Fundamental Principles

- **Definitions**

Mechanics of Materials : Is the branch of applied mechanics that deals with internal behavior of variously loaded solid bodies.

[Deals with the relationship between the external loads (forces and moments) and the internal forces and deformations induced in the body].

Fundamental Principles

Strength : measure the ability of the member to resist permanent deformation or fracture;

Stiffness : measure the ability of the member to resist deflection;

Stability : measure the ability of the member to retain its equilibrium configuration.

Failure : Is any action that results in an inability on the part of the structure to function in the manner intended.

Fundamental Principles

External Forces: All forces acting on a body, including the reactive forces caused by supports.

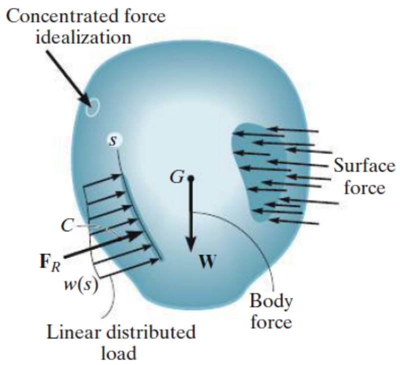


Fig. 1–1

Surface Forces :

Concentrated acts at a point; or
Distributed over a finite area

Body Forces :

acts on a volumetric element and is attributable to fields such as gravity and magnetism.

Fundamental Principles

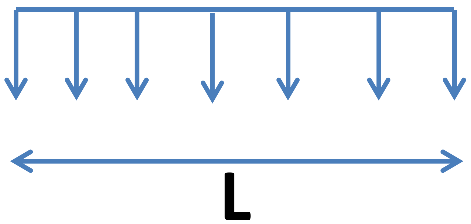
Internal Forces : The forces of interaction between the constituent material particles of the body.

Load Classification :

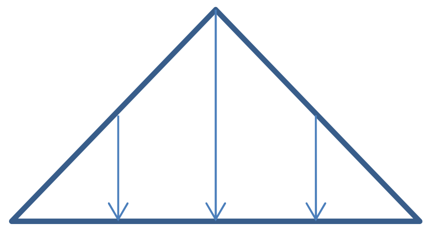
Concentrated Load : Any Force applied to an area is relatively small compared with the size of the loaded structural member. Draw as ↓ and measured usually by **N** or **kN**.

Fundamental Principles

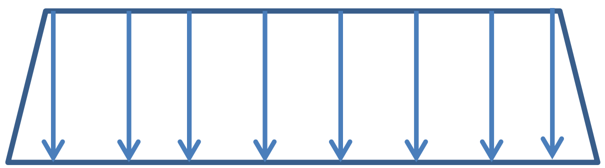
Distributed Load : A load distributed over a considerable length (or area) within the loaded structural member. Draw as



or

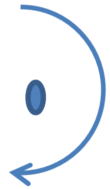


or



and measure usually by **N/m** or **kN/m** .

Fundamental Principles

Couples or Moments : Any load try to bend or twist the structural member. Draw as  and measure by N.m or kN.m.

Loads also classified as :

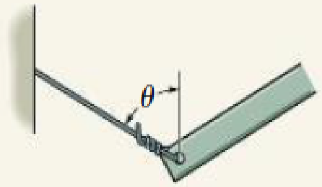
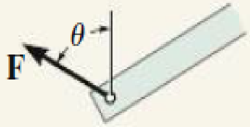
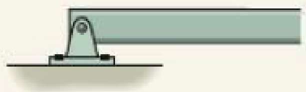
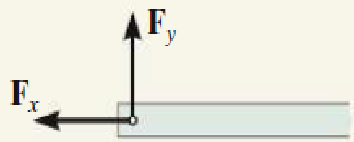
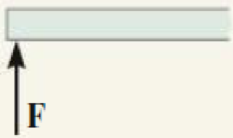
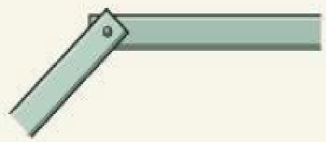
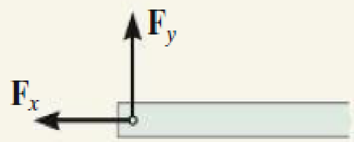
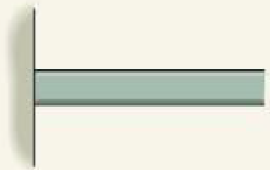
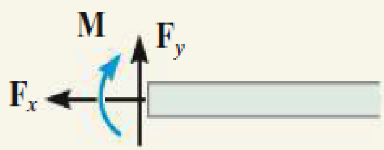
Static Load : A load slowly and steadily applied.

Impact Load : A rapidly applied load .

Repeated Load : Multiple application and removals of load, usually measured in thousands of episodes or more.

Support Reactions

TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
 Cable	 One unknown: F	 External pin	 Two unknowns: F_x, F_y
 Roller	 One unknown: F	 Internal pin	 Two unknowns: F_x, F_y
 Smooth support	 One unknown: F	 Fixed support	 Three unknowns: F_x, F_y, M

Equations of Equilibrium. Equilibrium of a body requires both a ***balance of forces***, to prevent the body from translating or having accelerated motion along a straight or curved path, and a ***balance of moments***, to prevent the body from rotating. These conditions can be expressed mathematically by two vector equations

$$\Sigma \mathbf{F} = \mathbf{0} \qquad \Sigma \mathbf{M}_O = \mathbf{0} \tag{1–1}$$

In case of three dimensions, the equilibrium equations will be

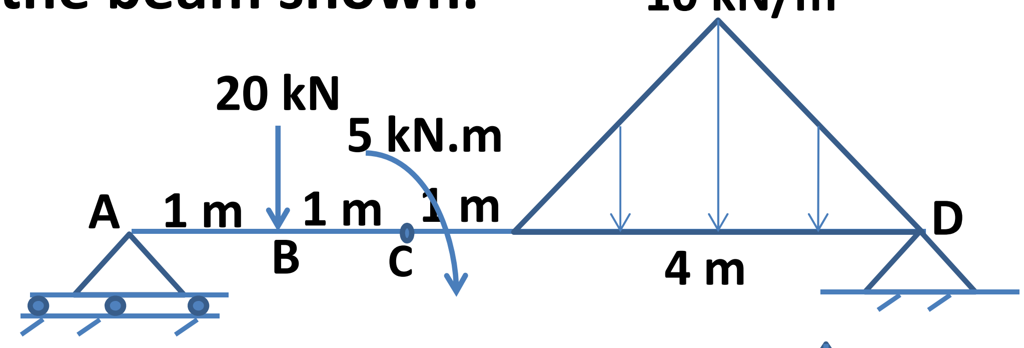
$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$
$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0 \tag{1–2}$$

In case of two dimensions, the equilibrium equations will be

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0 \tag{1–3}$$

Fundamental Principles

Ex. **Find the reactions for the beam shown.**



Solution:

$\sum M_D = 0$ **+ve**

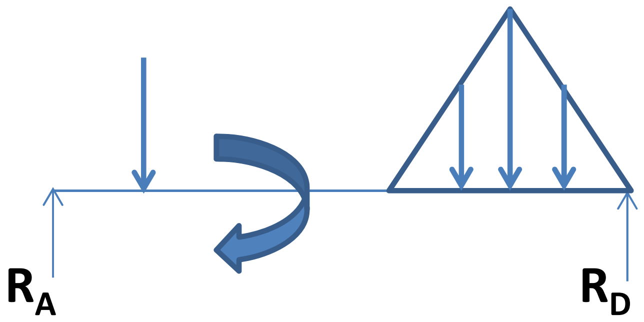
$$R_A \times 7 - 20 \times 6 + 5 - 0.5 \times 4 \times 10 \times 2 = 0$$

$$R_A = 22.143 \text{ kN} \uparrow$$

$\sum M_A = 0$ **+ve**

$$-R_D \times 7 + 20 \times 1 + 5 + 0.5 \times 4 \times 10 \times 5 = 0$$

$$R_D = 17.857 \text{ kN} \uparrow$$



F.B.D

Check: $\sum F_y = 0$

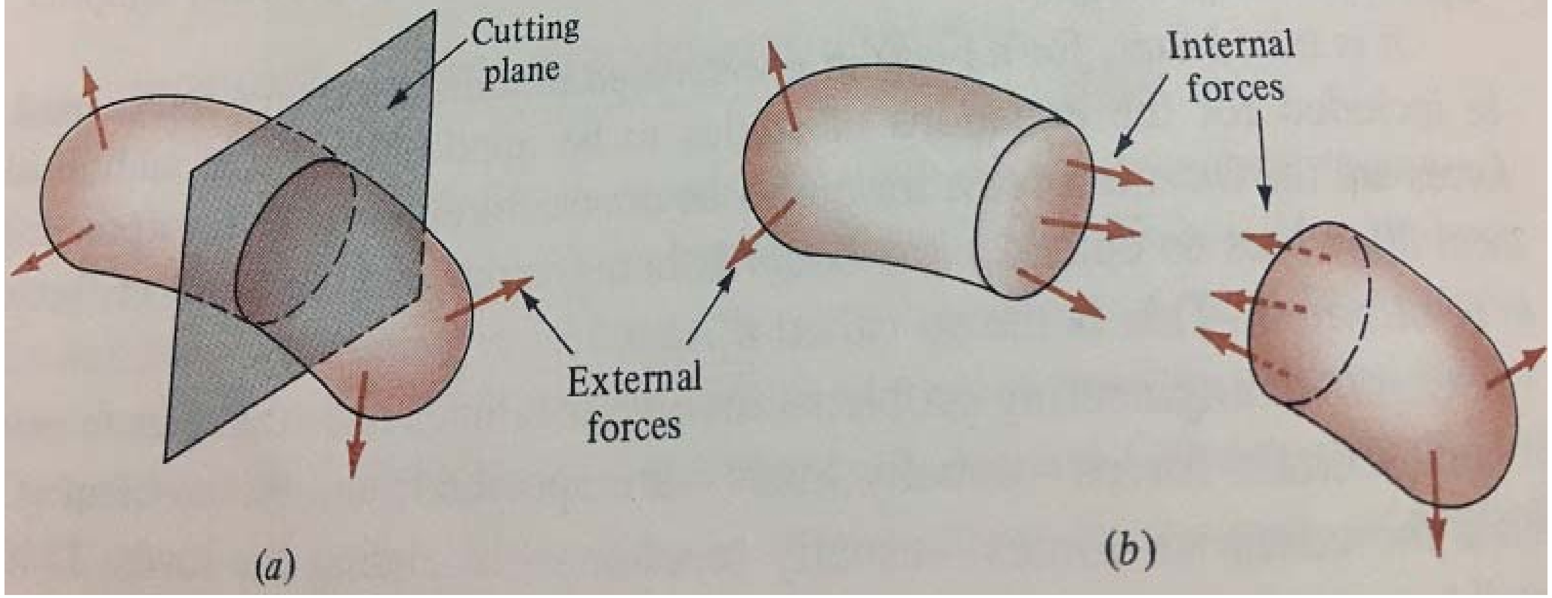
$R_A + R_D - 20 - 0.5 \times 4 \times 10 = 0$

$22.143 + 17.857 - 20 - 20 = 0$ O.K.

Fundamental Principles

Analysis of Internal Forces : Method of Sections

If the entire body is in equilibrium, any part of it must be in equilibrium. The external forces are balanced by internal forces.



Fundamental Principles

Components of Internal – Force Resultants

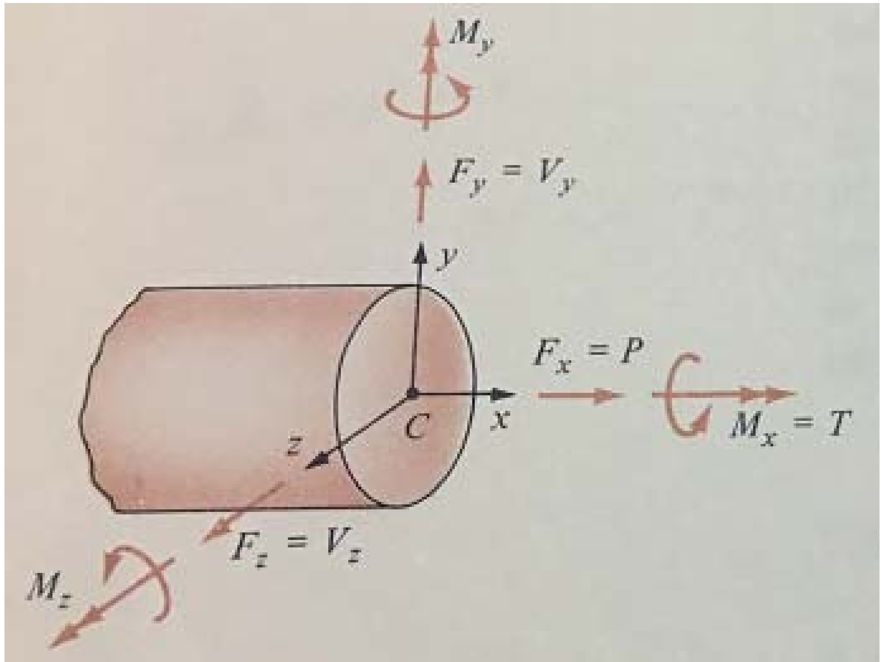
Fig. (a) :

A statically equivalent set of force vector and moments or couples vector acting at the centroid C of the cross section.

X-axis : the length of the member;

Y-axis :upward axis (e.x. thickness);

Z-axis: normal to reader (e.x. width).



(a)

Fundamental Principles

The internal forces and moments can be defined according to their effects on the member:

Axial Force F_x tends to elongate (or contract) the member and is often identified by the letter **P**. If the force acts away from the cut, it is termed as **axial tension**; if toward the cut, it is called **axial compression**.

The shear forces F_y and F_z tend to shear one part of the member relative to the adjacent part and are often designated as V_y , V_z or **V**.

Fundamental Principles

The twisting moment or **torque** M_x is responsible for twisting the member about its axis and is identified as **T**.

The bending moments M_y and M_z cause the member to bend and are often designated as **M**.

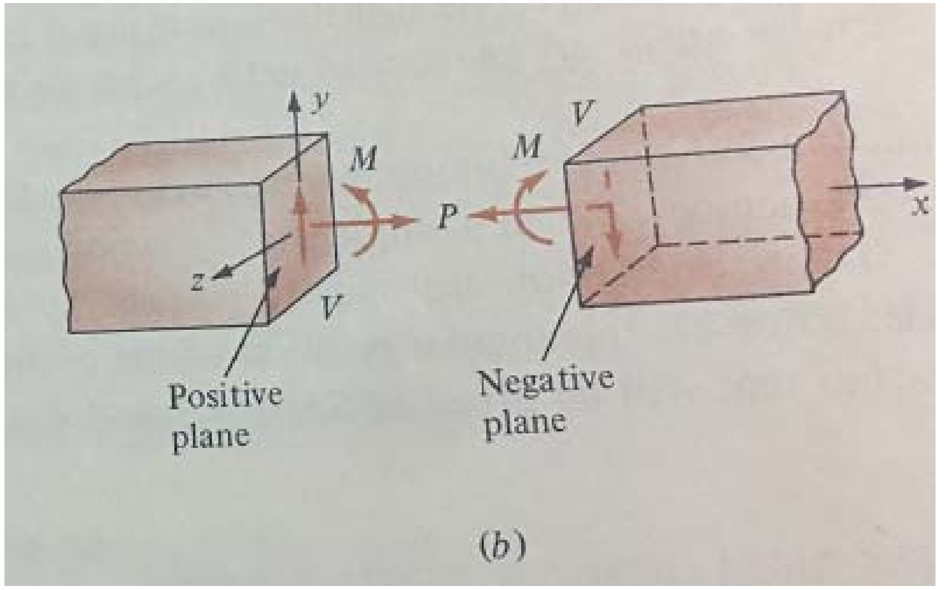
Note *:* A structural member may subject to any combination of or all of these four modes of force. The modes are usually treated separately and the results are combined to obtain the final results.

Fundamental Principles

For a components in two dimensions (xy plane):

Fig. (b):

- **Positive direction (+ve) when directed with +ve coordinate system and vice versa.**



- **Axial forces, shear forces, and bending moments acting on the faces (planes) at a cut section are equal and opposite.**

Normal force, N. This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of the body.

Shear force, V. The shear force lies in the plane of the area, and it is developed when the external loads tend to cause the two segments of the body to slide over one another.

Torsional moment or torque, T. This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.

Bending moment, M. The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.

Concept of Stress

2-1 . Simple Stress

The unit strength of a material is usually defines as the stress in the material; **σ**

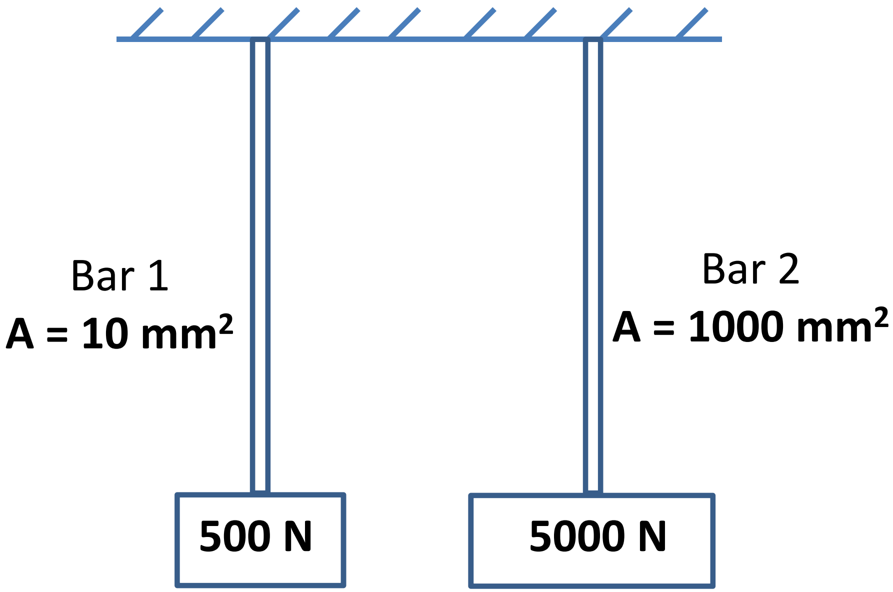
$\sigma = P / A$ [**P** = Applied Load; **A** = Cross-sectional Area]

[Units : **Pa = N/m^2**, **MPa = N/mm^2 or MN/m^2**]

[**σ** also called **Normal Stress** since **P** is normal to **A**]

Concept of Stress

Ex:



Bar 1:

$\sigma_1 = 500\ N / 10\ mm^2$

$= 50\ N/mm^2 = 50\ MPa$

or ;

$\sigma_1 = 500\ N / 10\ mm^2$

$= 500\ N / 10 \times 10^{-6}\ m^2 = 50 \times 10^6\ N/m^2 = 50\ MN/m^2$

Bar 2: $\sigma_2 = 5000\ N / 1000\ mm^2 = 5\ N/mm^2 = 5\ MPa$

$\sigma_2 = 5000\ N / 1000 \times 10^{-6}\ m^2 = 5 \times 10^6\ N/m^2 = 5\ MN/m^2$

Concept of Stress

2-2 . Average Shear Stress

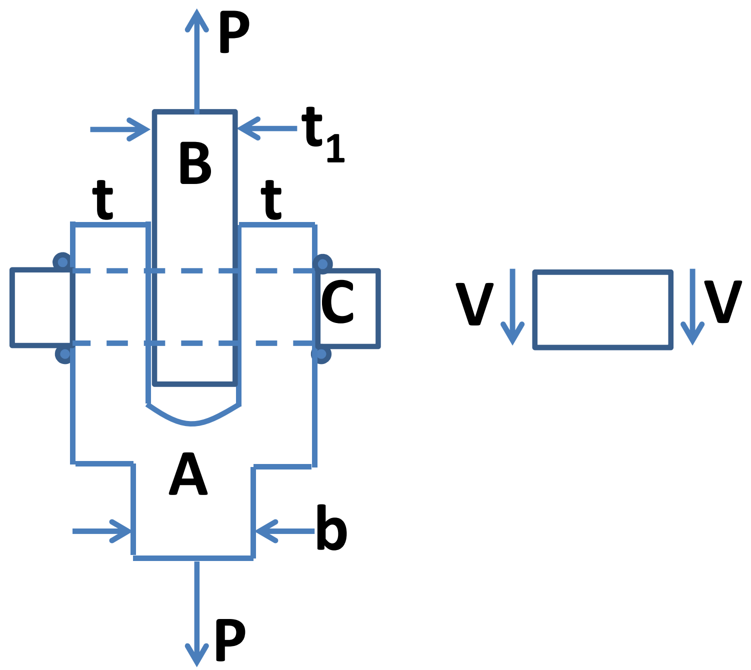
Shearing stress occur whenever the applied forces cause one section of a body to tend to slide past its adjacent section.

