>Answer</u>

4. Velocities:

- From the velocity diagram below:



Where:

V: cutting velocity,

V_f: frictional velocity or chip velocity,

 V_s : shearing velocity at which shearing happens along the shear plane.

- Using the sine rule:

$$\frac{V_s}{\sin(90-\alpha)} = \frac{V}{\sin(90+\alpha-\phi)}$$
$$V_s = \frac{V\sin(90-\alpha)}{\sin(90+\alpha-\phi)}$$
$$\therefore V_s = \frac{V\cos\alpha}{\cos(\phi-\alpha)}$$

Also

$$\frac{V_f}{\sin \emptyset} = \frac{V}{\sin(90 + \alpha - \emptyset)}$$
$$\therefore V_f = \frac{V \sin \emptyset}{\cos(\emptyset - \alpha)}$$

Where:

$$\sin(90 - \alpha) = \sin 90 \cos \alpha - \cos 90 \sin \alpha = \cos \alpha$$

 $\sin(90 + \alpha - \emptyset) = \sin[90 - (\emptyset - \alpha)] = \sin 90 \cos(\emptyset - \alpha) - \cos 90 \sin(\emptyset - \alpha)$ $= \cos(\emptyset - \alpha)$

5. Forces Relationships Metal Cutting:

- The forces acting on the chip during orthogonal cutting can be shown in figure (4-30):



Figure (4-30) Forces in metal cutting: (a) forces acting on the chip in orthogonal cutting, (b) forces acting on the cutting tool that can be measured

a. <u>The Forces Applied Against the Chip by the Cutting Tool:</u>

There are two perpendicular components:

- Friction Force (F): is the frictional force resisting the flow of the chip along the rake face of cutting tool.
- Normal Force to Friction (N): is perpendicular to the friction force.

Coefficient of Friction between Cutting Tool and Chip:

$$\mu = \frac{F}{N}$$

Friction Angle (β):

- From figure (4-30a), the resultant (*R*) of *F* and *N* forces is oriented at angle (β) called friction angle.
- The friction angle is related to coefficient of friction as:

$$\mu = \tan \beta$$

b. The Forces Applied by the w.p. on the Chip:

- Shear Force (F_s) : is the force that causes shear deformation to occur in the shear plane.
- Normal Force to Shear (F_n) : is perpendicular to the shear force.
- Based on F_s , the shear stress that acts along the shear plane between w.p. and chip can be written as:

$$\tau = \frac{F_s}{A_s}$$

Where:

 τ : the level of stress required to perform the machining operation. Therefore, this stress is equal to the shear strength of the w.p. ($\tau = S$) under the conditions at which cutting occurs.

 A_s : area of shear plane,



Where:

 A_c : cross-sectional area of uncut chip.

- The normal stress on the chip can be written as:

$$\boldsymbol{\sigma} = \frac{F_n}{A_s}$$

- From figure (4-30a), in order for the forces acting on the chip to be in balance, the resultant (R') of F_s and F_n , must be equal in magnitude, opposite in direction, and collinear (lye on the same line or on parallel lines) with resultant R.

Note:

None of the four forces F, N, F_s , and F_n can be directly measured in a machining operation, because the directions in which they are applied vary with different cutting tool geometries and cutting conditions.

c. <u>The Forces Acting Against the Cutting Tool:</u>

- The Cutting Force (F_c) : is in the direction of cutting, the same direction as the cutting speed
- The Thrust Force (F_t): is perpendicular to the cutting force and is associated with the chip thickness before the cut t_o .
- The F_c and F_t are shown in figure (4-30b) with their resultant (R'').
- It is possible for the cutting tool to be instrumented using a force measuring device called dynamometer.
- The respective directions of F_c and F_t are known, so the force transducers in the dynamometer can be aligned accordingly.
- Figure (4-31) shows the force diagram for the forces F, N, F_s , F_n , F_c and F_t .



Figure (4-31) force diagram of F, N, F_s , F_n , F_c and F_t forces.

