Engineering Mechanics

Dynamics

Engineering Mechanics

University OF BASRAH
COLLAGE OF ENGINEERING NG
DEPARTMENT OF CIVIL ENGINEER

Lecture Notes in Engineering Mechanics

((Dynamics))

Engineering Mechanics

References

- Engineering Mechanics: Statics and Dynamics; by Archie Higdon and William B. Stiles.
- Theory and Problems of Engineering Mechanics: Statics and Dynamics; by Mclean and Nelson.
- Engineering Mechanics : Dynamics; 5th Edition by R. C. Hibbeler.
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Engineering Mechanics

Chapter One

Introduction

- **Dynamics** is a branch of the physical sciences that is concerned with the state of motion of bodies subjected to the action of forces.
- **Dynamics**, deals with the accelerated motion of a body.

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- The subject of dynamics will be presented in two parts:

  1. **Kinematics** : treats only the geometric aspects of the motion, or deals with the motion of particles, lines, and bodies without consideration of the forces required to produce or maintain the motion.

Note: Knowledge of the relationships between position, time, velocity, acceleration, displacement, and distance traveled for particles, lines, and bodies is essential to the study of the effects of unbalanced force systems on bodies.

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2. Kinetics : deals with the force system that produce accelerated motion of bodies, the inertial properties of the bodies, and the resulting motion of the bodies.

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- **Definitions:**
- **Length (Space):** Length is used to locate the position of a point in a space and thereby describe the size of a physical system;
- **Time:** measure of succession of events → basic quantity in Dynamics;
- **Mass:** quantity of matter in a body that is used to compare the action of one body with that of another. Provides a measure of inertia of a body (its resistance to change in velocity);

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- **Force:** represents the action of one body on another → characterized by its magnitude, direction of its action, and its point of application, it's a vector quantity.
- <u>**Notes:**</u>

1. Length, Time, and Mass are absolute concepts independent of each other;

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2. Force is a derived concept

not independent of the other fundamental concepts. Force acting on a body is related to the mass of the body and the variation of its velocity with time.

3. Mass is a property of matter that does not change from one location to another.

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3. Weight refers to the gravitational attraction of the earth on a body or quantity of mass. Its magnitude depends upon the elevation at which the mass is located;

Weight of a body is the gravitational force acting on it.

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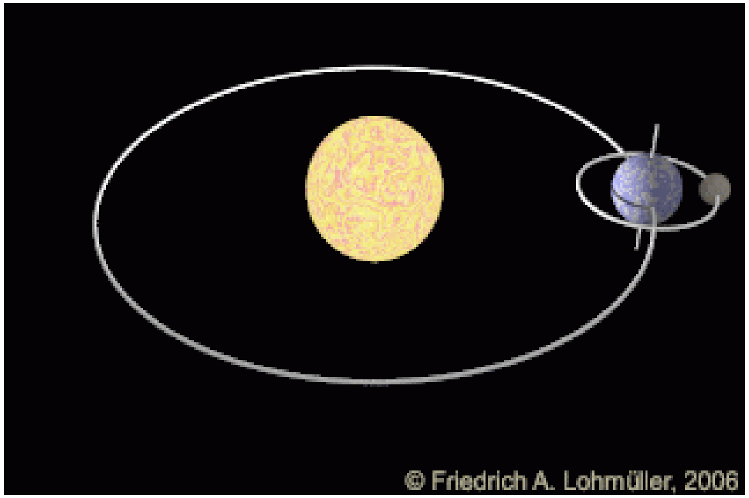
Idealization :

Particle: A body with mass but with dimensions that can be neglected.



Size of earth is insignificant compared to the size of its orbit. Earth can be modeled as a particle when studying its orbital motion.

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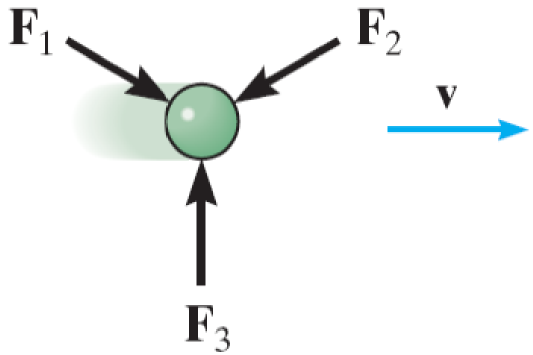
Newton’s Three Laws of Motion

- **First Law:** A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

First law contains the principle of the equilibrium of forces → main topic of concern in Statics



Equilibrium

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- **Second Law:** A particle of mass "m" acted upon by an unbalanced force "F" experiences an acceleration "a" that has the same direction as the force and a magnitude that is directly proportional to the force.

$$F = ma$$

Accelerated motion

Second Law forms the basis for most of the analysis in Dynamics

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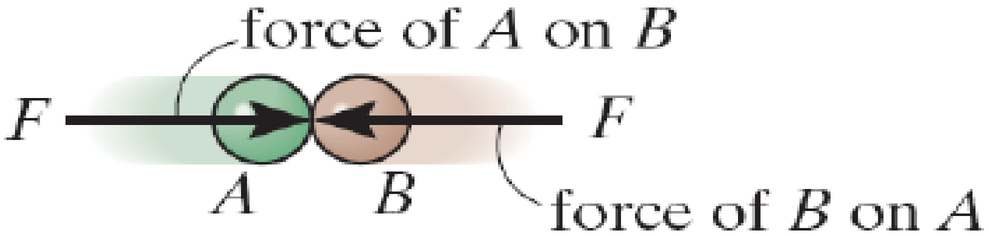
Third Law: The mutual forces of action and reaction between two particles are equal, opposite, and collinear.



Action – reaction

Third law is basic to our understanding of force. Forces always occur in pairs of equal and opposite forces.

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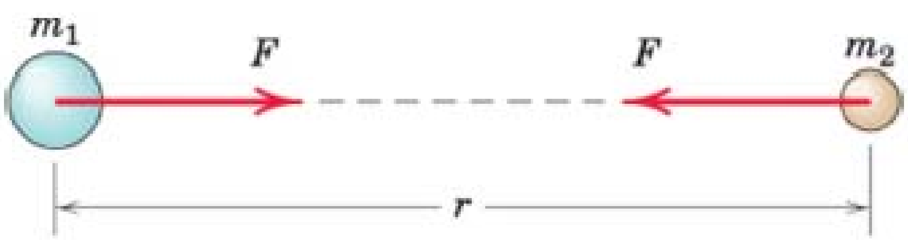
- **Weight of a body** (gravitational force acting on a body) is required to be computed in Statics as well as Dynamics.

This law governs the gravitational attraction between any two particles.

$$F = G\frac{m_1 m_2}{r^2}$$



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- ***F* = *mutual force of attraction between two particles;***
- ***G* = *universal constant of gravitation***

  Experiments ***G*** $= 6.673 \times 10^{-11}\ m^3/(kg.s^2)$;
- Rotation of Earth is not taken into account;
- ***m1*, *m2* = *masses of two particles;***
- ***r* = *distance between two particles.***

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- **Weight of a Body:** If a particle is located at or near the surface of the earth, the only significant gravitational force is that between

  the earth and the particle;

  Weight of a particle having mass ***m1 = m :***

  Assuming earth to be a nonrotating sphere of constant density and having mass ***m2 = Me***

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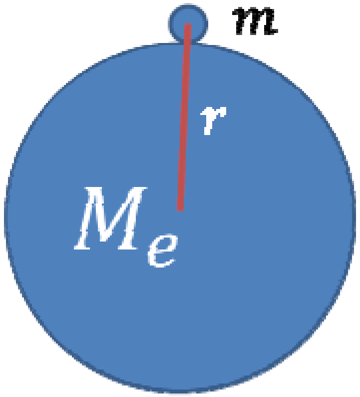
$$W = G\frac{mM_e}{r^2}$$

r = distance between the earth's center and the particle



Let $g = G\,M_e/r^2$ = acceleration due to gravity (9.81m/s^2)

$$W = mg$$

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Four Fundamental Quantities

Quantity	Dimensional Symbol	SI UNIT	
		Unit	Symbol
Mass	M	Kilogram	Kg
Length	L	Meter	M
Time	T	Second	s
Force	F	Newton	N

Basic Unit

$$F = ma \quad \rightarrow \quad N = kg.m/s^2$$

1 Newton is the force required to give a mass of 1 kg an acceleration of 1 m/s²

$$W = mg \quad \rightarrow \quad N = kg.m/s^2$$

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	Exponential Form	Prefix	SI Symbol
Multiple			
1 000 000 000	10^9	giga	G
1 000 000	10^6	mega	M
1 000	10^3	kilo	k
Submultiple			
0.001	10^{-3}	milli	m
0.000 001	10^{-6}	micro	μ
0.000 000 001	10^{-9}	nano	n

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2.1 Rectilinear Kinematics: Continuous Motion

This chapter deals with the kinematics of a particle that moves along a rectilinear or straight line path.

Rockets, Projectiles, or vehicles can be considered as a particle.

Any rotation of the body is neglected.

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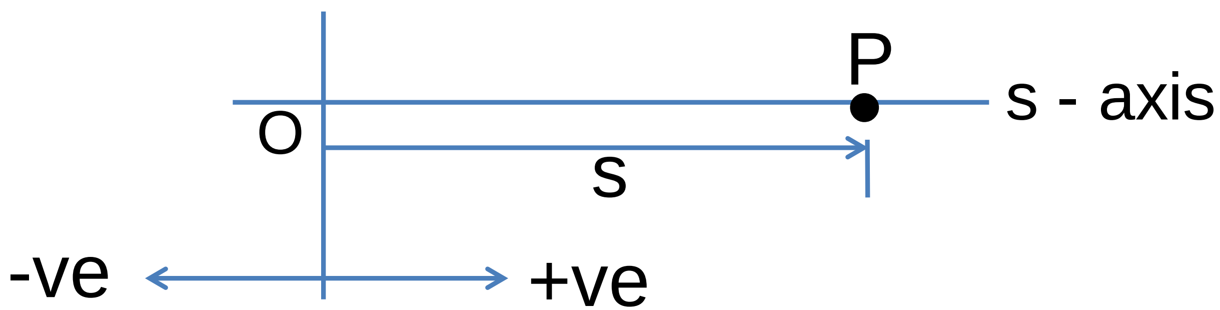
Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

(a) Position:

- The position of paritcle **P** at any time **t** is expressed in terms of its distance **x** from origin **O** on the x axis. It's a vector quantity. Units : millimetre (**mm**), centimetre (**cm**), and meter (**m**).



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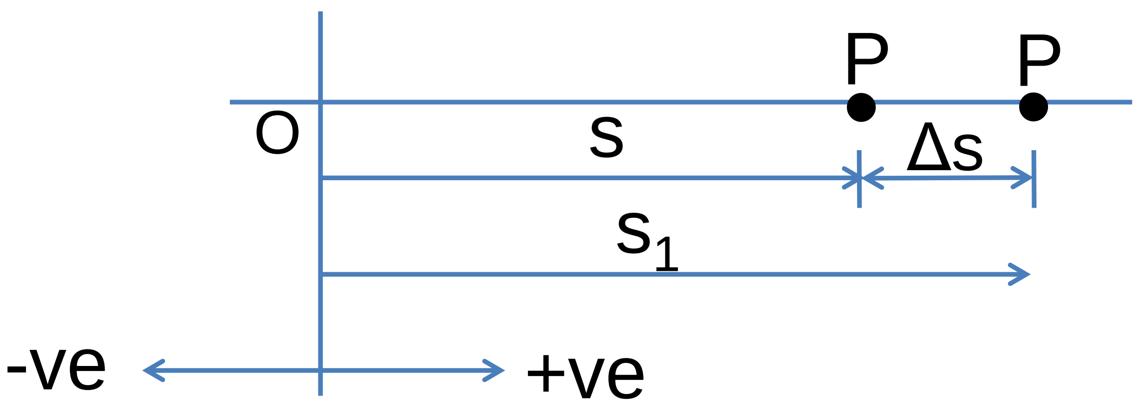
(b) **Displacement**: The displacement of the particle **P** is defined as the change in its position.

For example, if the particle moves from one point to another, the displacement is :

$$s = \Delta s = s_1 - s$$



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Δs : +ve if [$s_1 > s$] [i.e. the particle's final position is to the right of its initial position];

Δs : -ve if [$s_1 < s$] [i.e. the particle's final position is to the left of its initial position];

***Note:** distance traveled is a positive scalar that represents the total length of path over which the particle travels, [i.e. Total distance travel, $s^T = s + \Delta s$]*

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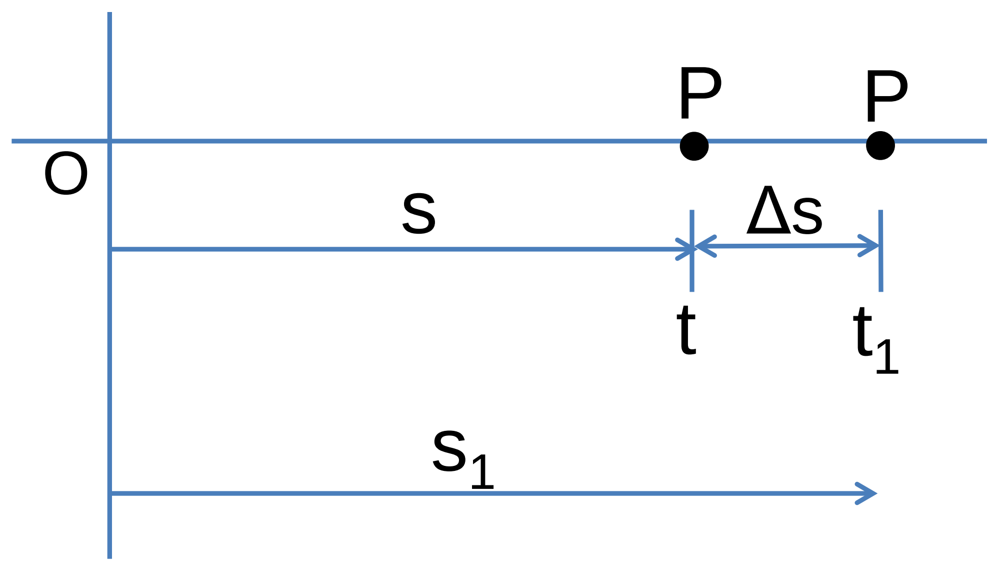
(c) **The average velocity v_{av}** of a particle P during the time interval **t** and **t+Δt** during which its position changes from **s** to **s+Δs** is :

$$\Delta x = s_1 - s$$

$$\Delta t = t_1 - t$$

$$v_{av} = \Delta x / \Delta t$$



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(d) **The instantaneous velocity *v*** of a particle **P** at time **t** is the limit of the average velocity as the increment time **Δt** approaches zero as a limit.

$$\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}\text{, or} \qquad (\overset{\rightarrow}{+}) \qquad v = \frac{ds}{dt}$$

Speed : is the magnitude of the velocity.

Average Speed : always a positive scalar and is defined as the total distance traveled by a particle, s_T divided by the elapsed time Δt,

$$(v_{sp})_{ave} = \frac{s_T}{\Delta t}$$

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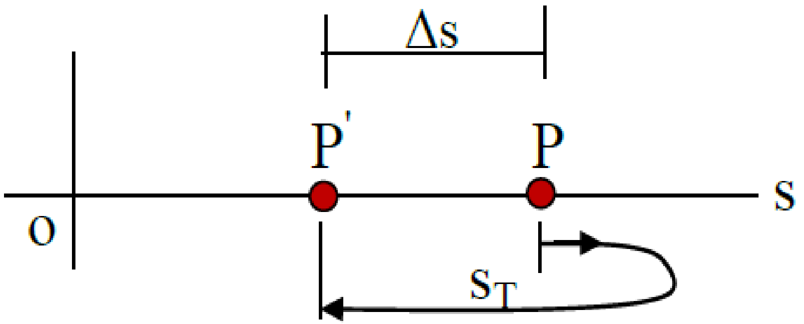
Example : the particle in Fig. below travels along the path of length s_T in a time Δt

$$(v_{sp})_{ave} = \frac{s_T}{\Delta t} \;\; ; \quad v_{ave} = \frac{-\Delta s}{\Delta t}$$



(e) The average acceleration a_{av} of a particle P during the time interval t and **t+Δt** during which its velocity changes from **v** to **v+Δv** is :

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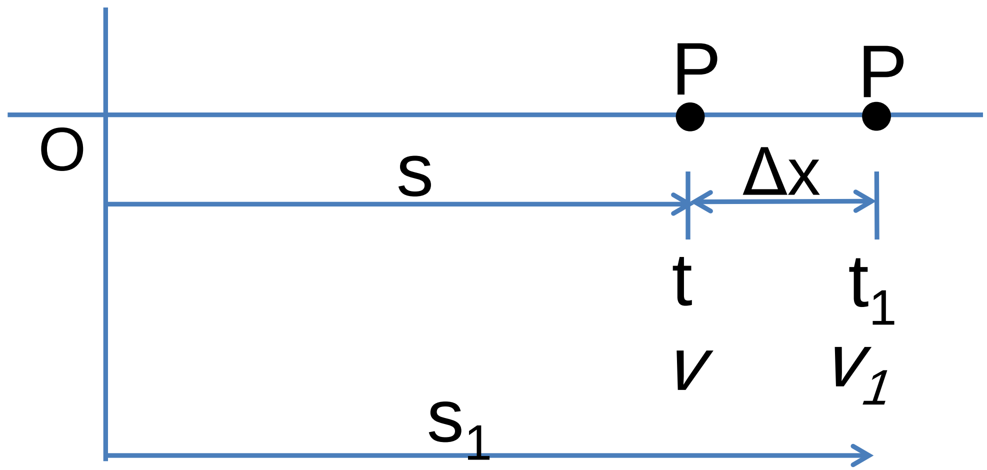
$\Delta v = v_1 - v$

$a_{ve} = \Delta v / \Delta t$



(f) **The instantaneous acceleration** ***a*** of a particle **P** at time **t** is the limit of the average acceleration as the increment time **Δt** approaches zero as a limit.

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$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2} \qquad (\overrightarrow{+})$$

Also; $$a = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2} \qquad (\overrightarrow{+})$$

Notes:

1. Both *average and instantaneous acceleration can be either +ve or -ve .*
2. when the particle is slowing down, or its speed is decreasing, the particle is said to be ***decelerating, [i.e. $v_1 < v$, and a is negative and in opposite sense to v].***

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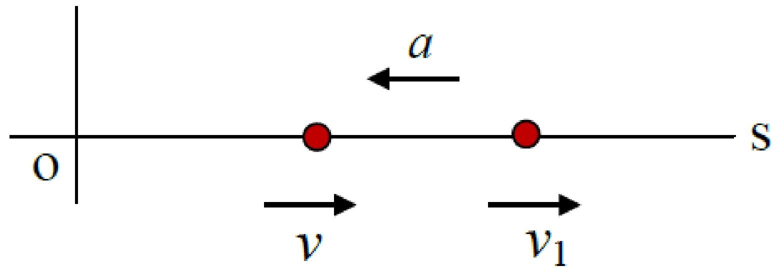

Deceleration

3. when the velocity is constant, the acceleration is **zero** since
$\Delta v = v_1 - v = 0$.

4. Units of acceleration are **m / s^2 or ft / s^2.**

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Important Relations

1. **Since** $v = \dfrac{ds}{dt}$ $\quad \gg\gg\gg \quad$ $dt = \dfrac{ds}{v}$ **and**

   $a = \dfrac{dv}{dt}$ $\quad \gg\gg\gg \quad$ $dt = \dfrac{dv}{a}$ **Therefore;**

$$\frac{ds}{v} = \frac{dv}{a} \quad \Rightarrow \quad \boxed{a\,ds = v\,dv}$$

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(g) For constant acceleration   Let : $a = a_c$

1. Velocity as a function of time; Assume $v = v_o$ *when* $t = 0$

Since; $a_c = dv / dt \Rightarrow dv = a_c \, . \, dt \Rightarrow \int_{v_o}^{v} dv = \int_{0}^{t} a_c \, dt$

Then: $\boxed{v = v_o + a_c t}$ $(\xrightarrow{+})$

2. Position as a function of time; Assume $s = s_o$ **when** $t = 0$

Since; $v = ds / dt \Rightarrow ds = v \, . \, dt = (v_o + a_c t) \, dt$

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Then; $$\int_{s_0}^{s} ds = \int_0^t (v_o + a_c t)\, dt$$

$$\boxed{s = s_o + v_o t + 0.5\, a_c t^2} \qquad (\overset{+}{\rightarrow})$$

3. Velocity as a function of position; Assume ***$v = v_o$* and *$s = s_o$* when *$t = 0$* ;**

Since: $a_c ds = v\, dv$ $\Rightarrow$ $$\int_{s_0}^{s} a_c\, ds = \int_{v_0}^{v} v\, dv$$

Then; $$\boxed{v^2 = v^2{}_o + 2\, a_c (s - s_0)} \qquad (\overset{+}{\rightarrow})$$

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<u>Ex:</u> A point P moves along a straight line according to the equation $x = 4t^3 + 2t + 5$; where x is in meters, t in seconds. Determine the displacement, velocity, and acceleration when t= 3 s. What is the average acceleration during the fourth second?

<u>Solution:</u>

$x_{(3\,s)} = 4t^3 + 2t + 5 = 4\,(3)^3 + 2\,(3) + 5 =$ **119 m**

$v = dx/dt = 12t^2 + 2$

$v_{(3\,s)} = 12\,(3)^2 + 2 =$ **110 m/s**

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$a = dv/dt = 24\ t$

$a_{(3\ s)} = 24\ (3) =$ **$72\ m/s^2$**

$v_{(4\ s)} = 12\ (4)^2 + 2 = 194\ m/s$

$\Delta t = 4 - 3 = 1\ sec$

$\Delta v = v_{(4\ s)} - v_{(3\ s)} = 194 - 110 = 84\ m/s$

So ;

$a_{av} = \Delta v / \Delta t = 84 / 1 =$ **$84\ m/s^2$**

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Ex: The magnitude of the linear acceleration of a point moving along a vertical path is given by the equation **a= 6t – 24**; where **a** is in m/s^2 and t is in second. The acceleration is upward when t = 5 sec, the point is 4m below the origin when t=0 and 23 m above the origin when t=3 sec. Determine **(a)** the velocity when t=3 sec; **(b)** the displacement during the time interval from t=0 to t= 4 sec; **(c)** the total distance travelled during the time interval t=0 to t=4 sec.

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Solution:

Given: a= 6t – 24

a is upward at t = 5 sec

s is -4m at t = 0

s is 23m at t = 3 sec

(a) $v_{(3sec)}$??

$\mathbf{a_{(5\ sec)} = 6\ (5) - 24 = +\ 6\ m/s^2}$

$\mathbf{a = dv / dt \Rightarrow dv = a\ dt}$

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And $v = \int a \, dt = \int (6t - 24) \, dt = 3t^2 - 24t + C_1$

Also; $v = ds/dt \Rightarrow ds = v \, dt$

$$s = \int v \, dt = \int (3t^2 - 24t + C_1)dt$$

$$= t^3 - 12t^2 + C_1 . t + C_2$$

$s_{(t = 0)} = -4 = 0 - 0 + 0 + C_2 \Rightarrow C_2 = -4$

$s_{(t=3 \text{ sec})} = 23 = (3)^3 - 12 (3)^2 + C_1 (3) - 4 \Rightarrow C_2 = 36$

So; $v = 3t^2 - 24t + 36$

$$s = t^3 - 12t^2 + 36t - 4$$

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$v_{(t=\ 3\ sec)} = 3(3)^2 - 24(3) + 36 = -9\ m\ /\ s$

$= $ **9 m/s (↓) downward**

(b) Δs during t=0 to t=4 sec

$s_{(t=0)} = -4\ m$ downward (given)

$s_{(t=4\ sec)} = (4)^3 - 12(4)^2 + 36\ (4) - 4 = 12\ m$

$\Delta s = 12 - (-4) = $ **16 m upward (↑)**

(c) The total distance travelled during t=0 to t= 4 sec:

To determine the total distance travelled, its important to know the actual path traversed during time.

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$v = 3t^2 - 24t + 36 = 3\,(t-6)(t-2)$

Therefore;

$v_{(t=\,0)} = 36$ m/s upward

$v_{(t=\,2\,sec)} = 0$

$v_{(t=\,4\,sec)} = -12$ m/s downward

$v_{(t=\,6\,sec)} = 0$ (ignored since the required interval between t=0 and t=4sec);

$s_{(t=\,0)} = -4$ m downward (given)

$s_{(t=2\,sec)} = (2)^3 - 12(2)^2 + 36\,(2) - 4 = 28$ m upward

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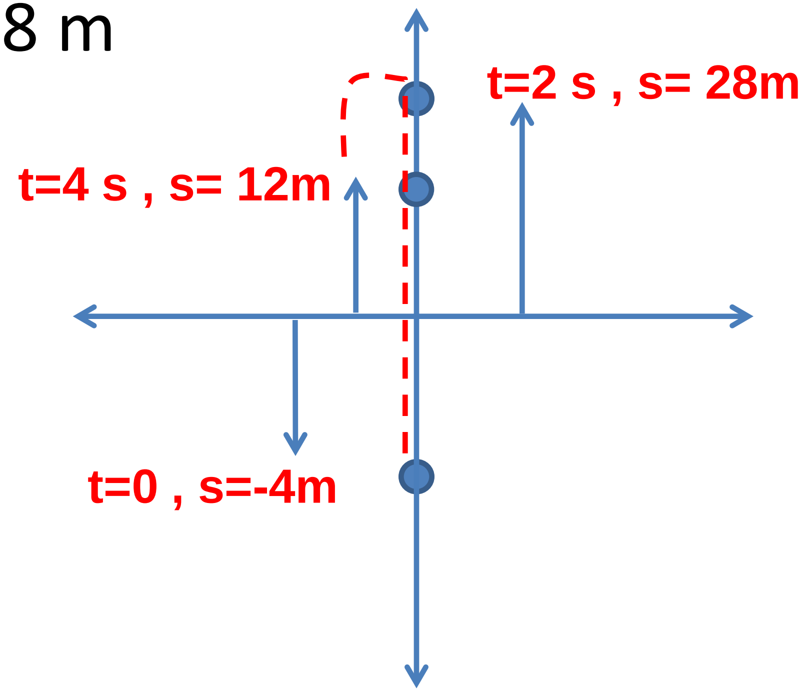
$s_{(t=4\ sec)} = (4)^3 - 12(4)^2 + 36\ (4) - 4 = 12\ m$ upward

Therefore;

Total distance = 4 + 28 + 16= 48 m



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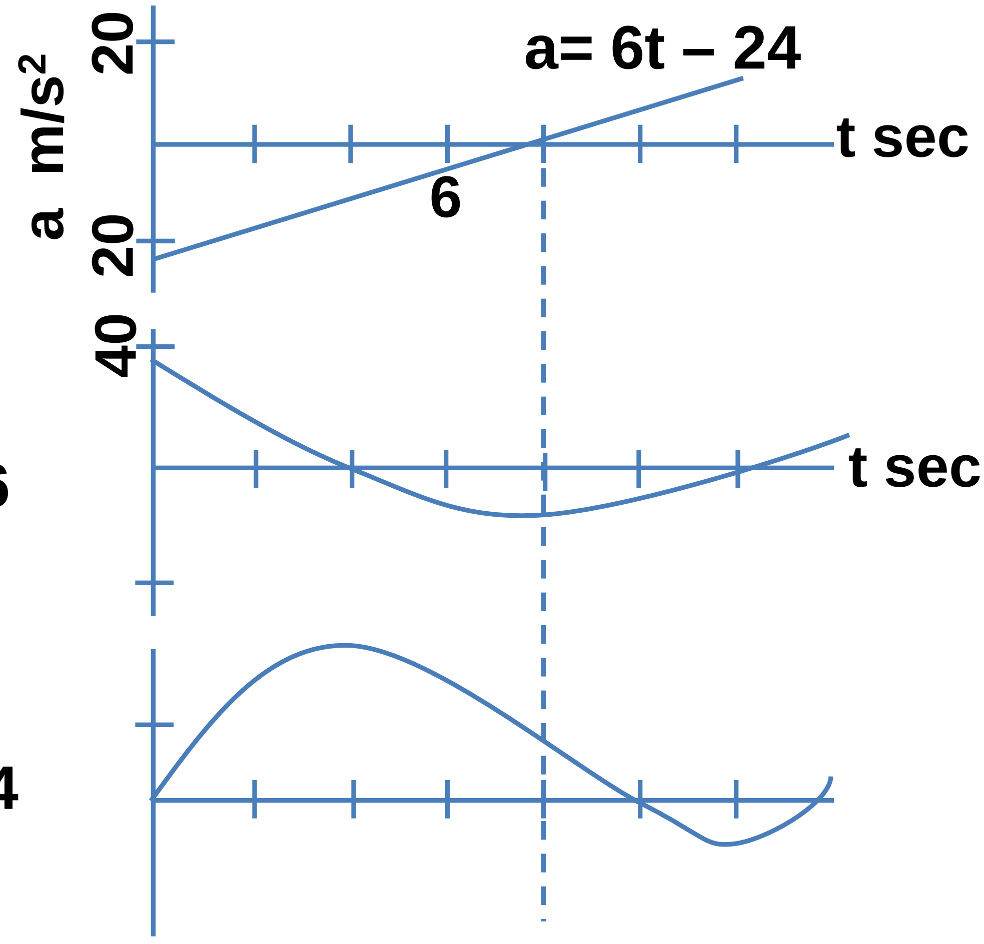
Solution 2: Use the a-t, v-t, and s-t curves.