Ex. A clevis A, Bracket B and a pin C;

Pin is in double shear;

Shear occur over an area parallel to the applied load (direct Shear).

V = P/2



Concept of Stress

Ex: Single Shear of a rivet

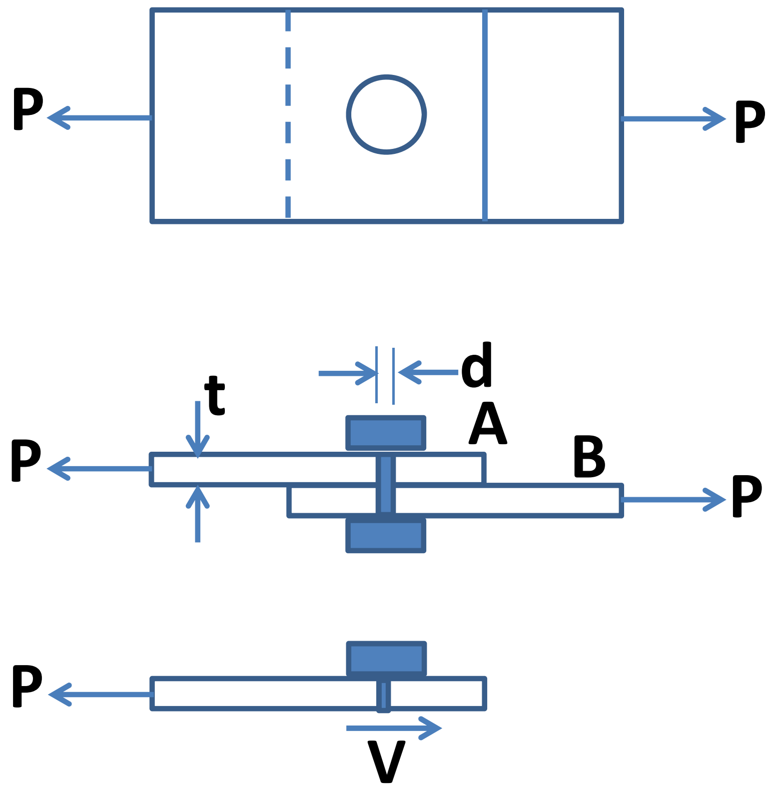
Two plates A and B are joined by a rivet.

The rivet is in single shear.

$V = P$

$\tau_{ave.} = P / (\pi.d^2 / 4)$

$\tau_{ave.} = V / (\pi.d^2 / 4)$

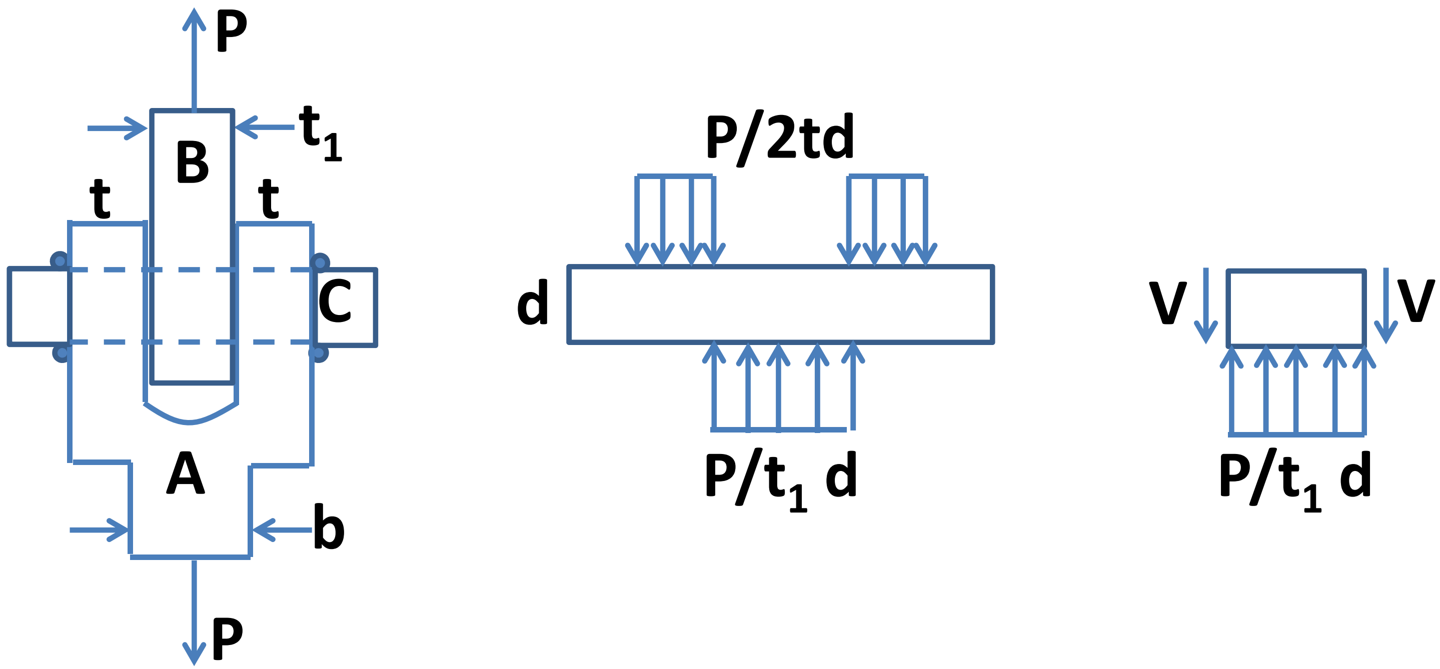


Concept of Stress

2-3 . Bearing Stress

Is the contact pressure between separate bodies.

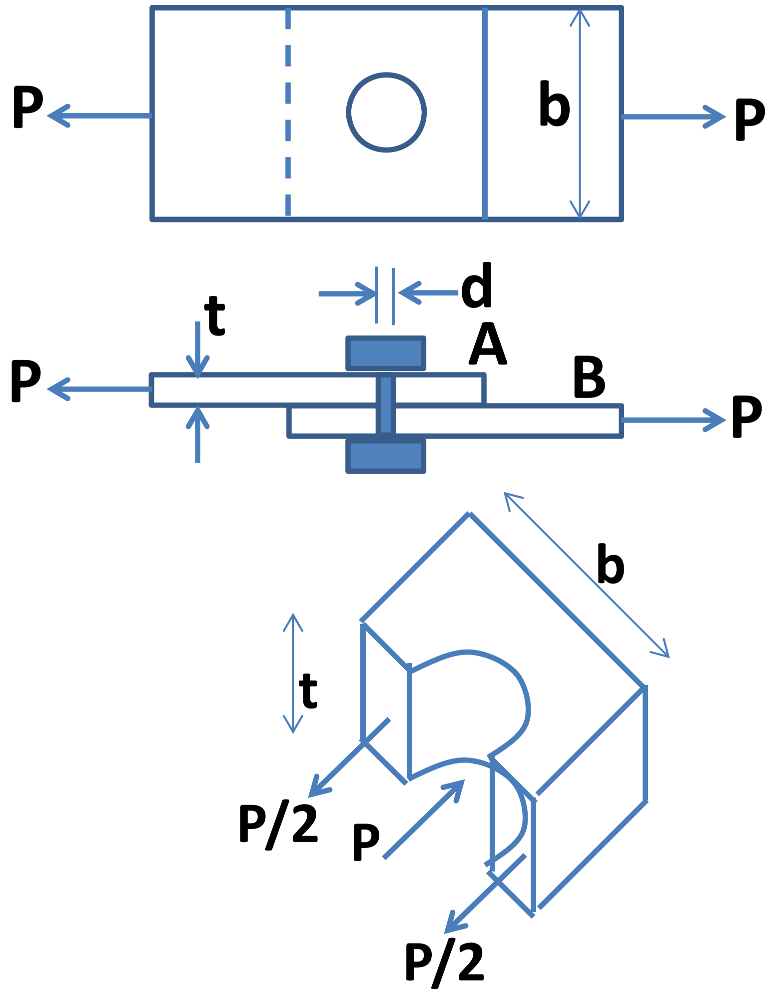
clevis : $\sigma_b = P/2td$ Bracket : $\sigma_b = P/t_1 d$



Concept of Stress

Bearing stress

$\sigma_b = P/td$



Note:

For normal Stress

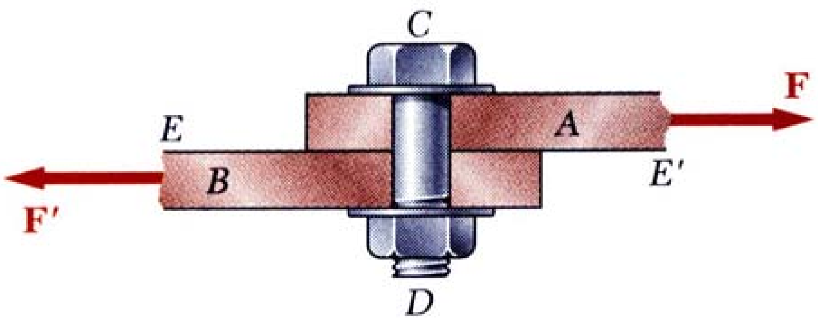
- At a section through the hole

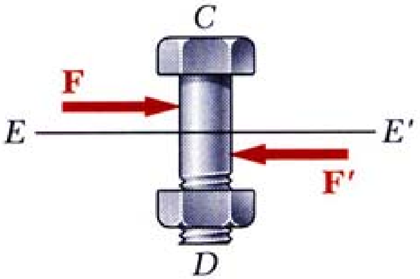
  $\sigma = P / [\ t\ (b-d)\ ]$

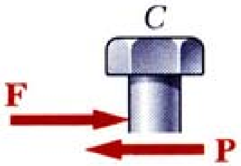
- At any other section

  $\sigma = P / (t\ b)$

Single Shear

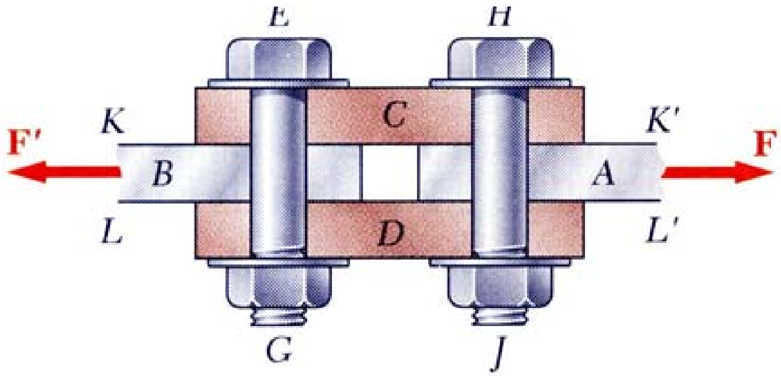


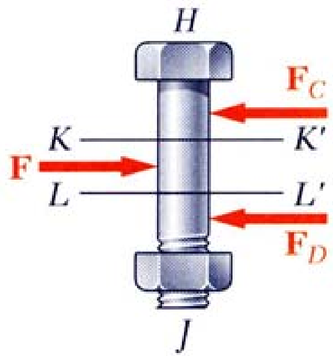


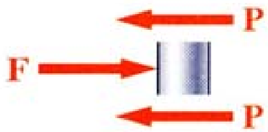


$$\tau_{ave} = \frac{P}{A} = \frac{F}{A}$$

Double Shear

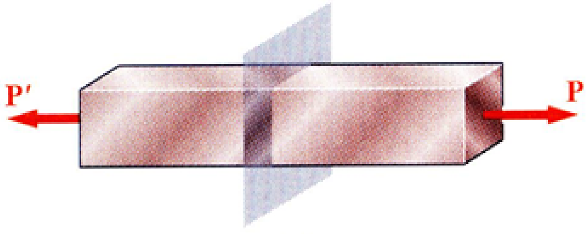




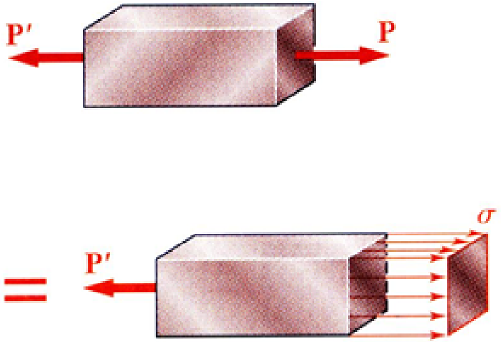


$$\tau_{ave} = \frac{P}{A} = \frac{F}{2A}$$

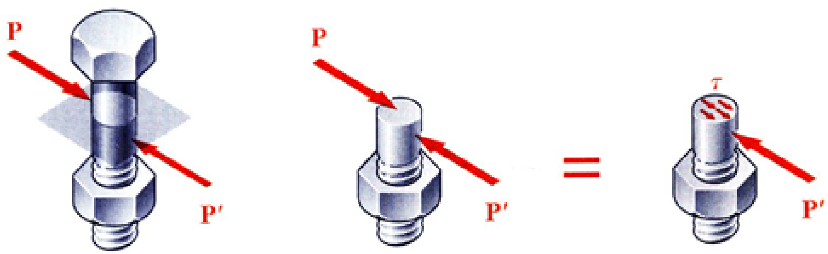
Stress in Two Force Members



(a)

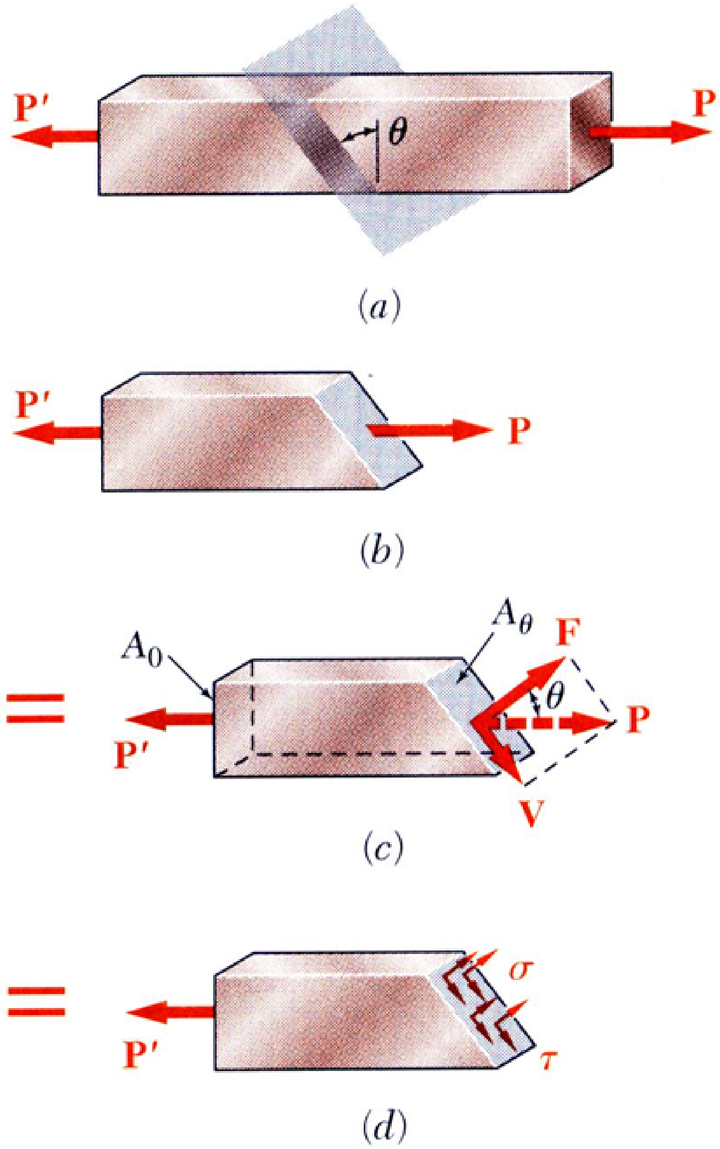


(b)



- Axial forces on a two force member result in only normal stresses on a plane cut perpendicular to the member axis.
- Transverse forces on bolts and pins result in only shear stresses on the plane perpendicular to bolt or pin axis.
- Will show that either axial or transverse forces may produce both normal and shear stresses with respect to a plane other than one cut perpendicular to the member axis.

Stress on an Oblique Plane



- Pass a section through the member forming an angle θ with the normal plane.

- From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force P.

- Resolve P into components normal and tangential to the oblique section,

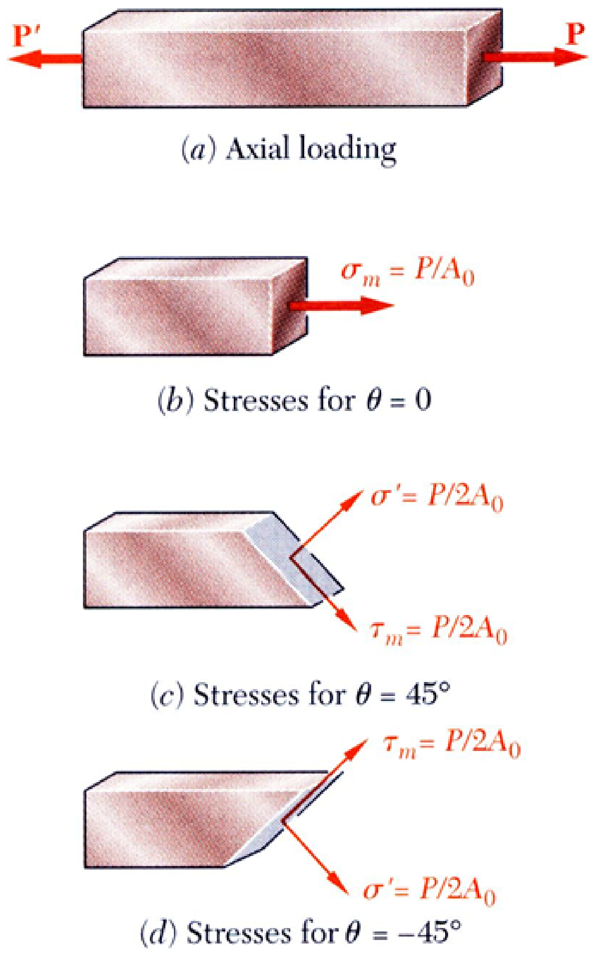
$$F = P\cos\theta \qquad V = P\sin\theta$$

- The average normal and shear stresses on the oblique plane are

$$\sigma = \frac{F}{A_\theta} = \frac{P\cos\theta}{A_0 / \cos\theta} = \frac{P}{A_0}\cos^2\theta$$

$$\tau = \frac{V}{A_\theta} = \frac{P\sin\theta}{A_0 / \cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta$$

Maximum Stresses



- Normal and shearing stresses on an oblique plane

$$\sigma = \frac{P}{A_0}\cos^2\theta \quad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

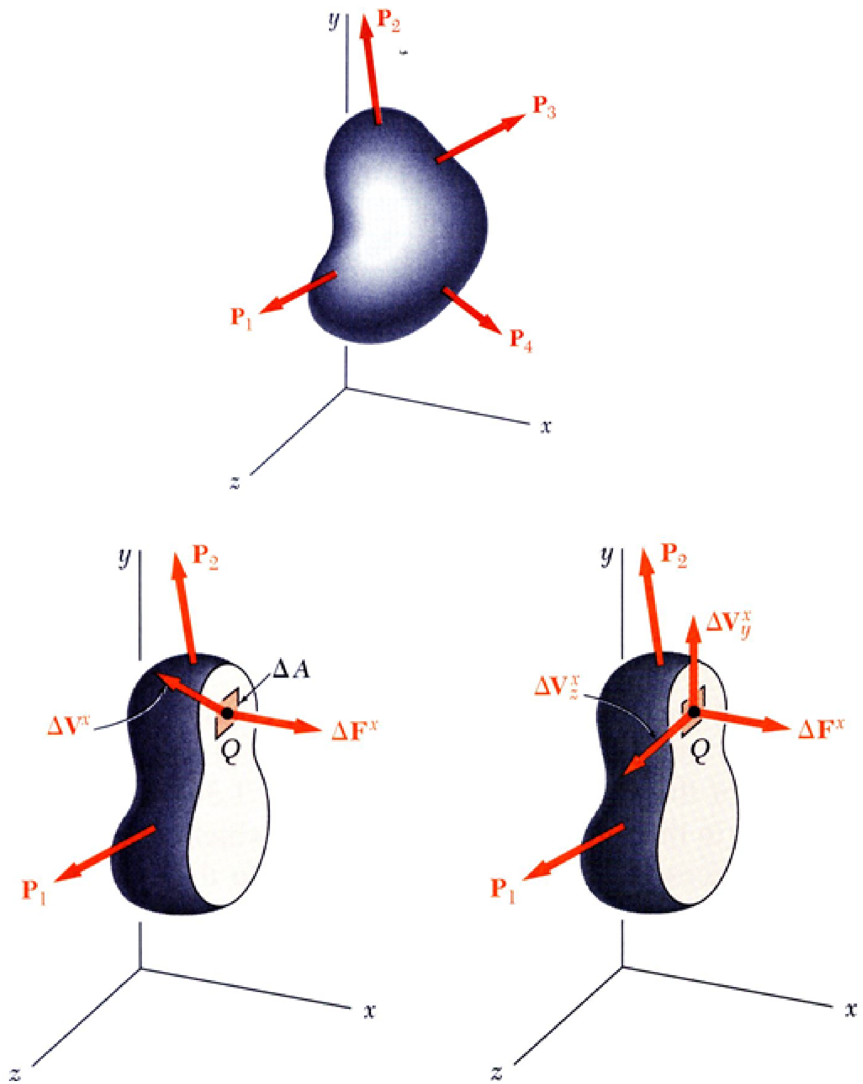
- The maximum normal stress occurs when the reference plane is perpendicular to the member axis,

$$\sigma_m = \frac{P}{A_0} \quad \tau' = 0$$

- The maximum shear stress occurs for a plane at $\pm$ 45° with respect to the axis,

$$\tau_m = \frac{P}{A_0}\sin 45 \cos 45 = \frac{P}{2A_0} = \sigma'$$

Stress Under General Loadings



- A member subjected to a general combination of loads is cut into two segments by a plane passing through Q