- Using figure (4-31) and trigonometric relationships, it can relate the four forces F, N, F_s , F_n , that cannot be measured to the two forces F_c , F_t that can be measured as follows:

 $F = F_c \sin \alpha + F_t \cos \alpha$ $N = F_c \cos \alpha - F_t \sin \alpha$ $F_s = F_c \cos \emptyset - F_t \sin \emptyset$ $F_n = F_c \sin \emptyset + F_t \cos \emptyset$

(Show that above equations)

- Total Resultant:

$$R = R' = R'' = \sqrt{F_c^2 + F_t^2} = \sqrt{F_s^2 + F_n^2} = \sqrt{F^2 + N^2}$$

- We can write (**µ** - Coefficient of Friction) as:

$$\mu = \frac{F}{N} = \frac{F_c \sin \alpha + F_t \cos \alpha}{F_c \cos \alpha - F_t \sin \alpha}$$

- By dividing the Numerator and Denominator on $\cos \alpha$:

$$\therefore \mu = \frac{F}{N} = \frac{F_t + F_c \tan \alpha}{F_c - F_t \tan \alpha}$$

Example (6):

In a machining operation, the cutting force and thrust force are measured during an orthogonal cutting operation: $F_c = 1559$ N and $F_t = 1271$ N. The width of the orthogonal cutting operation w = 3.0 mm. Based on these data, determine the shear strength of the work material. Take: the rake angle = 10°, the shear plane angle = 25.4° and the chip thickness before the cut $t_0 = 0.5$ mm.

Solution:

$$\tau = \frac{F_s}{A_s} , \quad A_s = \frac{t_o w}{\sin \phi} , \quad F_s = F_c \cos \phi - F_t \sin \phi$$

$$F_s = F_c \cos \phi - F_t \sin \phi = 1559 \cos 25.4 - 1271 \sin 25.4 = 863.1 N$$

$$A_s = \frac{t_o w}{\sin \phi} = \frac{0.5(3.0)}{\sin 25.4} = 3.497 mm^2$$

$$\tau = \frac{F_s}{A_s} = \frac{863.1}{3.497} = 246.8 Mpa \qquad \text{Answer}$$

d. The Cutting and Thrust Forces in Term of Shear Force:

- Recalling figure (4.31)



 $F_{c} = R \cos(\beta - \alpha) \dots (1)$ $F_{t} = R \sin(\beta - \alpha) \text{ or } F_{t} = F_{c} \tan(\beta - \alpha) \dots (2)$ $F_{s} = R \cos(\phi + \beta - \alpha) \dots (3)$ $(1) 180 - 90 - \beta = 90 - \beta$ $(2) 90 - \alpha - (90 - \beta) = \beta - \alpha$

From equations (1) and (3):

$$F_c = \frac{F_s \cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$$
 and

$$F_t = F_c \tan(\beta - \alpha) = \frac{F_s \cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)} \tan(\beta - \alpha) = \frac{F_s \sin(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$$

- These relations permit to estimate F_c and F_t in an orthogonal cutting operation if material shear strength is known.

The Merchant Equation:

- This equation is based one orthogonal cutting, but can extend to 3D machining operations.
- From the shear stress equation:

$$\tau = \frac{F_s}{A_s} = \frac{F_c \cos \emptyset - F_t \sin \emptyset}{(t_o w / \sin \emptyset)}$$

By taking the derivative of the shear stress equation above with respect to ϕ and setting the derivative to zero. Solving for ϕ , the relationship named after Merchant is:

- Among the assumptions in the Merchant equation is: Shear strength of w.p. is a constant, unaffected by strain rate, temperature and other factors.
- This assumption is violated in practical machining operations, therefore Merchant Eq. must be considered an approximate relationship rather than an accurate mathematical equation.
- Merchant equation defines relationship between rake angle (α), friction angle (β) and shear angle (ϕ).
- Shear angle (ϕ) is increased by (1) increasing α by proper tool design (2) decreasing β and μ by using a lubricant cutting fluid. Effect of shear angle is stated in Fig. (4-32).



Figure (4-32) Effect of shear angle (a) higher ϕ resulting in lower shear plane area (b) smaller ϕ resulting in larger shear plane area

- Since shear strength is applied across shear plane area, F_s required to form the chip will be decreased and a greater ϕ results in lower cutting energy, lower power and lower cutting temperature.