$v = 3t^2 - 24t + 36$

$s = t^3 - 12t^2 + 36t - 4$

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Ex: The magnitude of the acceleration of a point moving along a horizontal straight line varies according to the equation $\mathbf{a= 12\ s^{1/2}}$, where a is in m/s^2 and s is the distance of the point from the origin in meter. When the time t is 2 sec, the point is 16 m to the right of the origin and has a velocity of 32 m/s to the right and an acceleration of 48 m/s^2 to the right. Determine the velocity and acceleration of the point when the time is 3 sec.

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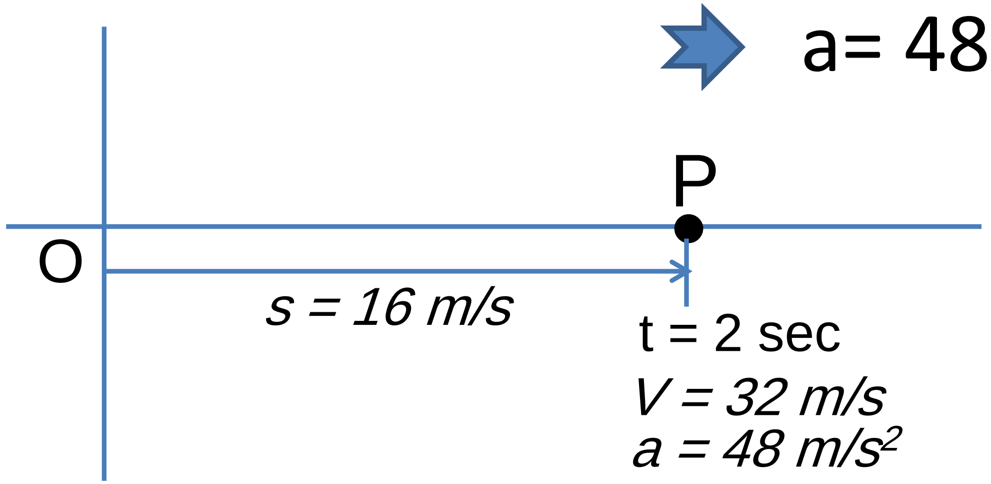
Solution:

Given : **a= 12 $s^{1/2}$**

at t = 2 sec ➔ s = 16 m to the right

➔ v= 32 m/s to the right

➔ a= 48 m/s^2 to the right



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Solution:

$a = v\,(dv/ds)$

$a\,ds = v\,dv$ ⇒ $\mathbf{12\,s^{1/2}}\,ds = v\,dv$

$8\,s^{3/2} = (v^2/2) + C_1$

When s=16 m , v= 32 m/s, then

$8\,(16)^{3/2} = (32^2/2) + C_1$ ⇒ $C_1 = 0$

So : $v^2 = 16\,s^{3/2}$ ⇒ $v = 4\,s^{3/4}$

But: $v = (ds/dt) = 4\,s^{3/4}$ ⇒ $(ds/s^{3/4}) = 4\,dt$ ⇒

$4s^{1/4} = 4t + C_2$

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When s= 16 m/s, t= 2 sec, then

$4\,(16)^{1/4} = 4\,(2) + C_2$ ⇒ $C_2 = 0$

So; $4s^{1/4} = 4t$

$s = t^4$ ⇒ $s_{(t=3sec)} = (3)^4 = 81$ m to the right

$v = 4t^3$ ⇒ $v_{(t=3sec)} = 4\,(3)^3 = 108$ m/s to the right

$a = 12t^2$ ⇒ $a_{(t=3\ sec)} = 12\,(3)^2 = 108$ m/s to the right

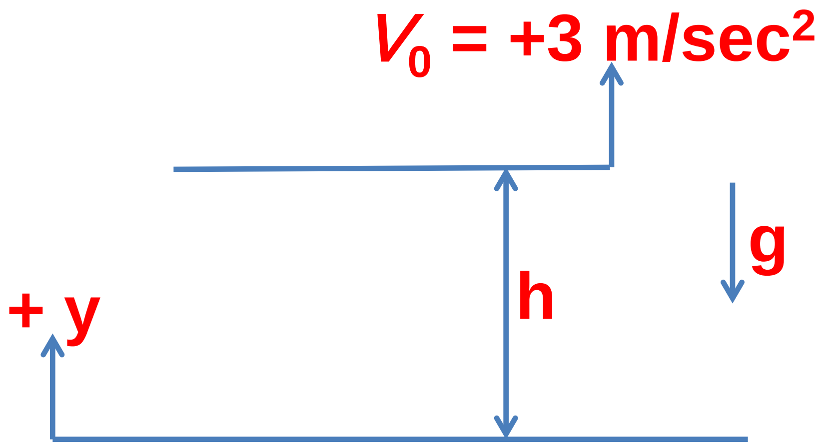
Ex: A particle falls from an elevator that is moving up with a velocity of **3 m/sec**. If the particle reaches the bottom in **2 sec**, how high above the bottom was the elevator when the particle started falling?

Solution:

Free falling

i.e. Constant acceleration

$a = g = -9.81\ m/sec^2$



$$y = y_0 + v_0\, t + 0.5\, a\, t^2$$

$$0 = h + 3 \times 2 + 0.5 \times (-9.81) \times 2^2$$

$$h = 13.62\ m \qquad \text{Ans.}$$

Ex: An automobile accelerators uniformly from rest on a straight level road. A second automobile starting from the same point **6 sec** later with zero initial velocity and accelerates at **6 m/sec^2** to overtake the first automobile **400 m** from the starting point. What is the acceleration of the first automobile.

Solution:

If the **1st** automobile need a time **t** to reach **400 m** from the starting point, the **2nd** automobile need a time **(t – 6)** to reach the same point. (i.e. **Difference in time**)

$$y = y_0 + v_0\, t + 0.5\, a\, t^2$$

2nd automobile $v_0 = 0$ $s_0 = 0$ $s = 400$ m

$400 = 0+0+ 0.5 \times 6 \times (t - 6)^2$ $t = 17.547$ sec

1st automobile $v_0 = 0$ $s_0 = 0$ $s = 400$ m

$400 = 0+0+ 0.5 \times a \times 17.547^2$ $a = 2.598$ m/sec^2 **Ans.**

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2.2 Rectilinear Kinematics: Erratic Motion

When a particle has erratic or changing motion then its position, velocity, and acceleration *cannot be described by a single continuous mathematical function along the entire path. Instead, a series of functions will be required to specify the motion at different intervals. For this reason, it is convenient to represent the motion as a graph. If a graph of the motion that relates any two of the variables* ***s****,* ***v****,* ***a****,* ***t*** *can be drawn, then this graph can be used to construct subsequent graphs relating two other variables since the variables are related by the differential relationships* ***v = ds/dt****, or* ***a ds = v dv****.*

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The *s - t, v - t, and a - t Graphs.*

Used to

Given : ***s – t curve*** $\Longrightarrow$ ***v – t curve***

Construct

$$v = \frac{ds}{dt}$$

Slope of $s - t$ graph = Velocity

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Used to

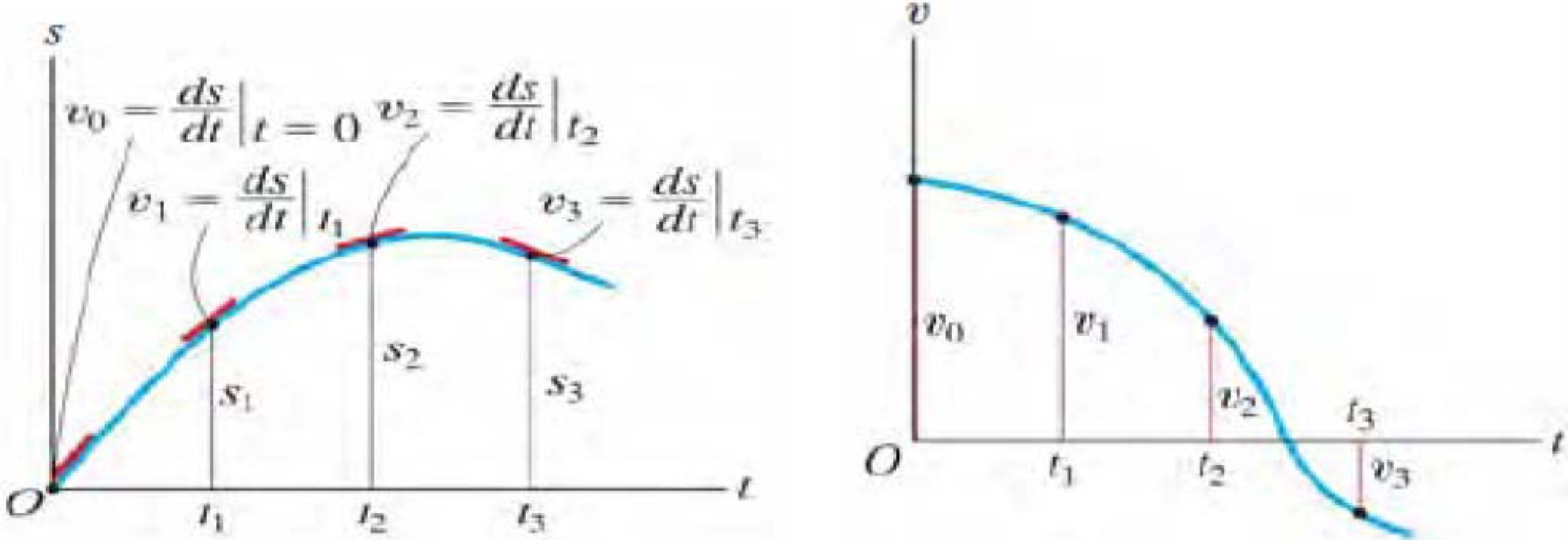
Given : *v – t curve* ⟹ ***a – t curve***

Construct

$$a = \frac{dv}{dt}$$

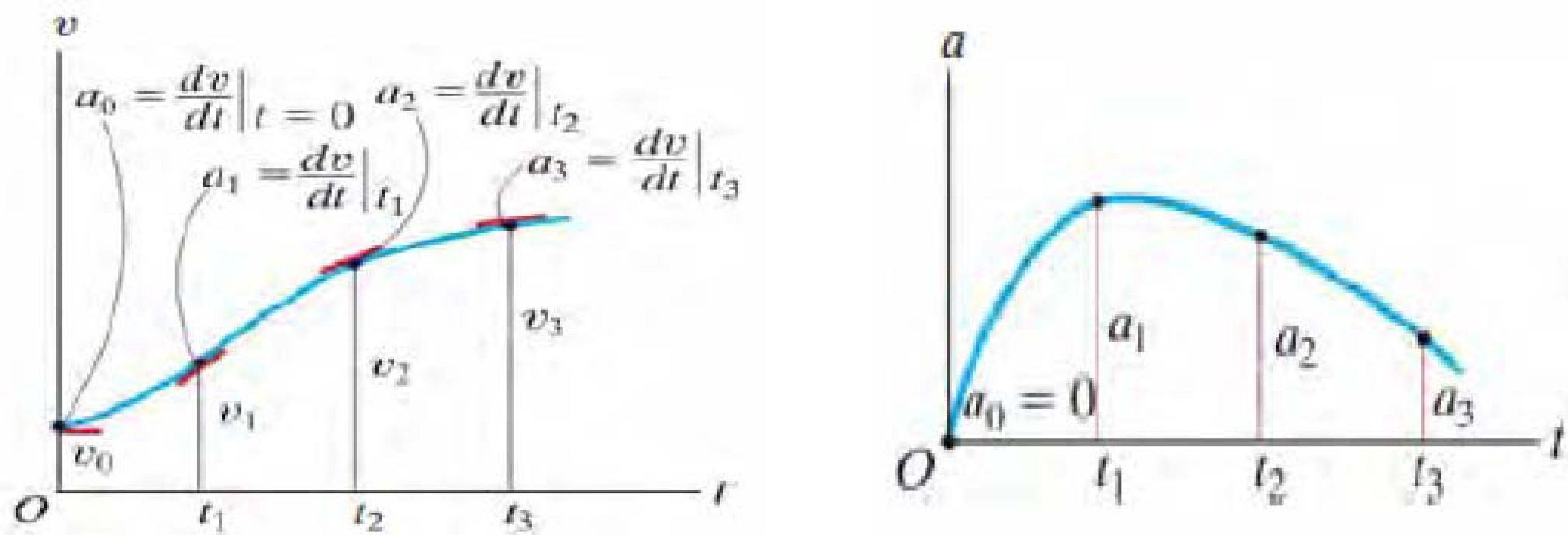
Slope of $v - t$ graph = Acceleration

Note *: If s – t curve is parabolic (2^{nd} degree curve) ; the v – t graph will be a sloping line (1^{st} degree curve), and the a – t graph will be a constant or a horizontal line (zero degree curve).*



s
$v_0 = \frac{ds}{dt}\Big|_{t=0}$
$v_2 = \frac{ds}{dt}\Big|_{t_2}$
$v_1 = \frac{ds}{dt}\Big|_{t_1}$
$v_3 = \frac{ds}{dt}\Big|_{t_3}$
s_1
s_2
s_3
O
t_1
t_2
t_3
t
v
v_0
v_1
v_2
v_3
O
t_1
t_2
t_3
t

(a) (b)



v
$a_0 = \frac{dv}{dt}\Big|_{t=0}$
$a_2 = \frac{dv}{dt}\Big|_{t_2}$
$a_1 = \frac{dv}{dt}\Big|_{t_1}$
$a_3 = \frac{dv}{dt}\Big|_{t_3}$
v_0
v_1
v_2
v_3
O
t_1
t_2
t_3
t
a
$a_0 = 0$
a_1
a_2
a_3
O
t_1
t_2
t_3
t

(a) (b)

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Used to

Given : ***a – t curve*** → ***v – t curve***

Construct

$$\Delta v = \int a\, dt$$

Change in velocity = area under a – t graph

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Used to

Given : ***v – t curve*** ➡ ***s – t curve***

Construct

$$\Delta s = \int v\, dt$$

Change in velocity = area under a – t graph

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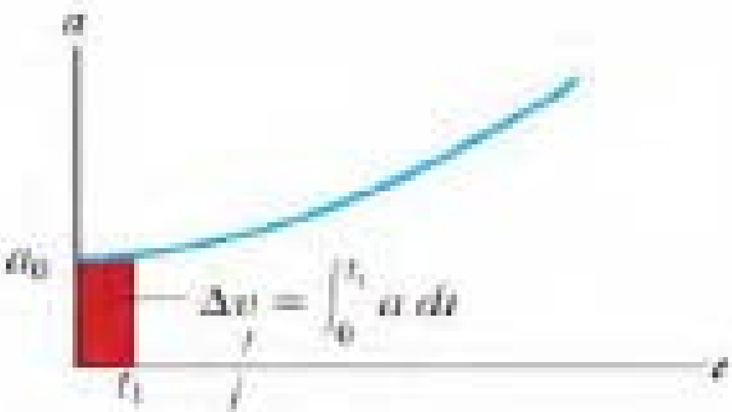
<u>Notes</u>:

1. The area above t *axis is +ve (an increase in* a *or* v *); and the area below the* t *axis is –ve (a decrease in* a *or* v *);*

2. If the initial velocity is v_o $\Rightarrow$ $v_1 = v_o + \Delta v$ *(algebraic sum);*

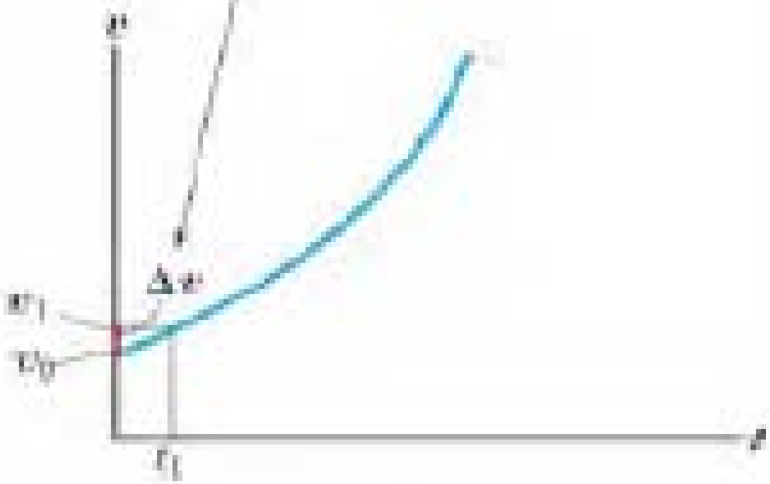
3. If the initial distance is s_o $\Rightarrow$ $s_1 = s_o + \Delta s$ *(algebraic sum)*

4. If $a - t$ *is linear (1st degree curve)* $\Rightarrow$ $v - t$ *curve is a parabolic (2nd* degree curve) $\Rightarrow$ $s - t$ *curve is cubic (3rd degree curve).*



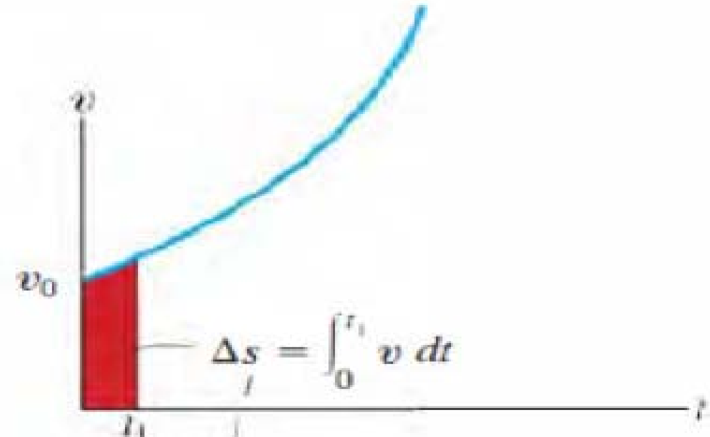
a
a_0
$\Delta v = \int_0^{t_1} a \, dt$
t_1
t

(a)



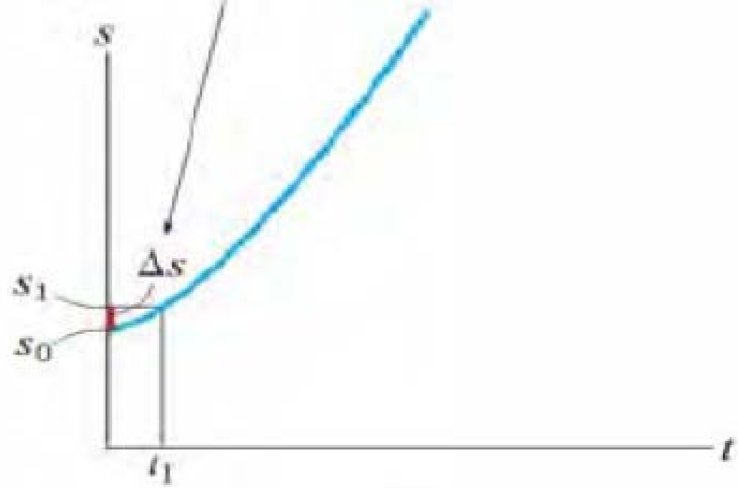
v
Δv
v_1
v_0
t_1
t

(b)



v
v_0
$\Delta s = \int_0^{t_1} v \, dt$
t_1
t

(a)



s
Δs
s_1
s_0
t_1
t

(b)

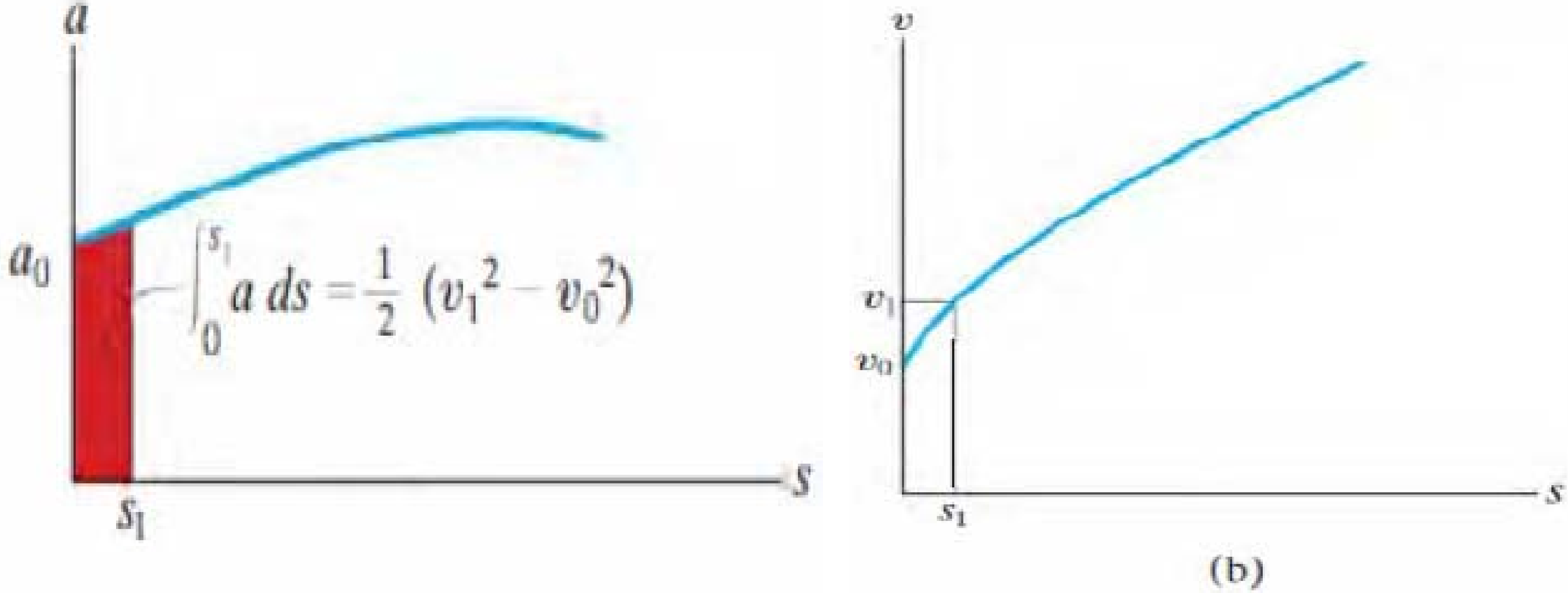
The *v - s and a – s Graphs.*

Given : *a – s curve* → **Used to Construct** → ***v – s curve***

$$\int_{v_0}^{v_1} v\,dv = \int_{s_0}^{s_1} a\,ds$$

$$\frac{1}{2}(v_1^2 - v_o^2) = \int_{s_0}^{s_1} a\,ds$$

area under $a - s$ graph



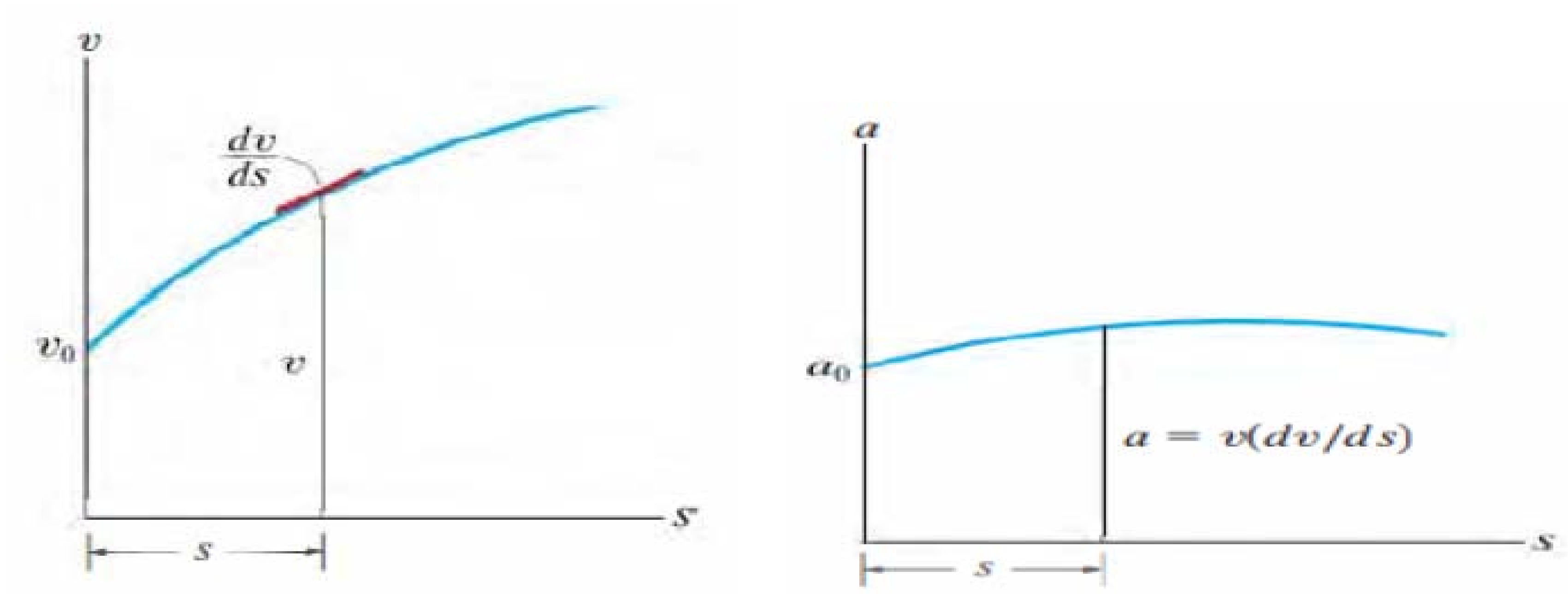
(a) (b)

Given : *v – s curve* $\xrightarrow[\text{Construct}]{\text{Used to}}$ *a – s curve*

$$a\,ds = v\,dv$$

$$a = v\left(\frac{dv}{ds}\right)$$

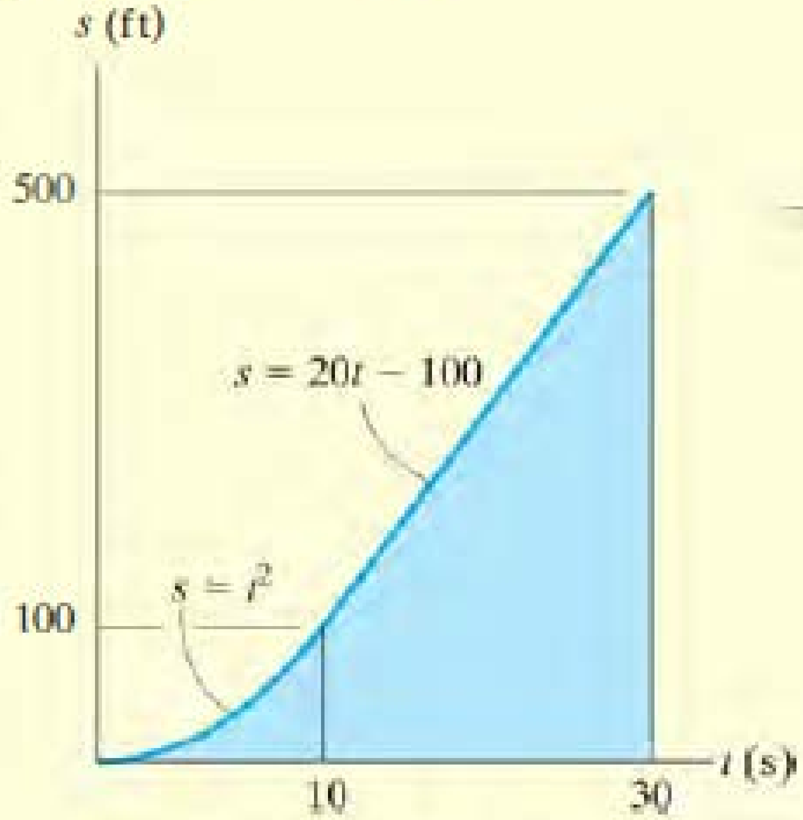
Acceleration = Velocity × slope of $v - s$ graph



(a) (b)

Ex. Hibbeler Pag.22 Ex.12.6

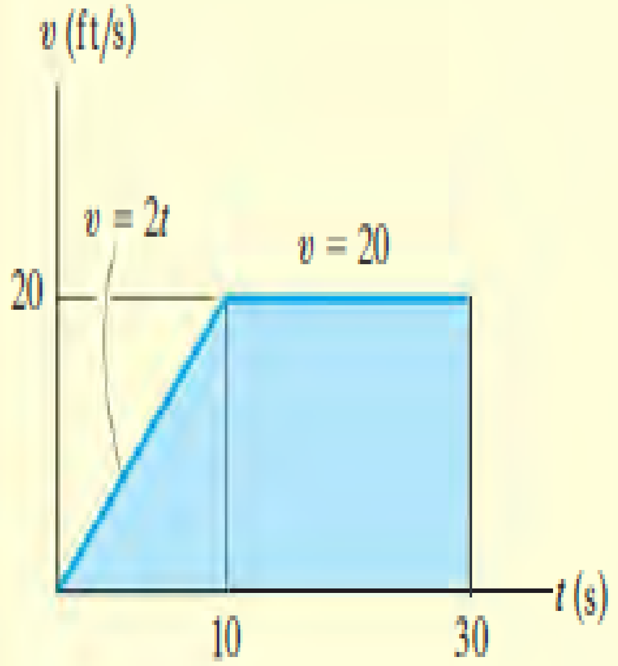
A bicycle moves along a straight road such that its position is described by the graph shown in Fig. 12–13*a*. Construct the v–t and a–t graphs for $0 \leq t \leq 30$ s.