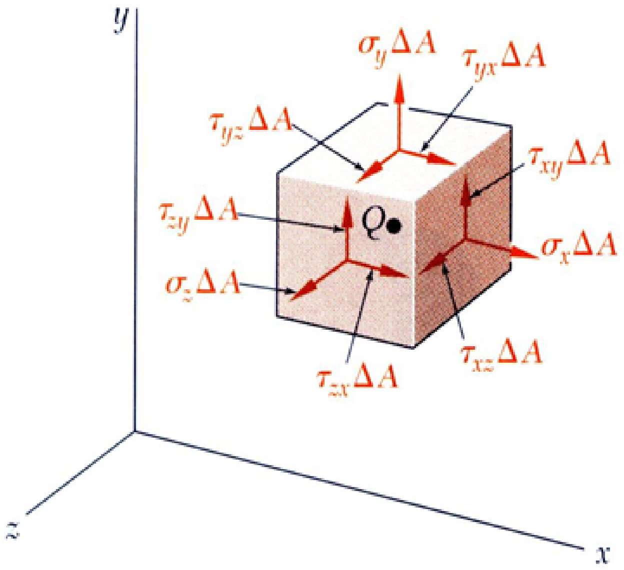
- The distribution of internal stress components may be defined as,

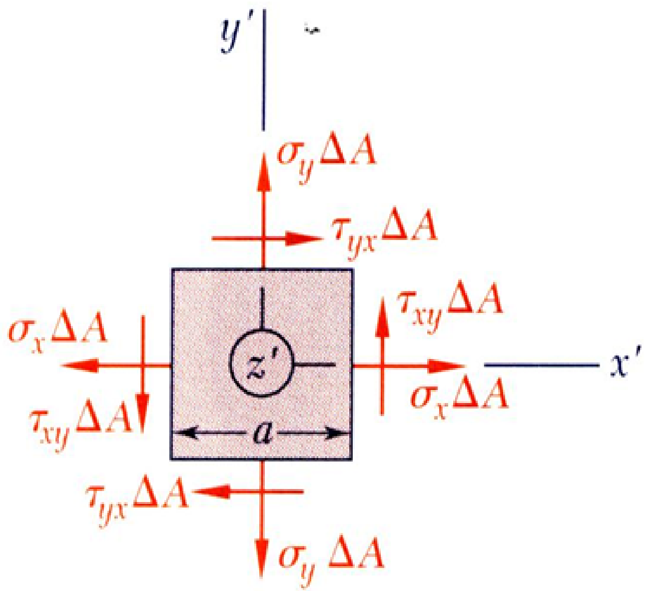
$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

- For equilibrium, an equal and opposite internal force and stress distribution must be exerted on the other segment of the member.

State of Stress





- Stress components are defined for the planes cut parallel to the *x*, *y* and *z* axes. For equilibrium, equal and opposite stresses are exerted on the hidden planes.

- The combination of forces generated by the stresses must satisfy the conditions for equilibrium:

$$\sum F_x = \sum F_y = \sum F_z = 0$$

$$\sum M_x = \sum M_y = \sum M_z = 0$$

- Consider the moments about the *z* axis:

$$\sum M_z = 0 = (\tau_{xy} \Delta A)a - (\tau_{yx} \Delta A)a$$

$$\tau_{xy} = \tau_{yx}$$

similarly, $\tau_{yz} = \tau_{zy}$ and $\tau_{yz} = \tau_{zy}$

- It follows that only 6 components of stress are required to define the complete state of stress

Factor of Safety

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

FS = Factor of safety

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to structures integrity
- risk to life and property
- influence on machine function

Simple stress and strains

- ***Introduction***

  \- In the preceding chapters, the **strength** of the material was discussed, i.e., the relations between load, area and stress.

  \- In this chapter, **deformations** is the major concern.

  \- **Deformation** is the change in shape that accompany a loading.

  \- **Axially loaded bodies** will only be studied.

Simple stress and strains

- **Elasticity**

  - when a force (or a system of forces) acts on a body, it undergoes some deformations and the molecules offer some resistance to the deformations.

  - When the external force is removed, the force of resistance also vanishes; and the body returns to its original shape only when the deformation is within a certain limit. Such limit is called ***elastic limit***.

  - ***Elastic limit*** is the stress beyond which the material will not return to its original shape when unloaded but will retain a permanent deformation.

Simple stress and strains

- **Elasticity** is the property of certain materials of returning back to their original position, after removing the external force.
- **Perfectly elastic body** is that body which returns back completely to its original shape and size, after the removal of external forces.
- **Partially elastic body** is that body which does not return back completely to its original shape and size, after the removal of external forces (**When the force causes deformation within the elastic limit**).
- ***Note:*** Beyond the elastic limit (**When the force causes deformation beyond the elastic limit**), the body will not return to its original shape and size (the body gets into the **plastic stage**), and some residual deformation to the body will remain permanently.

Simple stress and strains

- ***Stress***

  Is the resistance (**of molecules of the body**) per unit area to deformation.

  Mathematically stress is the force per unit area. i.e.

  $\sigma = P / A$

  Where **P**= Load or force acting on the body, and

  A= Cross sectional area of the body.

In **S.I. system**, the unit of stress is **Pascal** (**Pa**) (**Pa= 1 N/m2**). Bigger units is (**MPa = N/mm2**) and (**GPa= kN/mm2**)

Simple stress and strains

- ***Strain***

  Is the deformation per unit length.

  Mathematically **$\varepsilon = \delta L / L$**

  Where **δL** = Change in length of the body, and

  L = Original length of the body.

Notes:

1. When a body is subjected to two **opposite tensile forces**, the length of the body will increase. The induced stress is called a **tensile stress** and the corresponding strain is called **tensile strain**.
2. When a body is subjected to two **opposite compressive forces**, the length of the body will decrease. The induced stress is called a **compressive stress** and the corresponding strain is called **compressive strain**.

Simple stress and strains

- ***Hook's Law***

  It states, " **when a material is loaded, within its elastic limit, the stress is proportional to the strain**''.
  Mathematically,

  (stress / strain)= **E** = constant

  or; **σ** α **ε** ⟹ **σ** = **E ε**

  and **E** = (**σ / ε**) = A constant of proportionality Known as modulus of elasticity or Young's modulus. Numerically, it is the tensile stress, which when applied to a uniform bar will increase its length to double the original length if the material of the bar could remain perfectly elastic throughout such an excessive strain.

Hooke's Law: Modulus of Elasticity

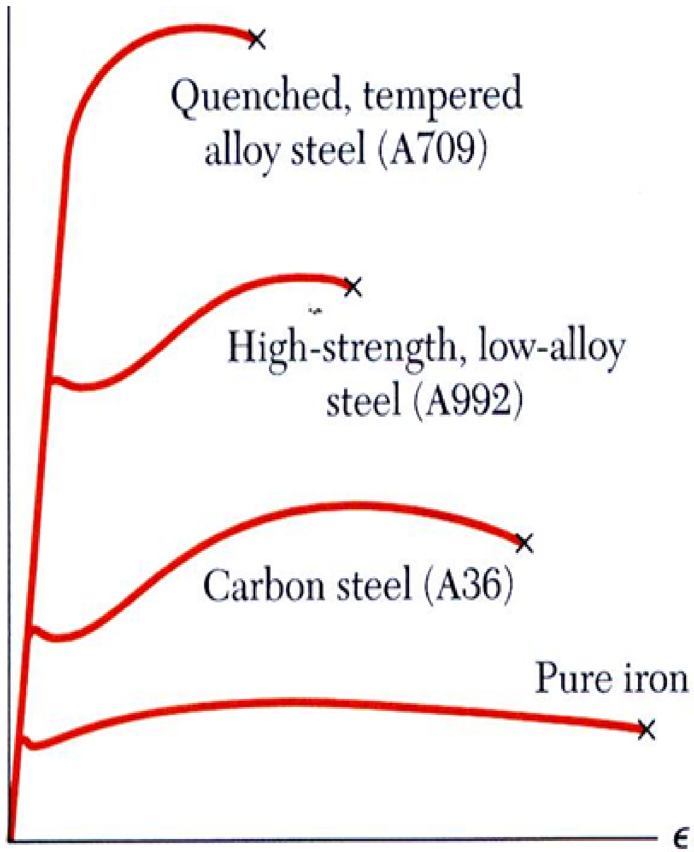


Fig. 2.16 Stress-strain diagrams for iron and different grades of steel.

- Below the yield stress

  $\sigma = E\varepsilon$

  E = Youngs Modulus or Modulus of Elasticity

- Strength is affected by alloying, heat treating, and manufacturing process but stiffness (Modulus of Elasticity) is not.

Elastic vs. Plastic Behavior

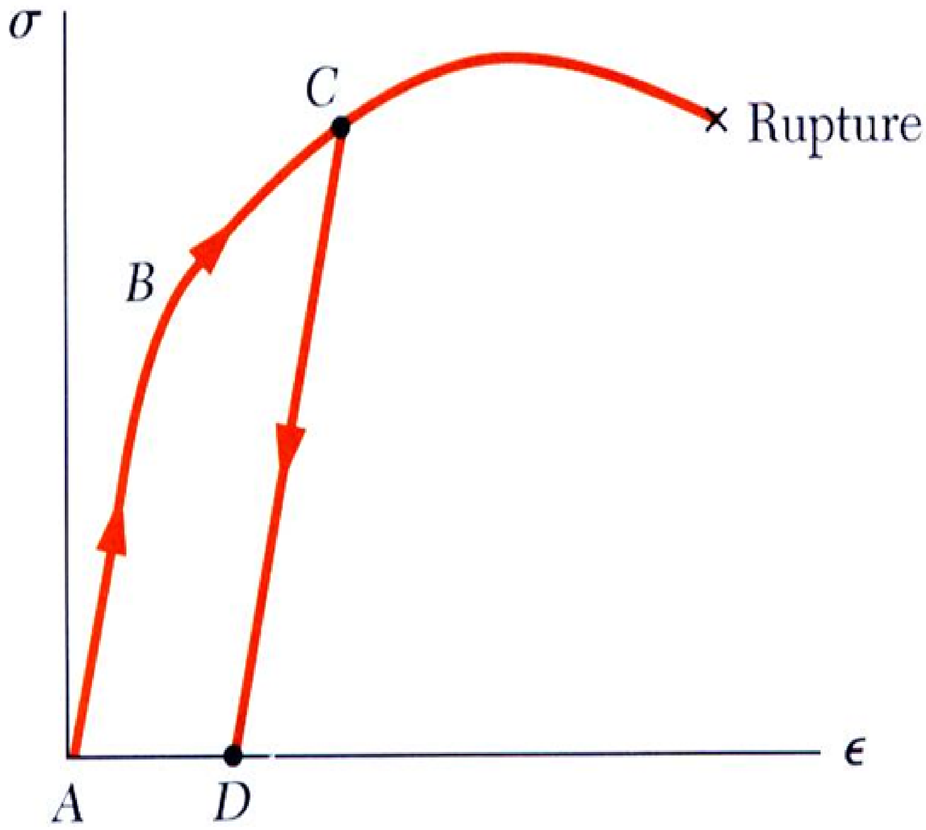


Fig. 2.18

- If the strain disappears when the stress is removed, the material is said to behave *elastically.*
- The largest stress for which this occurs is called the *elastic limit.*
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically.*

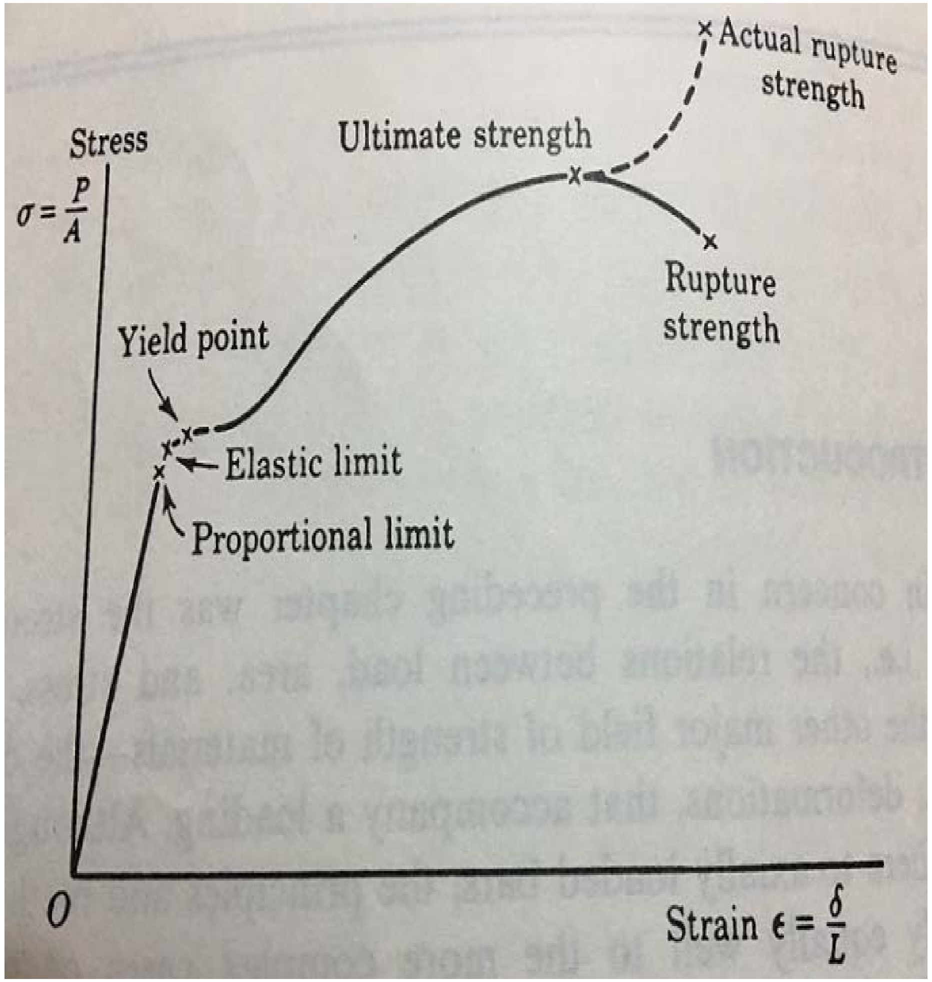
Simple stress and strains

The values of E (*i.e.*, modulus of elasticity) of materials, in every day use, are given below :

S.No.	*Material*	*Modulus of elasticity (E) in GPa i.e. GN/m² or kN/mm²*
1.	Steel	200 to 220
2.	Wrought iron	190 to 200
3.	Cast iron	100 to 160
4.	Copper	90 to 110
5.	Brass	80 to 90
6.	Aluminium	60 to 80
7.	Timber	10

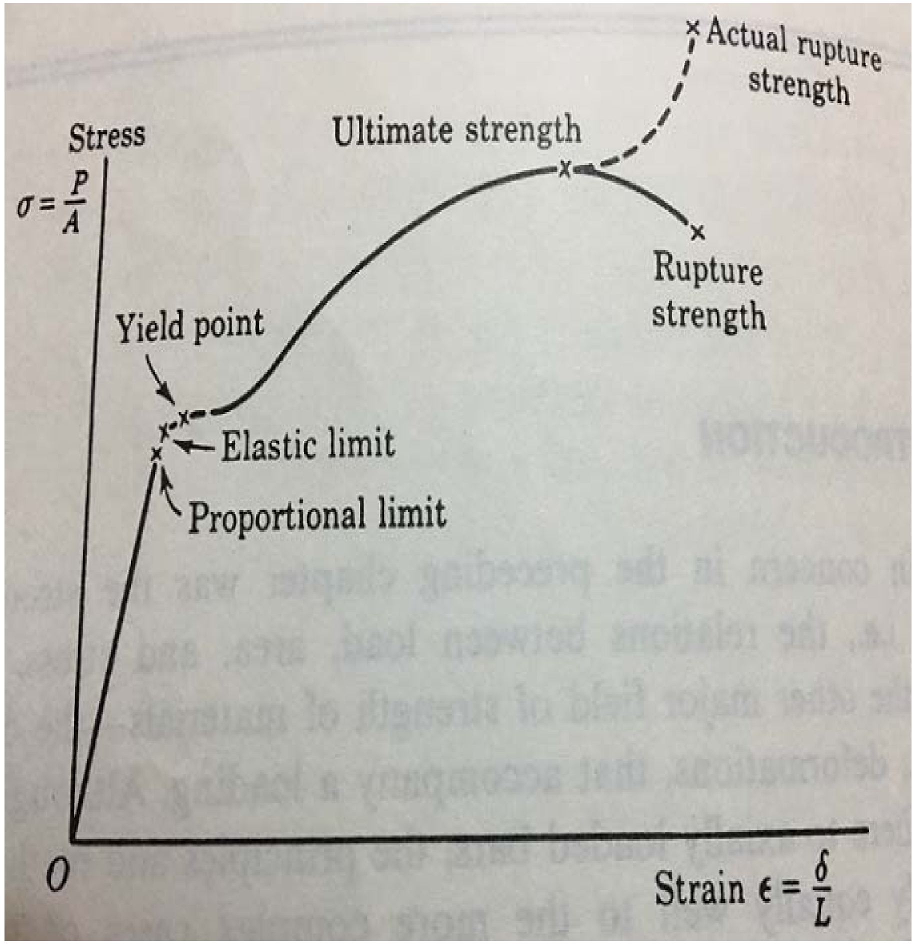
Simple stress and strains

- ***Stress Strain Diagram***
- A standard specimen (a mild steel bar) is subjected to a gradually increasing pull (as applied by universal testing machine).
- Stress-strain diagram is a straight line between O and the point of **proportional limit**, i.e., the stress is proportional to strain.



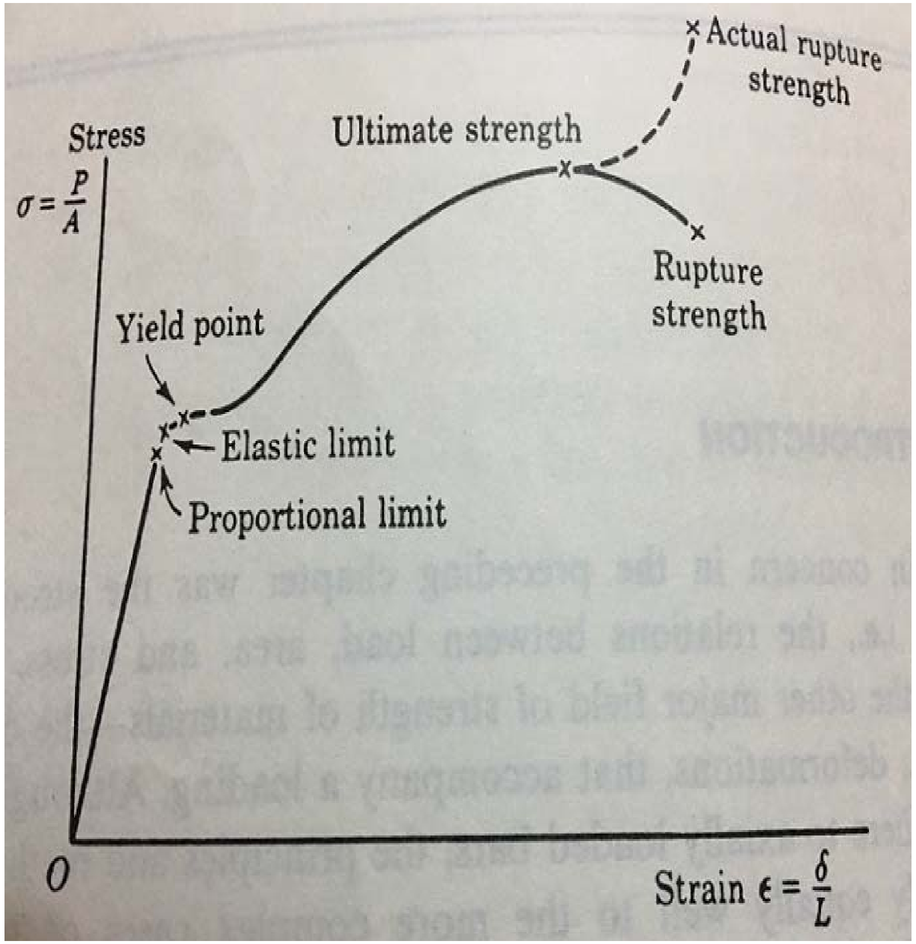
Simple stress and strains

- **The proportional limit** is important because all subsequent theory involving the behaviour of elastic bodies is based upon a stress-strain proportionality.
- Beyond the point of proportional limit, the stress is no longer proportional to strain.