Approximation of Turning by Orthogonal Cutting:

- The *orthogonal model* can be used to approximate *turning* and certain other single-point machining operations so long as the *feed* in these operations is small relative to *depth of cut*.
- Thus, most of the cutting will take place in the direction of the feed, and cutting on the point of the tool will be negligible.
- Figure (4-33) indicates orthogonal model compared to turning model.



Figure (4-33) Approximation of turning by the orthogonal model: (a) turning; and (b) the corresponding orthogonal cutting

- Table below summarizes the conversions key between turning and orthogonal cutting.

Conversion key: turning operation vs. orthogonal cutting.	
Turning Operation	Orthogonal Cutting Model
Feed $f =$ Depth $d =$ Cutting speed $v =$ Cutting force $F_c =$ Feed force $F_f =$	Chip thickness before cut t_o Width of cut w Cutting speed v Cutting force F_c Thrust force F_t

Power and Energy Relationships in Machining:

- There are some reasons necessitate us calculate the power consumed in machining operations:
- 1. How fast we can cut,
- 2. How large the motor on a machine must be.
- There are three types of power consumed to perform the cutting process:
 - a. Shearing Power,
 - b. Friction Power,
 - c. Cutting Power or Total Power

a. Shearing Power

It is required for shearing along the shear plane:

$$P_s = \frac{F_s V_s}{33000} \ (hp_s) \ F_s \ in \ Ib_f \ , V_s \ in \ ft/min$$

Or

$$P_s = \frac{F_s V_s}{60000} \quad (kW) \quad F_s \text{ in } N, V_s \text{ in } m/min$$

Or

 $P_s = F_s V_s (W) F_s in N, V_s in m/s$

- It is often useful to convert power into power per unit volume rate of metal cut. This is called the unit power in shear, P_{us} (or unit horsepower in shear, HP_{us}), defined:

$$P_{us} = \frac{P_s}{R_{MR}} \quad or \qquad HP_{us} = \frac{hp_s}{R_{MR}} \quad , \qquad R_{MR} \quad in \left(\frac{mm^3}{s}, \frac{ft^3}{min}, \frac{in^3}{min}\right)$$

- Unit power in shear is also known as the specific energy in shear U_s as follows:

$$U_s = \frac{P_s}{R_{MR}} = \frac{F_s V_s}{R_{MR}}$$
 $(\frac{N.m}{m^3}, \frac{in.lb}{in^3}, \frac{ft.lb}{ft^3})$

In turning operation (orthogonal cutting):

$$R_{MR} = Vfd = wt_oV = wt_cV_f$$

R_{MR}: material removal rate (metal flow rate)

t_o: feed in other cases

w: depth of cut in other cases

wto: projected area of cut

In other machining operations like drilling and milling, R_{MR} will be taken as stated there.

b. Friction Power

It is required to overcome friction at the tool-chip interface:

$$P_f = \frac{F V_f}{33000} (hp_f) F in Ib_f, V_f in ft/min$$

Or

$$P_f = \frac{F V_f}{60000} \ (kW) \ F \ in \ N \ V_f \ in \ m/min$$

Or

$$P_f = F V_f(W) F in N, V_f in m/s$$

- Then, unit power in friction, P_{uf} (or unit horsepower in friction, HP_{uf}):

$$P_{uf} = \frac{P_f}{R_{MR}} \quad or \quad HP_{uf} = \frac{hp_f}{R_{MR}} , \quad R_{MR} \quad in \left(\frac{mm^3}{s}, \frac{ft^3}{min}, \frac{in^3}{min}\right)$$

- Specific energy in friction U_f is as follows:

$$U_f = \frac{P_f}{R_{MR}} = \frac{F V_f}{R_{MR}} \qquad (\frac{N.m}{m^3}, \qquad \frac{in.Ib}{in^3}, \qquad \frac{ft.Ib}{ft^3})$$

c. Total Cutting Power

It is required to perform the cutting process (machining operation):

$$P_c = \frac{F_c V}{33000} \ (hp_c) \ F_c \ in \ Ib_f \ , V \ in \ ft/min$$

Or

$$P_c = \frac{F_c V}{60000} \ (kW) \ F_c \ in \ N \ , V \ in \ m/min$$

Or

$$P_c = F_c V(W) F_c \text{ in } N, V \text{ in } m/s$$

- Then, unit power in cutting, P_{uc} (or unit horsepower in cutting, HP_{uc}):

$$P_{uc} = \frac{P_c}{R_{MR}} \quad or \qquad HP_{uc} = \frac{hp_c}{R_{MR}} \quad , \qquad R_{MR} \quad in \ (\frac{mm^3}{s}, \frac{ft^3}{min}, \frac{in^3}{min})$$