(a)

Solution:

SOLUTION



(b)

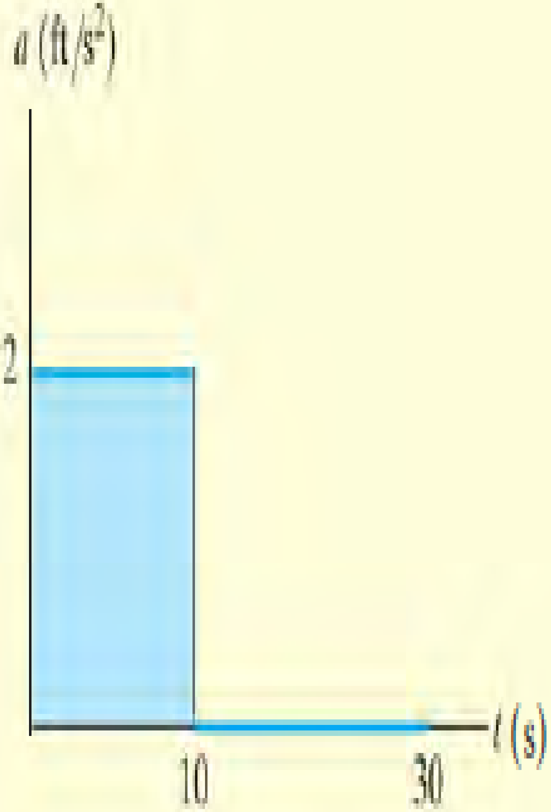
v–t Graph. Since $v = ds/dt$, the v–t graph can be determined by differentiating the equations defining the s–t graph, Fig. 12–13a. We have

$$0 \le t < 10\text{ s}; \qquad s = (t^2)\text{ ft} \qquad v = \frac{ds}{dt} = (2t)\text{ ft/s}$$

$$10\text{ s} < t \le 30\text{ s}; \qquad s = (20t - 100)\text{ ft} \qquad v = \frac{ds}{dt} = 20\text{ ft/s}$$

The results are plotted in Fig. 12–13b. We can also obtain specific values of v by measuring the *slope* of the s–t graph at a given instant. For example, at $t = 20$ s, the slope of the s–t graph is determined from the straight line from 10 s to 30 s, i.e.,

$$t = 20\text{ s}; \qquad v = \frac{\Delta s}{\Delta t} = \frac{500\text{ ft} - 100\text{ ft}}{30\text{ s} - 10\text{ s}} = 20\text{ ft/s}$$



(c)

Fig. 12–13

a–t Graph. Since $a = dv/dt$, the a–t graph can be determined by differentiating the equations defining the lines of the v–t graph. This yields

$$0 \le t < 10\,\text{s}; \qquad v = (2t)\ \text{ft/s} \qquad a = \frac{dv}{dt} = 2\ \text{ft/s}^2$$

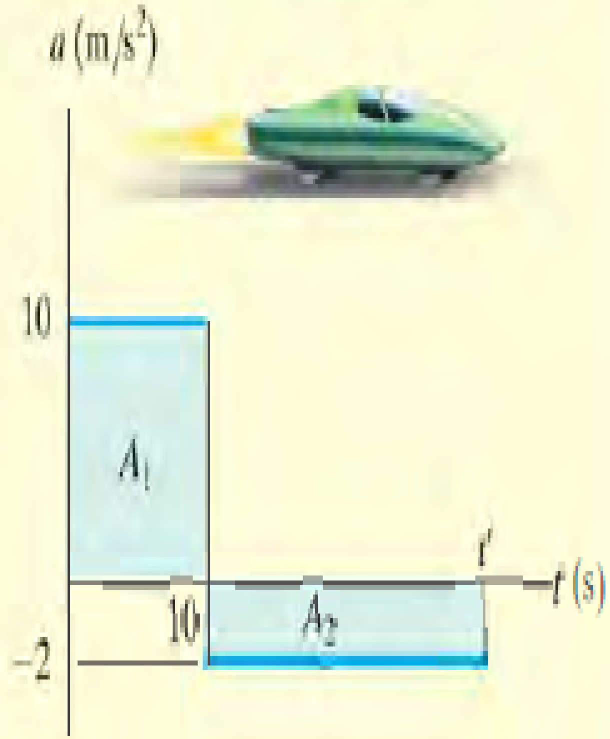
$$10 < t \le 30\,\text{s}; \qquad v = 20\ \text{ft/s} \qquad a = \frac{dv}{dt} = 0$$

The results are plotted in Fig. 12–13c.

NOTE: Show that $a = 2\ \text{ft/s}^2$ when $t = 5$ s by measuring the slope of the v–t graph.

Ex. Hibbeler Pag.23 Ex.12.7

The car in Fig. 12–14*a* starts from rest and travels along a straight track such that it accelerates at $10\ m/s^2$ for 10 s, and then decelerates at $2\ m/s^2$. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?



(a)

SOLUTION

v–t Graph. Since $dv = a\,dt$, the v–t graph is determined by integrating the straight-line segments of the a–t graph. Using the *initial condition* $v = 0$ when $t = 0$, we have

$$0 \le t < 10\ s; \quad a = (10)\ m/s^2; \quad \int_0^v dv = \int_0^t 10\,dt, \qquad v = 10t$$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

Solution:

$10\text{ s} < t \leq t'$; $a = (-2)\text{ m/s}^2$; $\displaystyle\int_{100\text{ m/s}}^{v} dv = \int_{10\text{ s}}^{t} -2\,dt$, $v = (-2t + 120)\text{ m/s}$

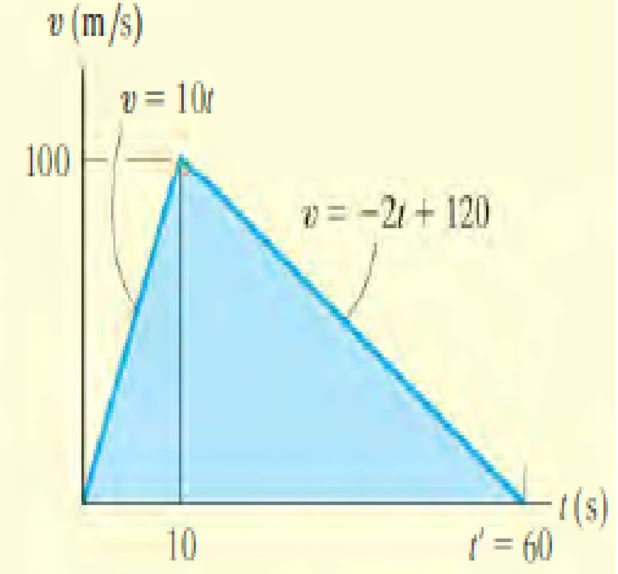
When $t = t'$ we require $v = 0$. This yields, Fig. 12–14b,

$$t' = 60\text{ s} \qquad \textit{Ans.}$$

A more direct solution for t' is possible by realizing that the area under the a–t graph is equal to the change in the car's velocity. We require $\Delta v = 0 = A_1 + A_2$, Fig. 12–14a. Thus

$$0 = 10\text{ m/s}^2(10\text{ s}) + (-2\text{ m/s}^2)(t' - 10\text{ s})$$

$$t' = 60\text{ s} \qquad \textit{Ans.}$$



(b)

s–t Graph. Since $ds = v\,dt$, integrating the equations of the v–t graph yields the corresponding equations of the s–t graph. Using the *initial condition* $s = 0$ when $t = 0$, we have

$0 \leq t \leq 10\text{ s}$; $\quad v = (10t)\text{ m/s}$; $\displaystyle\int_0^s ds = \int_0^t 10t\,dt$, $\qquad s = (5t^2)\text{ m}$

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

$10\text{ s} \le t \le 60\text{ s}$; $v = (-2t + 120)\text{ m/s}$; $\displaystyle\int_{500\text{ m}}^{s} ds = \int_{10\text{ s}}^{t} (-2t + 120)\, dt$

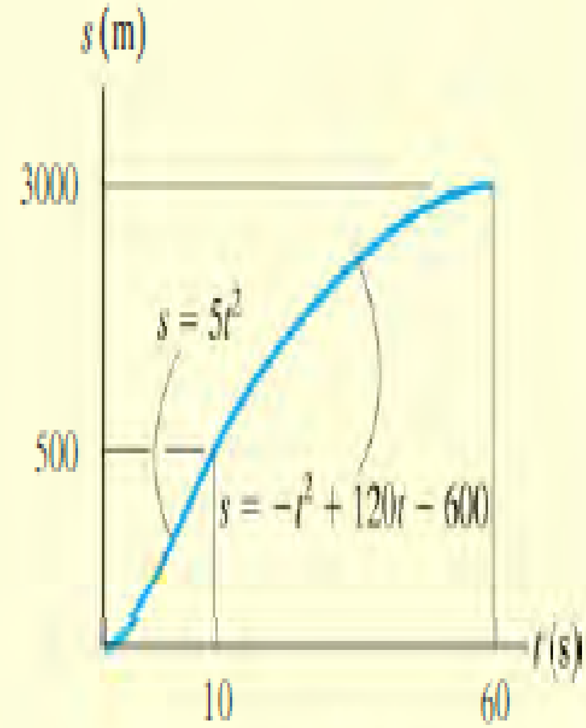
$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

$$s = (-t^2 + 120t - 600)\text{ m}$$

When $t' = 60$ s, the position is

$$s = -(60)^2 + 120(60) - 600 = 3000\text{ m} \qquad \textit{Ans.}$$

The s–t graph is shown in Fig. 12–14c.



(c)

Fig. 12–14

NOTE: A direct solution for s is possible when $t' = 60$ s, since the *triangular area* under the v–t graph would yield the displacement $\Delta s = s - 0$ from $t = 0$ to $t' = 60$ s. Hence,

$$\Delta s = \tfrac{1}{2}(60\text{ s})(100\text{ m/s}) = 3000\text{ m} \qquad \textit{Ans.}$$

Engineering Mechanics

Dynamics

Engineering Mechanics

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Lecture Notes in Engineering Mechanics

((Dynamics))

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3.1 Coplanar Angular Motion of a Line

A line has angular motion when the angle between it and a fixed reference line changes.

Particles are dimensionless, and any angular motion they might have cannot be measured or described, ***therefore, angular motion will be considered a property restricted to lines and bodies.***

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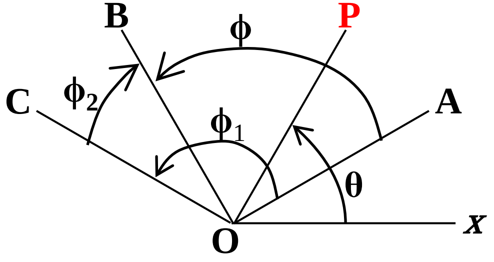
In Fig. below, the angle between the fixed x axis and the moving line OP varies with time and completely defines the angular position of OP at any instant.

ϑ is the angular position function and is a scalar function of time.

+ve sense is often with clockwise direction unless it described.

Angular displacement of a line during any time interval is the change of angular position of the line during that time interval.



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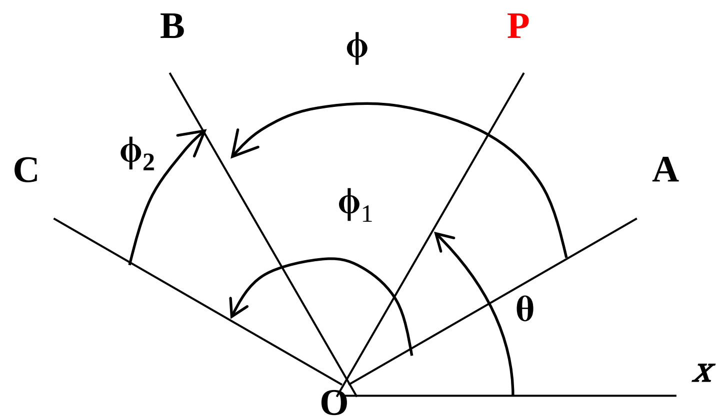

when the line OP turns from OA to OB during a certain time interval.

The angular displacement is ϕ (counter clockwise).

The total angle turned is ϕ .

when the line OP turns from to OC and then back to position OB during a time interval •

The angular displacement is ϕ (counter clockwise).

The total angle turned is $\phi_1 + \phi_2$.

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Notes:

1. For motion involving more than one revolution, values of **ϕ** and **θ** will continue to increase, [i.e. for two revolutions **θ** will be **2л** and is ***not zero***.
2. Units for angular measurements are radians (rad), revolutions (rev) and degrees.

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Notes:

1. For motion involving more than one revolution, values of **φ** and **θ** will continue to increase, [i.e. for two revolutions **θ** will be **2л** and is ***not zero***.
2. Units for angular measurements are radians (rad), revolutions (rev) and degrees.

*The **angular velocity**, **ω**, of a line is the time rate of change of the angular position of the line.*

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*The **angular velocity**, **ω**, of a line is the time rate of change of the angular position of the line.*

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Units:

rad per sec,

rev per min. (rpm)

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*The **angular acceleration**, **α**, of a line is the time rate of change of the angular velocity of the line.*

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\alpha\, d\theta = \frac{d\omega}{dt}\, d\theta = \frac{d\theta}{dt}\, d\omega = \omega\, d\omega$$

$$\alpha = \omega \frac{d\omega}{dt}$$

Units:

rad per sec^2,

rev per min per sec. (rpm per sec)

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Constant Angular Acceleration When $\alpha = \alpha_c$

Rectilinear Motion	Angular Motion
$v = v_o + a_c \cdot t$	$w = w_o + \alpha_c \cdot t$
$s = s_o + v_o \cdot t + 0.5\, a_c \cdot t^2$	$\theta = \theta_o + w_o \cdot t + 0.5\, \alpha_c \cdot t^2$
$v^2 = v_o^2 + 2\, a_c\, (s - s_o)$	$w^2 = w_o^2 + 2\, \alpha_c\, (\theta - \theta_o)$

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<u>Note</u> : If P is a point travels along a circular path of radius r with a centre at O

<u>Velocity</u>

The velocity vector is always tangent to the path.

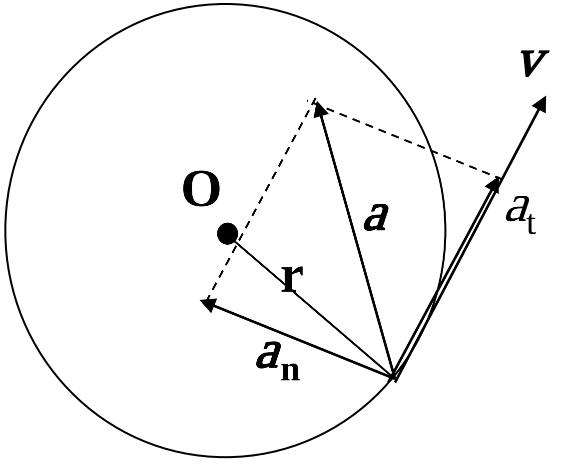
$\boldsymbol{v} = \omega \cdot r$ +

<u>Acceleration</u>

$a_t = \alpha \cdot r$ + **Tangential component**

$a_n = \omega^2 \cdot r$ + **Normal component**

$a = \sqrt{a_t^2 + a_n^2}$ +



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Ex. A line rotates in a vertical plane according to the law $\theta = t^3 - 2t^2 - 2$; where θ gives the angular position of the line in radians and t is the time in seconds. The line is turning clockwise when $t=1$ sec. Determine

(a) the angular acceleration when $t = 2$ sec;

(b) the value of t when the angular velocity is zero;

(c) the total angle turned through the time $t=1$ sec to $t=3$ sec.

Solution:

Check the +ve direction;

$\Theta_{(1\ sec)} = 1^3 - 2(1^2) - 2 = -3$ rad clockwise;

So the +ve direction is counterclockwise.

(a) $\omega = d\theta / dt = 3t^2 - 4t$

$\alpha = d\,\omega / dt = 6t - 4$

$\alpha = 6\,(2) - 4 = 8\ \text{rad/sec}^2$ counterclockwise

(b) $\omega = 3t^2 - 4t = 0$ $t = 0$ or $t = 1.333$ sec

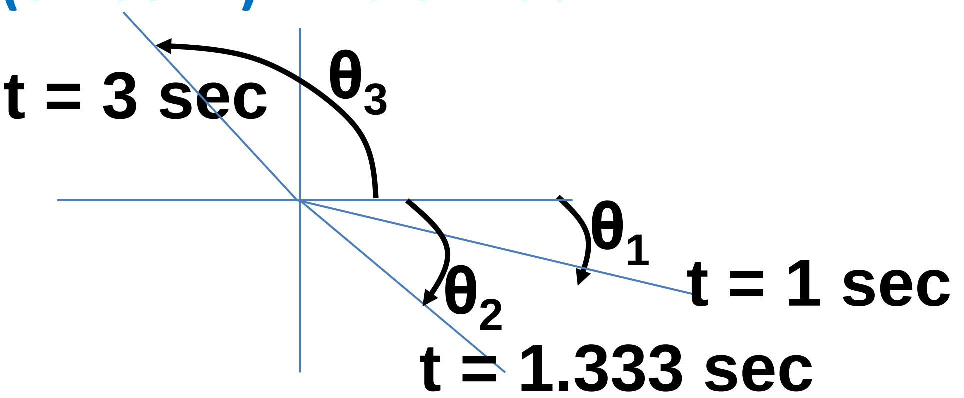
$t = 0$ is the initial condition

(c) Find t when $\omega = 0$ $t = 1.333$ sec from (b)

$\Theta_{1\,(1\ \text{sec})} = -3$ rad $\Theta_{2\,(1.333\ \text{sec})} = -3.185$ rad

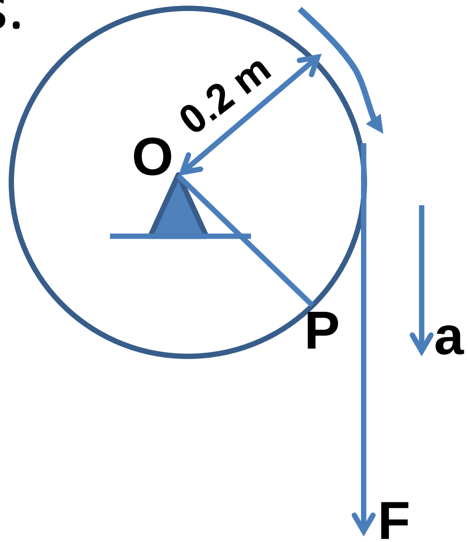
$\Theta_{3\,(3\ \text{sec})} = 7$ rad

Total angle turned = (3.185 – 3)+(3.185+7) = 10.37 rad



Ex: A cord is wrapped around a wheel in Fig. Below which is initially at rest when $\theta=0$. If a force is applied to the cord and gives it an acceleration $a = 4t\ m/s^2$, where t is in seconds. Determine as a function of time:

1. the angular velocity of the wheel, and
2. The angular position of the line OP in radians.



Solution: Since P on the cord and under the force P which tangent to the wheel (tangential component).

1. $(a_p)_t = \alpha \cdot r \Rightarrow 4t = 0.2\,\alpha \Rightarrow \alpha = 20t$

$\alpha = d\omega/dt \Rightarrow d\omega = \alpha\, dt = 20t\, dt \Rightarrow \omega = 10\,t^2 + C_1$

At $t = 0 \Rightarrow \omega = 0 \Rightarrow C_1 = 0$

So, $\omega = 10\,t^2$ +

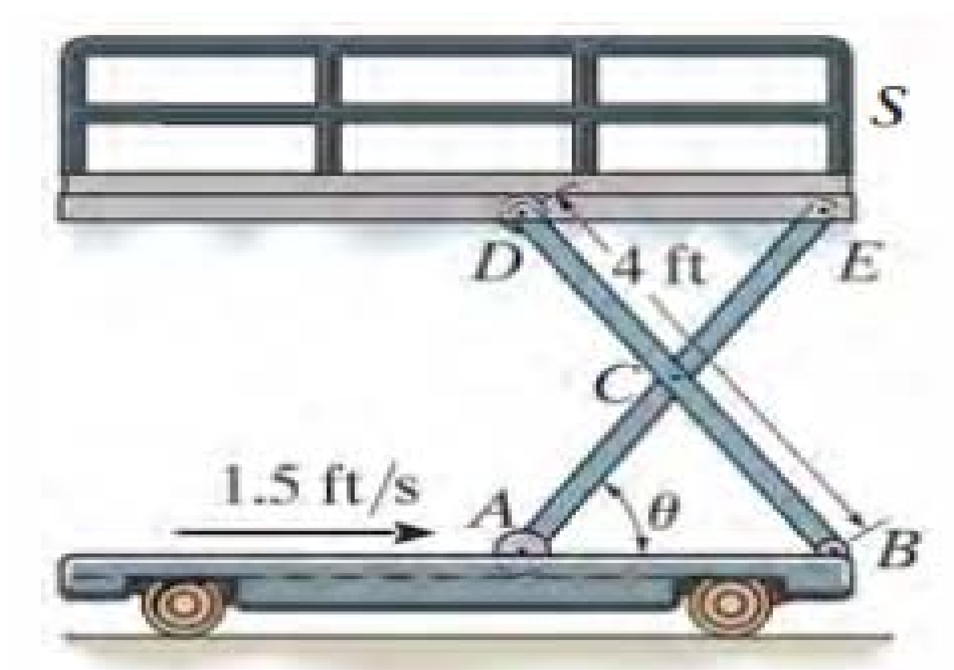
2. $\omega = d\theta / dt \Rightarrow \theta = (10/3)\,t^3 + C_2$

at $t=0 \Rightarrow \theta = 0 \Rightarrow C_2 = 0$

$\theta = (10/3)\,t^3$ rad.

Ex: Prob. 16.37 Hibbler Page 333

The scaffold S is raised by moving the roller at A toward the pin at B. If A is approaching B with a speed of 1.5 ft/s, determine the speed at which the platform rises as a function of θ. The 4ft links are pin connected at their midpoint.



$\text{Cos}\ \theta = x / 4$

$x = 4\ \text{Cos}\ \theta$

$dx//dt = -4\ \sin\theta\ (d\theta/dt)$

$v_x = -4\ \sin\theta\ .\ \omega$

$\omega = -\ v_x / 4\sin\theta \quad .....\ (1)$

$\sin\theta = y / 4$

$y = 4 \sin\theta$

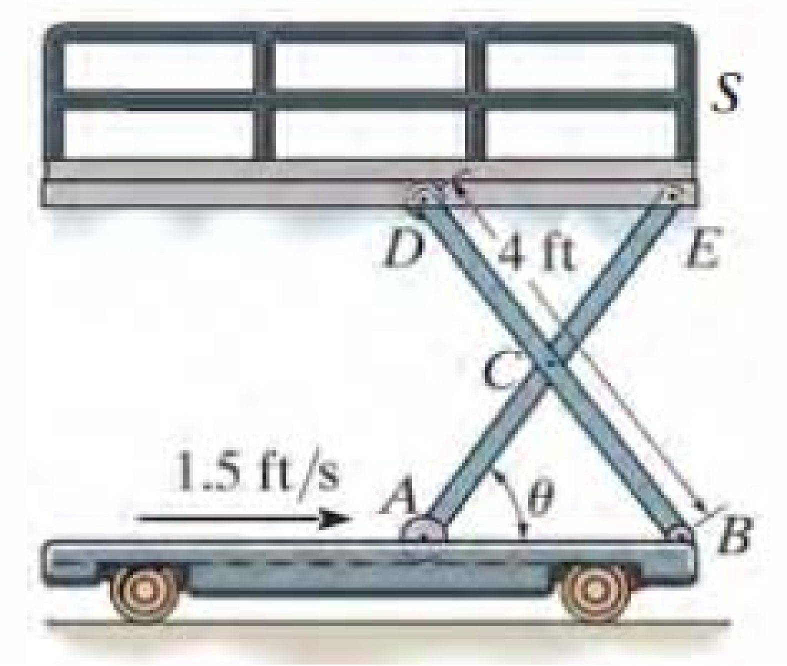
$dy//dt = 4\cos\theta\ (d\theta/dt)$

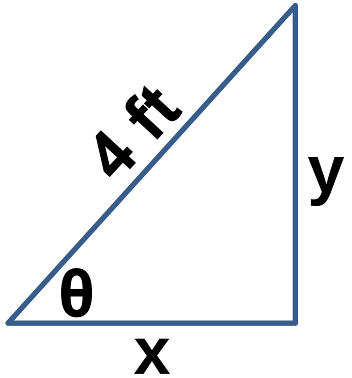
$v_y = 4\cos\theta\ .\ \omega \quad ....\ (2)$

Sub. (1) into (2);

$v_y = 4\cos\theta\ .\ (-\ v_x / 4\sin\theta\ ) = -\ v_x \cot\theta$

So, Speed = $v_x \cot\theta = 1.5 \cot\theta$





Ex: A line rotate in a plane with a constant angular acceleration of 2 rad/sec^2. During a certain interval, the line has an angular displacement of 2 rad clockwise while turning through a total angle of 10rad. Determine

1. The angular velocity 0.5 sec after the beginning of the time interval;
2. The length of the time interval.

Solution: Assume $t_o = 0$ $\theta_o = 0$ $\omega_0 = 0$

the +ve direction is clockwise with the direction of acceleration,

Displacement : $\Delta\theta = 2 = \theta_2 - \theta_1$ $\theta_2 = \theta_1 + 2$

Total angle turned: $\theta_T = 10 = \theta_2 + \theta_1$ $10 = \theta_1 + 2 + \theta_1$

$\theta_1 = 4$ rad $\theta_2 = 6$ rad

For constant angular acceleration :

$$\theta = \theta_o + w_o \cdot t + 0.5\, \alpha_c \cdot t^2$$

$$4 = 0 + 0 + 0.5 \times 2 \times t_1^2$$

$$t_1 = 2 \text{ sec}$$

$$6 = 0 + 0 + 0.5 \times 2 \times t_2^2 \qquad t_2 = 2.449 \text{ sec}$$

1. $\omega = \omega_o + \alpha_c \cdot t$

   $= 0 + 2\,(2 + 0.5) = 5 \text{ rad/sec}$

2. $T = 2 + 2.449 = 4.449$ sec

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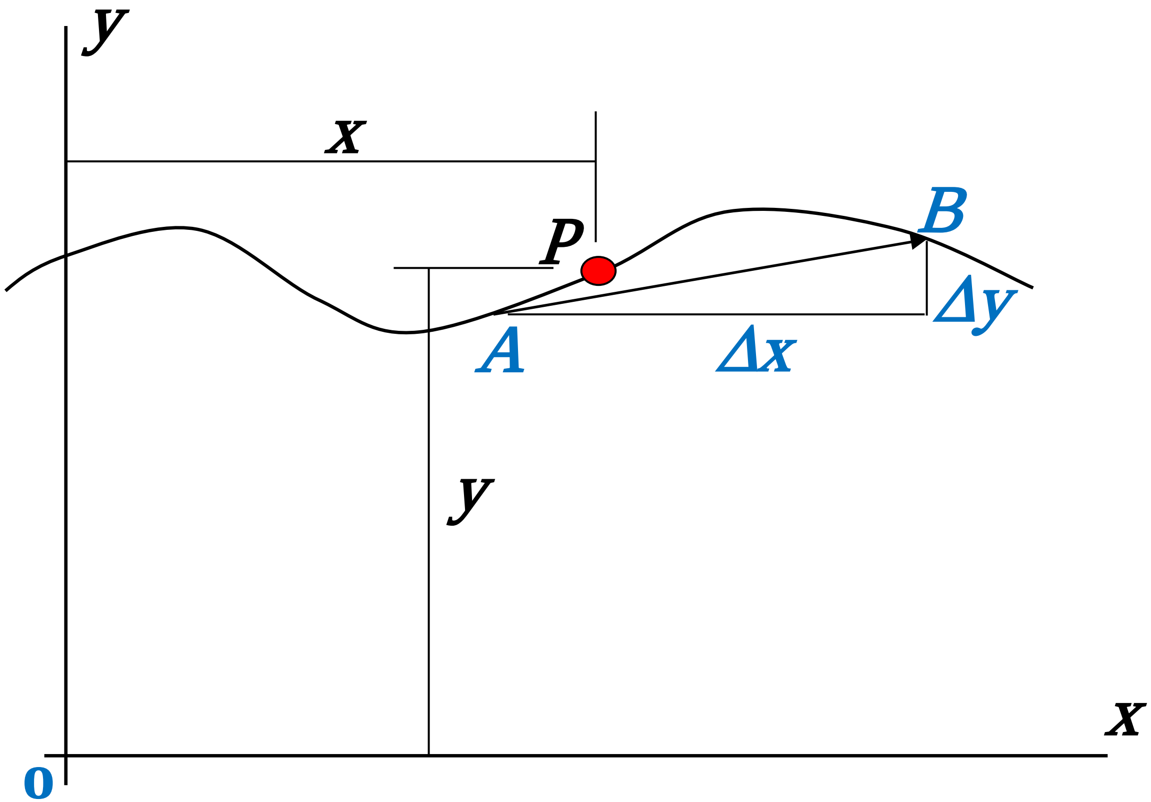
Coplanar Curvilinear Motion of a Particle Using Rectangular Components

Curvilinear motion occurs when a particle moves along a curved path.

When a particle moves along a curved path in space, ***its position at any instance is completely determined by its rectangular coordinates when they are given as a function of time***.