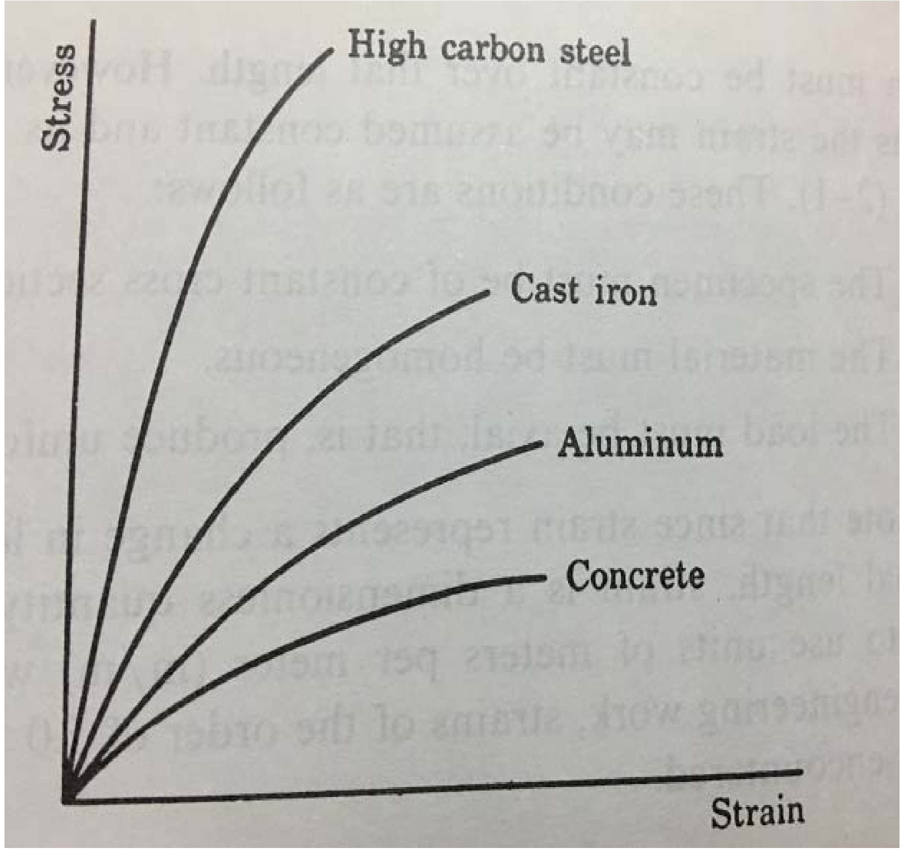
Simple stress and strains

- ***Elastic limit*** is the stress beyond which the material will not return to its original shape when unloaded but will retain a permanent deformation.
- ***Yield Point*** is the point at which there is an appreciable elongation or yielding of the material without any corresponding increase of load.



Simple stress and strains

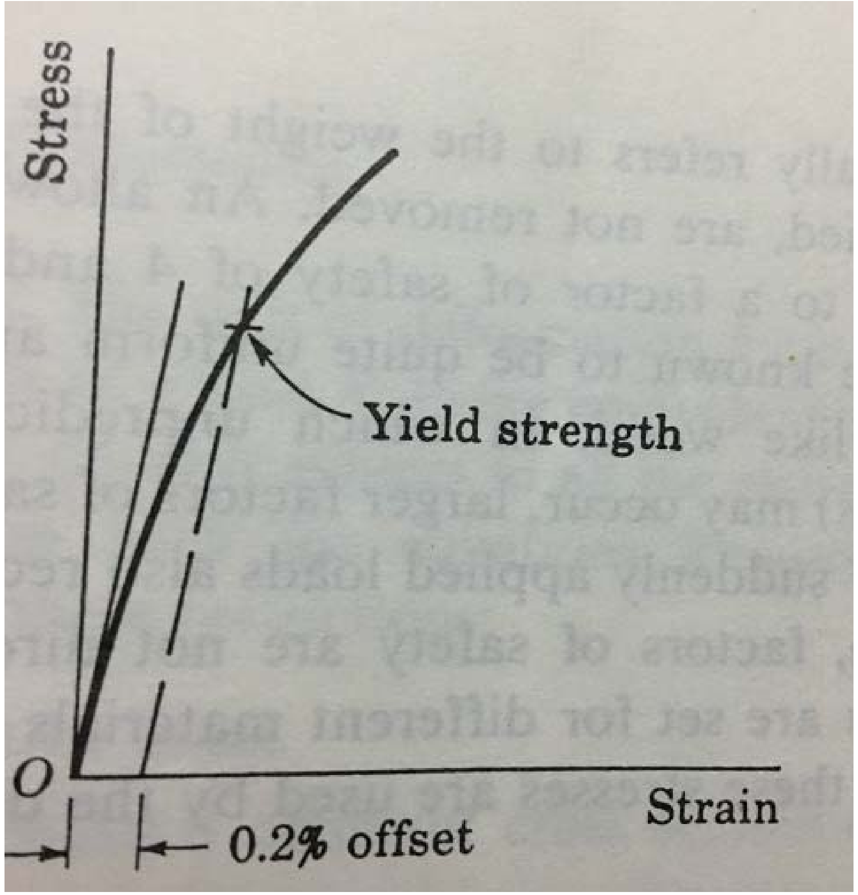
- The phenomenon of yielding is peculiar to structural steel; other grades and steel alloys or other materials do not possess it.
- For materials with no yield point, the yield strength is determined by the ***offset method***.



Simple stress and strains

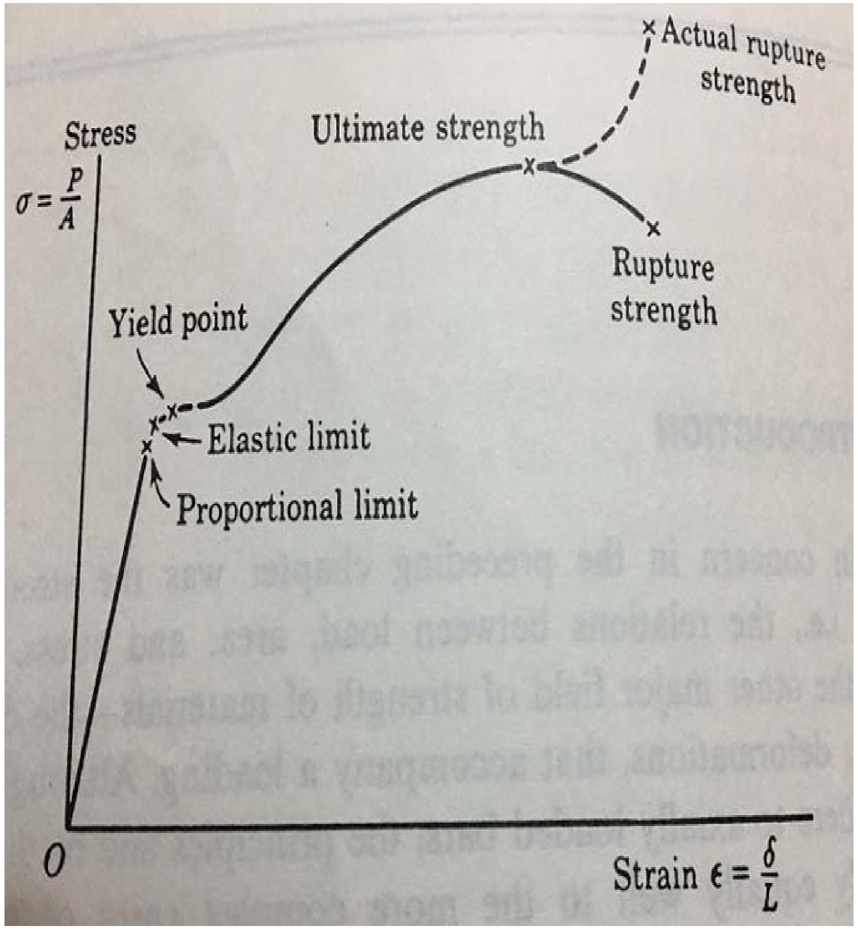
- ***Offset Method***

  Consist of drawing a line parallel to the initial tangent of the stress–strain curve, at an arbitrary offset strain, usually of **0.2% or 0.002 m/m**. The intersection of this line with the stress–strain curve is called **yield strength**.



Simple stress and strains

- **Ultimate stress** or **ultimate strength** is the highest ordinate on the stress-strain curve.
- **Rupture strength** or the **stress at failure**; is usually lower than the ultimate strength because it is obtained by dividing the rupture load by the original cross-sectional area, **which is incorrect**. This is caused by a phenomenon known as **necking**.

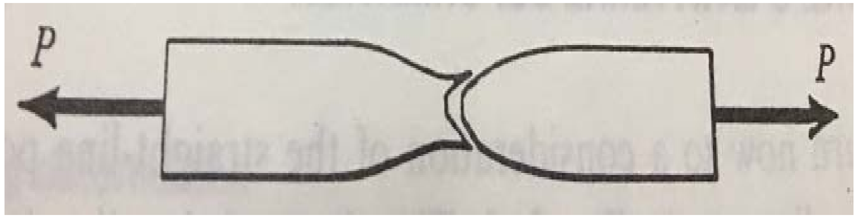


Simple stress and strains

- ***Necking Phenomenon***

  As failure occurs, the material stretches very rapidly and simultaneously narrows down, so that the rapture load is actually distributed over a smaller area. If the rupture area is measured after failure occurs, and divided into the rupture load, the result is a true value of the actual failure stress.

Note: the ultimate strength is commonly taken as the maximum stress of the material.



Stress-Strain Test

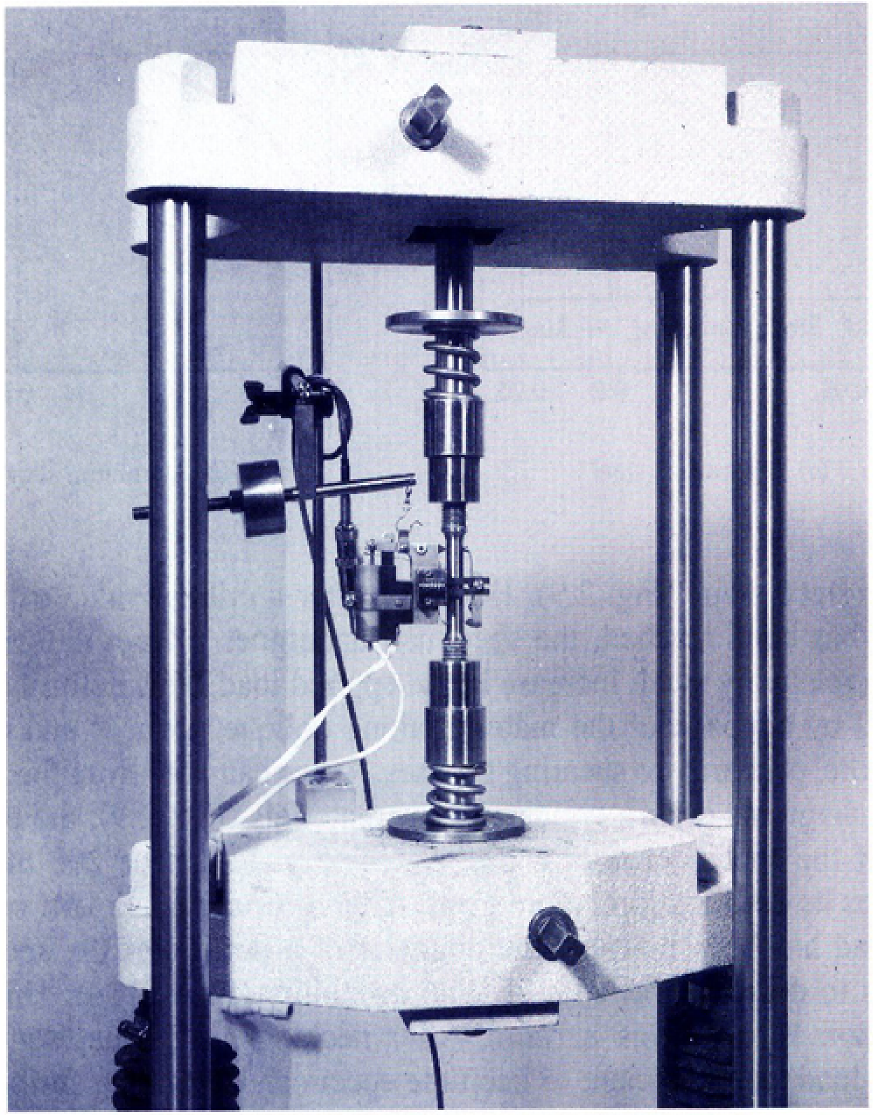


Fig. 2.7 This machine is used to test tensile test specimens, such as those shown in this chapter.

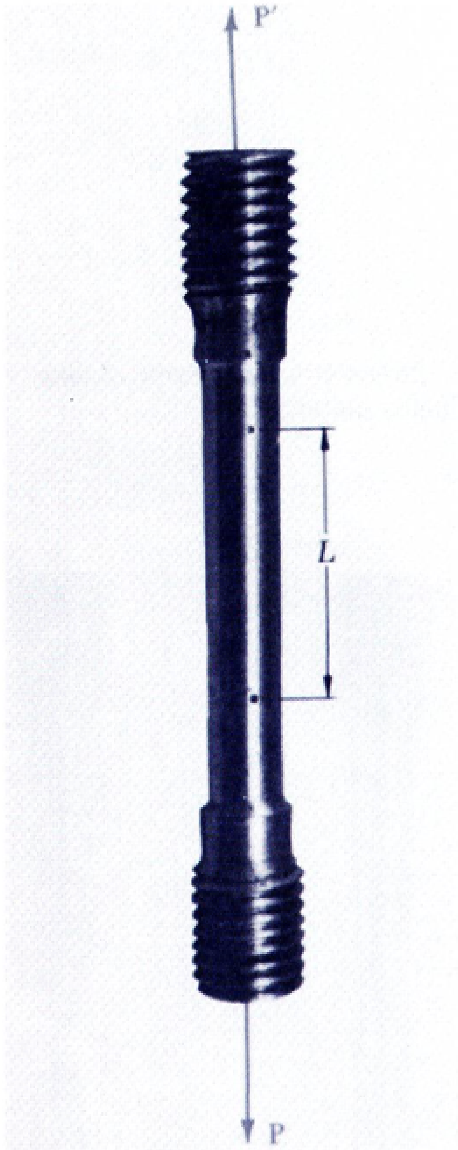
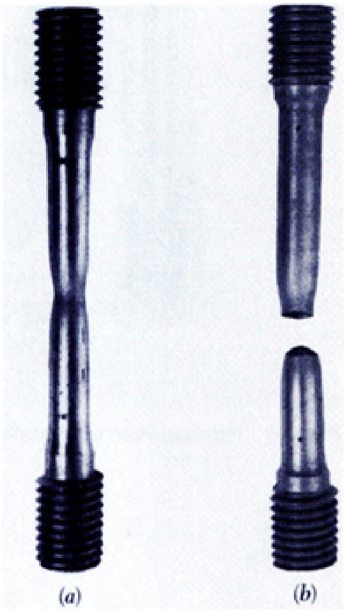
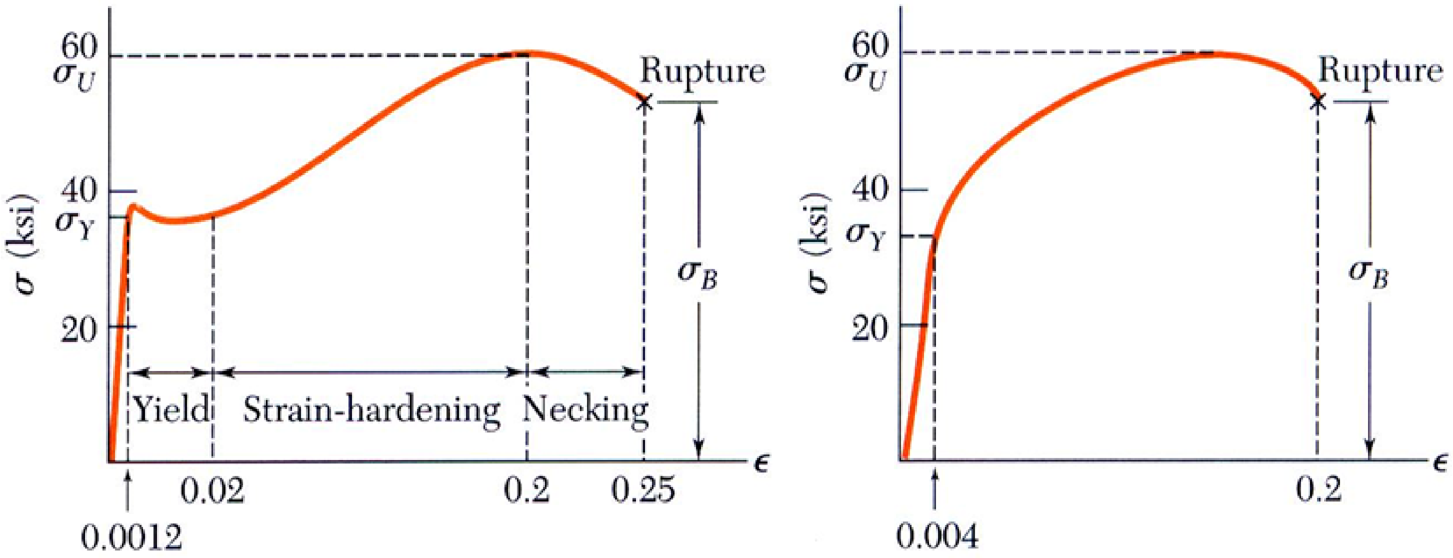


Fig. 2.8 Test specimen with tensile load.

Stress-Strain Diagram: Ductile Materials





(a) Low-carbon steel

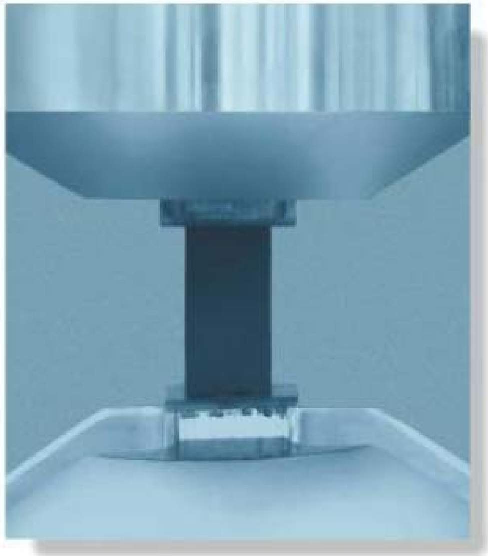
(b) Aluminum alloy

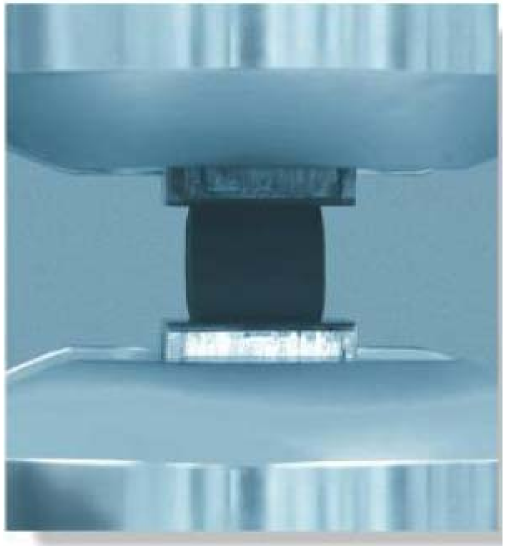
Poisson's Ratio

When a deformable body is subjected to an axial tensile force, not only does it elongate but it also contracts laterally. For example, if a rubber band is stretched, it can be noted that both the thickness and width of the band are decreased. Likewise, a compressive force acting on a body causes it to contract in the direction of the force and yet its sides expand laterally.