- Specific energy in cutting U_c is as follows:

$$U_c = \frac{P_c}{R_{MR}} = \frac{F_c V}{R_{MR}} \qquad (\frac{N.m}{m^3}, \qquad \frac{in.Ib}{in^3}, \qquad \frac{ft.Ib}{ft^3})$$

$$P_c = P_s + P_f$$

And

 $U_c = U_s + U_f$

In General:

$$Pr_{sc} = \frac{Energy Consumed}{Volume of Metal Removed}$$

$$Pr_{sc} = \frac{P_c}{wt_o V} = \frac{F_c V}{wt_o V} = \frac{F_c}{wt_o}$$

Pr_{sc} : Specific cutting pressure.

Gross Cutting Power (Pgc)

- P_{gc} required to operate the machine tool is greater than the power delivered to the cutting process because of mechanical losses in the powertrain which includes the losses in machine motor and drivetrain (group of components of machine motor that deliver power to the cutting elements).

Then,

$$P_{gc} = \frac{P_c}{E}$$
 also $HP_{gc} = \frac{hp_c}{E}$

 HP_{gc} : gross cutting horsepower

E: mechanical efficiency of machine tool. Typical values of E for machine tools are around 90%.

Example (6):

In an orthogonal cutting process, take the following data:

 α = 15°, w= 0.25", V=250ft/min, F_c=375 Ibf, F_t=125 Ibf, Feed=t_o=0.0125", t_c=0.0375". Where: α : rake angle, w: width of chip. Find: ϕ , F, N, F_s, V_f, μ , R_{MR}

Solution:

- 1. $\tan \phi = \frac{r_c \cos \alpha}{1 r_c \sin \alpha}$ $r_c = \frac{t_o}{t_c} = \frac{0.0125}{0.0375} = 0.333$ $\therefore \phi = \frac{0.333 \cos 15}{1 - 0.333 \sin 15} = 19.4^\circ$ Answer
- 2. $F = F_c \sin \alpha + F_t \cos \alpha = 375 \sin 15 + 125 \cos 15 = 218 \, Ibf$ Answer
- 3. $N = F_c \cos \alpha F_t \sin \alpha = 375 \cos 15 125 \sin 15 = 330 \, Ibf$ Answer
- 4. $F_s = F_c \cos \phi F_t \sin \phi = 375 \cos 19.4 125 \sin 19.4 = 312.2 \, Ibf$ Answer

5.
$$V_f = \frac{V \sin \phi}{\cos(\phi - \alpha)} = \frac{250 \sin 19.4}{\cos(19.4 - 15)} = 83.3 \ ft/min$$
 Answer

- 6. $\mu = \frac{F}{N} = \frac{218}{330} = 0.661$ <u>Answer</u>
- 7. $R_{MR} = wt_o V = \left(\frac{0.25}{12}\right) \left(\frac{0.0125}{12}\right) (250) = 0.00543 \frac{ft^3}{min} x \ 12^3$ $R_{MR} = 9.38 \ in^3/min \ Answer$

Example (7):

Tube having a 1.5" outside diameter turned in lathe. The following data were recorded:

 $\alpha=35^o$, V=3fpm , F_c=433 Ibf , F_t=166 Ibf , Feed=t_o=0.005". If the length of continuous chip for one revolution of w.p. is L_c=2.5" (1)(rev/min)= 2.5in/min. Find: V_f and V_s.

Solution:

1.
$$V_f = \frac{V \sin \phi}{\cos(\phi - \alpha)}$$
, $\tan \phi = \frac{r_c \cos \alpha}{1 - r_c \sin \alpha}$

$$r_c = \frac{t_o}{t_c} = \frac{L_c}{L_o}$$

L_o: length of chip before cutting.

L_c: length of chip removed.

N: rpm = 1 rpm as given.