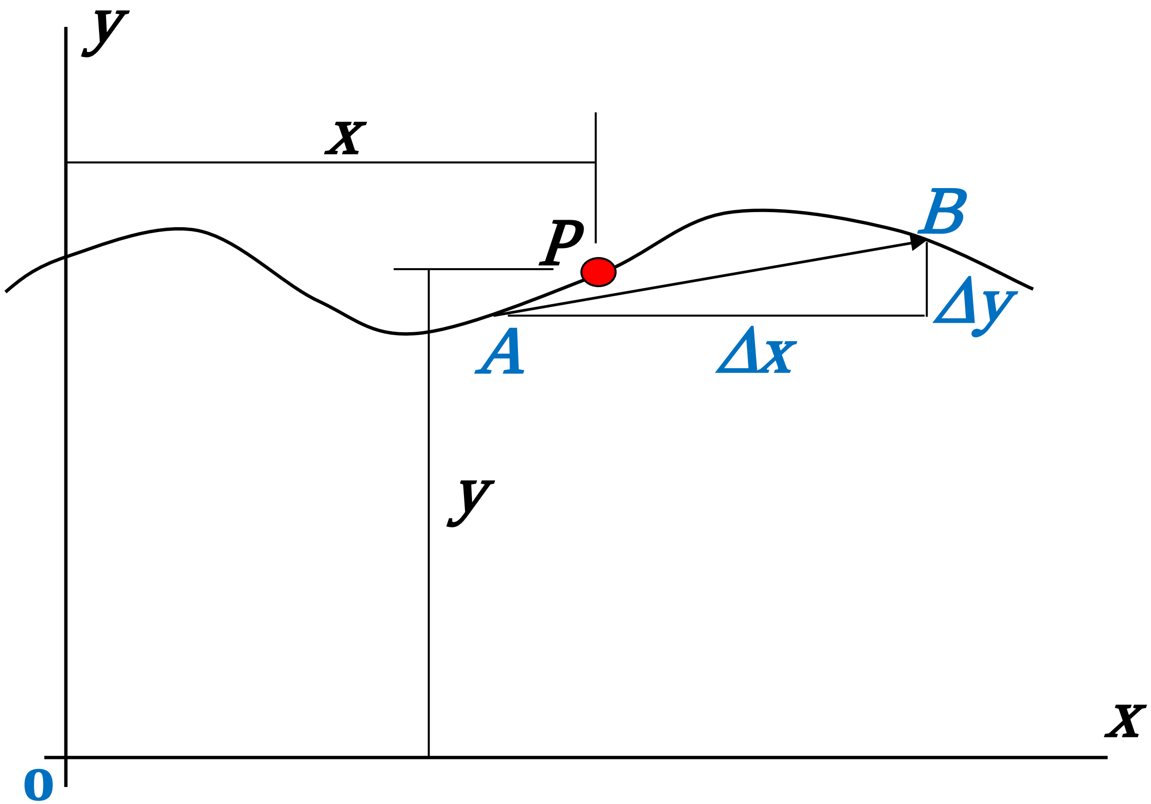
In fig. below, the position of the particle as it moves in the xy plane is given by its x and y coordinates expressed as a function of time.

$s = f(x, y)$ $x = f(t)$ $y = f(t)$



The linear displacement of the particle ***P*** in fig. above as it moves from position ***A*** to ***B*** is the vector from ***A*** to ***B***, which can be expressed in term of the change in the ***x*** and ***y*** coordinates of ***P*** as:

$$\Delta s = \Delta s_x + \Delta s_y = [\,(\Delta x)^2 + (\Delta y)^2\,]^{1/2}$$



The total distance travel s_T, in curvilinear motion is the total accumulated length of path traversed.

The linear velocity of the particle P in fig. is a vector quantity, is the vector sum of the time rates of change of its coordinates.

The x component of velocity = $v_x = dx / dt$

The y component of velocity = $v_y = dy / dt$

The *curvilinear* velocity $= v = [\,(v_x)^2 + (v_y)^2\,]^{1/2}$

The linear acceleration of the particle *P* in fig. is a vector quantity, is the vector sum of the time rates of change of its velocities.

The *x* component of acceleration = $a_x = dv_x / dt$

The *y* component of acceleration = $a_y = dv_y / dt$

The *curvilinear* acceleration = $a = [\,(a_x)^2 + (a_y)^2\,]^{1/2}$

Notes:

1. in ***curvilinear motion***, always $\Delta s < s_T$.

2. Curvilinear motion **can cause changes in both the magnitude and direction of the position, velocity, and acceleration vectors.**

3. The velocity vector **is always directed tangent to the path.**

4. If the motion is described using *rectangular coordinates*, then the components along each of the axes do not change direction, only their magnitude and sense (algebraic sign) will change.

Example: If the x & y components of a particle's velocity are $v_x = 32t$ and $v_y = 8$, where v in **m/sec**, determine the equation of the path $y = f(x)$. Note $x=0$, $y=0$ when $t=0$.

Solution:

$v_x =$ dx/dt = 32t $\Rightarrow$ dx = 32t . dt

$x = 16t^2 + C_1$

At t=0 x=0 $C_1 = 0$

So; $x = 16t^2$ ..... (1)

$v_y =$ dy/dt = 8 $\Rightarrow$ dy = 8 . dt $\Rightarrow$ y = 8t+C_2

At t=0 y=0 $C_2 = 0$

So; $y = 8t$ ..... (2)

$x = 16t^2$ ..... (1)

$y = 8t$ ..... (2)

From Equ. (2) ⇨ $t = y/8$ .... (3)

Sub. Equ. (3) into Eq.(1)

$x = 16\ (y/8)^2$

$x = 16\ y^2/64$ and

$y^2 = 4x$

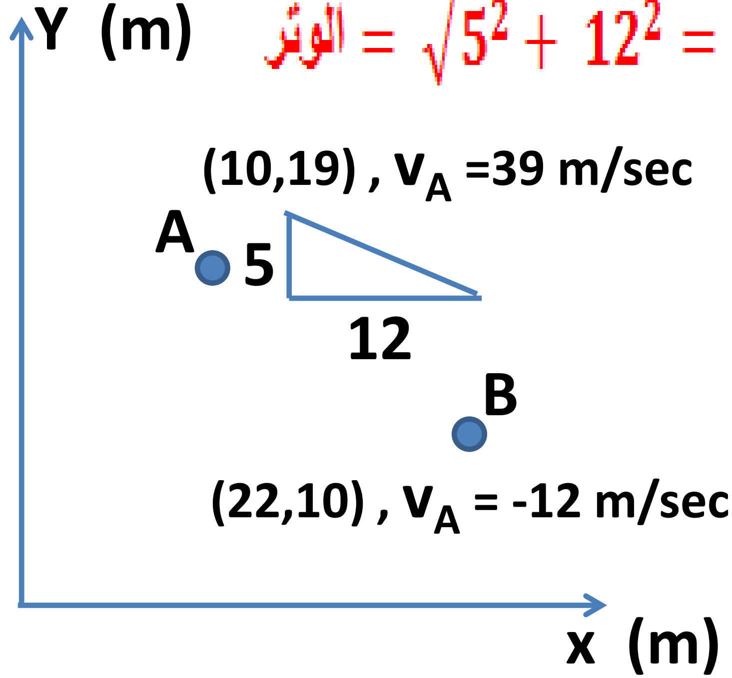
Example: A particle moves from point **A** to point **B** in **3 sec**. The **x** and **y** coordinates (**in meters**) of point **A** are (**10,19**), and those of point **B** are (**22,10**). The velocity of the particle at **A** is **39 m/sec** with a slope of **12** to the right and **5** upward; at **B** the velocity is **12 m/sec** vertically down.

(a) Determine the average velocity of the particle as it moves from A to B.

(b) Determine the average acceleration of the particle as it moves from A to B.

$$v_{ave} = \Delta s / \Delta t$$

$$\text{الوتر} = \sqrt{5^2 + 12^2} = 13$$



$\Delta t = 3$ sec

$\Delta s_x = 22-10 = 12$ m

$\Delta s_y = 10-19 = -9$ m

$$\Delta s = \sqrt{12^2 + (-9)^2} = 15\, m$$

$v_{ave} = 15/3 = 5$ m/sec **Ans.**

$\theta v = \tan^{-1}(\Delta s_y / \Delta s_x) = \tan^{-1}(9/12) = 36.896°$ **Ans.**

$a_{ave.} = \Delta v \;/\; \Delta t$

$v_{Ax} = 39 \times (12/13) = 36$ m/sec

$v_{Ay} = 39 \times (5/13) = 15$ m/sec

$v_{Bx} = 0$ ⟷ velocity of B is downward only

$v_{By} = 12$ m/sec ↓

$v_x = v_{Ax} + v_{Bx} = 36+0 = 36$ m/sec →

$v_y = v_{Ay} + v_{By} = 15+12 = 27$ m/sec ↓

السرعة في A و B
بنفس الاتجاه (للاسفل)

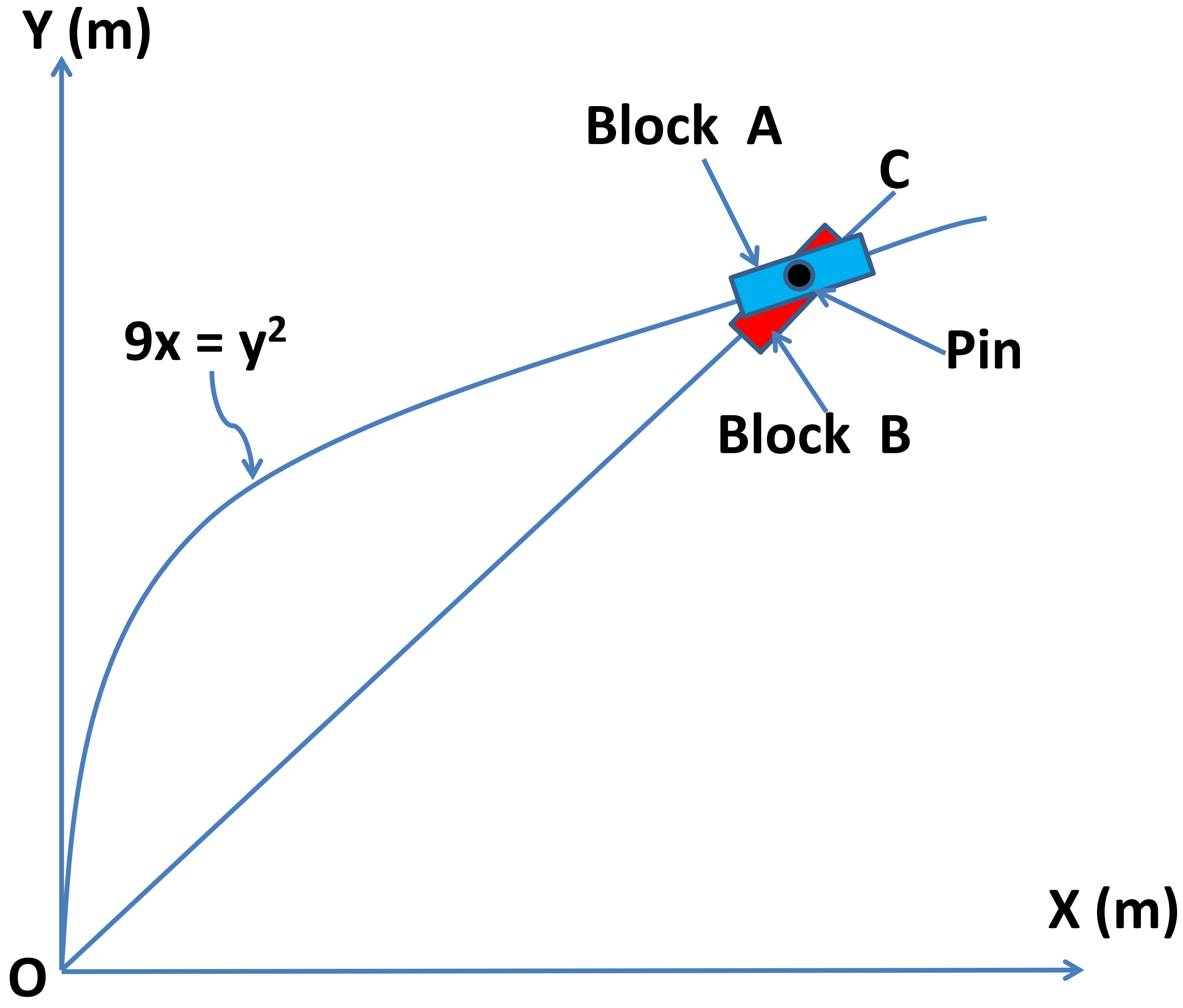
$$\Delta v = \sqrt{v_x^2 + v_y^2} = \sqrt{36^2 + 27^2} = 45\ m/sec$$ ↘

$$a_{avg.} = \frac{\Delta v}{\Delta t} = \frac{45}{3} = 15\ m/sec^2$$

Example: the block A and B shown below are pinned together and while block B slides on the rotating arm OC, block A slides along the curved rod having the shape $9x = y^2$:

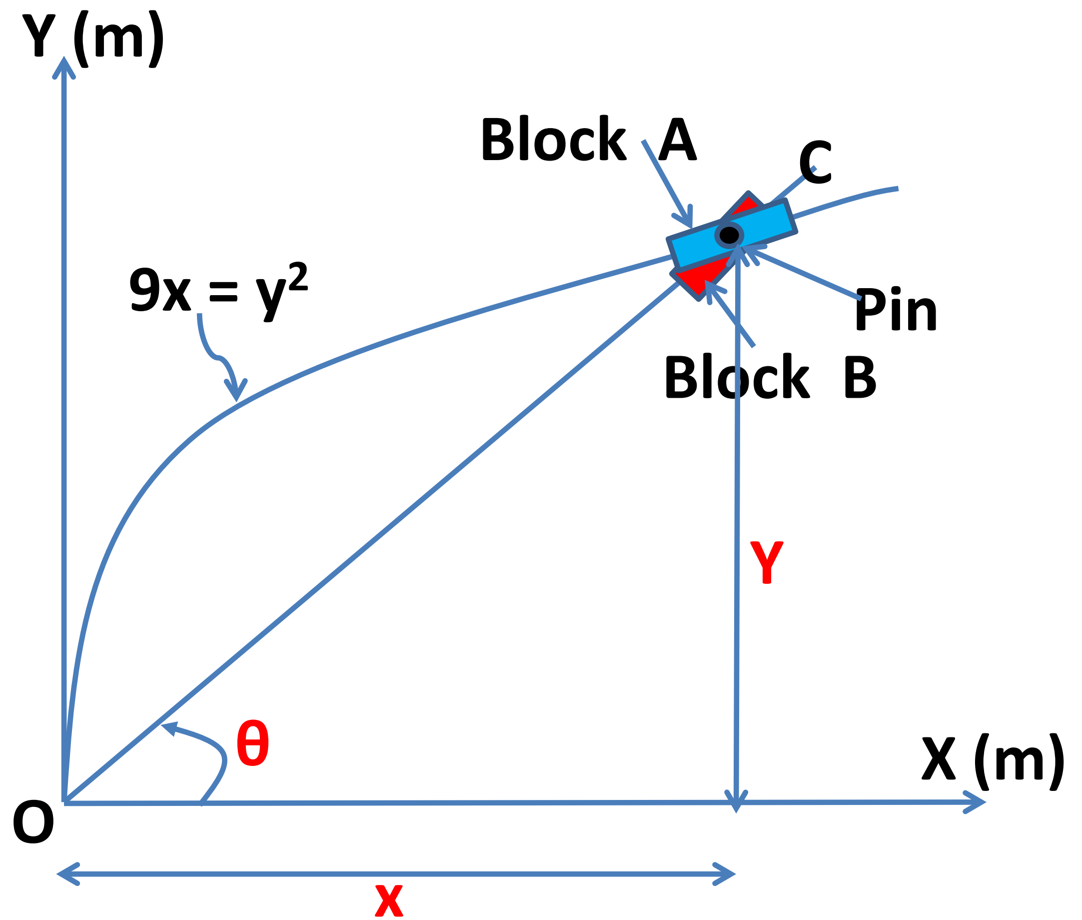
1. **Write an expression for the x coordinate of the position of the pin as a function of the angle θ, and an expression for the x component of velocity.**

2. **Using the result of (1), determine the x component of the linear velocity of the when of the pin when the arm OC has a slope of 3 vertical to 4 horizontal and an angular velocity of 3 rad/sec clockwise.**



Solution:

The **pin** is a point on the **line OC** and the **curve** and its coordinate **x** and **y** can be obtained from equation of line or equation of the curve.



(1) $9x = y^2$

$$y = 3\sqrt{x}$$

$\tan\theta = y/x$

$$\tan\theta = \frac{3\sqrt{x}}{x} = \frac{3}{\sqrt{x}}$$

$$\sqrt{x} = \frac{3}{\tan\theta}$$

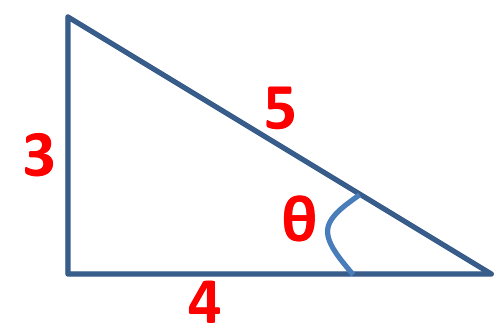
$$x = \frac{9}{\tan^2\theta}$$

$$\frac{dx}{dt} = v_x = \frac{-18\sec^2\theta}{\tan^3\theta}\frac{d\theta}{dt}$$

$d\theta/dt = \omega$

$$v_x = \frac{-18\sec^2\theta}{\tan^3\theta}\,\omega$$

(2) $\theta = \tan^{-1}\theta\frac{3}{4} = 36.869^\circ$



$$v_x = \frac{-18\sec^2(36.869)}{\tan^3(36.869)} \times 3 = -200\frac{m}{sec} \leftarrow$$

Dynamic Engineering Mechanics

Motion of Projectiles

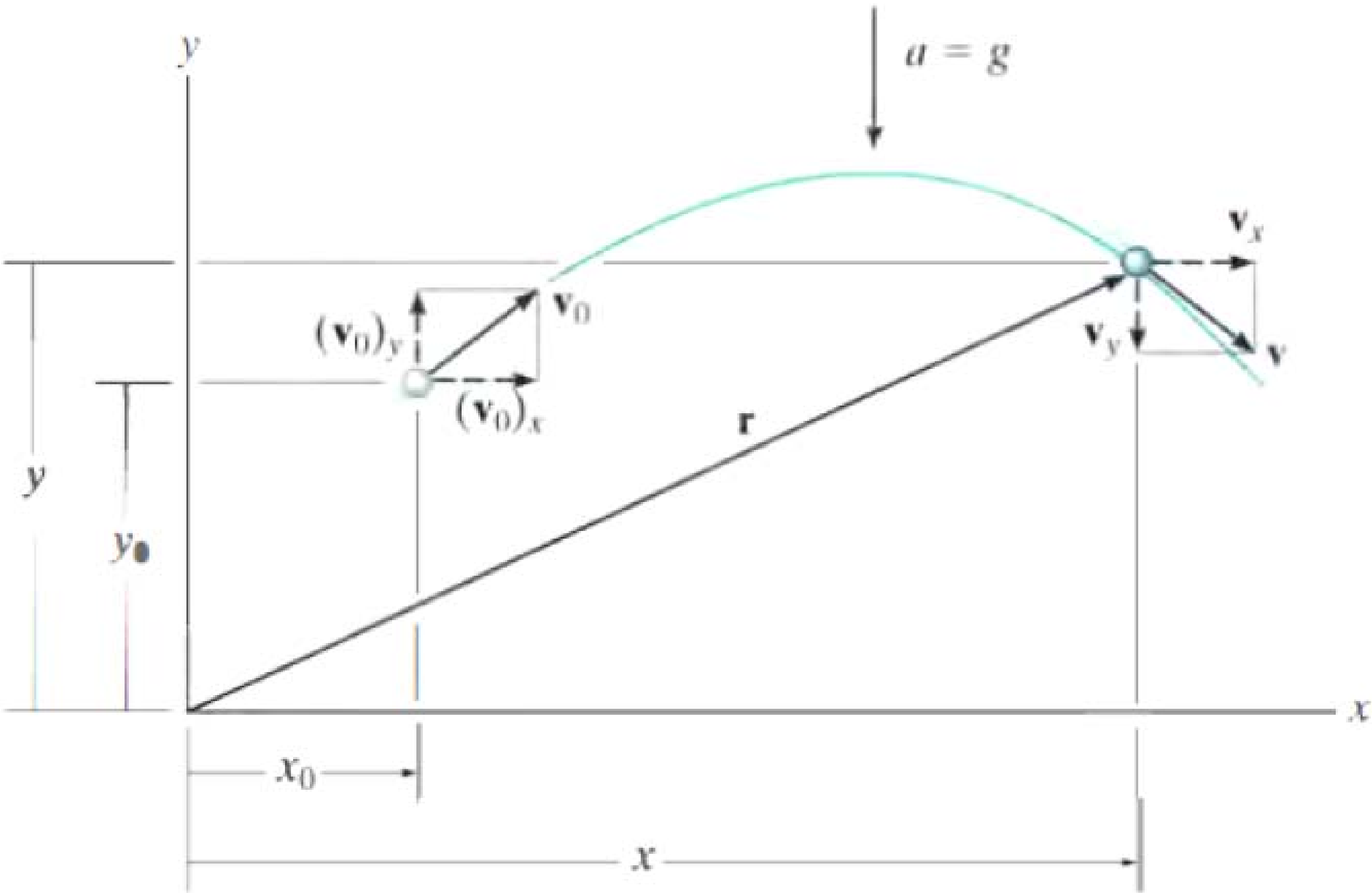
Motion of Projectiles

- The motion of a projectile in flight can usually be considered as curvilinear motion of a particle.

- When air resistance is neglected, the only force acting on the projectile is its weight.

- The horizontal component of acceleration is zero, and the vertical component of acceleration due to the weight of the body is approximately $9.81 m/sec^2$ or 32.2 ft/sec^2 directed vertically downward (constant acceleration).

Consider the projectile shown in Fig. below;



By Application of constant acceleration equations:

Horizontal motion : $a_x = a_c = 0;$

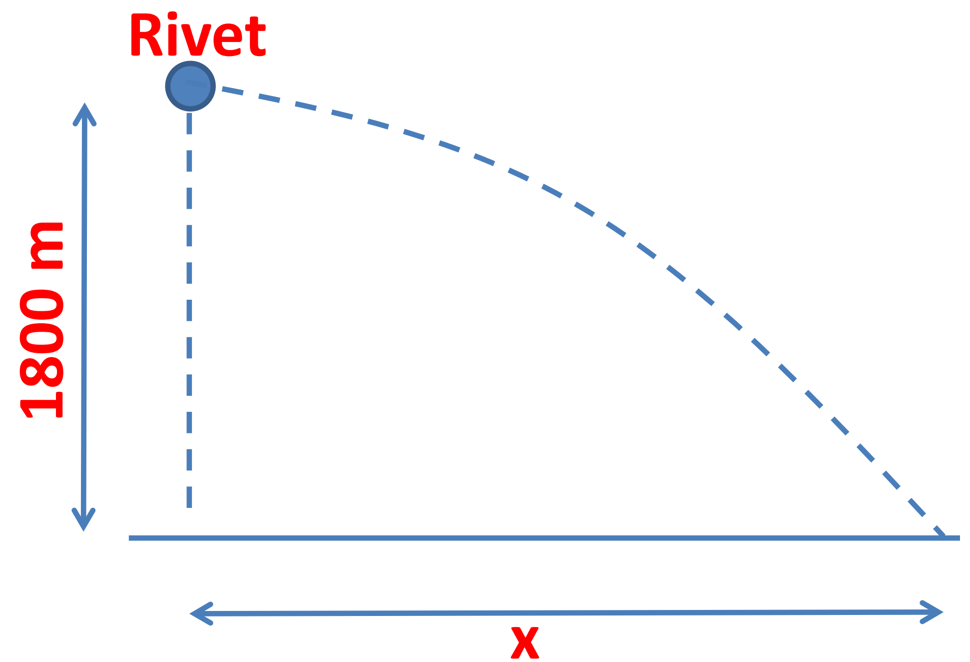
$(\overset{+}{\rightarrow})$	$v = v_0 + a_c t;$	$v_x = (v_0)_x$
$(\overset{+}{\rightarrow})$	$x = x_0 + v_0 t + \frac{1}{2} a_c t^2;$	$x = x_0 + (v_0)_x t$
$(\overset{+}{\rightarrow})$	$v^2 = v_0^2 + 2a_c(x - x_0);$	$v_x = (v_0)_x$

The first and last equations indicate that the horizontal components of velocity always remains constant during the motion.

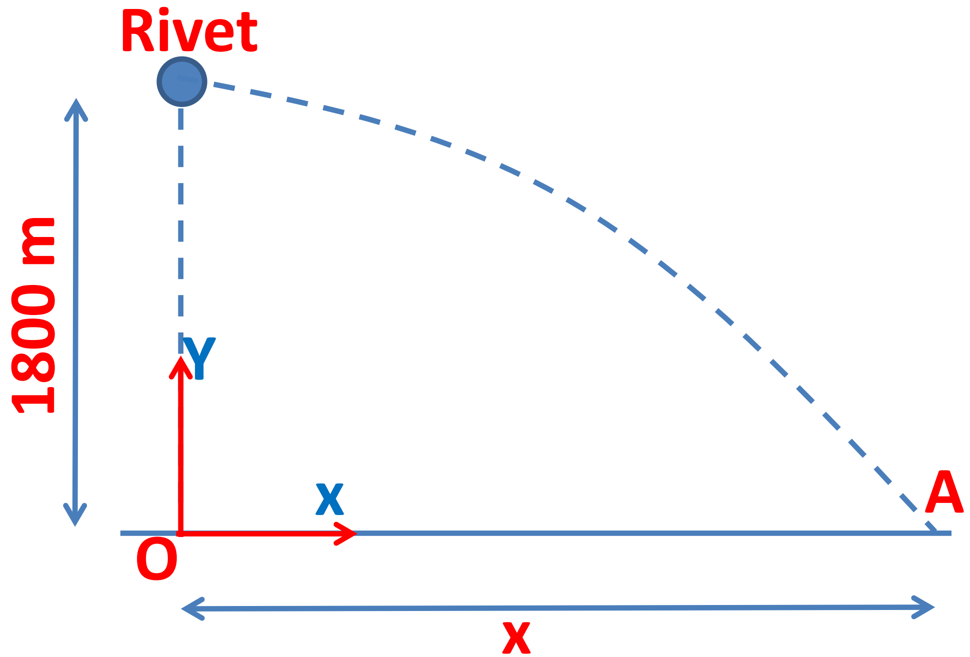
Vertical motion: $a_y = a_c = -g$;

$(+\uparrow)$	$v = v_0 + a_c t;$	$v_y = (v_0)_y - gt$
$(+\uparrow)$	$y = y_0 + v_0 t + \frac{1}{2} a_c t^2;$	$y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$
$(+\uparrow)$	$v^2 = v_0^2 + 2a_c(y - y_0);$	$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$

EX.1: **An airplane flying 400 km/hr horizontally accidentally losses a rivet when it is 1800 m above the ground. Determine the location where the rivet will land if air resistance is neglected.**



Solution : Choose the location of origin



At Origin

$x_o = 0$ $y_o = 1800$ m $v_{ox} = 400$ km/hr $= 111.111$ m/sec

$v_{oy} = 0$

At point A:

$X_A = ?$ $y_A = 0$

$$y = y_o + v_{oy} \cdot t + 0.5\, g\, t^2$$

$$0 = 1800 + 0.\, t - 0.5 \times 9.81 \times t^2$$

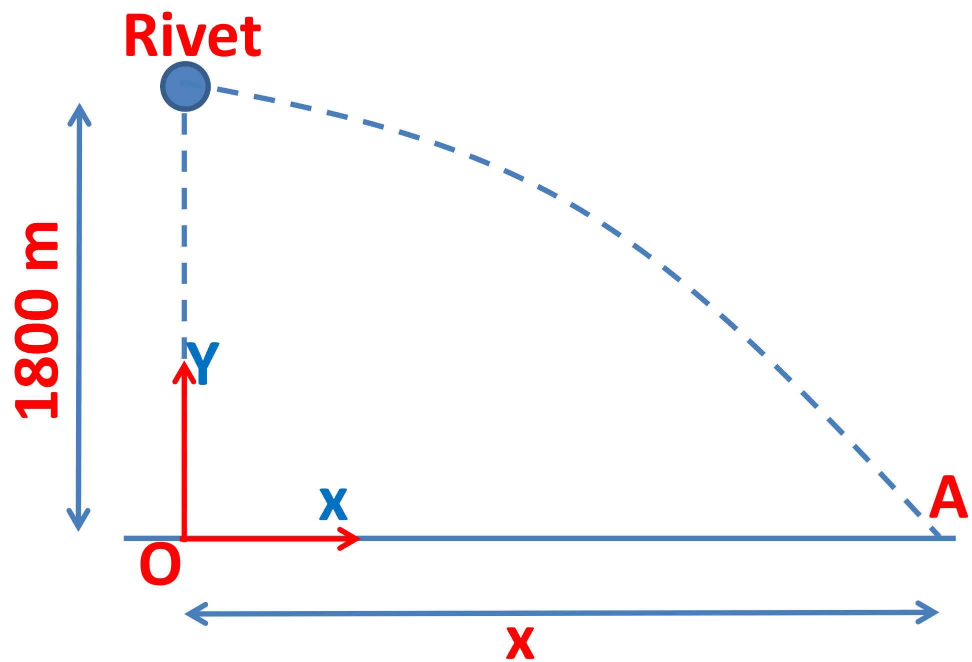
$$t = 19.157 \text{ sec}$$

$$x = x_o + v_{ox} \cdot t$$

$$x = 0 + 111.111 \times 19.157$$

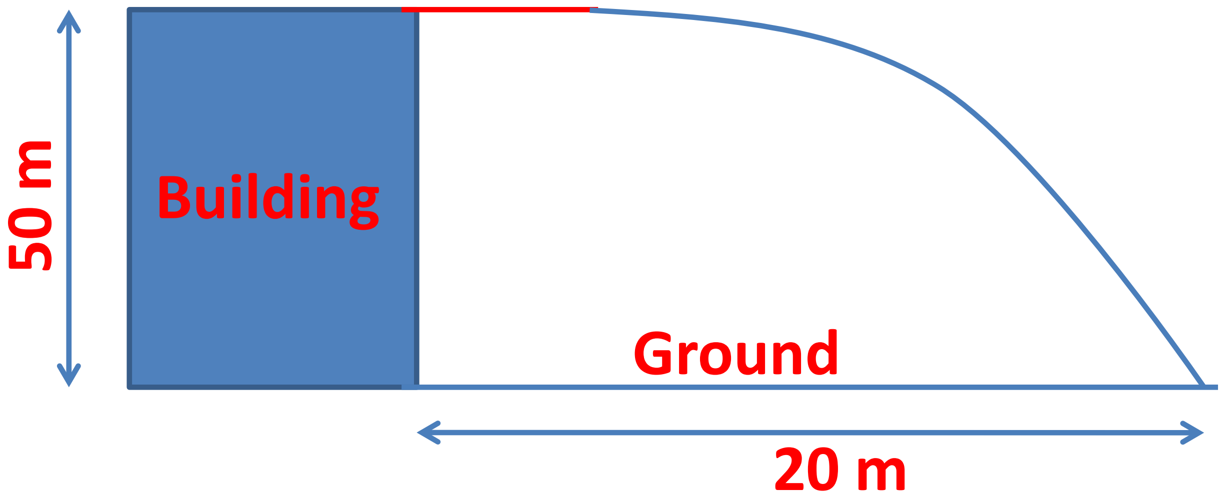
$$x = 2128.553 \text{ m}$$

H.W. Resolve by choosing the origin at the Rivet.