Consider a bar having an original radius r and length L and subjected to the tensile force P in Figure . This force elongates the bar by an amount δ, and its radius contracts by an amount δ'. Strains in the longitudinal or axial direction and in the lateral or radial direction are, respectively,

$$\epsilon_{\text{long}} = \frac{\delta}{L} \quad \text{and} \quad \epsilon_{\text{lat}} = \frac{\delta'}{r}$$





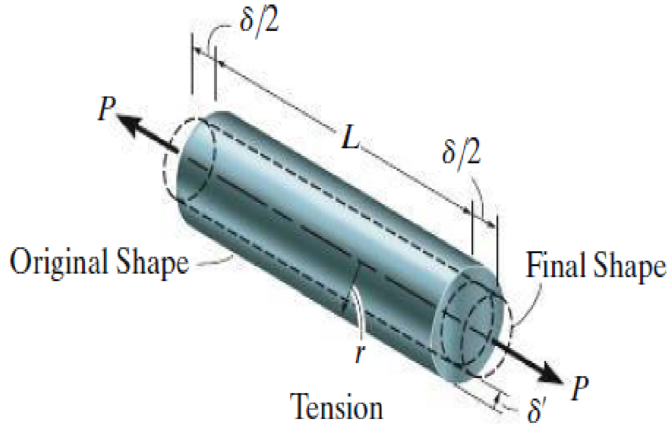


Fig.

In the early 1800s, the French scientist S. D. Poisson realized that within the *elastic range* the *ratio* of these strains is a *constant*, since the deformations δ and δ' are proportional. This constant is referred to as ***Poisson's ratio***, ν (nu), and it has a numerical value that is unique for a particular material that is both *homogeneous and isotropic*. Stated mathematically it is

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} \tag{3–9}$$

The negative sign is included here since *longitudinal elongation* (positive strain) causes *lateral contraction* (negative strain), and vice versa. Notice that these strains are caused only by the axial or longitudinal force P; i.e., no force or stress acts in a lateral direction in order to strain the material in this direction.

Poisson's ratio is a *dimensionless* quantity, and for most nonporous solids it has a value that is generally between $\frac{1}{4}$ and $\frac{1}{3}$. Typical values of ν for common engineering materials are listed on the inside back cover. For an "ideal material" having no lateral deformation when it is stretched or compressed, Poisson's ratio will be 0. Furthermore, it will be shown in Sec. 10.6 that the *maximum* possible value for Poisson's ratio is 0.5. Therefore $0 \le \nu \le 0.5$.

Simple stress and strains

- ***Working Stress and Safety Factor***

working stress: is the actual stress the material has when under load.

allowable stress : Is the maximum safe stress a material may carry.

working stress = **allowable stress** = σ_w

$\boldsymbol{\sigma_w}$ is obtained by dividing the yield stress or the ultimate stress by a suitable number N, called the safety factor.

$\sigma_w = \sigma_{yp} / N_{yp}$ or $\sigma_w = \sigma_{ult} / N_{ult}$

$\mathbf{N_{yp}}$ is the safety factor against yielding, and $\mathbf{N_{ult}}$ is the safety factor against ultimate stress.

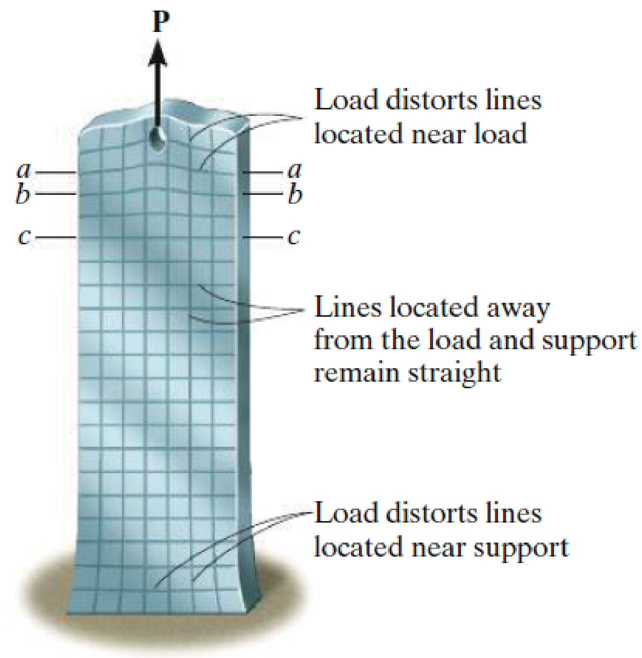
Axially Loaded Members

CHAPTER OBJECTIVES

- In Chapter 1, we developed the method for finding the normal stress in axially loaded members. In this chapter we will discuss how to determine the deformation of these members, and we will also develop a method for finding the support reactions when these reactions cannot be determined strictly from the equations of equilibrium. An analysis of the effects of thermal stress, stress concentrations, inelastic deformations, and residual stress will also be discussed.

Saint-Venant's Principle

In the previous chapters, we have developed the concept of *stress* as a means of measuring the force distribution within a body and *strain* as a means of measuring a body's deformation. We have also shown that the mathematical relationship between stress and strain depends on the type of material from which the body is made. In particular, if the material behaves in a linear elastic manner, then Hooke's law applies, and there is a proportional relationship between stress and strain.



(a)

Fig. 4–1

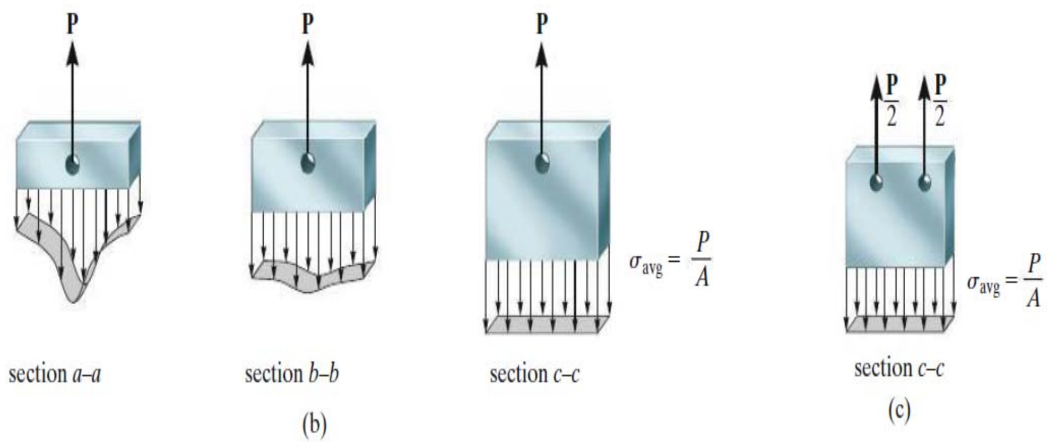


Fig. 4–1 (cont.)

Elastic Deformation of an Axially Loaded Member

Using Hooke's law and the definitions of stress and strain, we will now develop an equation that can be used to determine the *elastic* displacement of a member subjected to axial loads. To generalize the development, consider the bar shown in Fig. 4–2*a*, which has a cross-sectional area that *gradually* varies along its length L. The bar is subjected to concentrated loads at its ends and a variable external load distributed along its length. This distributed load could, for example, represent the weight of the bar if it is in the vertical position, or friction forces acting on the bar's surface. Here we wish to find the ***relative displacement*** δ (delta) of one end of the bar with respect to the other end as caused by this loading. We will *neglect* the localized deformations that occur at points of concentrated loading and where the cross section suddenly changes. From Saint-Venant's principle, these effects occur within small regions of the bar's length and will therefore have only a slight effect on the final result. For the most part, the bar will deform uniformly, so the normal stress will be uniformly distributed over the cross section.

Using the method of sections, a differential element (or wafer) of length dx and cross-sectional area $A(x)$ is isolated from the bar at the arbitrary position x. The free-body diagram of this element is shown in Fig. 4–2*b*. The resultant internal axial force will be a function of x since the external distributed loading will cause it to vary along the length of the bar. This load, $P(x)$, will deform the element into the shape indicated by the dashed outline, and therefore the displacement of one end of the element with respect to the other end is $d\delta$. The stress and strain in the element are

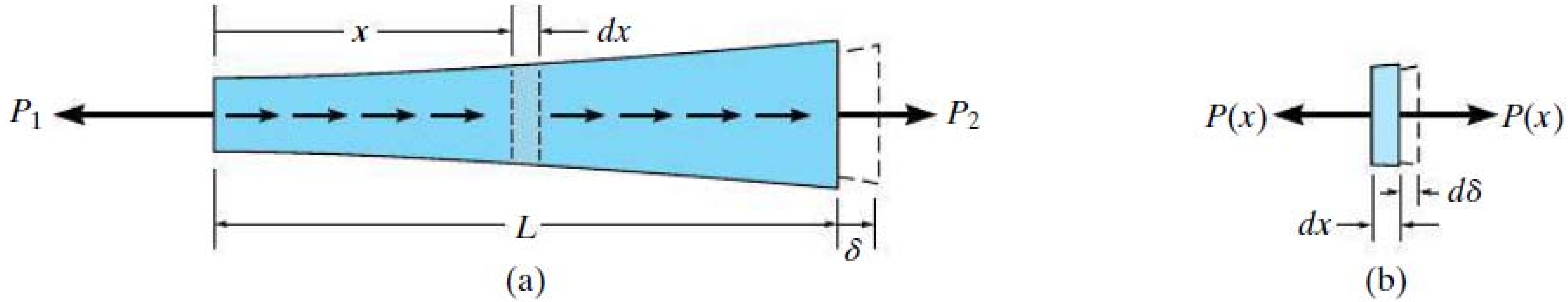


Fig. 4–2

$$\sigma = \frac{P(x)}{A(x)} \quad \text{and} \quad \epsilon = \frac{d\delta}{dx}$$

Provided the stress does not exceed the proportional limit, we can apply Hooke's law; i.e.,

$$\sigma = E(x)\epsilon$$

$$\frac{P(x)}{A(x)} = E(x)\left(\frac{d\delta}{dx}\right)$$

$$d\delta = \frac{P(x)dx}{A(x)E(x)}$$

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E(x)} \tag{4–1}$$

where

δ = displacement of one point on the bar relative to the other point
L = original length of bar
$P(x)$ = internal axial force at the section, located a distance x from one end
$A(x)$ = cross-sectional area of the bar expressed as a function of x
$E(x)$ = modulus of elasticity for the material expressed as a function of x.

Constant Load and Cross-Sectional Area. In many cases the bar will have a constant cross-sectional area A; and the material will be homogeneous, so E is constant. Furthermore, if a constant external force is applied at each end, Fig. 4–3, then the internal force P throughout the length of the bar is also constant. As a result, Eq. 4–1 can be integrated to yield

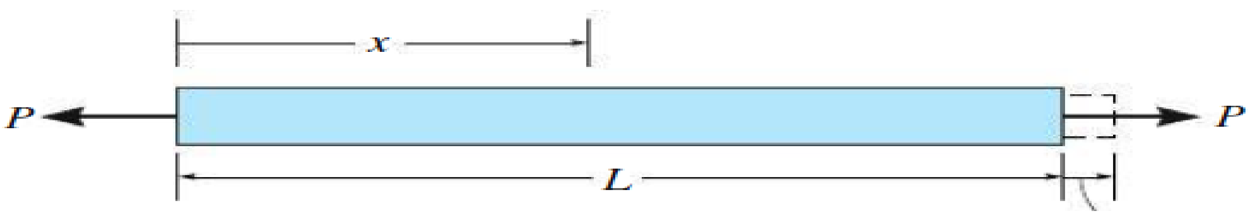
$$\delta = \frac{PL}{AE} \tag{4–2}$$

Constant Load and Cross-Sectional Area. In many cases the bar will have a constant cross-sectional area A; and the material will be homogeneous, so E is constant. Furthermore, if a constant external force is applied at each end, Fig. 4–3, then the internal force P throughout the length of the bar is also constant. As a result, Eq. 4–1 can be integrated to yield

$$\delta = \frac{PL}{AE} \tag{4–2}$$

If the bar is subjected to several different axial forces along its length, or the cross-sectional area or modulus of elasticity changes abruptly from one region of the bar to the next, the above equation can be applied to each *segment* of the bar where these quantities remain *constant*. The displacement of one end of the bar with respect to the other is then found from the *algebraic addition* of the relative displacements of the ends of each segment. For this general case,

$$\delta = \sum \frac{PL}{AE} \tag{4–3}$$



Sign Convention. In order to apply Eq. 4–3, we must develop a sign convention for the internal axial force and the displacement of one end of the bar with respect to the other end. To do so, we will consider both the force and displacement to be *positive* if they cause *tension and elongation*, respectively, Fig. 4–4; whereas a *negative* force and displacement will cause *compression* and *contraction*, respectively.

Positive sign convention for P and δ

Principle of Superposition

The principle of superposition is often used to determine the stress or displacement at a point in a member when the member is subjected to a complicated loading. By subdividing the loading into components, the ***principle of superposition*** states that the resultant stress or displacement at the point can be determined by algebraically summing the stress or displacement caused by each load component applied separately to the member.

The following two conditions must be satisfied if the principle of superposition is to be applied.

1. ***The loading must be linearly related to the stress or displacement that is to be determined.*** For example, the equations $\sigma = P/A$ and $\delta = PL/AE$ involve a linear relationship between P and σ or δ.

2. ***The loading must not significantly change the original geometry or configuration of the member.*** If significant changes do occur, the direction and location of the applied forces and their moment arms will change. For example, consider the slender rod shown in Fig. 4–9*a*, which is subjected to the load **P**. In Fig. 4–9*b*, **P** is replaced by two of its components, $\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2$. If **P** causes the rod to deflect a large amount, as shown, the moment of the load about its support, Pd, will not equal the sum of the moments of its component loads, $Pd \neq P_1d_1 + P_2d_2$, because $d_1 \neq d_2 \neq d$.

This principle will be used throughout this text whenever we assume Hooke's law applies and also, the bodies that are considered will be such that the loading will produce deformations that are so small that the change in position and direction of the loading will be insignificant and can be neglected.

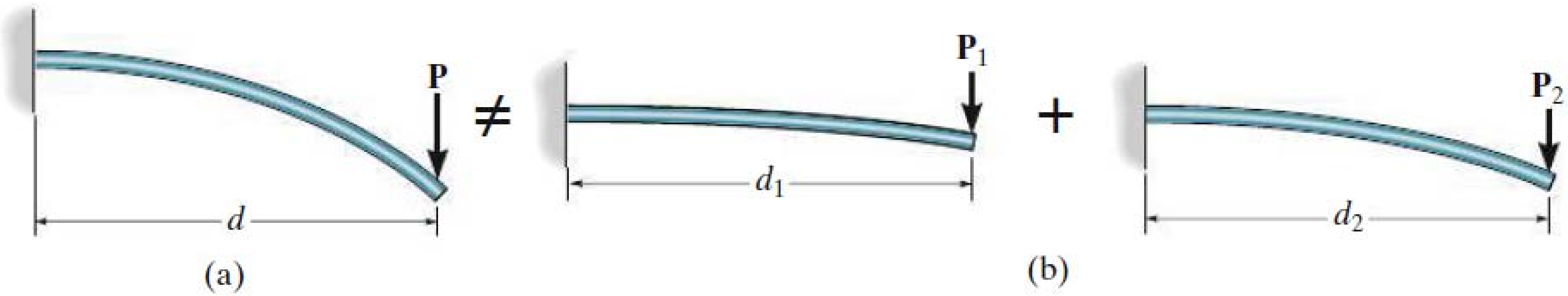


Fig. 4–9

Procedure for Analysis

The support reactions for statically indeterminate problems are determined by satisfying equilibrium, compatibility, and force-displacement requirements for the member.

Equilibrium.

- Draw a free-body diagram of the member in order to identify all the forces that act on it.
- The problem can be classified as statically indeterminate if the number of unknown reactions on the free-body diagram is greater than the number of available equations of equilibrium.
- Write the equations of equilibrium for the member.

Compatibility.

- Consider drawing a displacement diagram in order to investigate the way the member will elongate or contract when subjected to the external loads.
- Express the compatibility conditions in terms of the displacements caused by the loading.

Load-Displacement.

- Use a load–displacement relation, such as $\delta = PL/AE$, to relate the unknown displacements to the reactions.
- Solve the equations for the reactions. If any of the results has a negative numerical value, it indicates that this force acts in the opposite sense of direction to that indicated on the free-body diagram.

Simple stress and strains

- ***Axial Deformation***

From Hooke's law ; **E = σ / ε**

or, **σ = E ε** ⟶ $\frac{P}{A} = E\frac{\delta}{L}$ ⟶ $\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$

This Eq. is use only when :

1. The load must be axial
2. The bar must have a constant cross section and be homogenous.
3. The stress must not exceed the proportional limit.

Deformations Under Axial Loading

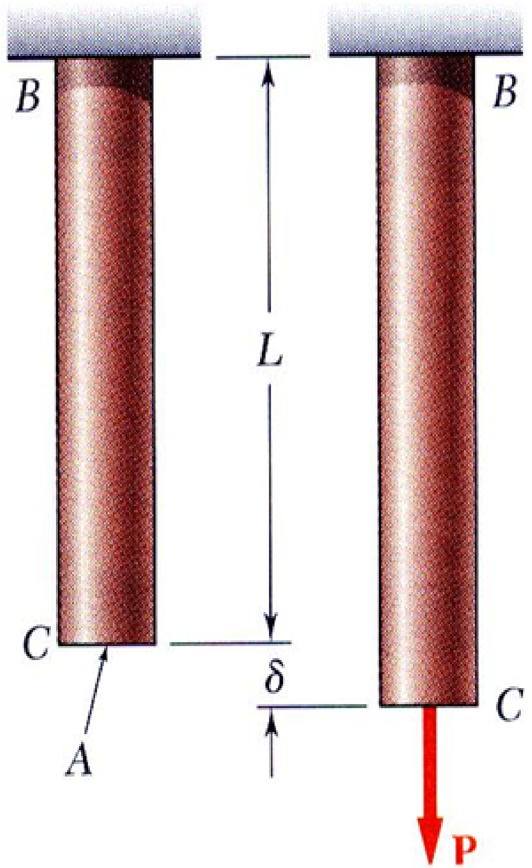


Fig. 2.22

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Simple stress and strains

- ***Shearing Deformation***

  Shearing forces causes a shearing deformation. An element subjected to shear undergoes **change in shape from a rectangle to parallelogram**, as shown in Fig. below.

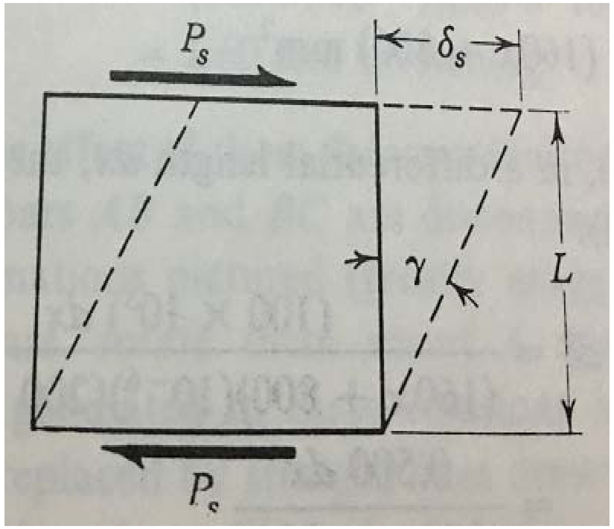
The average shearing strain = tan $\gamma = \dfrac{\delta_s}{L}$

since is usually very small, $\tan\gamma = \gamma$

then, $\gamma = \dfrac{\delta_s}{L}$

By Hooke's law; $\tau = G\gamma$

Where; G= modulus of elasticity in shear, or modulus of rigidity.