 $L_o = \pi DN$ (m/min, in/min, ft/min)

$$\therefore r_c = \frac{L_c}{L_o} = \frac{2.5}{\pi (1.5)(1)} = 0.531$$
$$\therefore \emptyset = tan^{-1} \left(\frac{r_c \cos \alpha}{1 - r_c \sin \alpha} \right) = 32^\circ$$
$$\therefore V_f = \frac{V \sin \emptyset}{\cos(\emptyset - \alpha)} = 18.6 fpm \quad \text{Answer}$$
$$2. V_s = \frac{V \cos \alpha}{\cos(\emptyset - \alpha)} = 28.71 fpm \quad \text{Answer}$$

Example (8):

In an orthogonal cutting process, the following data were recorded:

 $t_{o}{=}~0.005\,{}^{\prime\prime}{}$, V=400fpm , $\alpha{=}~10^{o}$, w=0.25 ${}^{\prime\prime}{}$, $t_{c}{=}0.009\,{}^{\prime\prime}{}$, $F_{c}{=}125Ib$, $F_{t}{=}50Ib.$

Calculate: $\frac{friction \ power}{total \ power}$ %

Solution:

$$\frac{P_f}{P_c} = \frac{F V_f}{F_c V}$$

$$F = F_c \sin \alpha + F_t \cos \alpha = 70.95 \, Ib$$

$$V_f = \frac{V \sin \phi}{\cos(\phi - \alpha)} , \quad \phi = tan^{-1} \left(\frac{r_c \cos \alpha}{1 - r_c \sin \alpha}\right) , \quad r_c = \frac{t_o}{t_c} = 0.556$$

$$\therefore \phi = 31.22^\circ$$

$$\therefore V_f = 222.4 \, fpm$$

$$\therefore \frac{P_f}{P_c} = \frac{(70.95)(222.4)}{(125)(400)} = 31.6\% \quad \text{Answer}$$

Example (9):

Determine the time required to turn a brass component 50mm in diameter and 100mm long at V=36m/min, feed=0.4mm/rev and only one cut taken.

Solution:

In turning operation: feed=to

time required (TR) =
$$\frac{\text{length of wp}}{(\text{feed})(N)}$$

 $N=V/\pi D$

 $\therefore TR = \frac{100\pi(50)}{(0.4)(36000/60)} = 65 \ sec \qquad \text{<u>Answer</u>}$

Chip Formation:

A. Discontinuous Chip:

When relatively brittle materials (e.g. cast irons) are machined at low cutting speeds, the chips often form into separate segments (sometimes the segments are loosely attached). This tends to impart an irregular texture to the machined surface. High tool-chip friction and large feed and depth of cut promote the formation of this chip type.

B. Continuous Chip:

When ductile work materials are cut at high speeds and relatively small feeds and depths, long continuous chips are formed. A good surface finish typically results when this chip type is formed. A sharp cutting edge on the tool and low tool-chip friction encourage the formation of continuous chips. Long continuous chips (as in turning) can cause problems with regard to chip disposal and/or tangling about the tool.

C. Continuous Chip with Built-up Edge:

When machining ductile materials at low-to-medium cutting speeds, friction between tool and chip tends to cause portions of the work material to adhere to the rake face of the tool near the cutting edge. This formation is called a builtup edge (BUE). The formation of a BUE is cyclical; it forms and grows, then becomes unstable and breaks off. Much of the detached BUE is carried away with the chip, sometimes taking portions of the tool rake face with it, which reduces the life of the cutting tool. Portions of the detached BUE that are not carried off with the chip become imbedded in the newly created work surface, causing the surface to become rough.

D. Serrated Chips:

The term shear-localized is also used for this fourth chip type. These chips are semi-continuous in the sense that they possess a saw-tooth appearance that is produced by a cyclical chip formation of alternating high shear strain followed by low shear strain. This fourth type of chip is most closely associated with certain difficult-to-machine metals such as titanium alloys, nickel-base superalloys, and austenitic stainless steels when they are machined at higher cutting speeds. However, the phenomenon is also found with more common metals (e.g., steels) when they are cut at high speeds.

Figure (4-34) shows the types of chip formation.



Figure (4-34) Four types of chip formation in metal cutting: (a) discontinuous, (b) continuous, (c) continuous with built-up edge, (d) serrated