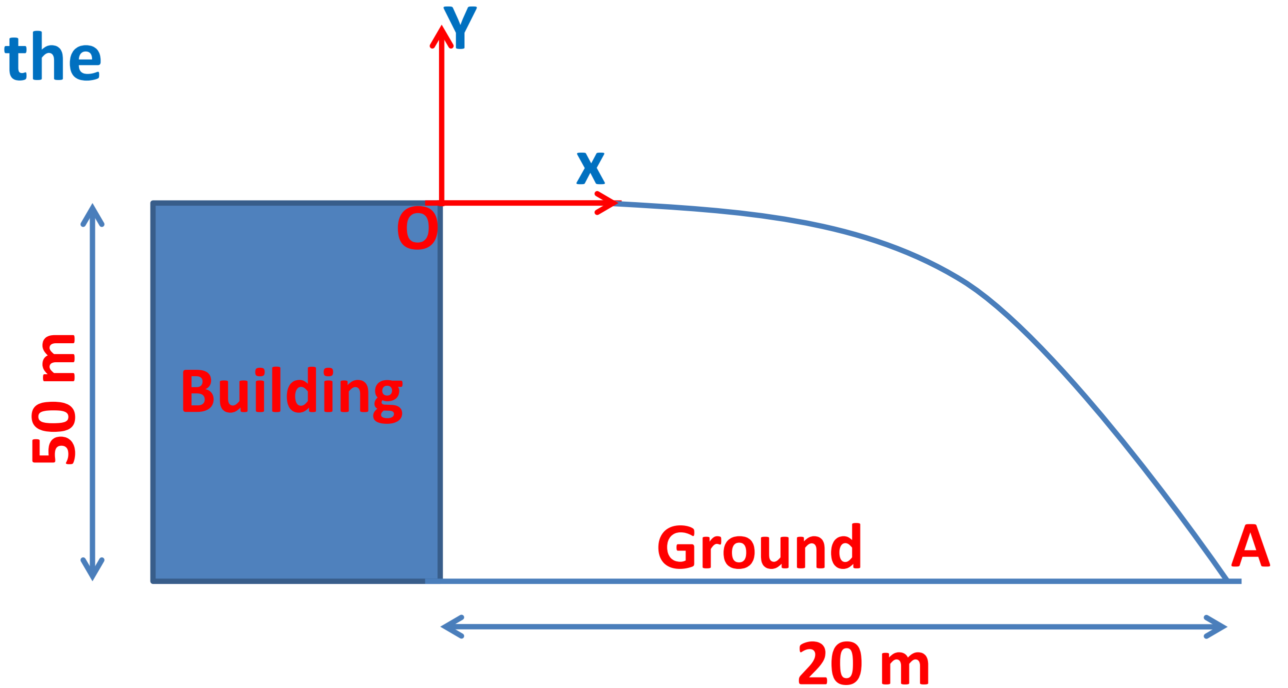


EX.2: A ball thrown horizontally from the top of a **50 m** high building hits the horizontal ground **20 m** from the base of the building. What was the initial velocity of the ball.

Solution:



Choose the origin at the top of the building



H.W. Resolve by choosing the origin at A.

$x_o = y_o = 0$ $\quad v_{ox} = v_x$ $\quad v_{oy} = 0$

At point A: $x_A = 20$ m $\quad y_A = -50$ m

$$y = y_o + v_{oy} . t + 0.5\, g\, t^2$$

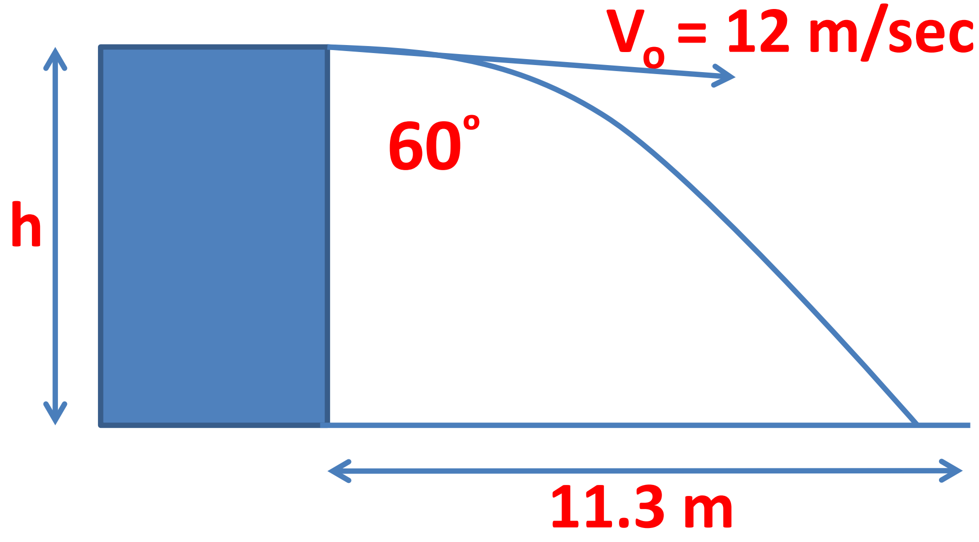
$$-50 = 0 + 0. t - 0.5 \times 9.81 \times t^2 \Rightarrow t = 3.193 \text{ sec}$$

$$x = x_o + v_{ox} \; . \; t \Rightarrow 20 = 0 + v_{ox} \times 3.193$$

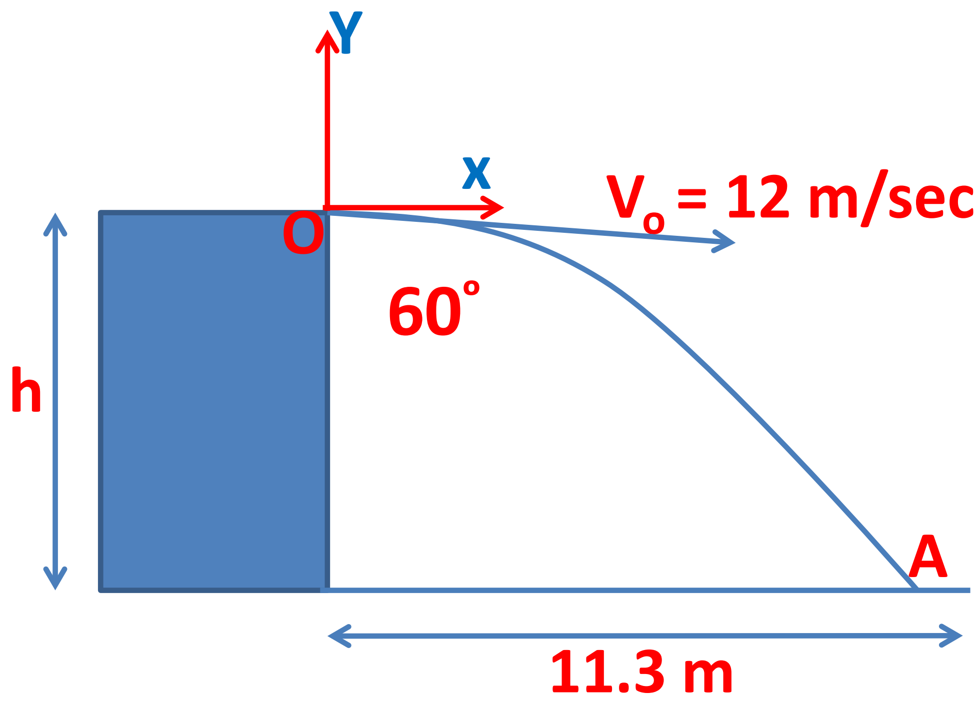
$$v_{ox} = 6.264 \text{ m/sec}$$

$$v_o = \sqrt{v_{ox}^2 + v_{oy}^2} = \sqrt{6.264^2 + 0} = 6.264\, m/\sec \quad horizontal$$

EX.3: A ball is thrown with a speed of **12 m/sec** at an angle of **60°** with the building and strike the ground **11.3 m** horizontally from the foot of the building. Determine the height of the building.



Solution : Choose the origin at the top of building



At origin: $x_o = y_o = 0$

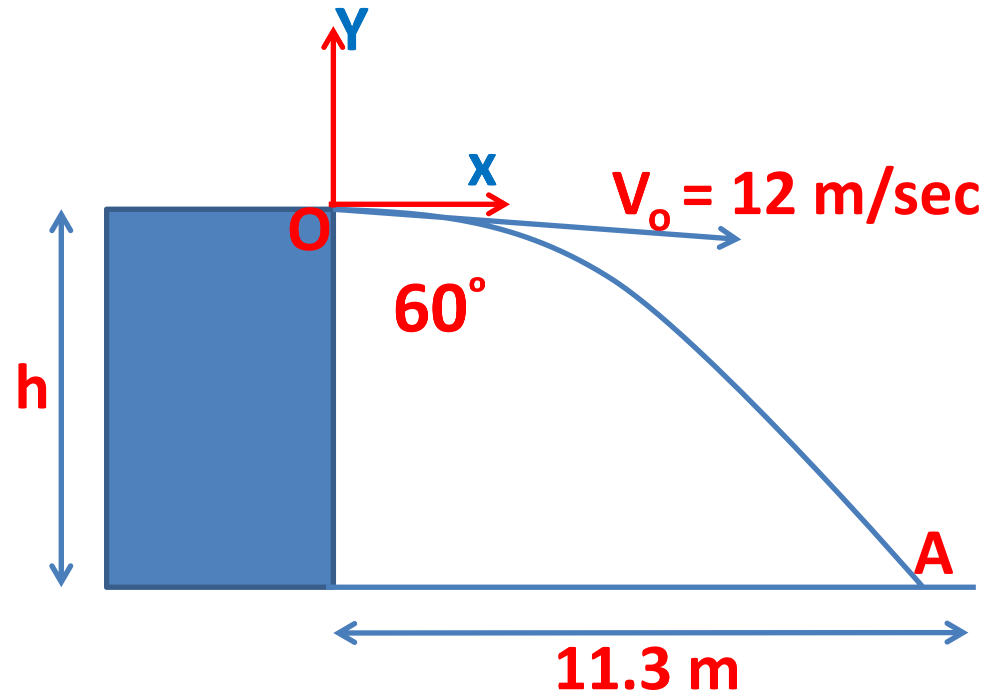
$v_{ox} = v_x = 12 \sin 60^\circ$ m/sec

$v_{oy} = -12 \cos 60^\circ$ m/sec

At Point A:

$x_A = 11.3m$

$Y_A = -h$



$$x = x_o + v_{ox} \; . \; t$$

$$11.3 = 0 + 12 \sin 60^\circ \times t$$

$$t = 1.087 \text{ sec}$$

$$y = y_o + v_{oy} \, . \, t + 0.5 \, g \, t^2$$

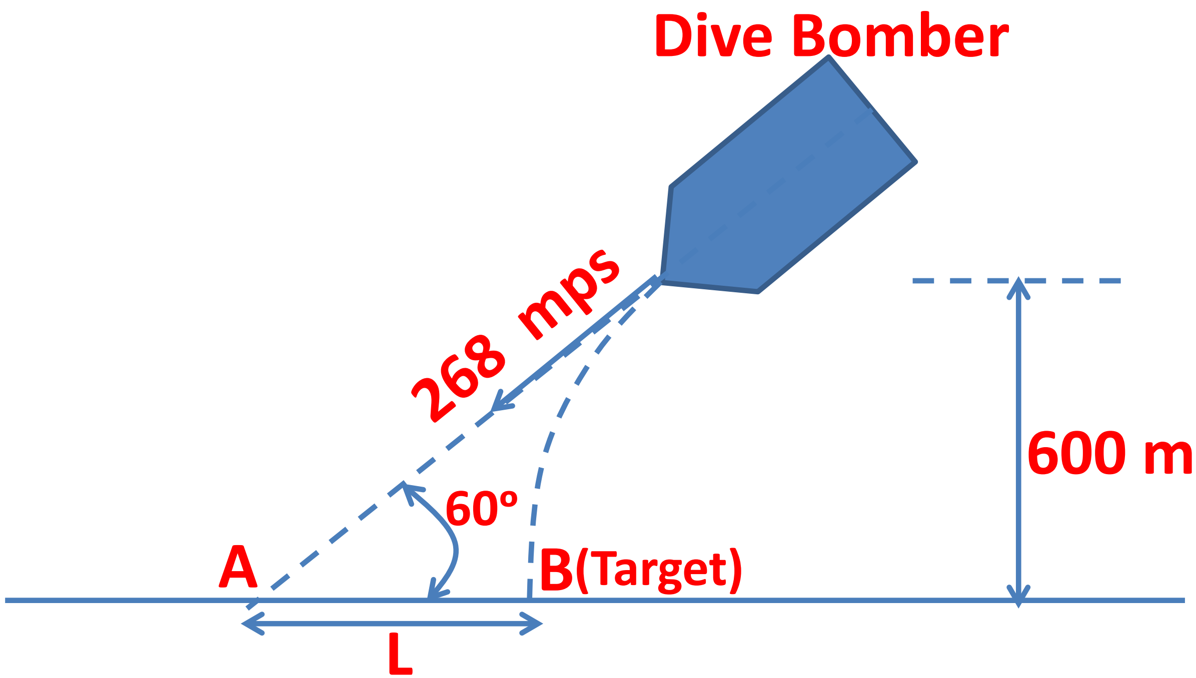
$$-h = 0 - 12 \cos 60^\circ \times t - 0.5 \times 9.81 \times 1.087^2$$

$$h = 12.318 \text{ m}$$

H.W.

1. Resolve with the origin at the top of the building with y—axis downward.
2. Resolve with the origin at the lower corner of the building.

EX.4: The dive bomber in Fig. below traveling at a velocity of **268 mile/sec** has a flight angle of **60°** when sighting on target. Determine the target lead **L**, which the pilot must allow when releasing a bomb from an altitude of **600 m**.



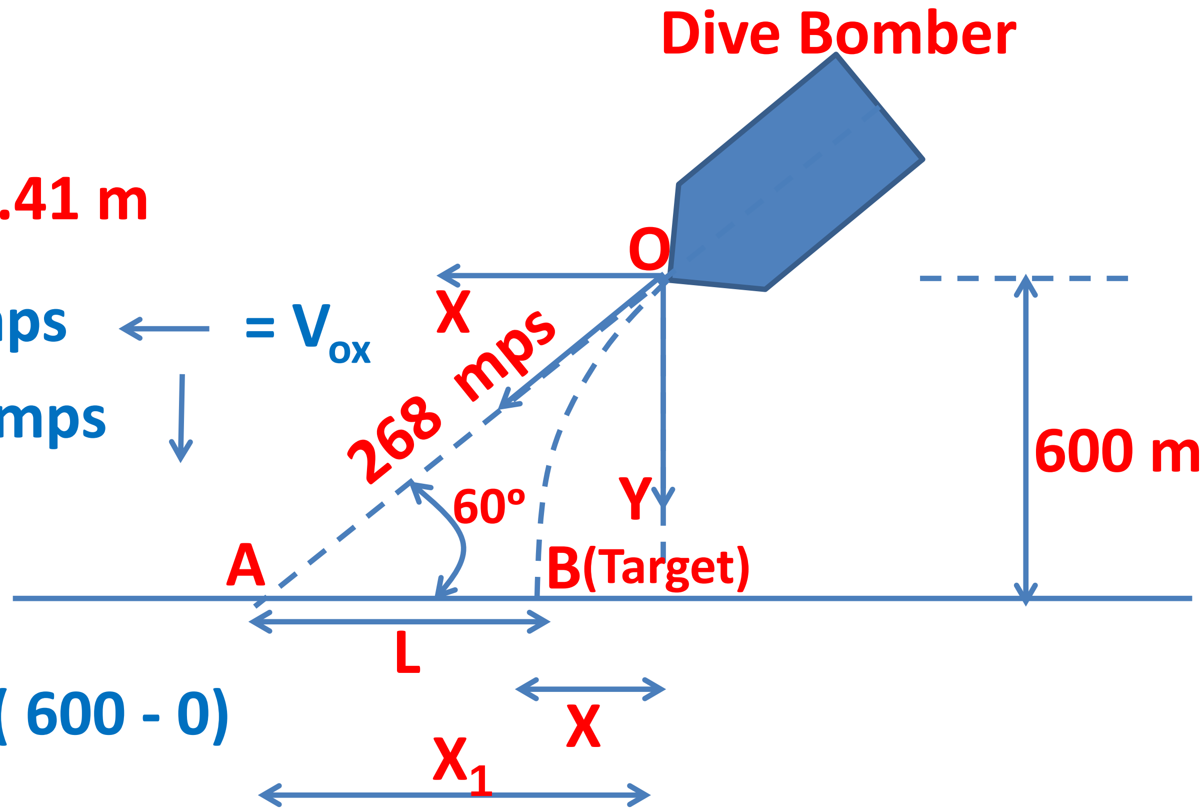
Solution: Choose the origin on the ground;

$L = X_1 - X$

$X_1 = 600 / \tan 60^\circ = 346.41 \text{ m}$

$V_x = 268 \cos 60^\circ = 134 \text{ mps} \leftarrow = V_{ox}$

$V_y = 268 \sin 60^\circ = 232.1 \text{ mps} \downarrow$



To Find X: For Point B:

$V^2_y = V^2_{oy} - 2g\,(y - y_o)$

$232.1^2 = V^2_{oy} - 2 \times 9.81 \times (600 - 0)$

$V_{oy} = 256.208 \text{ mps}$

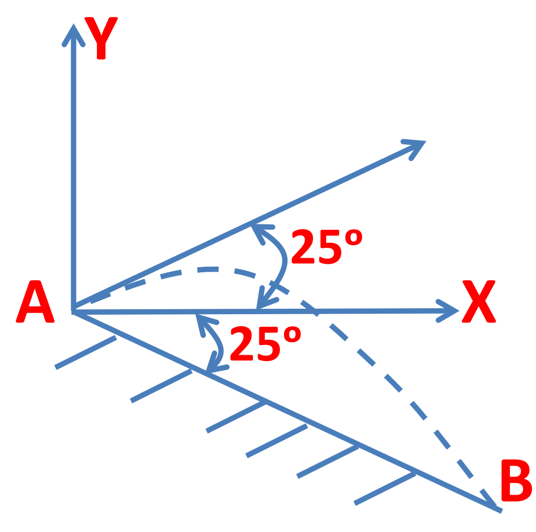
$V_y = V_{oy} - g\,.\,t \Rightarrow 232.1 = 256.208 - 99.81 \times t \Rightarrow t = 2.457 \text{ sec}$

$x = x_o + v_{ox} \times t \Rightarrow x = 0 + 134 \times 2.457 = 329.238 \text{ m}$

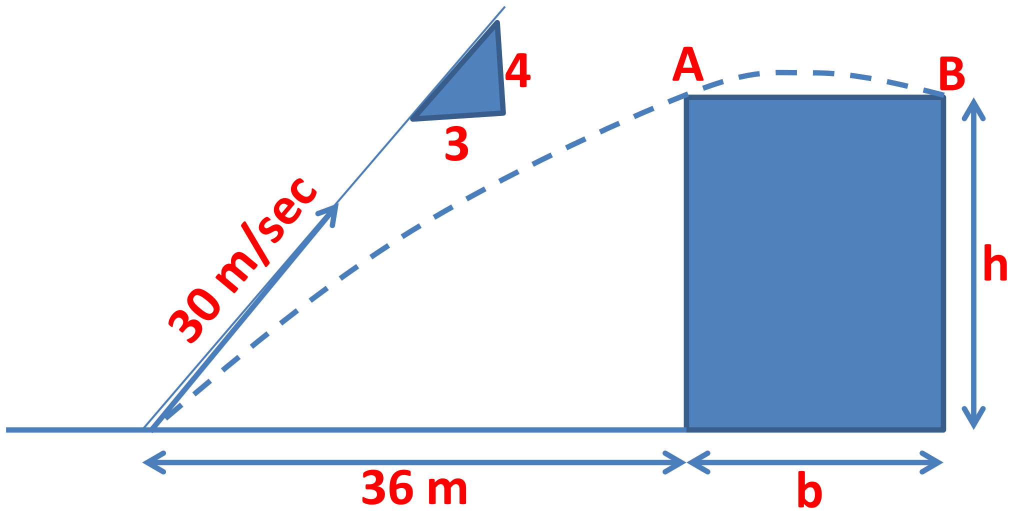
$L = X_1 - X = 346.41 - 329.238 = 17.172 \text{ m}$

Problem 1: A toy launcher projects a missile with an initial speed of 20 m/sec. If the missile lands 12 m away at the same elevation, what must have been the angle of elevation of the launcher?

Problem 2: A ball is projected from A with a speed of 3 m/sec at an angle of 25° as shown below. Determine the coordinates of point B at which the ball will hit the plane which is 25° below the horizontal.



Problem 3: A ball thrown with an initial velocity of **30 m/sec** as shown below, just clears the edges **A** and **B** of building **36m** away. Determine the height **h**, and the width **b**, of the building.

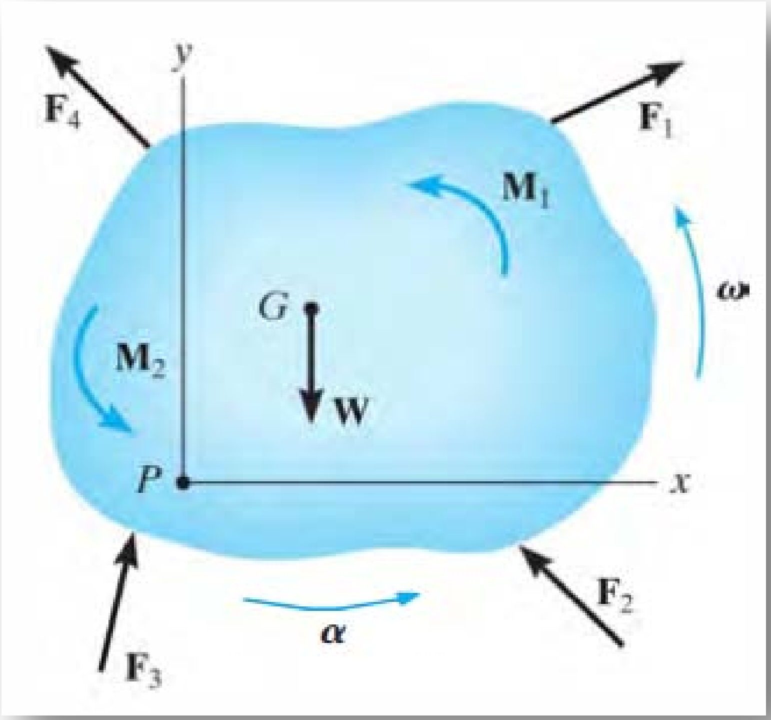


CHAPTER FOUR

Kinetics of Rigid Body

4.1 Planar Kinetics Equation of Motion

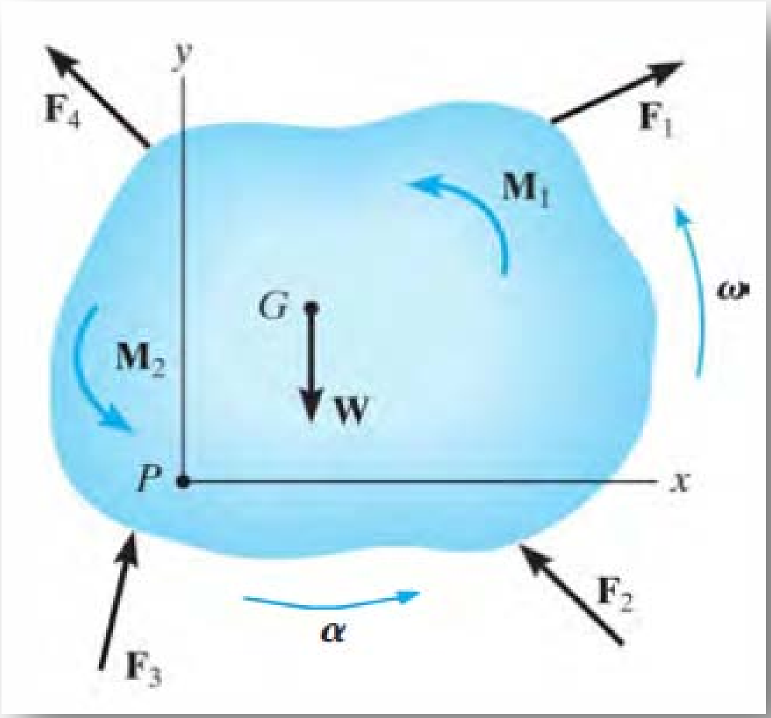
Consider an arbitrary rigid body of Fig. shown below. Here the inertial frame of reference x, y, z has its origin coincident with the arbitrary point P in the body. By definition, these axes do not rotate and are either fixed or translate with constant velocity.