Simple stress and strains

So, $\gamma = \frac{\tau}{G}$ → $\frac{\delta_s}{L} = \frac{\tau}{G}$

Then; $\delta_s = \frac{VL}{A_s G}$

Simple stress and strains

Summary

Axial Loading: $\sigma = P / A$ $\varepsilon = \delta L / L$ $\sigma = E\,\varepsilon$

Shear Loading : $\tau = V/A$ $\tau = G\gamma$ $\gamma = \dfrac{\delta_s}{L}$

Axial Deformation: $$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

Shear Deformation: $$\delta_s = \frac{VL}{A_s G}$$

Simple stress and strains

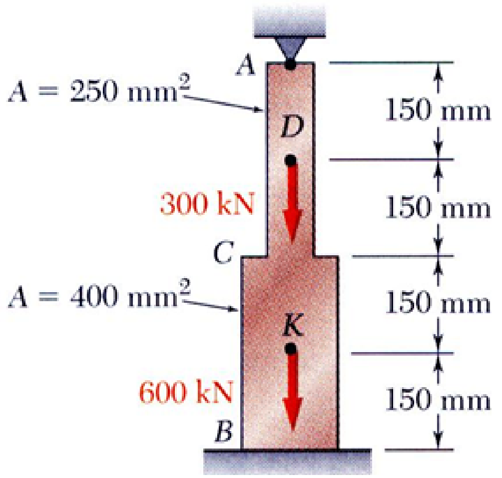
- Statically Indeterminate Members

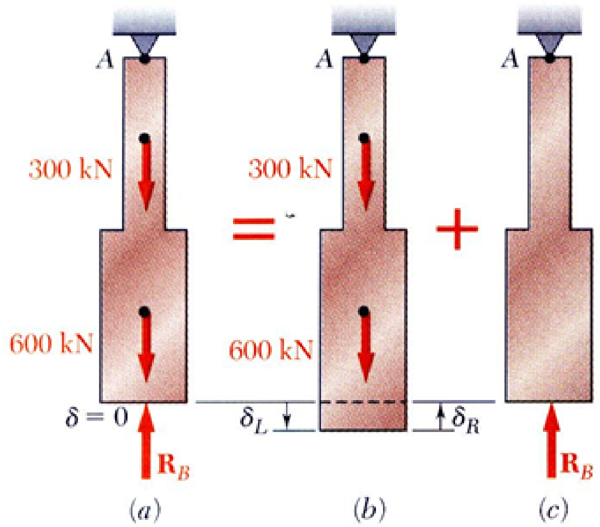
  Axially Loaded members in which the equations of equilibrium are not sufficient for the solution.

Two general principles may be followed:

1. To a FBD of the structure, or a part of it, apply the equations of static equilibrium.
2. Obtain additional equations from the geometric relations between the elastic deformations produced by the loads.

Static Indeterminacy

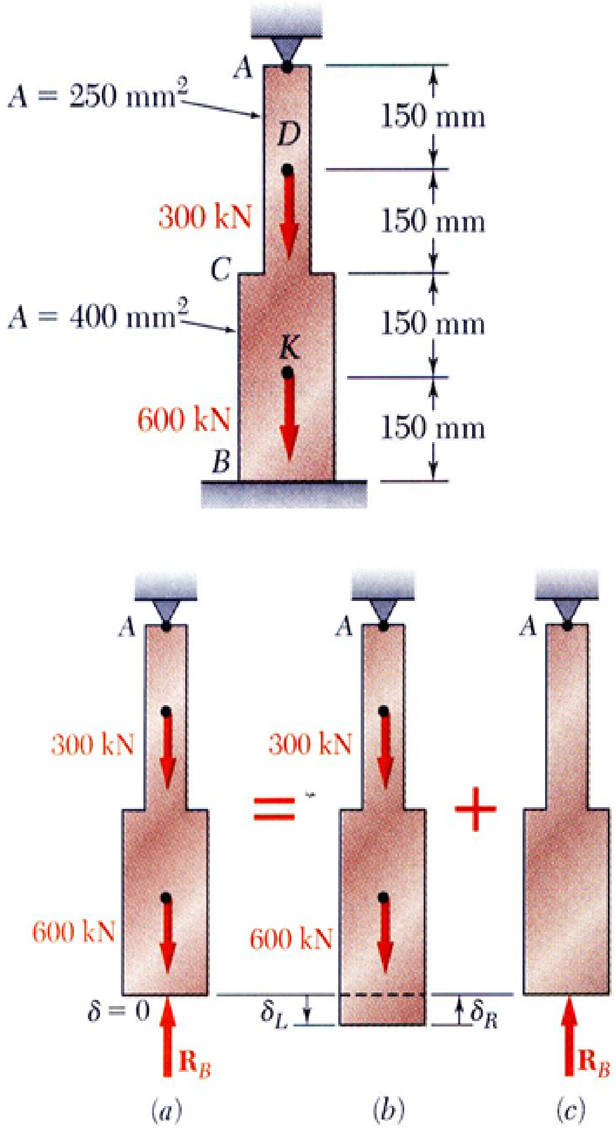




- Structures for which internal forces and reactions cannot be determined from statics alone are said to be *statically indeterminate*.
- A structure will be statically indeterminate whenever it is held by more supports than are required to maintain its equilibrium.
- Redundant reactions are replaced with unknown loads which along with the other loads must produce compatible deformations.
- Deformations due to actual loads and redundant reactions are determined separately and then added or *superposed*.

$$\delta = \delta_L + \delta_R = 0$$

Example 2.04

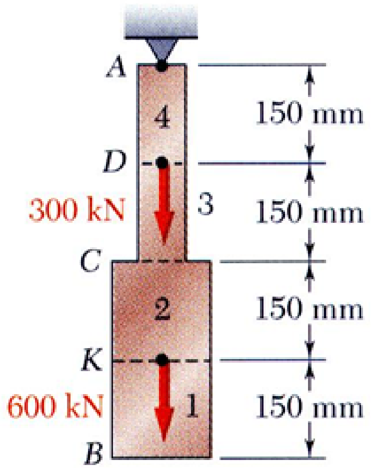


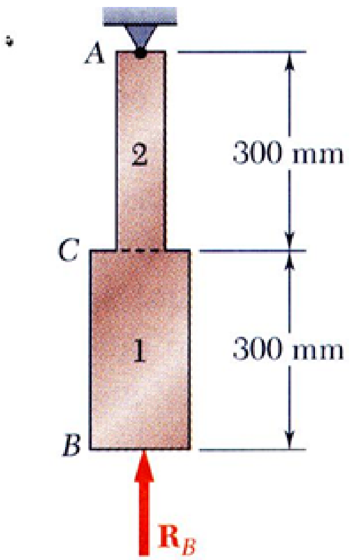
Determine the reactions at A and B for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.

SOLUTION:

- Consider the reaction at B as redundant, release the bar from that support, and solve for the displacement at B due to the applied loads.
- Solve for the displacement at B due to the redundant reaction at B.
- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.
- Solve for the reaction at A due to applied loads and the reaction found at B.

Example 2.04





SOLUTION:

- Solve for the displacement at B due to the applied loads with the redundant constraint released,

$$P_1 = 0 \quad P_2 = P_3 = 600\times10^3\,\text{N} \quad P_4 = 900\times10^3\,\text{N}$$

$$A_1 = A_2 = 400\times10^{-6}\,\text{m}^2 \quad A_3 = A_4 = 250\times10^{-6}\,\text{m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150\,\text{m}$$

$$\delta_L = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125\times10^9}{E}$$

- Solve for the displacement at B due to the redundant constraint,

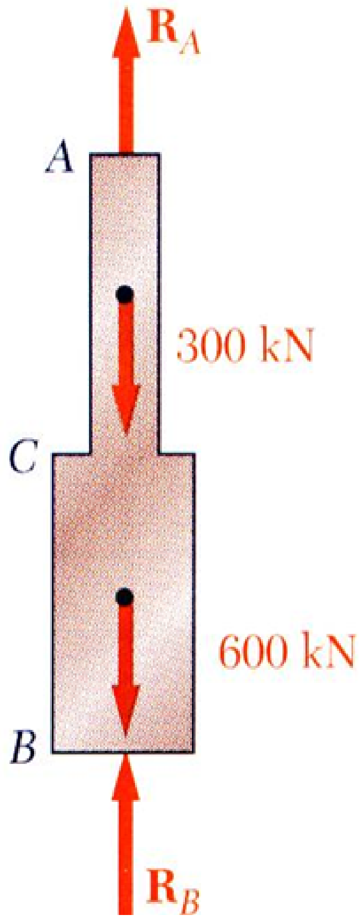
$$P_1 = P_2 = -R_B$$

$$A_1 = 400\times10^{-6}\,\text{m}^2 \quad A_2 = 250\times10^{-6}\,\text{m}^2$$

$$L_1 = L_2 = 0.300\,\text{m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{\left(1.95\times10^3\right)R_B}{E}$$

Example 2.04



- Require that the displacements due to the loads and due to the redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0$$

$$\delta = \frac{1.125\times10^9}{E} - \frac{\left(1.95\times10^3\right)R_B}{E} = 0$$

$$R_B = 577\times10^3\,\text{N} = 577\,\text{kN}$$

- Find the reaction at A due to the loads and the reaction at B

$$+\uparrow\sum F_y = 0 = R_A - 300\,\text{kN} - 600\,\text{kN} + 577\,\text{kN}$$

$$R_A = 323\,\text{kN}$$

$$R_A = 323\,\text{kN}$$

$$R_B = 577\,\text{kN}$$

Simple Stress

Thin Wall Cylinders

Thin Wall Cylinders

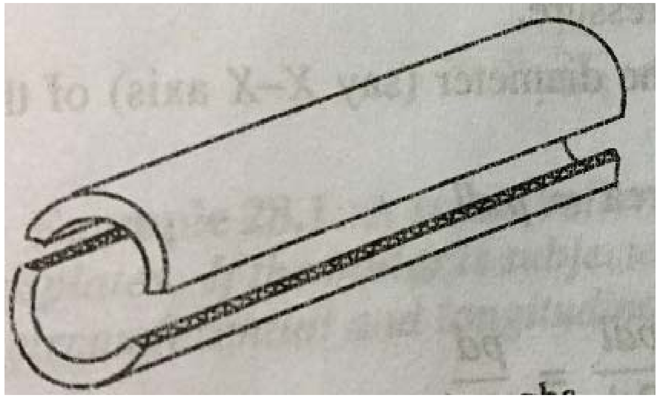
- In engineering field, cylindrical tank containing fluids such as tanks, boilers, compressed air receivers etc.
- These tanks, when empty, are subjected to atmospheric pressure internally as well as externally. In such a case, the resultant pressure on the wall of the shell is zero.
- But whenever a cylinder is subjected to internal pressure (due to stream, compressed air etc.) its wall are subjected to tensile stresses.

Thin Wall Cylinders

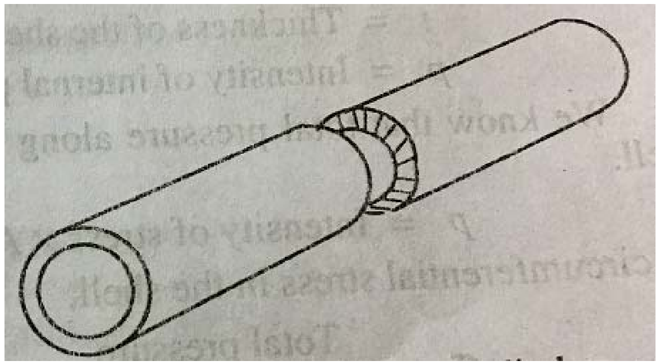
- For thin wall cylinders, the wall thickness (t) is less than (1/20) of the inner diameter.
- When the tensile stresses exceed the permissible limit, the cylinder is likely to fail in any of the following two ways:

  1. It may split up into two troughs, and
  2. It may split up into two cylinders.

Thin Wall Cylinders



Two Troughs



Two Cylinders

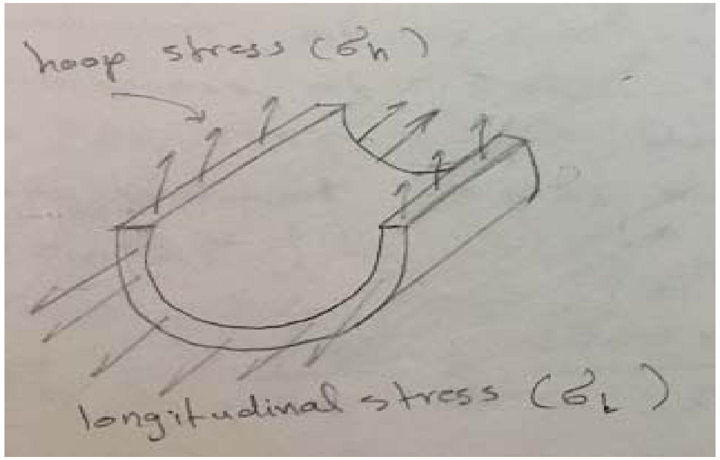
Thin Wall Cylinders

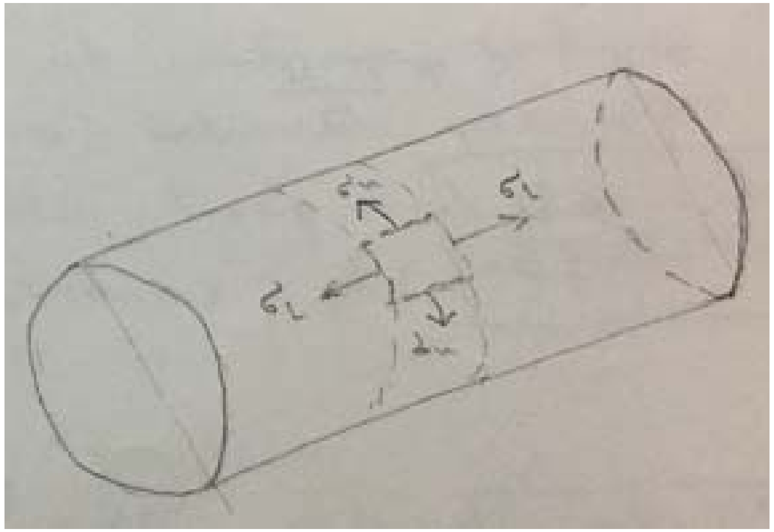
- Thin cylinders under internal pressure

The wall of the cylindrical shell will be subjected to the following two types of tensile stresses:

1. Circumferential stress (hoop or tangential stress), and
2. Longitudinal stress.

Thin Wall Cylinders



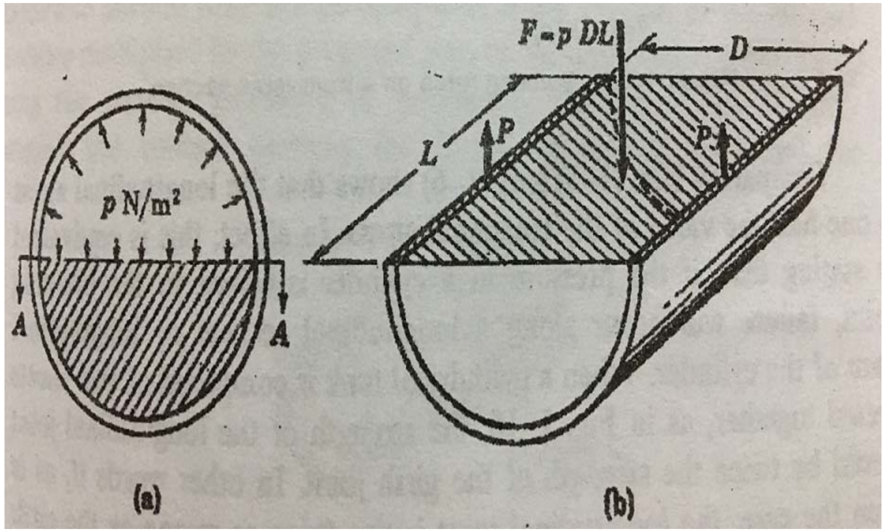


Thin Wall Cylinders

- <u>Hoop Stress</u>

  The lower half of the cylinder is occupied by a fluid as shown in fig. (a).

  A F.B.D of the half – cylinder isolated by the cutting plane A-A is shown in Fig. (b).



Thin Wall Cylinders

- Let $\mathcal{P}$ = internal pressure;

  F = bursting force ;

  P = Force acting on each cut surface of the cylinder wall;

  σ_h = hoop stress

  Then; $F = \mathcal{P}.D.L$

  $P = \sigma_h.t.L$

  $\sum Fy = 0$ → $2P = F$ → $2\,\sigma_h.t.L = \mathcal{P}.D.L$ →

$$\sigma_h = \mathcal{P}.D / 2t$$

Thin Wall Cylinders

- **Longitudinal Stress**

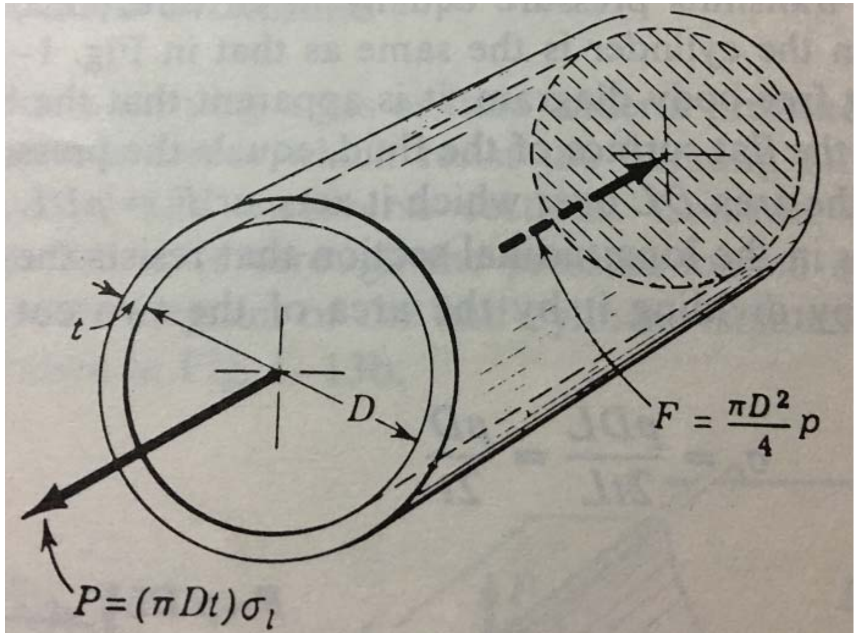
Consider the F.B.D of a transverse section :

Mean circumference = (D+t). π

In thin wall cylinders t is very small compared to D, therefore;

Mean circumference = D. π

So, area of Transverse section can be closely approximated by (π.D.t)



Thin Wall Cylinders

Then; F = bursting force = $\mathcal{P}$ ($\pi D^2/4$);

σ_L = Longitudinal stress;

P= resultant of the tearing forces = $\sigma_L \pi.Dt$

Since , F = P ⟹ $\mathcal{P}(\pi D^2/4) = \sigma_L \pi.Dt$ ⟹

$\sigma_L = \mathcal{P}.D / 4t$

Since $\sigma_h = \mathcal{P}.D / 2t$

Then : $\sigma_L = \sigma_h /2$

Thin Wall Cylinders

- Hoop and Longitudinal Strains
- ε = Strain = (change in dimension / original dimension);
  ν = Poisson's ratio; σ = Stress;

  E = Modulus of Elasticity.

$\nu = \varepsilon_{Lateral} / \varepsilon_{Longitudinal}$

$\boldsymbol{\varepsilon_{Lateral} = \nu \cdot \varepsilon_{Longitudinal}}$

$E = \sigma / \varepsilon \Rightarrow \varepsilon = \sigma / E$

$\varepsilon_h = (\sigma_h / E) - \nu (\sigma_L / E) \Rightarrow \varepsilon_h = (1/E)(\sigma_h - \nu \sigma_L)$

$\varepsilon_L = (\sigma_L / E) - \nu (\sigma_h / E) \Rightarrow \varepsilon_L = (1/E)(\sigma_L - \nu \sigma_h)$

Thin Wall Cylinders

Summary

Stresses

1. Hoop Stress : $\sigma_h = \frac{P\,D}{2t}$

2. Longitudinal Stress : $\sigma_L = \frac{P\,D}{4t}$

Strains

1. Hoop Strain : $\varepsilon_h = (1/E)\,(\sigma_h - \nu\,\sigma_L)$

2. Longitudinal Strain : $\varepsilon_L = (1/E)\,(\sigma_L - \nu\,\sigma_h)$

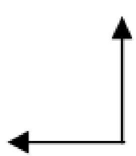
Plane Stress Analysis

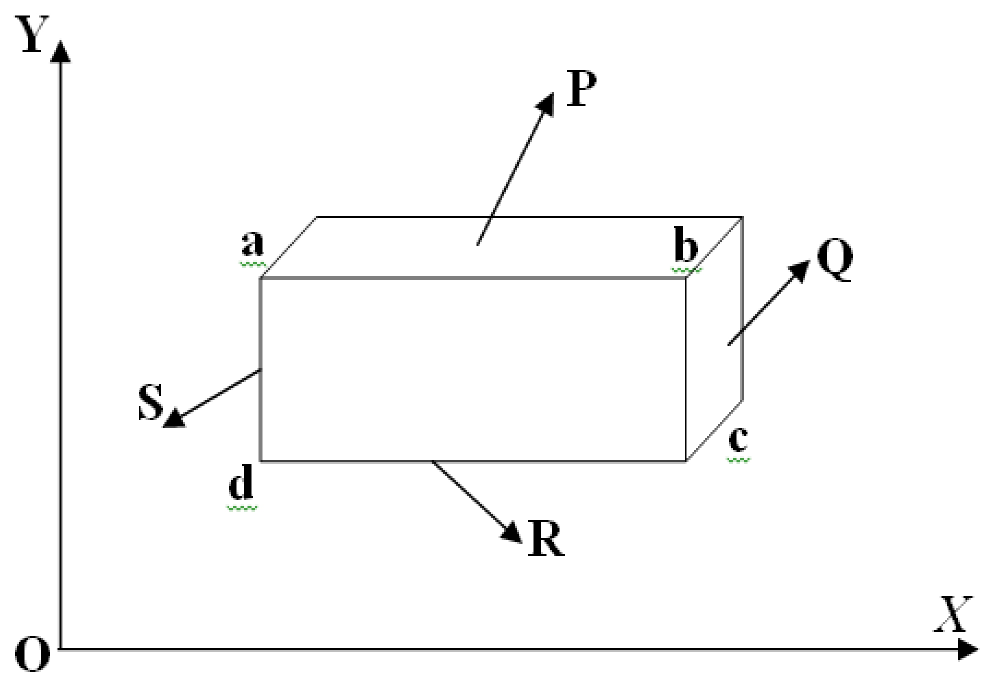
Plane Stress Analysis (Two Dimensional Stress Analysis)

A two dimensional stress analysis is one in which the stresses at any point in a body act in the same plane.