Equation of Translational Motion:

The external forces acting on the body in Fig. below represent the effect of different types of forces between adjacent bodies. So:

$$\sum F = m.a_G$$



The translational equation of motion for the mass center of a rigid body: **It states that the sum of all the external forces acting on the body is equal to the body's mass times the acceleration of its mass center G.**

For motion of the body in the x-y plane, the translational equation of motion may be written in the form of two independent scalar equations, namely,

$$\sum F_x = m.(a_G)_x \qquad \sum F_y = m.(a_G)_y$$

4.2 Equations of Motion: Translation

If a rigid body in Fig. below, has a motion of translation, the resultant of the external forces applied on the body must pass through its mass center and, therefore, the resultant moment of the external forces about the mass center must be zero. That is:

$$\sum(M_G)_x = 0 \qquad \sum(M_G)_y = 0 \qquad \sum(M_G)_z = 0$$

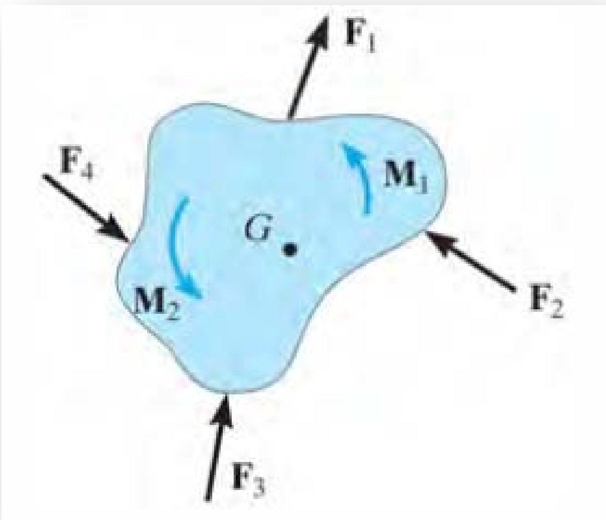
$\sum(M_G)_x$ = the moment of a force about an axis through the mass center in the x direction;

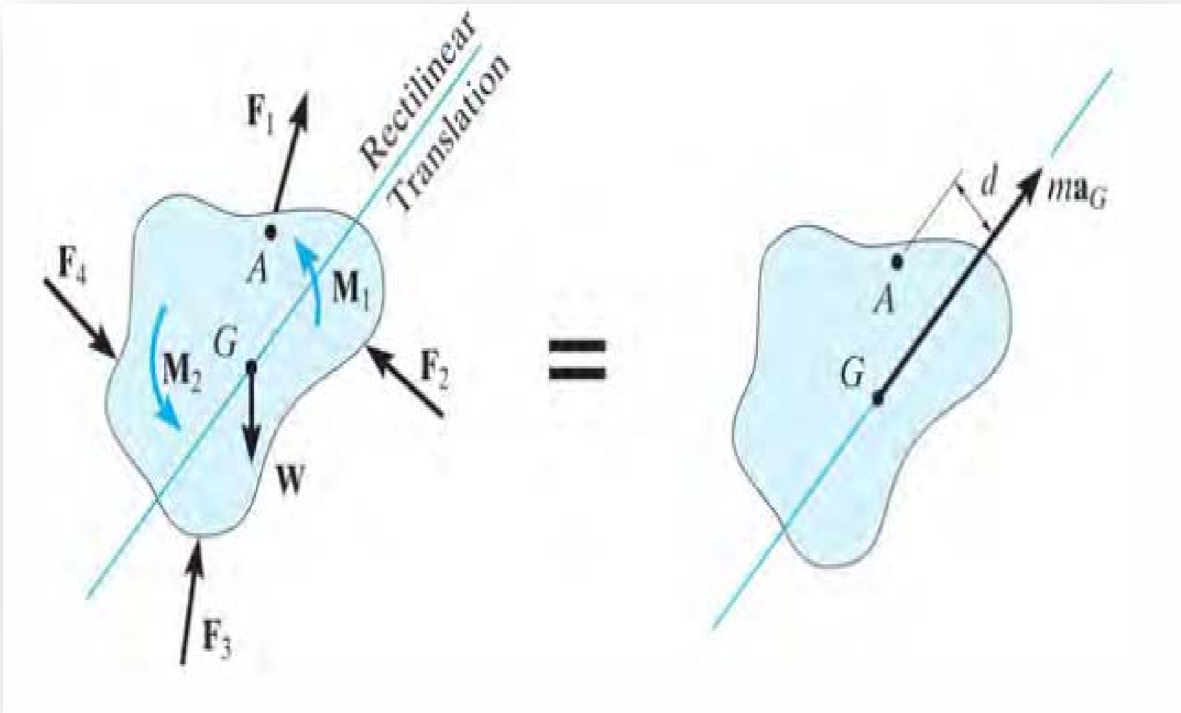
$\sum(M_G)_y$ = the moment of a force about an axis through the mass center in the y direction;

$\sum(M_G)_z$ = the moment of a force about an axis through the mass center in the z direction;

4.2.1 Rectilinear Translation

Consider the rigid body shown below:





If the mass center moves in the xy planes, then:

$\sum F_x = m\,(a_G)_x$ $\sum F_y = m\,(a_G)_y$ $\sum (M_G)_x = 0$

4.2.2 Curvilinear Translation

When a rigid body is subjected to curvilinear translation, all the particles of the body will travel along parallel curved paths. For analysis, its often convenient to use an internal coordinate system having an origin which coincides with the body's mass center at the instant considered, and axes which are oriented in the normal and tangential directions to the path of motion, Fig. below. The three scalar equations of motion are then

$$\sum F_n = m\,(a_G)_n$$
$$\sum F_t = m\,(a_G)_t$$
$$\sum M_G = 0$$

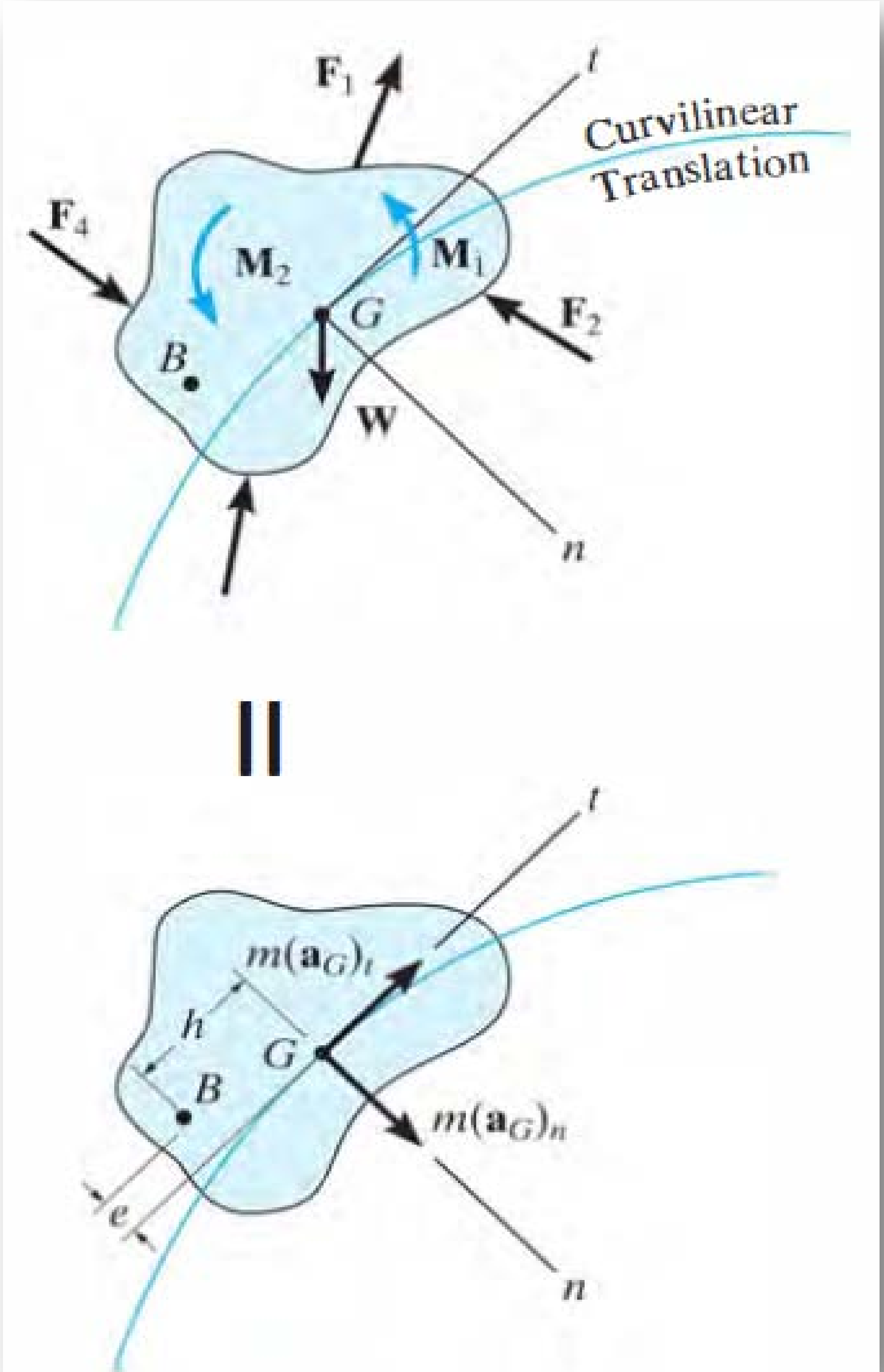
Note:

$$(a_G)_n = \frac{v_G^2}{r} = \omega^2 r$$

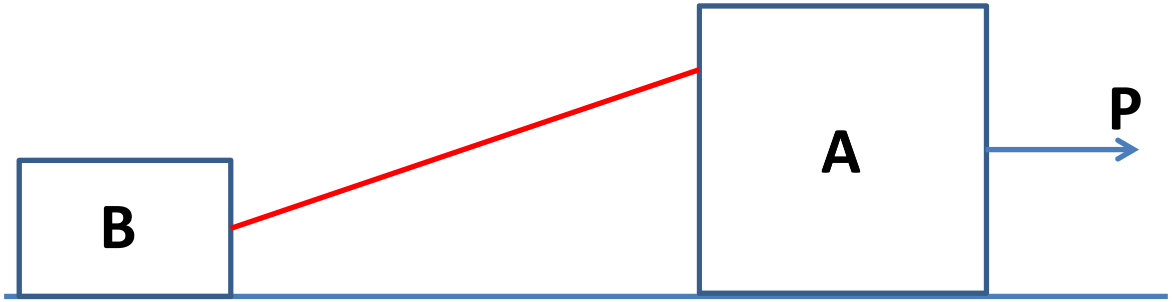
$$(a_G)_t = \frac{dv_G}{dt}$$

$$(a_G)_t\, ds_G = v_G\, dv_G$$

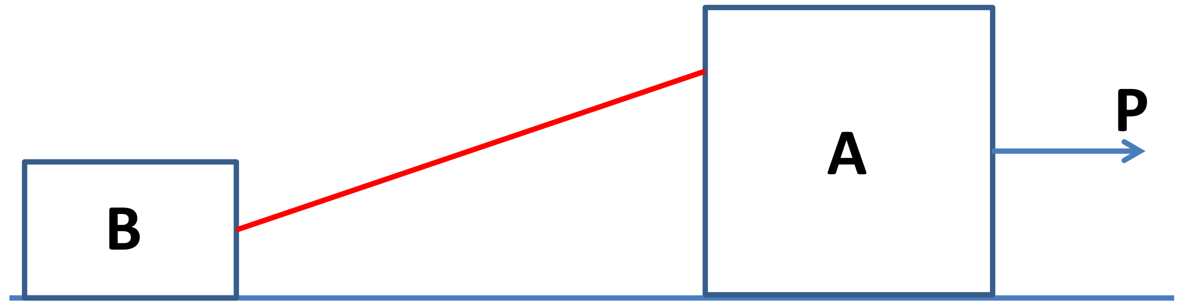
$$(a_G)_t = \alpha r$$



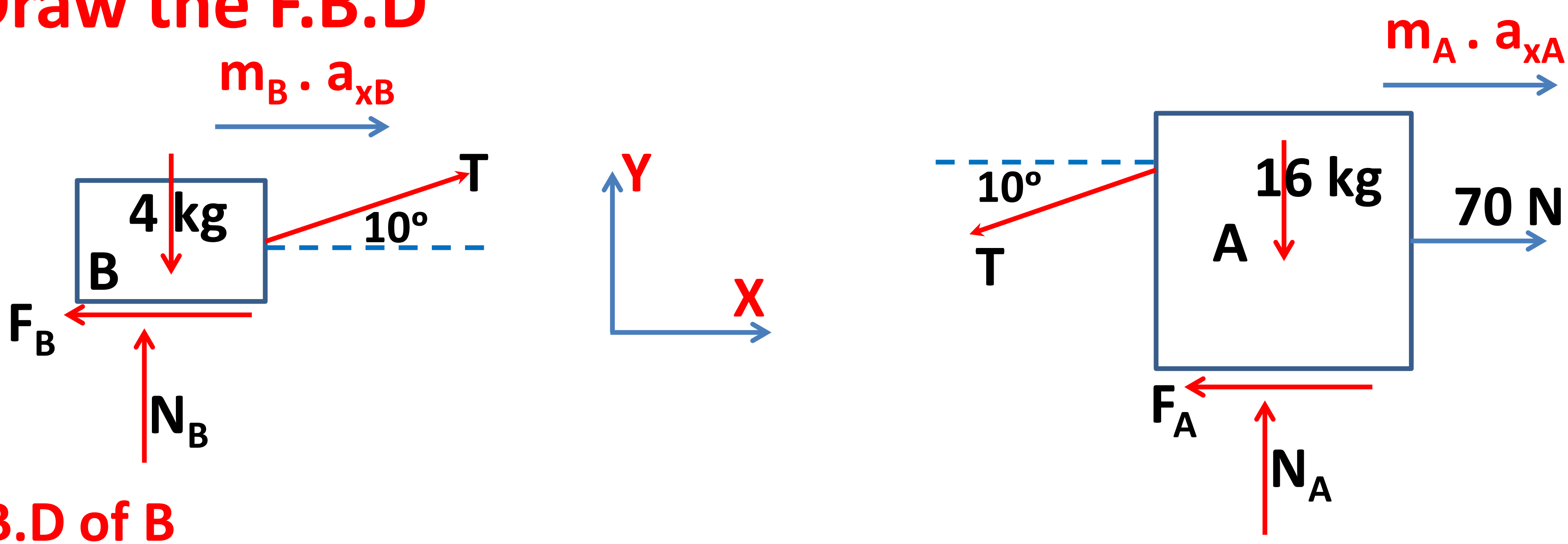
Ex.1: A horizontal force **P=70 N** is exerted on mass **A=16kg** as shown in Fig. below. The coefficient of friction between mass **A** and the horizontal plane is **0.25**. **B** has amass of **4 kg** and the coefficient of friction between it and the plane is **0.5**. the cord between the two masses makes an angle of **10°** with the horizontal. What is the tension in the cord.



Solution:



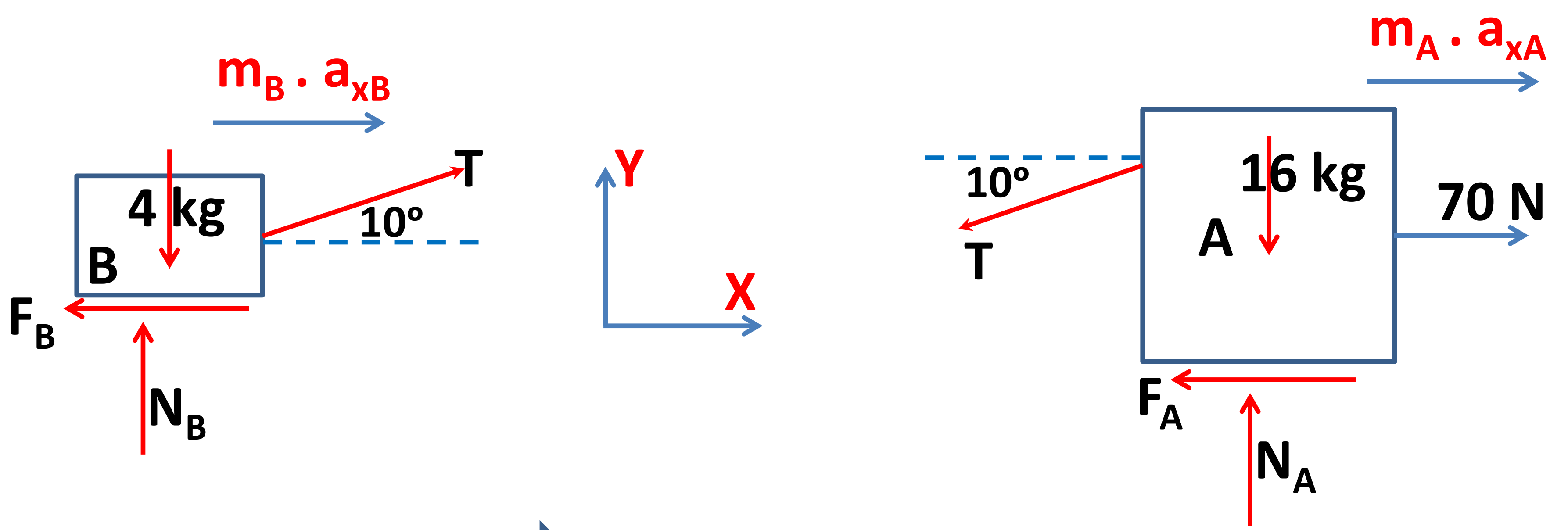
Draw the F.B.D



<u>F.B.D of B</u>

$$\sum F_y = m_B \cdot a_{yB} \quad \Rightarrow \quad N_B - 4 \times 9.81 + T \sin 10^\circ = 4 \times 0$$

$$N_B = 39.245 - T \sin 10^\circ \quad \Rightarrow \quad F_B = 0.5\, N_B = 0.5\,(39.24 - T \sin 10^\circ)$$



$\sum F_x = m_B \cdot a_{xB}$ ⇒ $T\cos 10° - F_B = 4\, a_{xB}$

$$T\cos 10° - 0.5\,(39.24 - T\sin 10°) = 4\, a_{xB}$$

$$a_{xB} = 0.268\,T - 4.905 \qquad \text{.......(1)}$$

F.B.D of A

$\sum F_y = m_A \cdot a_{yA}$ ⇒ $N_A - 16 \times 9.81 - T\sin 10° = 16 \times 0$

$$N_A = 156.96 + T\sin 10°$$

$$F_A = 0.25\,N_A = 0.25\,(156.96 + T\sin 10°)$$

$\sum F_x = m_A \cdot a_{xA} \Rightarrow 70 - F_A - T.\cos 10^\circ = 16\, a_{xA}$

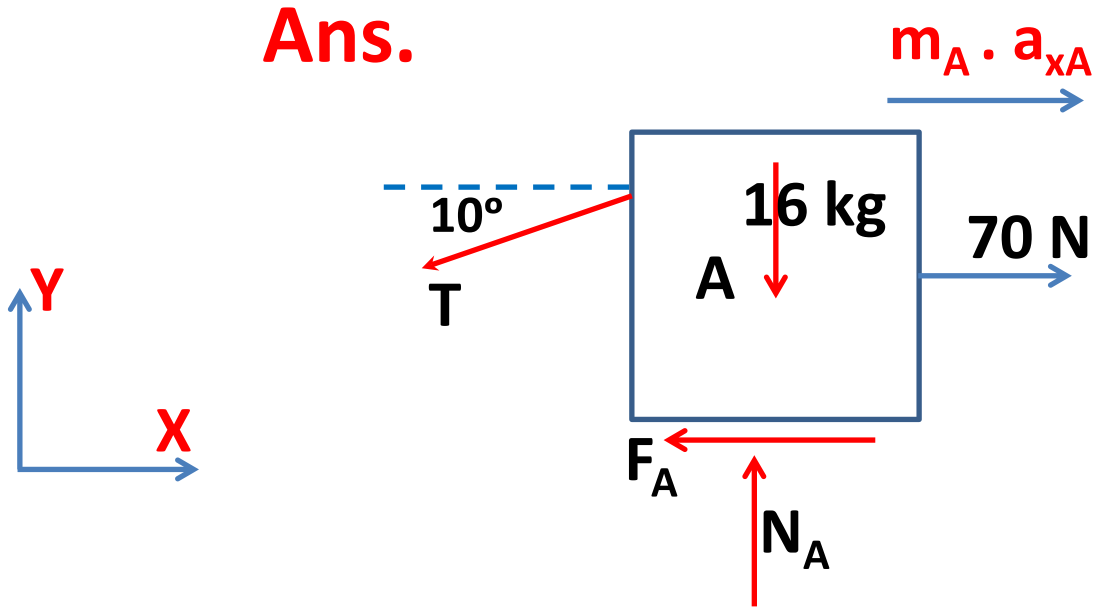
$70 - 0.25\,(156.96 + T \sin 10^\circ) - T.\cos 10^\circ = 16\, a_{xA}$

$a_{xA} = 1.923 - 0.0643\, T$ ....(2)

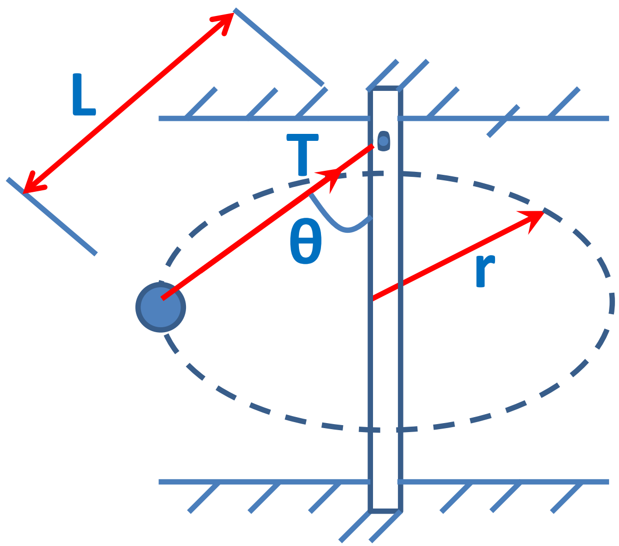
$a_{xA} = a_{xB}$

$1.923 - 0.0643\, T = 0.268\, T - 4.905$

$T = 20.548 \;\; N$ **Ans.**

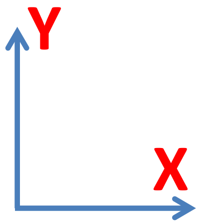


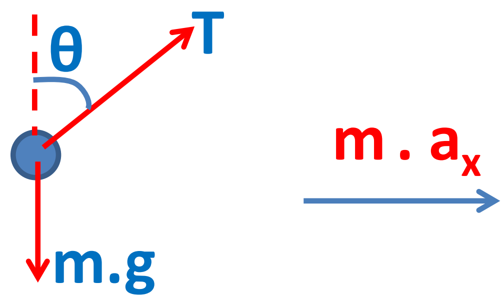
Ex.2: Fig. below indicates a particle of mass **m** which can move in a circular path about the **y** axis. The plane of the circular path is horizontal and perpendicular to the **y-axis**. As the angular velocity **ω** increases, however, the rises, which means that the radius **r** of its circular path also increases. Derive the relationship between **θ** and **ω** for constant angular velocity, and find the frequency in terms of **θ**.

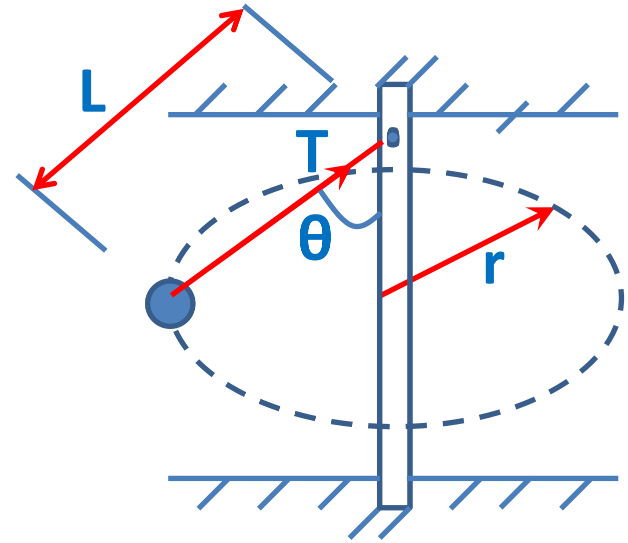


Solution:

Draw the F.B.D for The particle



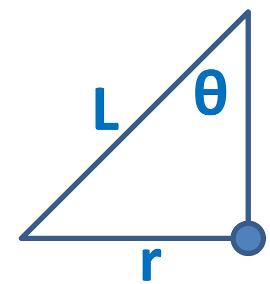




$$r = L \sin\theta$$

$$a_x = a_n = \text{normal acceleration} = \omega^2 \cdot r$$

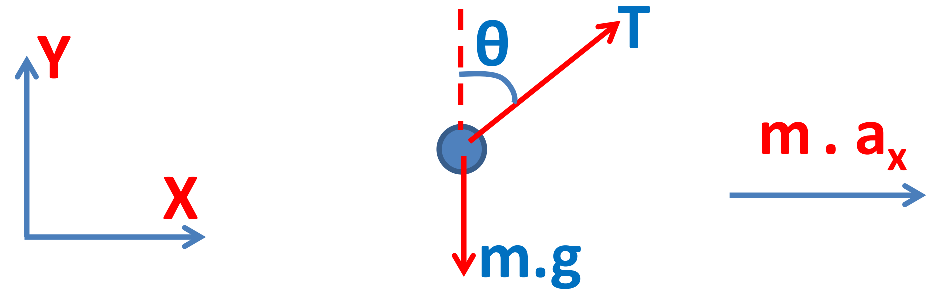
$$a_y = a_t = \alpha \cdot r = 0 \qquad \text{(constant } \omega\text{)}$$



$$\sum F_x = m \cdot a_x$$

$$T \cdot \sin\theta = m \cdot a_n = m \cdot \omega^2 \cdot r = m \cdot \omega^2 \cdot L \sin\theta$$

$$T = m \cdot \omega^2 \cdot L \qquad \text{.......... (1)}$$



$\sum F_y = m \, . \, a_y$

$T. \cos\theta - m. \, g = 0 \quad \Rightarrow \quad T = m \, . \, g \, / \cos\theta$ .....(2)

Equating eq.1 and eq.2

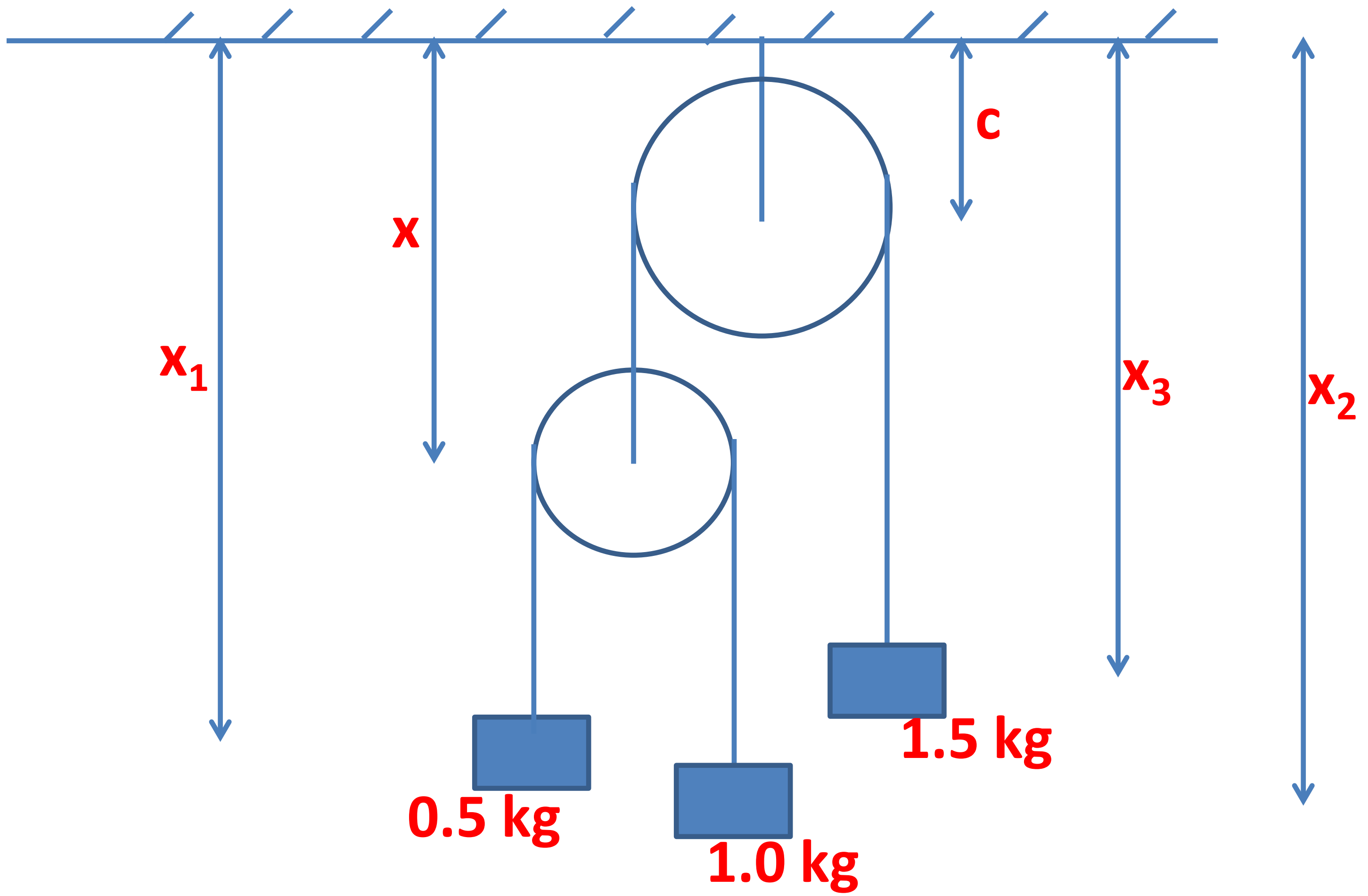
$m. \, \omega^2 \, . \, L = m \, . \, g \, / \cos\theta$ .... (1)

$$\omega = \sqrt{\frac{g}{L \, . \cos\theta}}$$

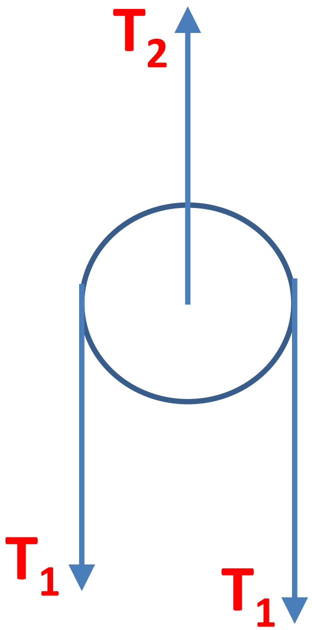
Frequency = f = $\dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L \, . \cos\theta}}$ Hz

About y-axis

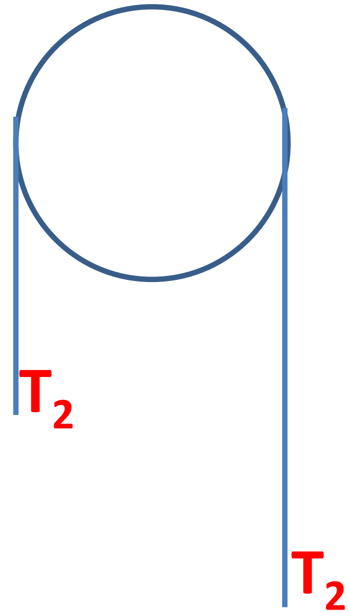
Ex.3: In the system below of pulleys and weights shown. Let x_1, x_2, x_3 be the position of the 0.5, 1.0, 1.5 kg masses respectively during any phase of the motion after the system is released. Neglect the masses of the pulleys and the cords, and assume no friction. Determine the tensions in the cords.



Solution: Draw F.B.D. for the pulleys



Pulley 1



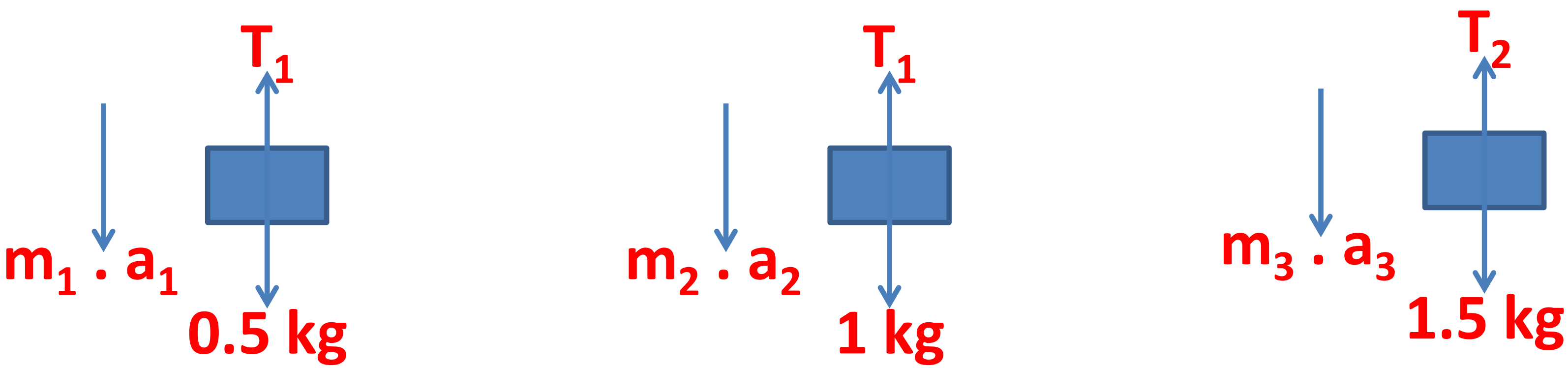
Pulley 2

__Pulley 1__

$\sum F_y = m \,.\, a \Rightarrow \sum F_y = 0$ (Mass is Neglected)

$T_2 - 2T_1 = 0 \Rightarrow T_2 = 2T_1$ ….. (1)

F.B.D of masses 0.5, 1.0 and 1.5 kg



$\sum F_y = m \,.\, a$

$0.5 \times 9.81 \;-\; T_1 = m_1 \,.\, a_1 \;\Rightarrow\; 0.5 \times 9.81 \;-\; T_1 = 0.5\, a_1$ ..... (2)

$1 \times 9.81 \;-\; T_1 = m_2 \,.\, a_2 \;\Rightarrow\; 9.81 \;-\; T_1 = a_2$ ..... (3)

$1.5 \times 9.81 \;-\; T_2 = m_3 \,.\, a_3 \;\Rightarrow\; 1.5 \times 9.81 \;-\; T_2 = 1.5\, a_3$ ..... (4)

Length of cord 1 remains constant.

$(x_1 - x) + (x_2 - x) = K_1 \;\Rightarrow\; x_1 + x_2 - 2x = K_1$

$v_1 + v_2 - 2v = 0 \;\Rightarrow\; a_1 + a_2 - 2a = 0$ ..... (5)

Length of cord 2 remains constant.

$$(x - c) + (x_3 – c) = K_2$$

$$x + x_3 – 2c = K_2$$

$$v + v_3 = 0$$

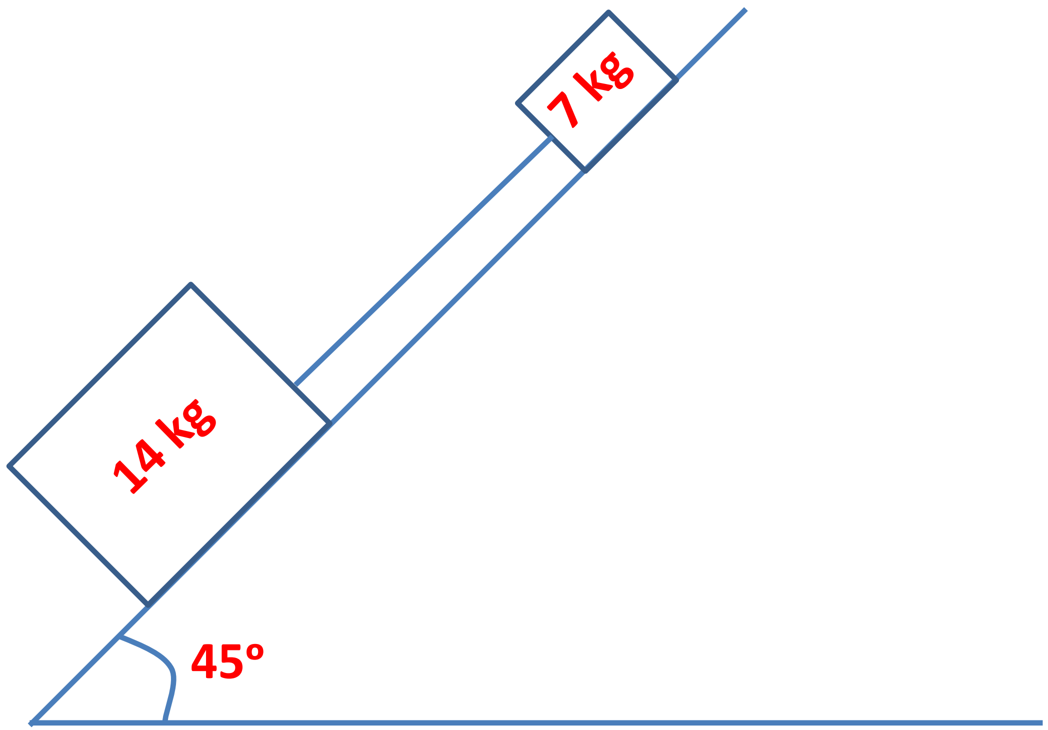
$$a + a_3 = 0 \qquad ..... (6)$$

By solving the above 6 eqs.

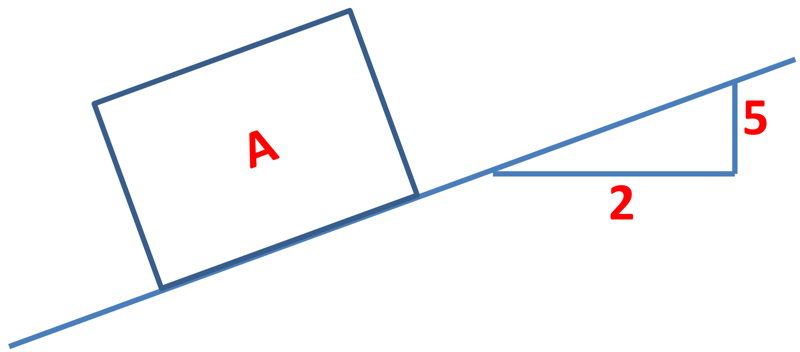
$$T_1 = 6.92 \quad N$$

$$T_2 = 13.85 \quad N$$

Problem 4: Two masses of 14 kg and 7 kg connected by a flexible inextensible cord rest on inclined plane shown below. When the masses are released what will be the tension T in the cord? Assume the coefficient of friction between the plane and the 14 kg mass is 0.25 and between the plane and 7 kg mass is 0.375.

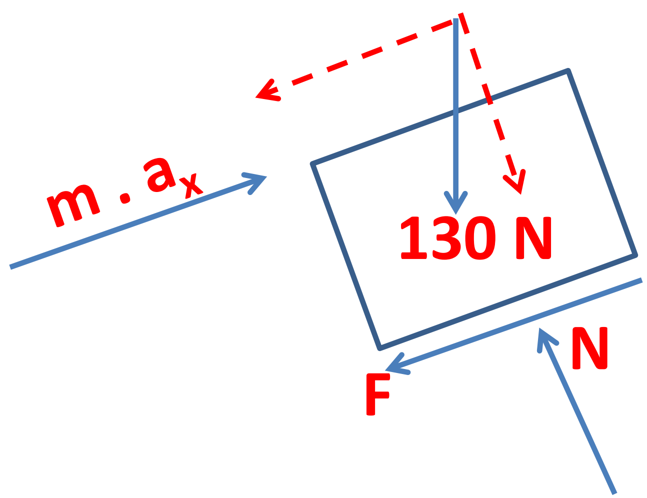


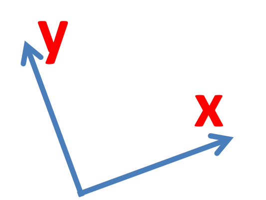
Ex. 5: The 130 N block A of fig. below has a velocity of 30 m/sec up the plane. the coefficient of friction between the block and the plane is 0.1. Determine how far up the plane the block will slide before it stops.

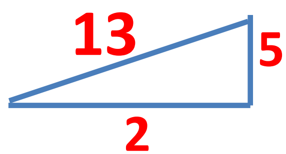


Solution:

Draw F.B.D



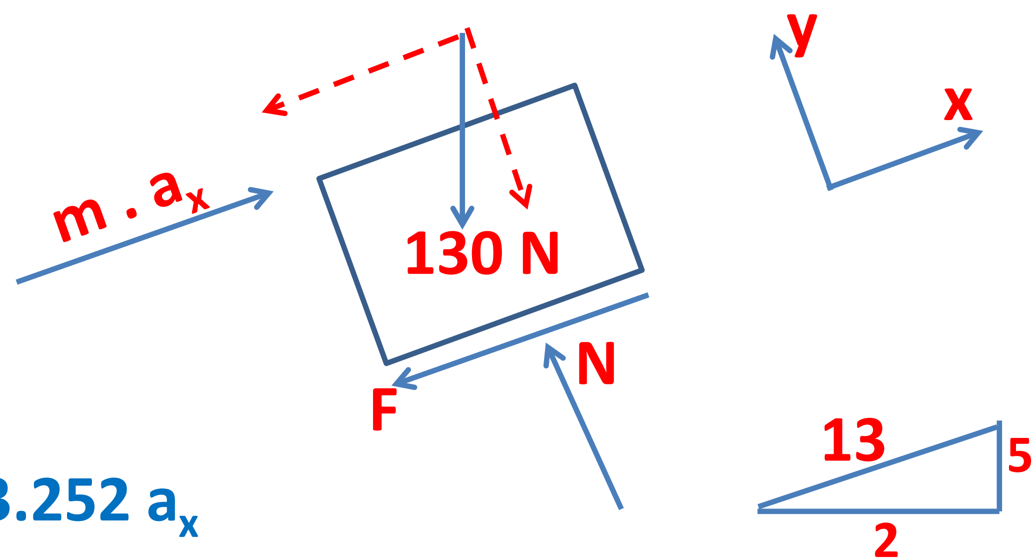




$m = 190/9.81 = 13.252$ kg

$$\sum F_y = m \cdot a_y$$

$N - 130\,(2/13) = 0$ $\Rightarrow$ $N = 120$ N



$$\sum F_x = m \, . \, a_x$$

$$-F - 130\,(5/13) = 13.252\, a_x$$

$$-0.1 \times 120 - 130\,(5/13) = 13.252\, a_x$$

$a_x = -\,4.679\ m/sec^2$ **decceleration**

The x-component of the weight is constant, so, a_x is constant.

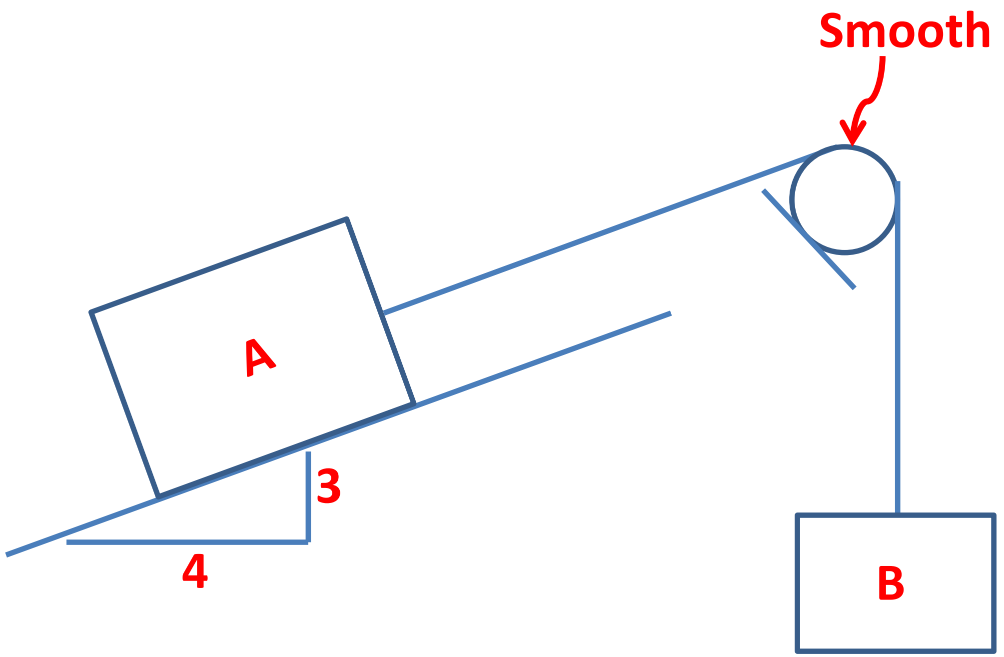
If the body slide till stop, $v = 0$.

$v = v_o + a_x \, . \, t$ ➡ $0 = 30 - 4.679 \times t$ ➡ $t = 6.412$ sec

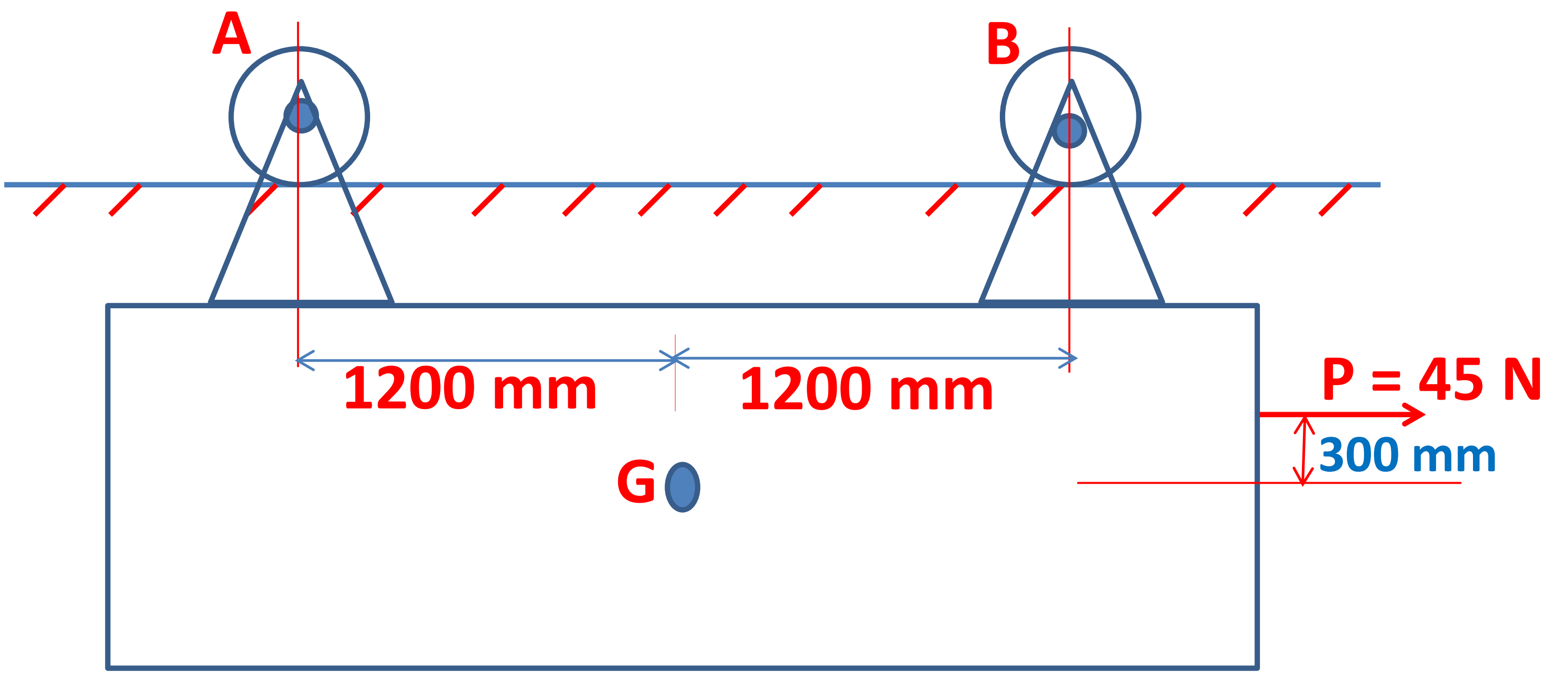
$$x = x_o + v_o \, . \, t + 0.5\, a_x \, . \, t^2$$

$x = 0 + 30 \times 6.412 + 0.5\,(-4.679) \times (6.412)^2$ ➡ $x = 96.174$ m

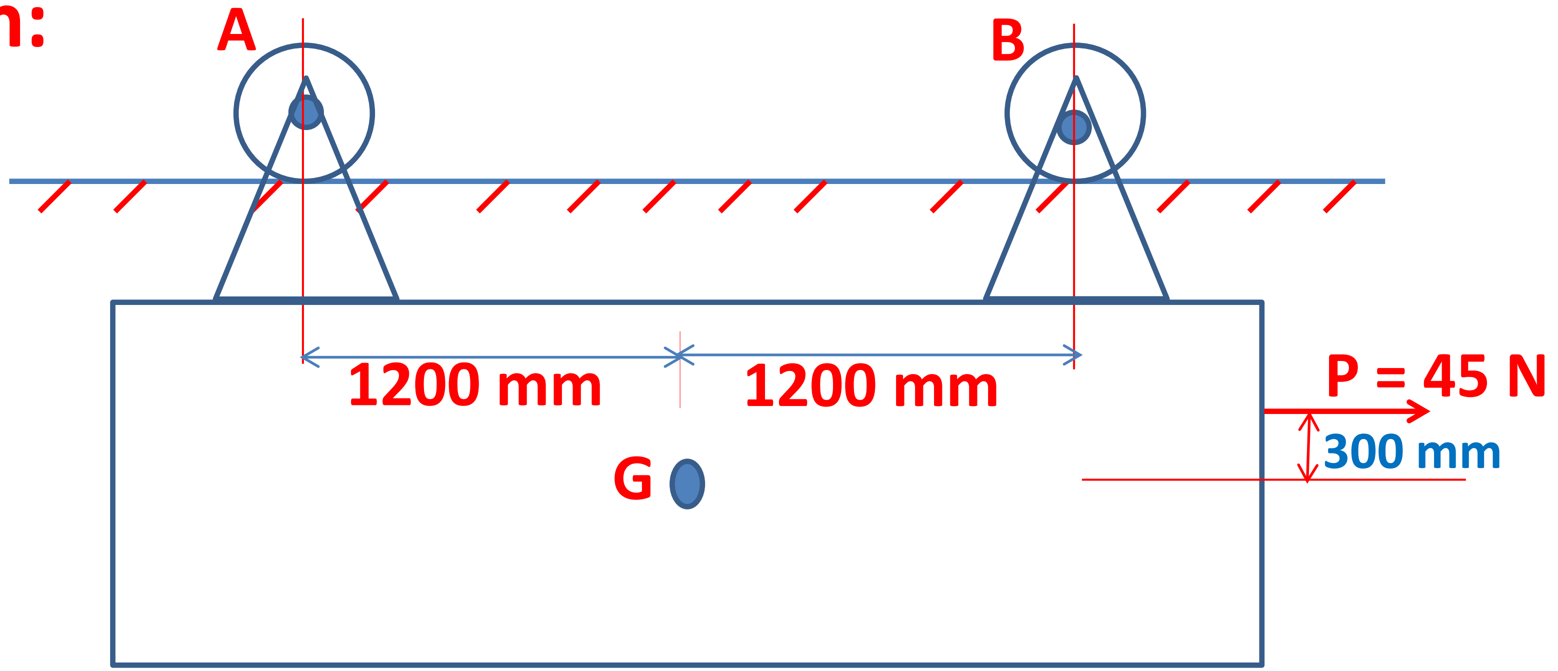
Problem.6: Block A of Fig. below weighs 100 N and B weighs 150 N. The coefficient of friction between A and the inclined plane is 0.2. Determine the acceleration of body A when it is moving up the plane.



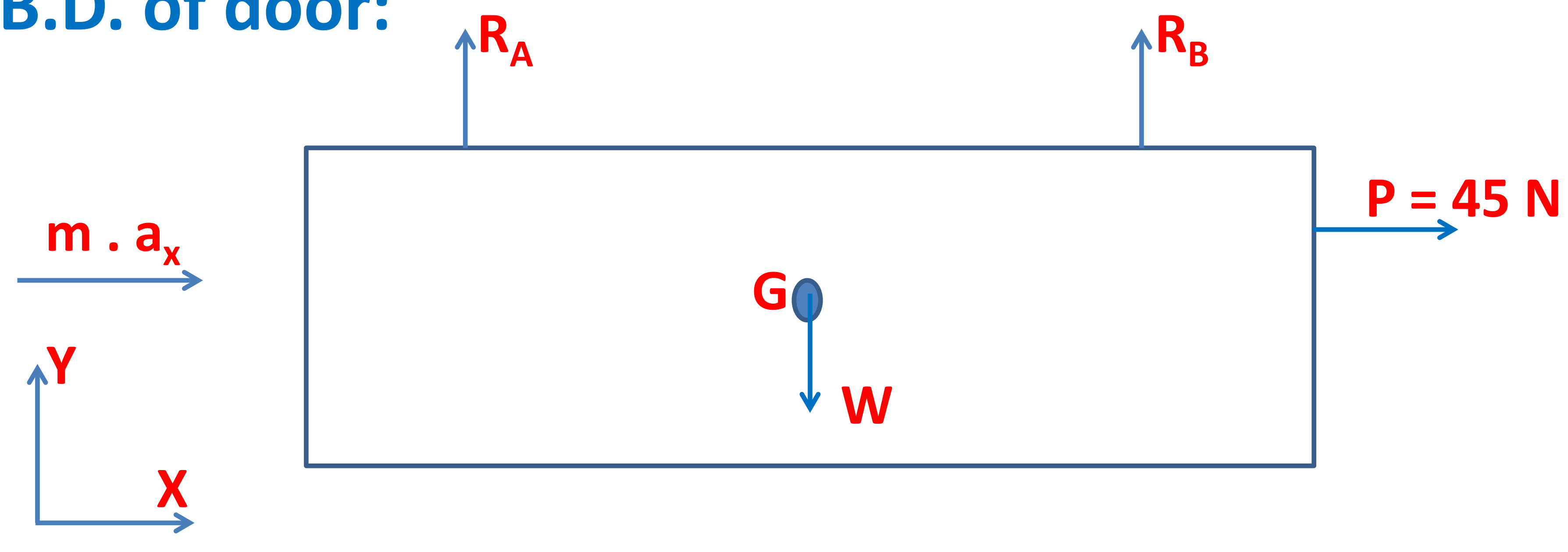
Ex.7: A 22.5 kg homogenous door is supported on frictionless rollers A and B resting on horizontal track, as shown below. A constant force P of 45 N is applied. What will be the velocity of the door 5 sec after starting from rest? What are the reactions of the rollers?



Solution:



F.B.D. of door:



$\sum F_x = m \, . \, a_x$ ...... (1)

$\sum F_y = m \, . \, a_y$ ...... (2)

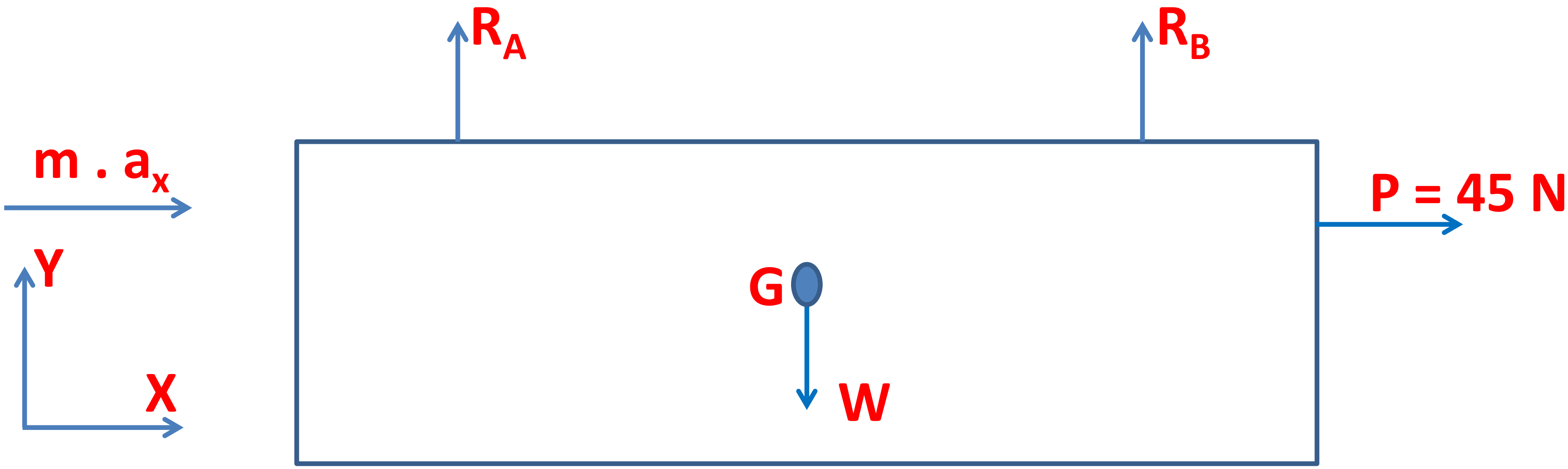
$\sum M_G = 0$ ......(3)

Eq.(1) $45 = 22.5 \, a_x$ ⇒ $a_x = 2 \ m/sec^2 \longrightarrow$

Eq.(2) $RA + RB - W = 0$ ⇒ $RA + RB - 22.5 \times 9.81 = 0$ ...(4)

Eq.(3) $-RA \times 0.12 + RB \times 0.12 - 45 \times 0.3 = 0$ .... (5)

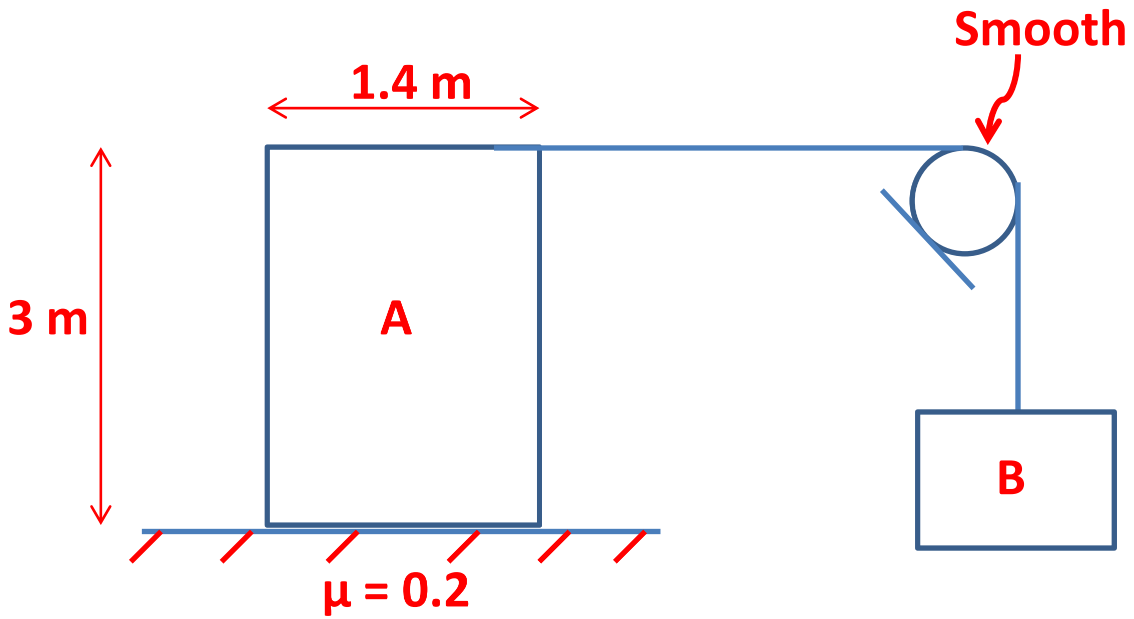
Solve Eqs. (4) & (5) ⇒ **RA = 104.75 N,** **RA = 116 N**



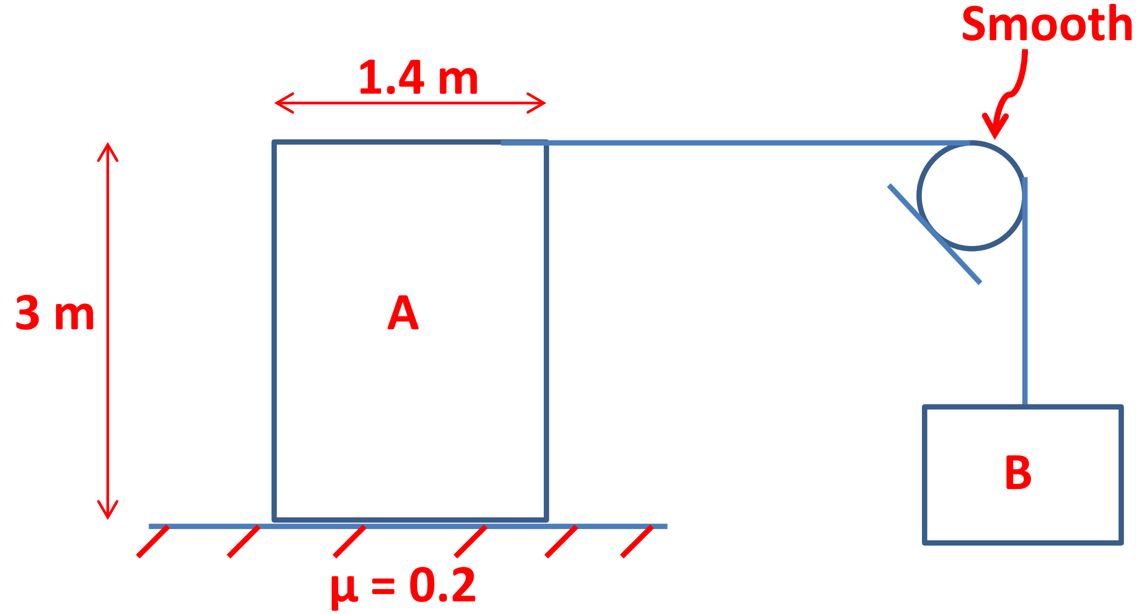
$$v_x = v_o + a_x \cdot t$$

$$v_x = 0 + 2 \times 5 = 10 \text{ m/sec} \quad \longrightarrow$$

Problem 7: The homogenous 10 kg body A of Fig. below is moving to the right with a velocity of 6 m/sec. Determine the max. weight of body B may have without causing A to tip.