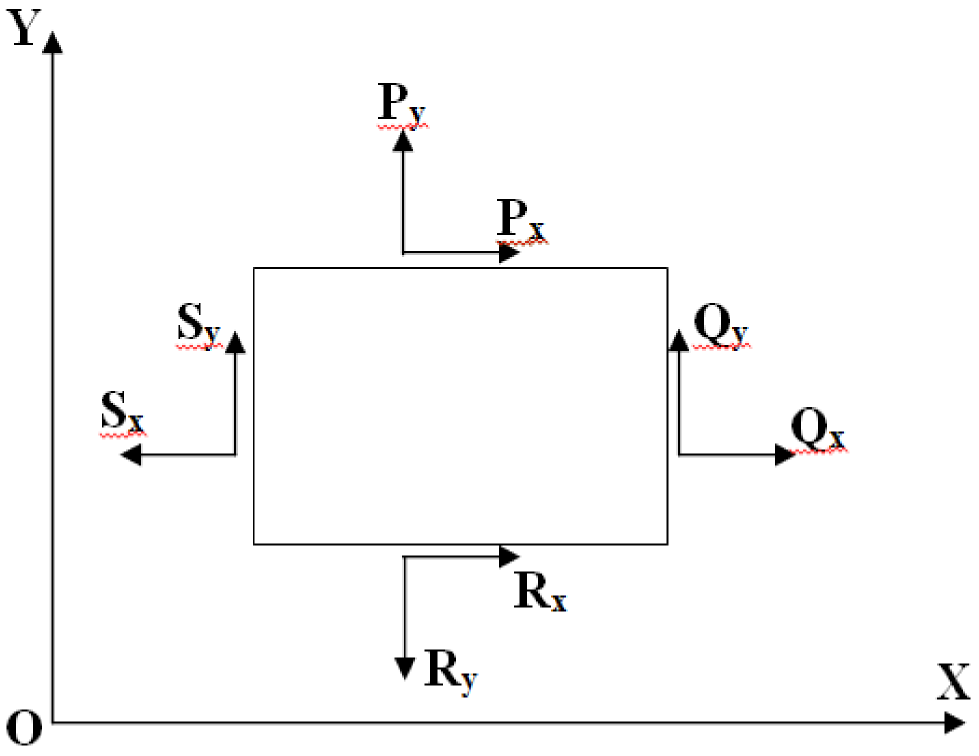
P, Q, R and S are stresses

In the x – y plane.



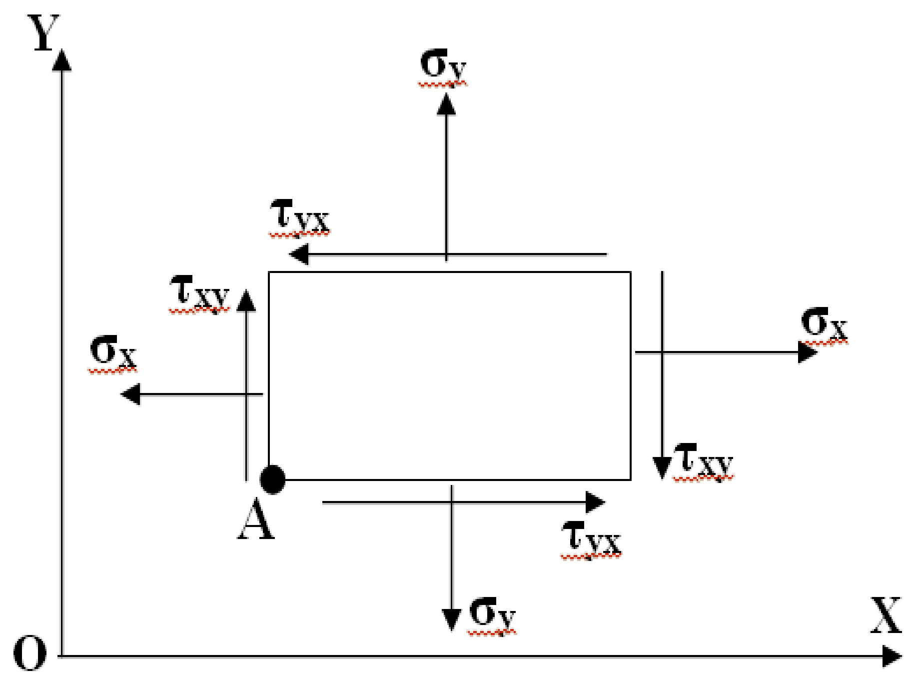


The perpendicular components introduce direct stresses, and the tangential components introduce shearing stresses.



Stress at a Point

The stress acting at a point is represented by the stresses acting on the faces of a differential element enclosing the point. The element is usually represented by four sides.



General Tow – Dimensional State of Stress at a point.

σ_x : Normal stress (direct stress) acting on a plane perpendicular to the x – axis (X – face);

σ_y : Normal stress (direct stress) acting on a plane perpendicular to the y – axis (Y – face);

τ_{xy} : Shearing stress acting on a plane perpendicular to the x – axis (X – face) and directed parallel to the y - axis; and

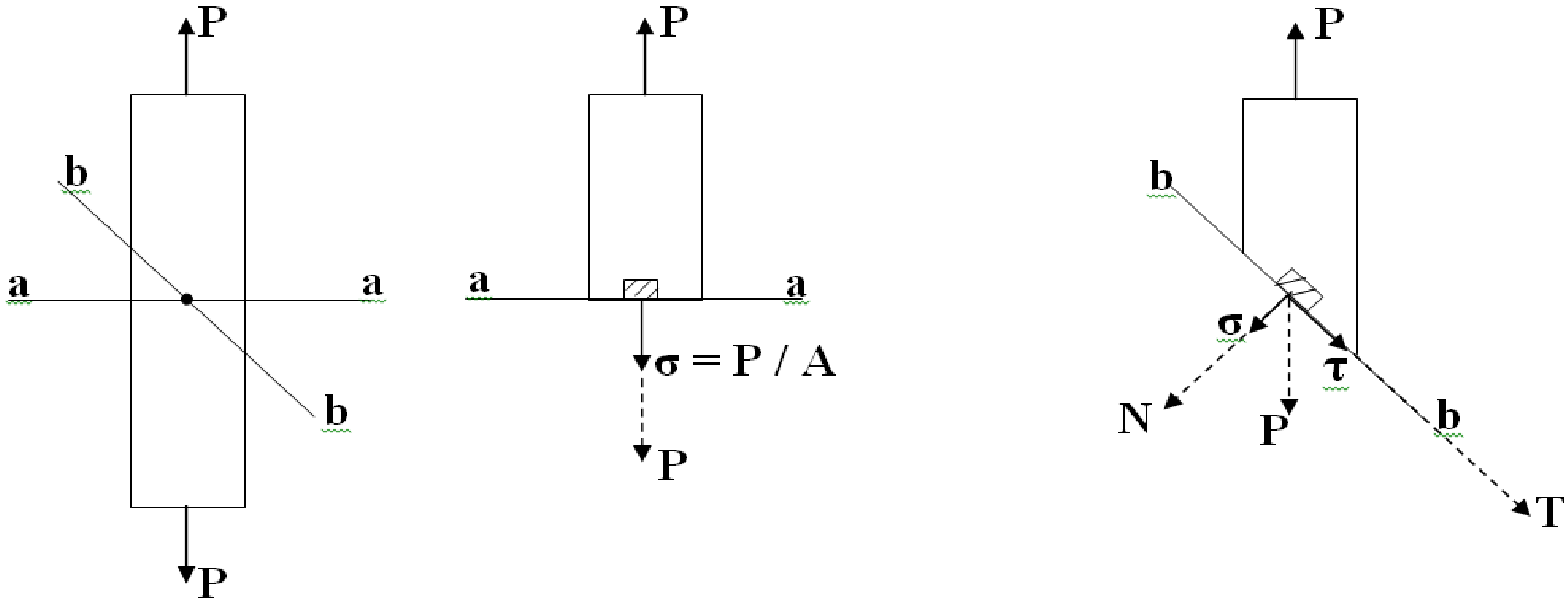
τ_{yx} : Shearing stress acting on a plane perpendicular to the y – axis (Y – face) and directed parallel to the x – axis.

Note: $\tau_{xy} = \tau_{yx}$ (Shearing stresses on perpendicular planes are equal).

$$\Sigma M_A = 0$$

Variation of stress at a point

The stresses on an element (point) vary with the orientation of the element.



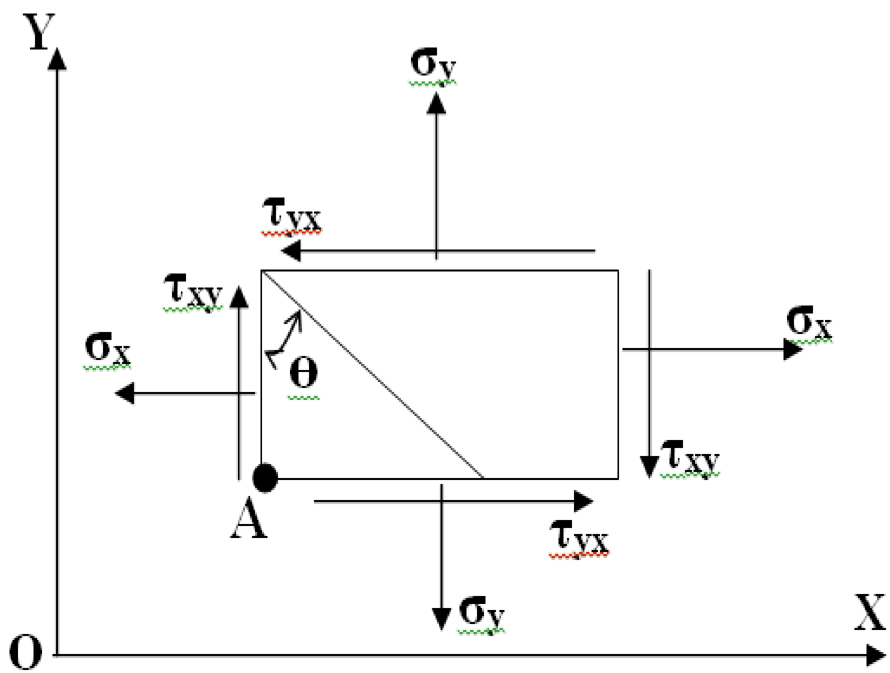
Stresses on an inclined plane

To find the stresses acting on an inclined plane, two methods may be used:

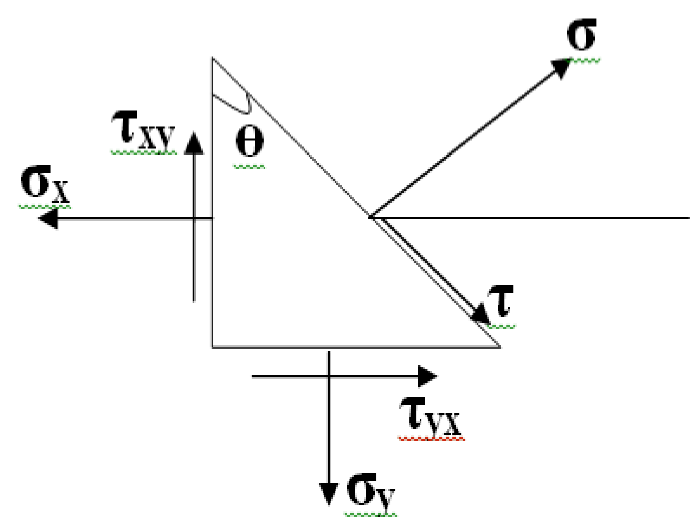
1. Analytical method;
2. Graphical method (Mohr's circle).

1. Analytical Method

In determining the stress variation analytically, a plane is passed that cuts the original element into two parts and the condition of equilibrium are applied to either part.

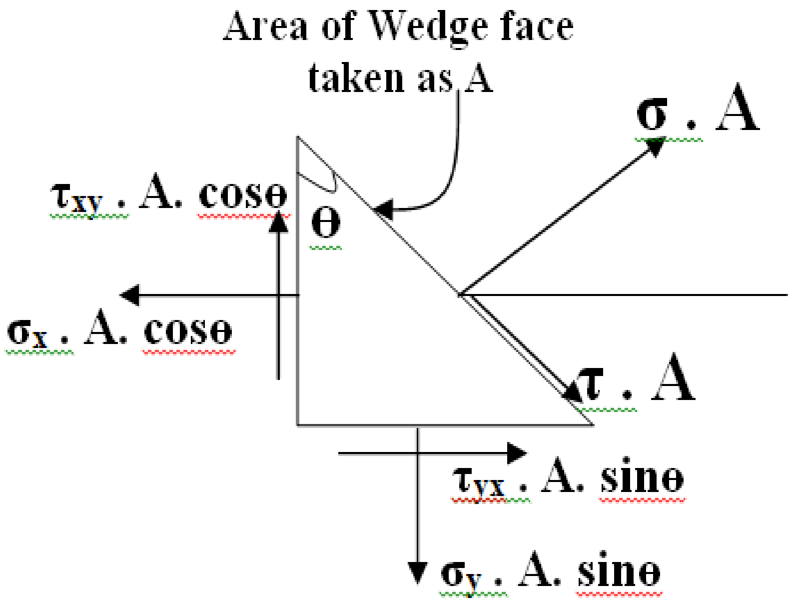


(a) Original State of stress

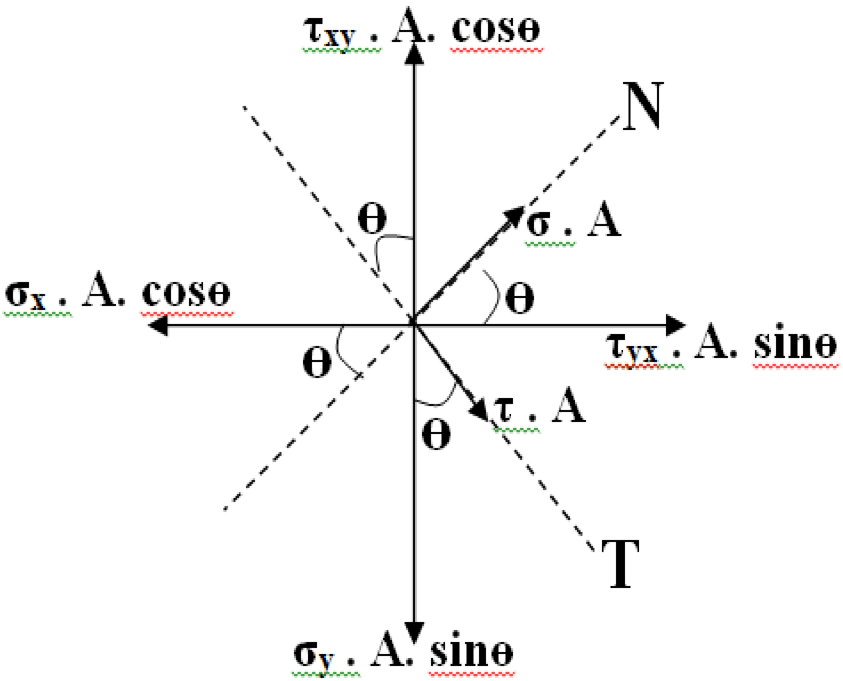


(b) Stresses acting on Wedge

Note: If A is the area of the inclined plane;



(c) F.B.D of Forces on Wedge

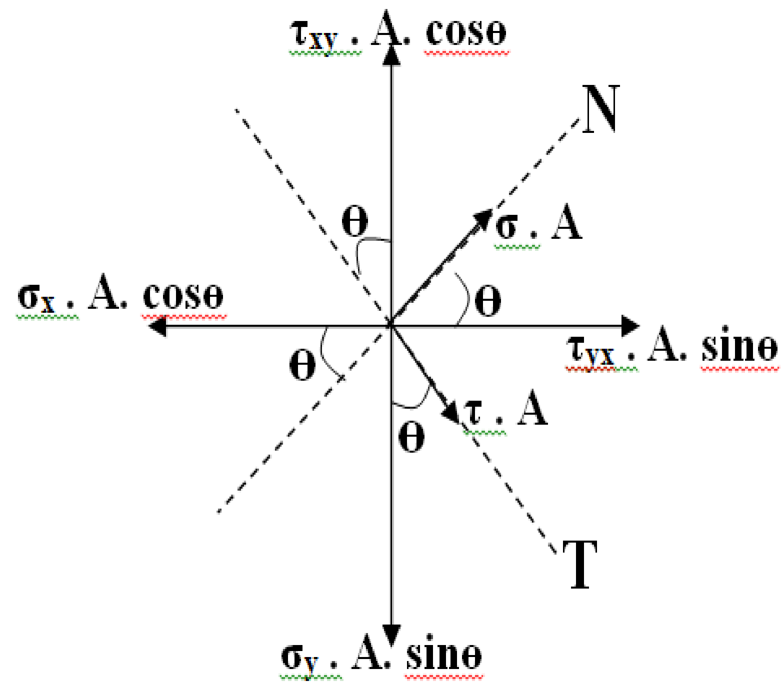


(d) Point Diagram of Forces

$$\Sigma F_N = 0$$

$$\sigma \cdot A + \tau_{xy} \cdot A \cdot \cos\theta \cdot \sin\theta + \tau_{yx} \cdot A \cdot \sin\theta \cdot \cos\theta =$$

$$\sigma_x \cdot A \cdot \cos\theta \cdot \cos\theta + \sigma_y \cdot A \cdot \sin\theta \cdot \sin\theta$$



(d) Point Diagram of Forces

A is cancelled form both sides;

$\tau_{xy} = \tau_{yx}$;

then

$$\sigma = \sigma_x \cdot \cos^2\theta + \sigma_y \cdot \sin^2\theta - 2\,\tau_{xy} \cdot \cos\theta \cdot \sin\theta$$

This equation may be written more conveniently;

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \quad ; \quad \sin^2\theta = \frac{1-\cos 2\theta}{2} \quad ; \quad 2\sin\theta \cdot \cos\theta = \sin 2\theta$$

$$\sigma = \sigma_x \cdot \frac{1+\cos 2\theta}{2} + \sigma_y \cdot \frac{1-\cos 2\theta}{2} - \tau_{xy} \cdot \sin 2\theta$$

$$\sigma = \frac{\sigma x}{2} + \frac{\sigma x}{2} \cos 2\theta + \frac{\sigma y}{2} - \frac{\sigma y}{2} \cos 2\theta - \tau_{xy} \cdot \sin 2\theta$$

$$\sigma = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta - \tau_{xy} . \sin 2\theta \qquad \text{.........} \ (1)$$

$$\Sigma F_T = 0$$

$$\tau . A + \tau_{yx} . A . \sin\theta . \sin\theta + \sigma_y . A . \sin\theta . \cos\theta = \tau_{xy} . A . \cos\theta . \cos\theta + \sigma_x . A . \cos\theta . \sin\theta$$

$$\tau = (\sigma_x - \sigma_y) \sin\theta . \cos\theta + \tau_{xy} (\cos^2\theta - \sin^2\theta)$$

Since , $\cos^2\theta - \sin^2\theta = \cos 2\theta$, then;

$$\tau = \frac{\sigma x - \sigma y}{2} \sin 2\theta + \tau_{xy} . \cos 2\theta \qquad \text{.........} \ (2)$$

Principle Stresses

The maximum and minimum normal stresses are known as **Principle Stresses**. The planes defining maximum and minimum normal stresses are found by differentiating Eq. 1 with respect to θ and setting the derivative equal to zero.

$$\sigma = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta - \tau_{xy} . \sin 2\theta \quad \ldots\ldots\ldots (1)$$

$$\frac{d\sigma}{d\theta} = 0 + \left(\frac{\sigma_x - \sigma_y}{2}\right)(-2 sin 2\theta) - 2\,\tau_{xy}\cos 2\theta = 0$$

$$(\sigma_x - \sigma_y)\sin 2\theta = -2\,\tau_{xy}\cos 2\theta \quad \rightarrow \quad \frac{\sin 2\theta}{\cos 2\theta} = -\frac{2\,\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta = \frac{-2\,\tau_{xy}}{\sigma_x - \sigma_y} \quad \ldots\ldots\ldots (3)$$

Since; $\tan\theta = \tan(\theta + 180^\circ)$ then;

$$(\sigma_x - \sigma_y) \sin 2\theta = -2\,\tau_{xy} \cos 2\theta \quad \rightarrow \quad \frac{\sin 2\theta}{\cos 2\theta} = -\frac{2\,\tau_{xy}}{\sigma_x - \sigma_y}$$

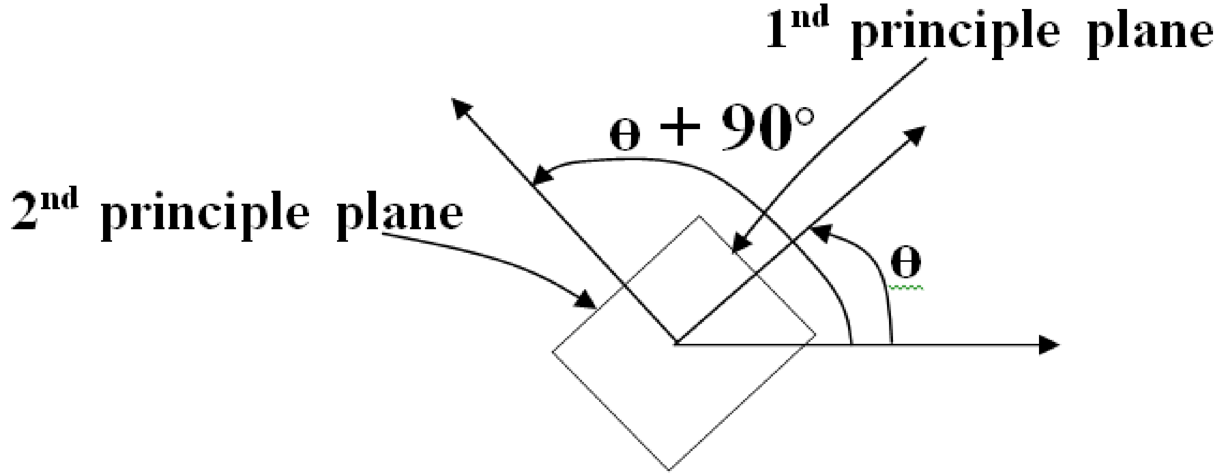
$$\tan 2\theta = \frac{-2\,\tau_{xy}}{\sigma_x - \sigma_y} \quad \ldots\ldots\ldots (3)$$

Since; $\tan\theta = \tan(\theta + 180^\circ)$ then;

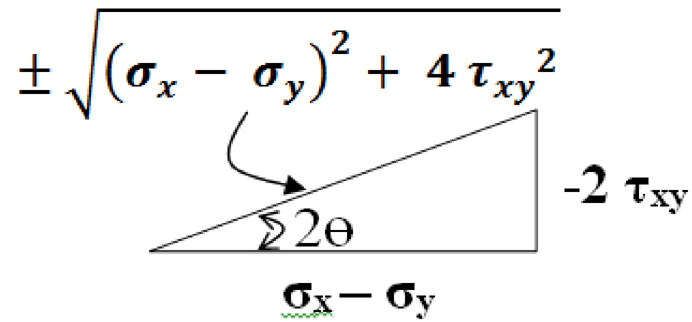
$$2\theta = \tan^{-1}\left(\frac{-2\,\tau_{xy}}{\sigma_x - \sigma_y}\right) + n\,.\,180^\circ$$

$$\theta = \left(\frac{1}{2}\right) \tan^{-1}\left(\frac{-2\,\tau_{xy}}{\sigma_x - \sigma_y}\right) + n\,.\,90^\circ \qquad \ldots\ldots\ldots (3')$$

Where: n = 0, 1, 2, 3, 4 ....



$$\tan 2\theta = \frac{-2\,\tau_{xy}}{\sigma_x - \sigma_y} \quad \dots\dots\dots (3)$$



From Eq. 1 ,

$$\sigma = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2}\cos 2\theta - \tau_{xy} .\ \sin 2\theta$$

$$= \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \times \frac{(\sigma_x - \sigma_y)}{\pm\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2}} - \tau_{xy} \times \frac{-2\,\tau_{xy}}{\pm\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2}}$$

$$= \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \times \frac{(\sigma_x - \sigma_y)}{\pm\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2}} - \tau_{xy} \times \frac{-2\,\tau_{xy}}{\pm\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2}}$$

$$= \frac{\sigma x + \sigma y}{2} + \frac{1}{\pm 2\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2}}\left[(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2\right]$$

$$\sigma_{1,2} = \frac{\sigma x + \sigma y}{2} \pm \frac{1}{2}\sqrt{\left[(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2\right]}$$

$$\sigma_{1,2} = \frac{\sigma x + \sigma y}{2} \pm \sqrt{\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2\right]} \quad \ldots\ldots\ldots\ldots (4)$$

Note : 1. σ_1 and σ_2 are principle stresses.

2. +ve sign for σ_1 and -ve sign for σ_2 .

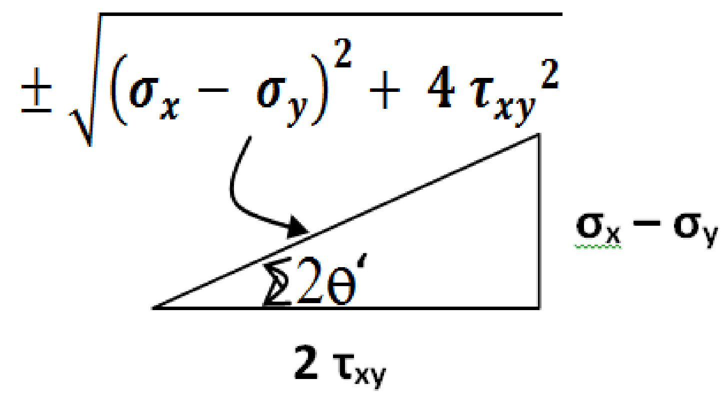
Maximum Shearing Stress

$$\tau = \frac{\sigma x - \sigma y}{2} \sin 2\theta + \tau_{xy} . \cos 2\theta \qquad \ldots\ldots\ldots (2)$$

Differentiate with respect to θ and set the result equal to zero;

$$\frac{d\tau}{d\theta} = \left(\frac{\sigma_x - \sigma_y}{2}\right)(2\cos 2\theta) - 2\,\tau_{xy}\,\sin 2\theta = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\sigma_x - \sigma_y}{2\,\tau_{xy}}$$



$$\tan 2\theta' = \frac{\sigma_x - \sigma_y}{2\,\tau_{xy}} \qquad \ldots\ldots\ldots (5)$$

$$\theta' = \left(\frac{1}{2}\right) \tan^{-1}\left(\frac{\sigma_x - \sigma_y}{2\,\tau_{xy}}\right) + n\,.\,90^\circ \qquad \ldots\ldots\ldots (5')$$

Substituting in Eq, 2;

$$\tau = \frac{\sigma x - \sigma y}{2} \times \frac{(\sigma_x - \sigma_y)}{\pm\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2}} + \tau_{xy} \times \frac{-2\,\tau_{xy}}{\pm\sqrt{(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2}}$$

$$\tau_{max} = \pm\sqrt{\left[\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}{}^2\right]} \;\ldots\ldots\ldots (6)$$

Planes of Zero Shear Stress

$$\tau = \frac{\sigma x - \sigma y}{2} \quad \sin 2\theta + \tau_{xy} . \ \cos 2\theta$$

$$\tan 2\theta = \frac{-2\,\tau_{xy}}{\sigma_x - \sigma_y} \quad \dots\dots\dots (7)$$

Eq. (7) is similar to Eq. (3), and this means that, on planes on which maximum normal stresses act, there are no shearing stresses.

Notes:

$$\tan 2\theta = \frac{-2\,\tau_{xy}}{\sigma_x - \sigma_y} \quad \dots\dots\dots (3)$$

$$\tan 2\theta' = \frac{\sigma_x - \sigma_y}{2\,\tau_{xy}} \quad \dots\dots\dots (5)$$

$$\tan 2\theta' = \frac{\sigma_x - \sigma_y}{2\,\tau_{xy}} \quad \dots\dots\dots (5)$$

So; $\tan 2\theta = -\cot 2\theta'$ ……. (a)

Since; *$\tan(\beta + 90°) = -\cot\beta$* *Engineering identical*

Then; $\tan(2\theta' + 90°) = -\cot 2\theta'$ ……. (b)

From Eqs. (a) and (b) ⇨ $\tan 2\theta = \tan(2\theta' + 90°)$

So; $2\theta = 2\theta' + 90°$ ⇨ $\theta = \theta' + 45°$

This mean that the angles that locate the planes of maximum or minimum shearing stress form angles of 45° with the planes of the principle stresses.

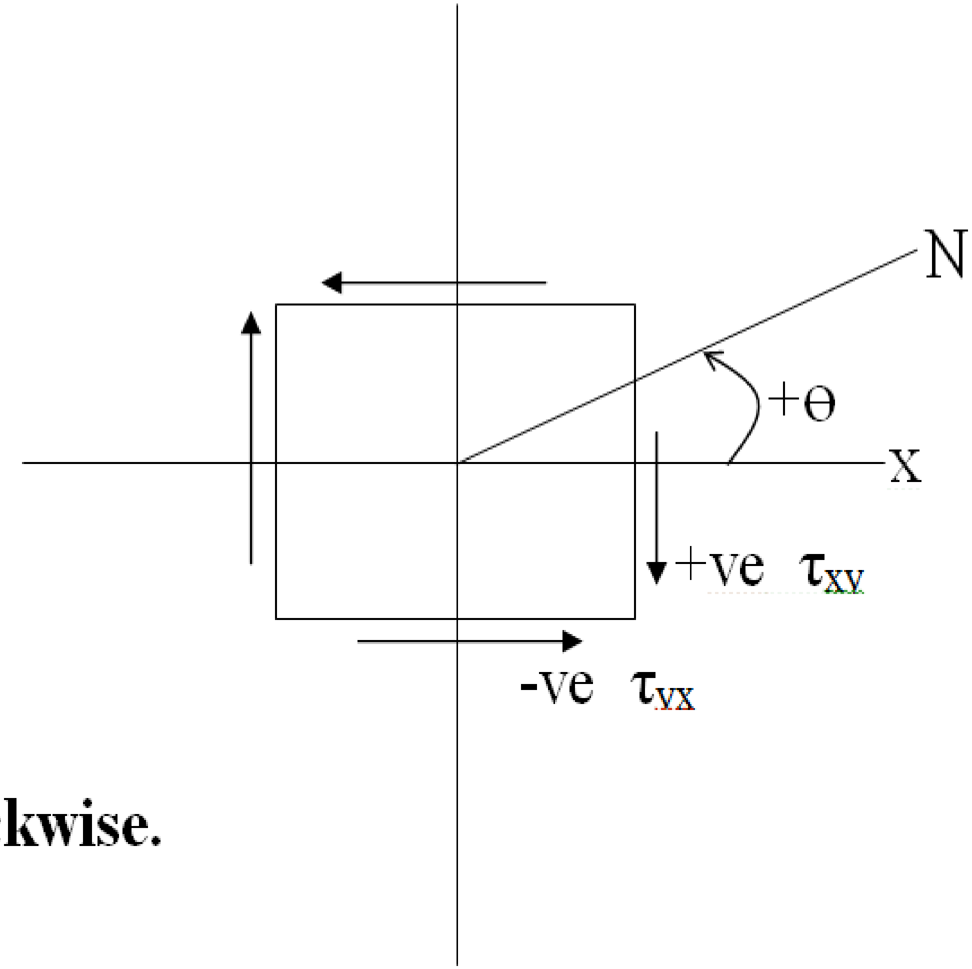
Sign Convention

Tension +ve

Compression -ve

θ : anti – clockwise +ve

τ : +ve if its moment about the centre of the element is clockwise.



2. Graphical Method (Mohr's Circle)

- The Equations developed in the Analytical method may be used for any case of two – dimensional stress;
- In 1882 the German engineer Otto Mohr devised a visual interpretation for analytical equations.

  This interpretation eliminates the necessity for remembering the analytical equations.

  In this interpretation a circle is used, and the construction is called Mohr's Circle.

  If this circle is plotted to a scale, the results can be obtained graphically.

  We have:

We have:

$$\sigma = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta - \tau_{xy} . \sin 2\theta \quad \ldots\ldots\ldots (1)$$

can be written as :

$$\sigma - \frac{\sigma x + \sigma y}{2} = \frac{\sigma x - \sigma y}{2} \cos 2\theta - \tau_{xy} . \sin 2\theta \quad \ldots\ldots\ldots (1')$$

Also;

$$\tau = \frac{\sigma x - \sigma y}{2} \sin 2\,\theta + \tau_{xy} . \cos 2\theta \ \ldots . \ \ldots . \ (2)$$

By squaring both these equations (1' & 2), adding the results and simplifying, we obtain:

$$\left(\sigma - \frac{\sigma x + \sigma y}{2} \right)^2 + \tau^2 = \left(\frac{\sigma x - \sigma y}{2} \right)^2 + (\tau_{xy})^2 \ \ldots\ldots \ (8)$$

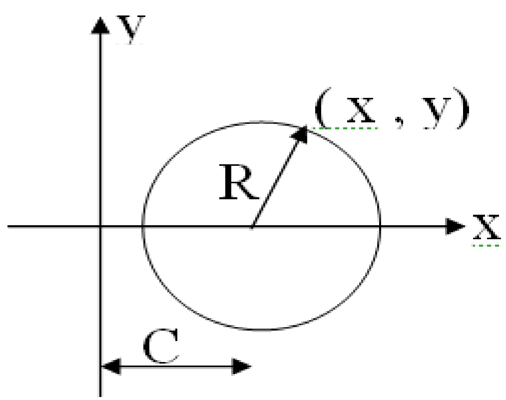
Notes: **1.** σ_x , σ_y and τ_{xy} are known constants defining the specified state of stress, whereas σ and τ are variables.

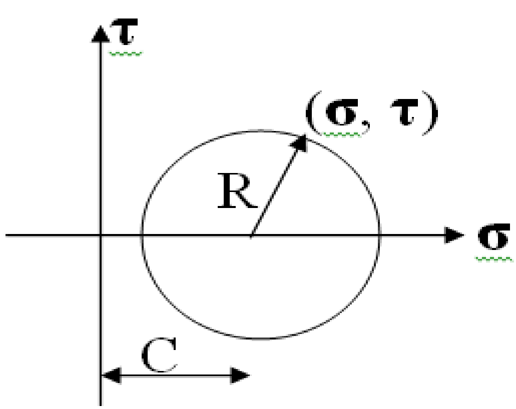
2. $\dfrac{\sigma x + \sigma y}{2} = constant = C$

3. $\left(\dfrac{\sigma x - \sigma y}{2}\right)^2 + (\tau_{xy})^2 = constant = R^2$

Eq. (8) can be written as: $(\sigma - C)^2 + \tau^2 = R^2$

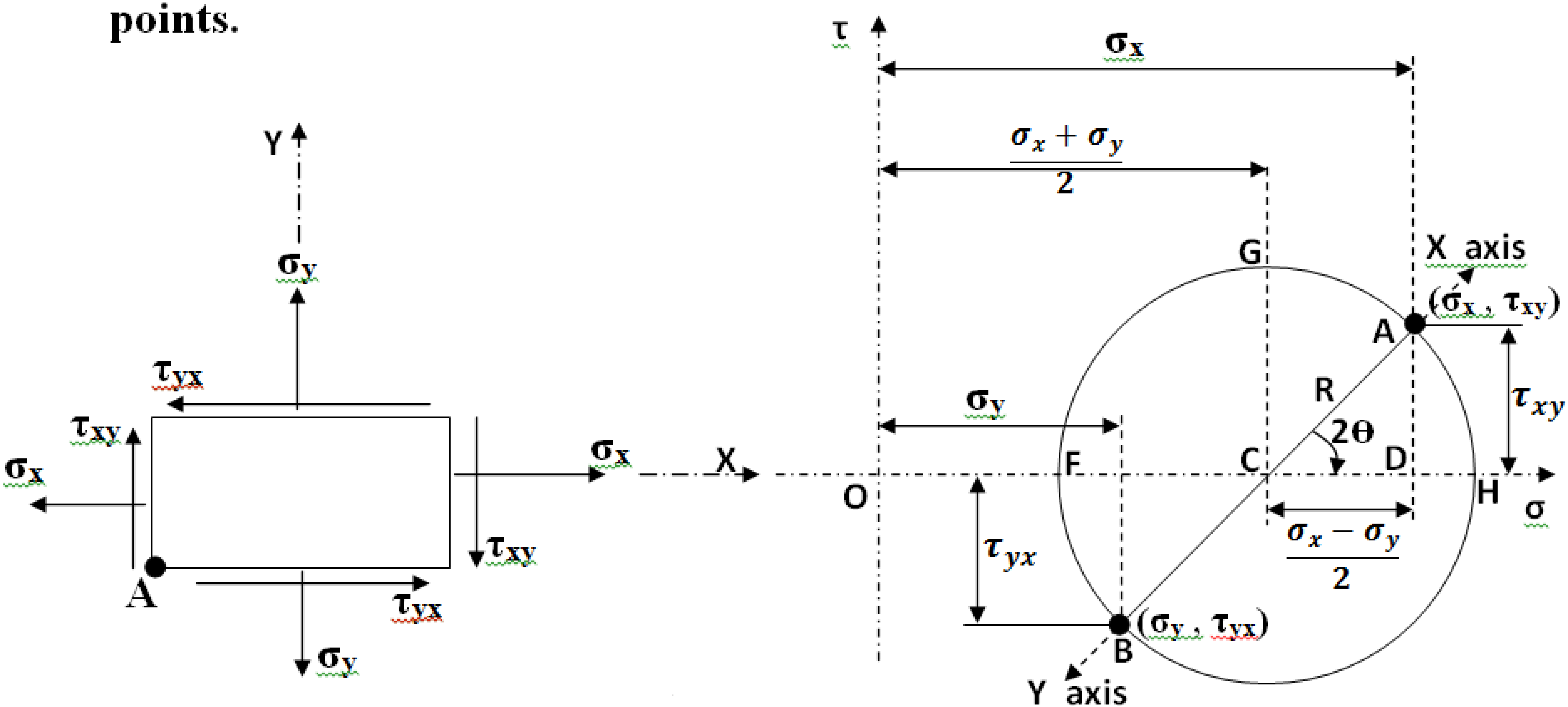
Comparing with the equation of the circle of the form $(x - C)^2 + y^2 = R^2$





Procedure for drawing Mohr's Circle

1. On rectangular $\sigma - \tau$ axes, plot points having the coordinates (σ_x , τ_{xy}) and (σ_y , τ_{yx}). The above sign convention will be used in plotting these points.



Mohr's Circle for General State of Stress

2. Join the points just plotted by a straight line. This line is the diameter of a circle whose center is on the σ – axis then:

$$CD = \frac{\sigma_x - \sigma_y}{2} \qquad OC = \sigma_y + \frac{\sigma_x - \sigma_y}{2} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

$$OH = OC + R = \frac{\sigma_x + \sigma_y}{2} + \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \sigma_1$$

$$OF = OC - R = \frac{\sigma_x + \sigma_y}{2} - \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2} = \sigma_2$$

3. The radius of the circle to any point on its circumference represents the axis directed normal to the plane whose stress components are given by the coordinates of that plane.

4. The angle between the radii to selected points on Mohr's circle is twice the angle between the normal to the actual planes represented by these points, i.e., if the N axis is actually at a counterclockwise angle θ from the X axis, then on Mohr's circle the N radius is laid off at a counterclockwise angle 2θ from the X radius.