Solution:



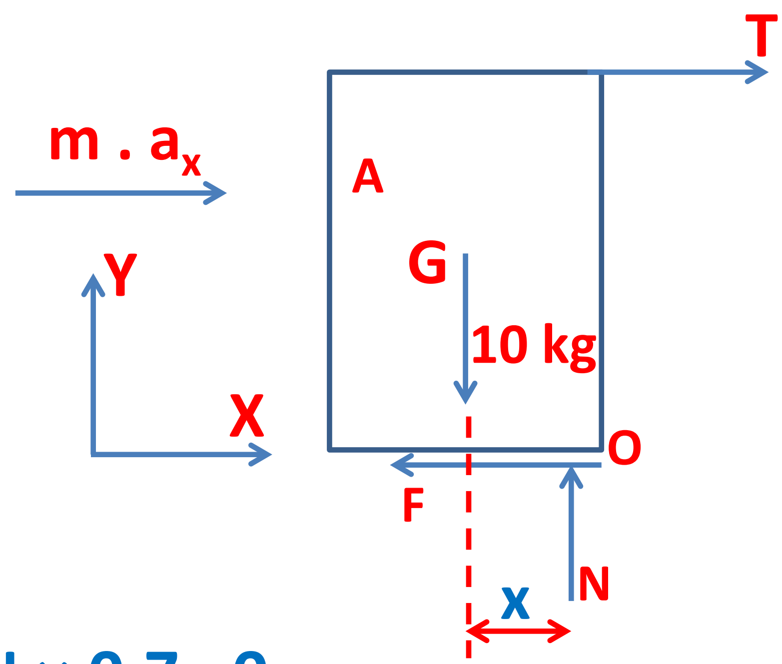
F.B.D of A

$$\sum F_y = m \cdot a_y$$

$$N - 10 \times 9.81 = 0$$

$$N = 98.1 \text{ N}$$

$$F = 0.2\, N = 19.62 \text{ N}$$



Since body A may tip about point O, so, maximum value of x before tipping is 0.7 m

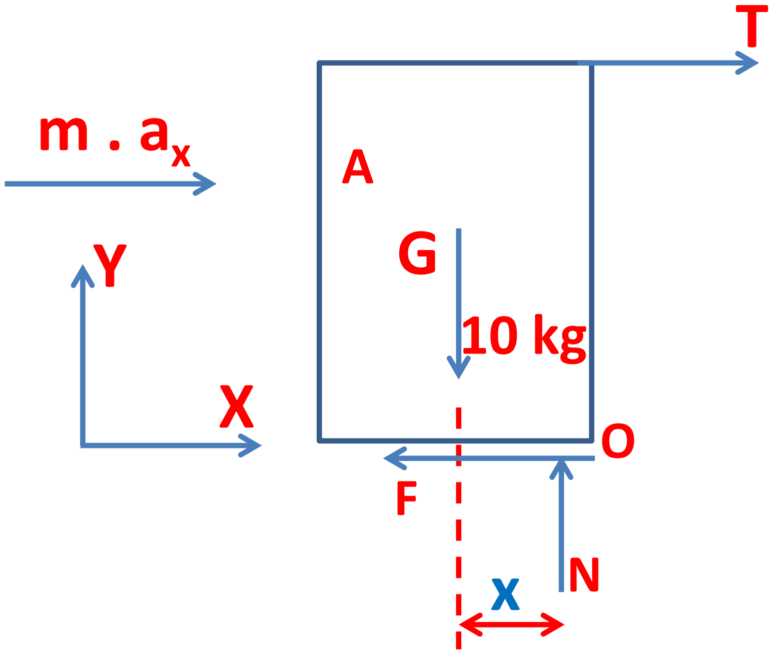
$$\sum M_G = 0 \quad \Rightarrow \quad T \times 1.5 + F \times 1.5 - N \times 0.7 = 0$$

$$T = 26.16\ N$$

$$\sum F_x = m \cdot a_x$$

$$T - F = 10 \times a_x$$

$$a_x = 0.654\ \ m/sec^2$$



F.B.D of B

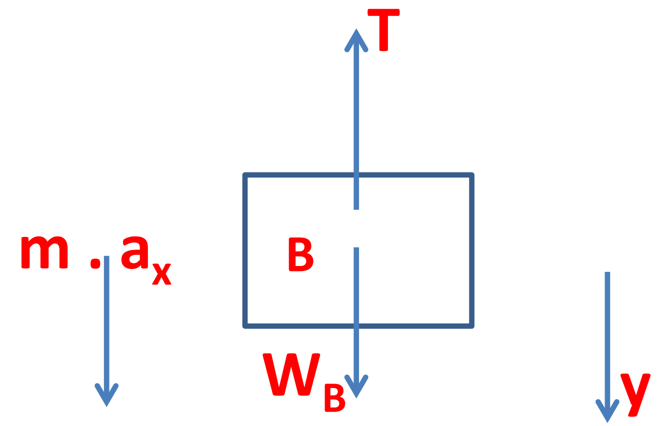
$$a_x = a_y\ \ = 0.654\ \ m/sec^2$$

$$\sum F_y = m \cdot a_y$$

$$W_B - T = (W / 9.81) \times 0.654$$

$$W_B = 28.029\ N$$

اقصى قيمة لوزن الجسم B
الذي لا يسبب انقلاب الجسم A



Chapter Five

Work and Energy

Work and Energy

1. The Work of a Force

A force F will do *work* on a particle only when the particle undergoes a *displacement in the direction of the force*.

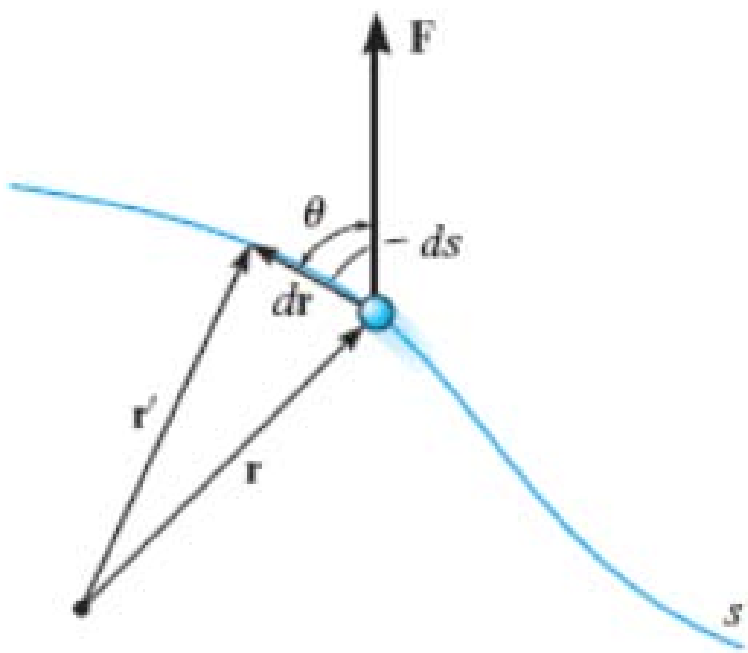


Figure 1

For example, if the force **F** in Fig. 1 below causes the particle to move along the path *s* from position **r** to a new position **r′**, the displacement is then ***d*r** = **r′** - **r**. The magnitude of ***d*r** is *ds*, the length of the differential segment along the path.

If the angle between the tails of ***d*r** and **F** is **θ** then the work done by **F** is a ***scalar quantity***, defined by:

$$dU = F\,ds\cos\theta$$

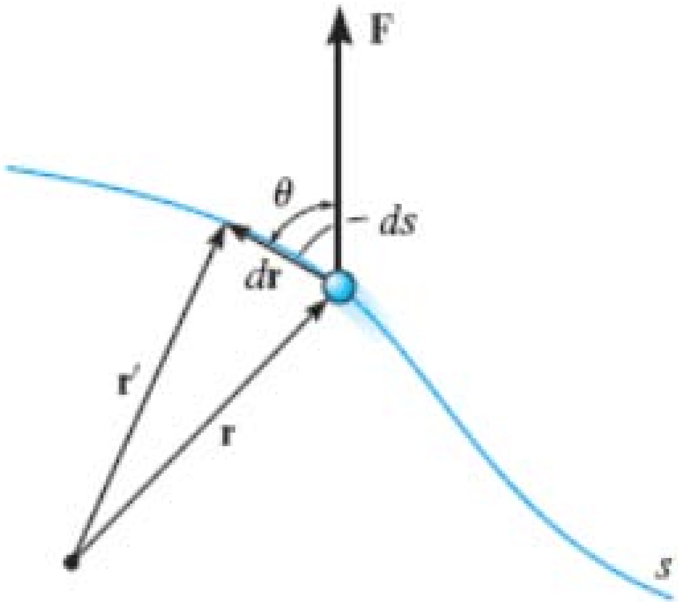


Figure 1

For : $0^o < \theta \leq 90^o$ ➡ ***dU* is +ve.**

i.e. **the force component and the displacement have the same sense.**

For : $90^o < \theta \leq 180^o$ ➡ ***dU* is -ve.**

i.e. **the force component and the displacement have opposite sense.**

For : $\theta = 90^o$ ➡ ***dU* is zero**

i.e. **the force is perpendicular to displacement, since cos 90^o = 0, or if the force is applied at a fixed point, in which case the displacement is zero.**

Notes:

1. **Work done by a one - newton force when it moves through a distance of one meter in the direction of the force (1 J = 1 N. m).**

2. **work is measured in units of foot-pounds (ft. lb), which is the work done by a one-pound force acting through a distance of one foot in the direction of the force.**

Work of a Variable Force. If the particle acted upon by the force **F** undergoes a finite displacement along its path from **r1** to **r2** or **s1** to **s2** , **Fig. 2a**, the work of force **F** [$U_{1\text{-}2}$ is the total work done by force **F**] is determined by integration. Provided **F** and **θ** can be expressed as a function of position, then

$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F \cos\theta \, ds \qquad 1$$

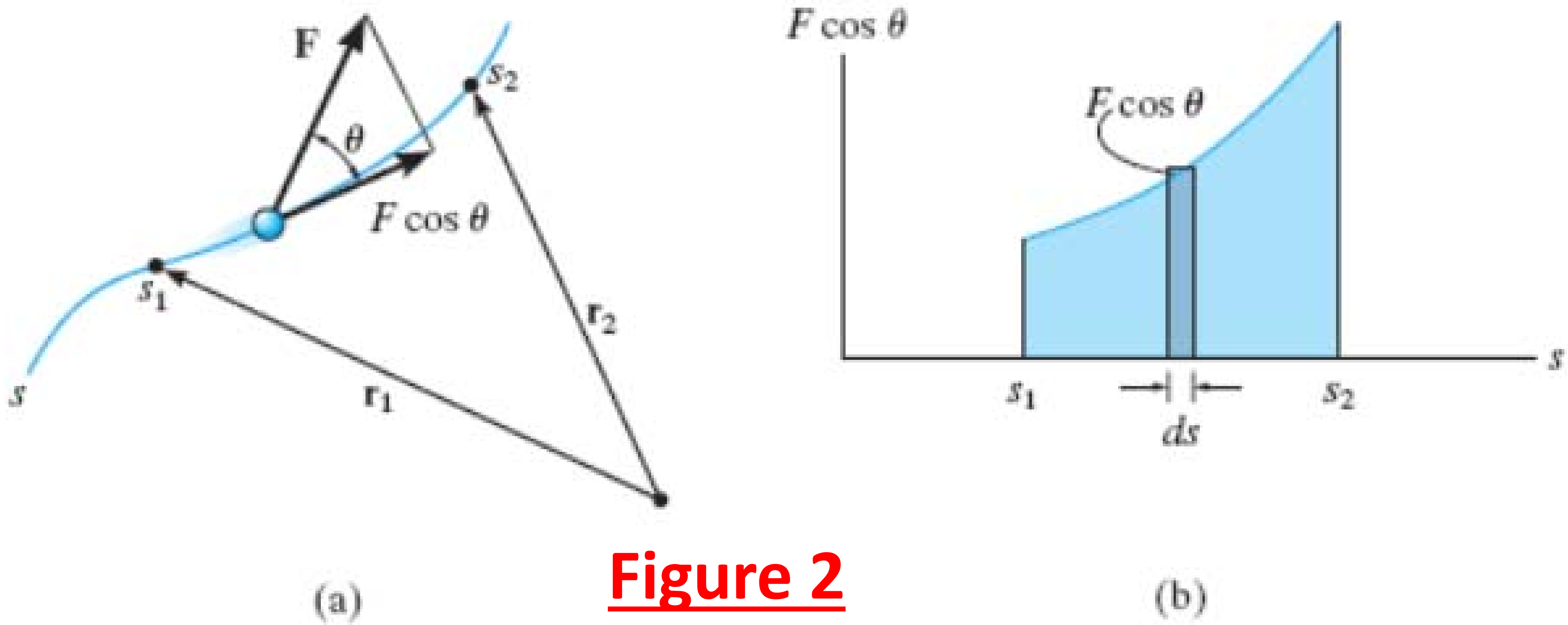


Figure 2

Work of a Constant Force Moving Along a Straight Line.

If the force F_c has a constant magnitude and acts at a constant angle θ from its straight-line path, Fig.3a, then the component of F_c in the direction of displacement is always $F_c .\cos\theta$. The work done by F_c when the particle is displaced from s1 to s2 is determined from Eq. 1, in which case:

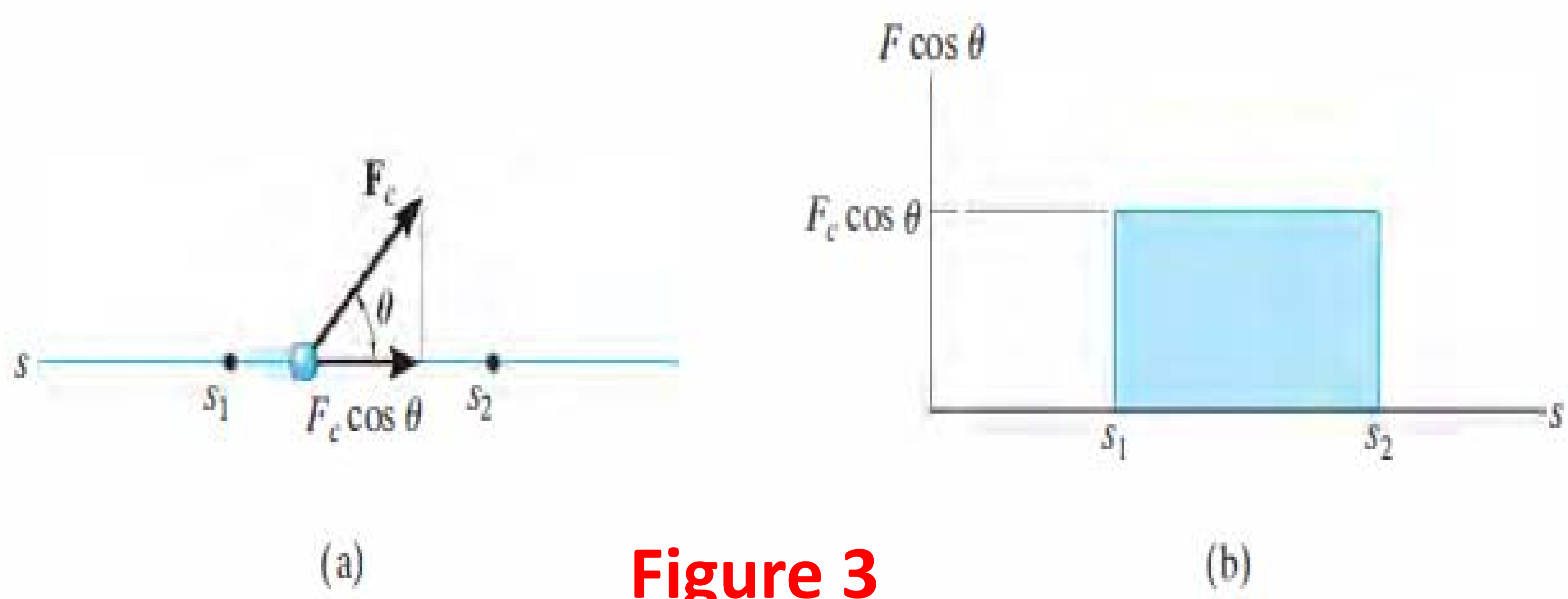


Figure 3

$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

or

$$\boxed{U_{1-2} = F_c \cos\theta (s_2 - s_1)} \qquad 2.$$

Here the work of Fc represents the area of the rectangle in Fig. 3b.

Work of a Weight. **Consider a particle of weight** W, **which moves up along the path s shown in** Fig. 4 **from position** s1 **to position** s2 **. At an intermediate point, the displacement** dr = dxi + dyj + dzk. **Since** W = -Wj **, applying** Eq.1 **we have**

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W\,dy = -W(y_2 - y_1)$$

or

$$\boxed{U_{1-2} = -W\,\Delta y}$$

3

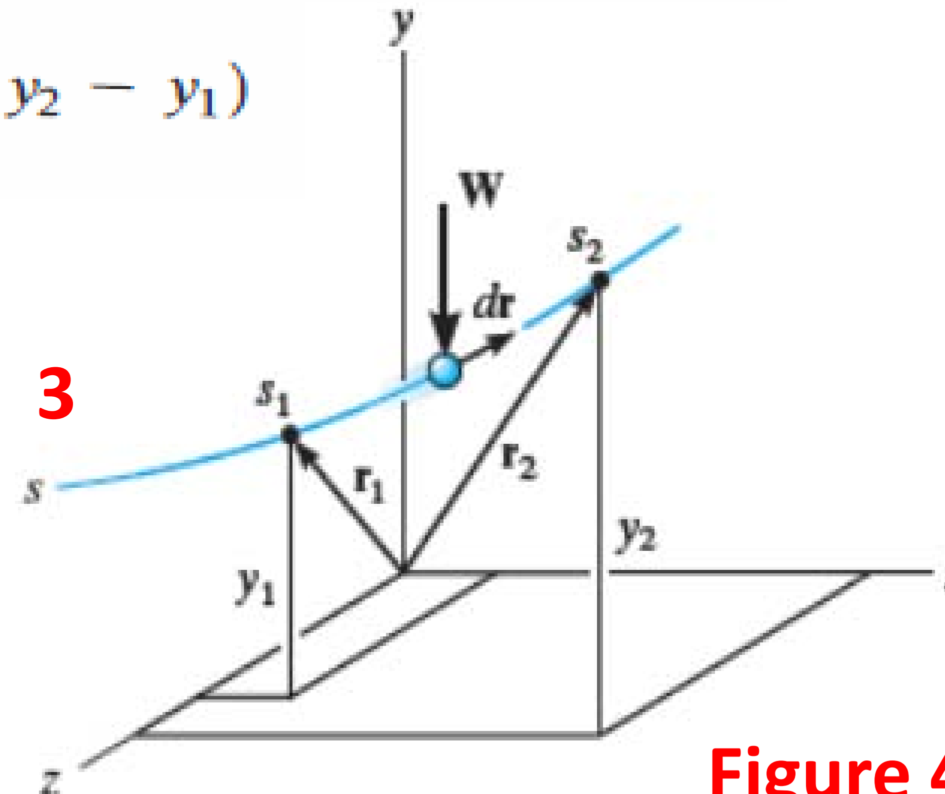


Figure 4

Thus, the work is independent of the path and is equal to the magnitude of the particle's weight times its vertical displacement. In the case shown in Fig.4 the work is negative, since W is downward and Δy is upward.

Note, however, that if the particle is displaced downward (-Δy), the work of the weight is positive.

$$U_{1-2} = -W\,\Delta y$$

3

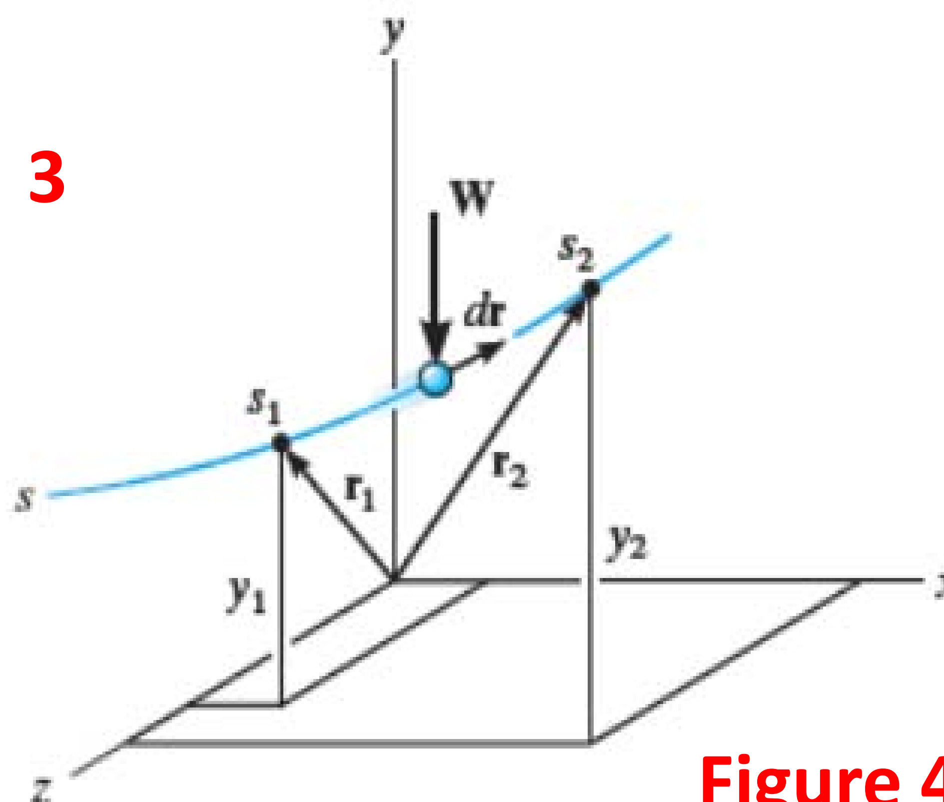


Figure 4

Work of a Spring Force. If an elastic spring is elongated a distance ds, Fig.5a, then the work done by the force that acts on the attached particle is $dU = -F_s\ ds = -k\ .\ s\ ds$. The work is negative since F_s acts in the opposite sense to ds. If the particle displaces from s1 to s2, the work of Fs is then

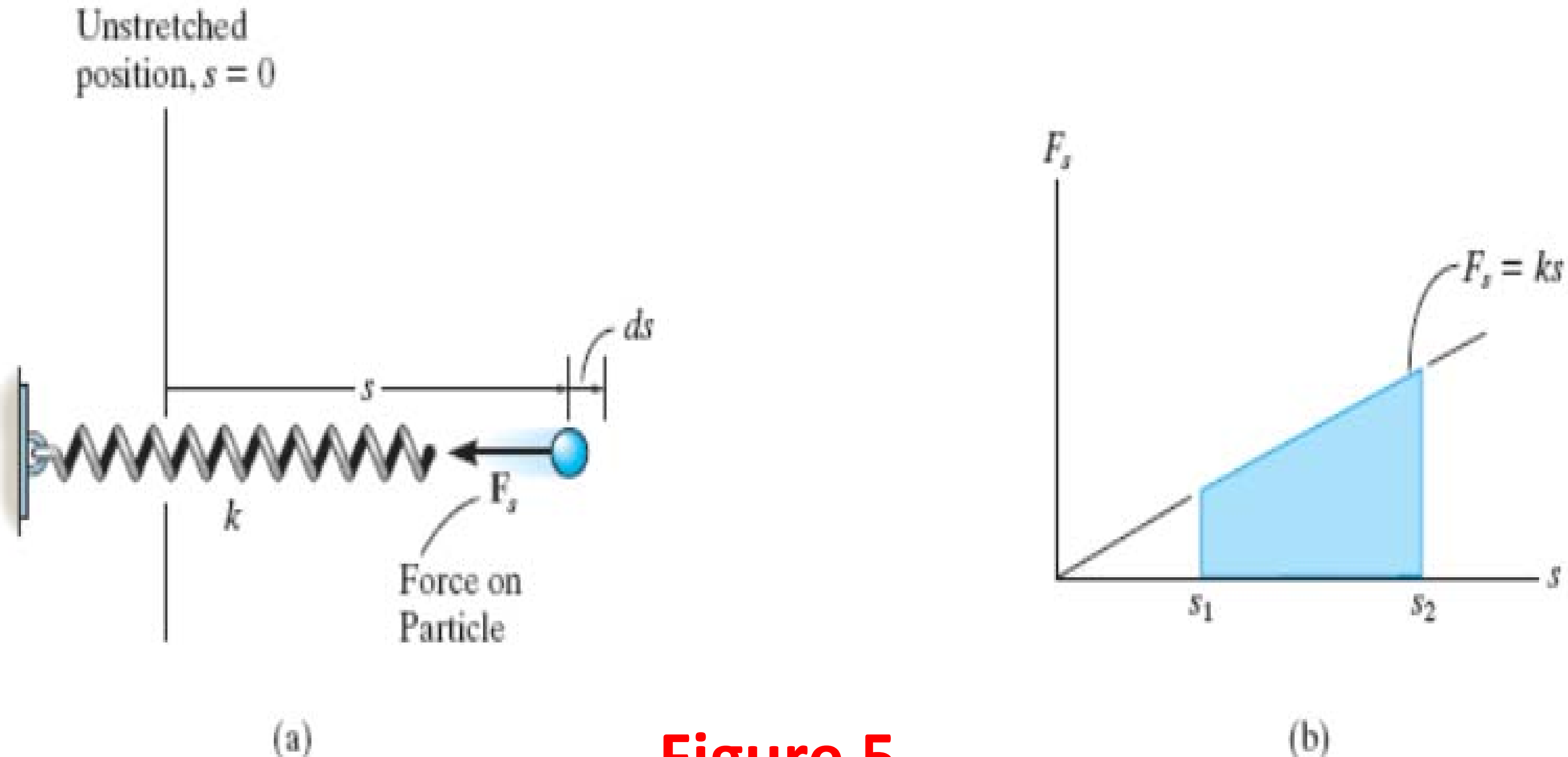
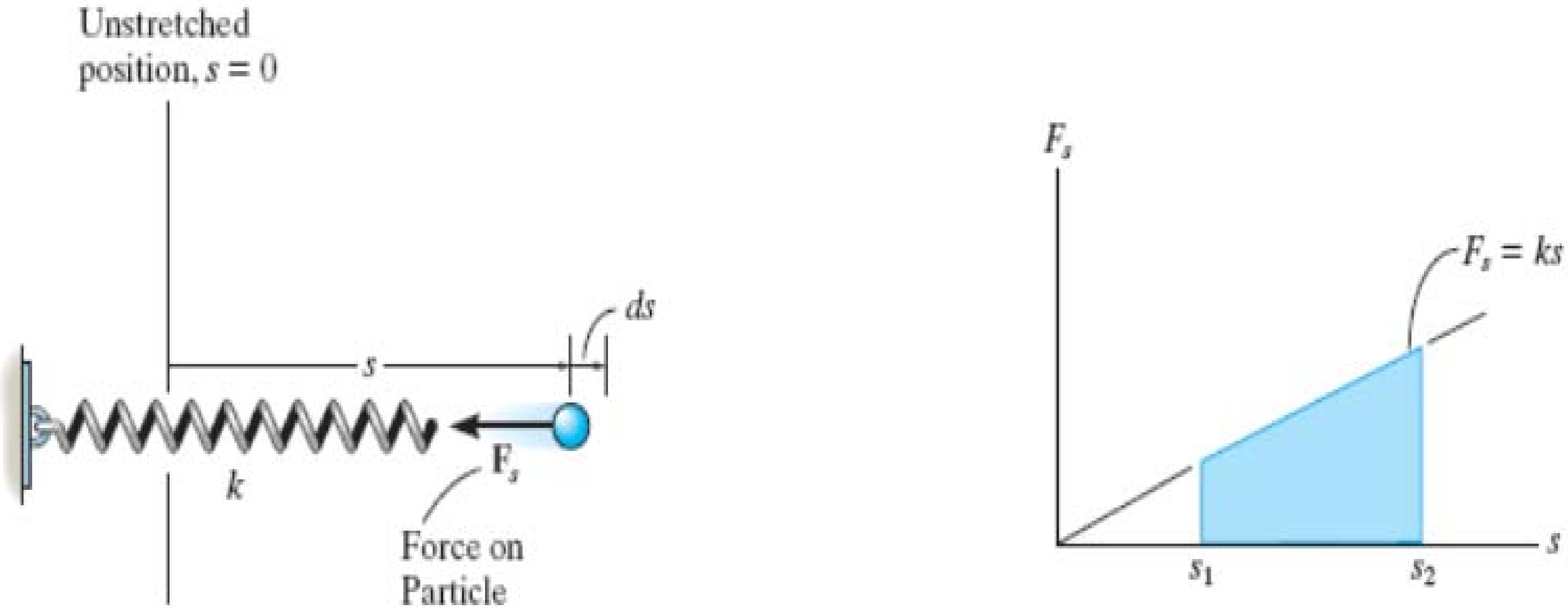


Figure 5



(a) **Figure 5** (b)

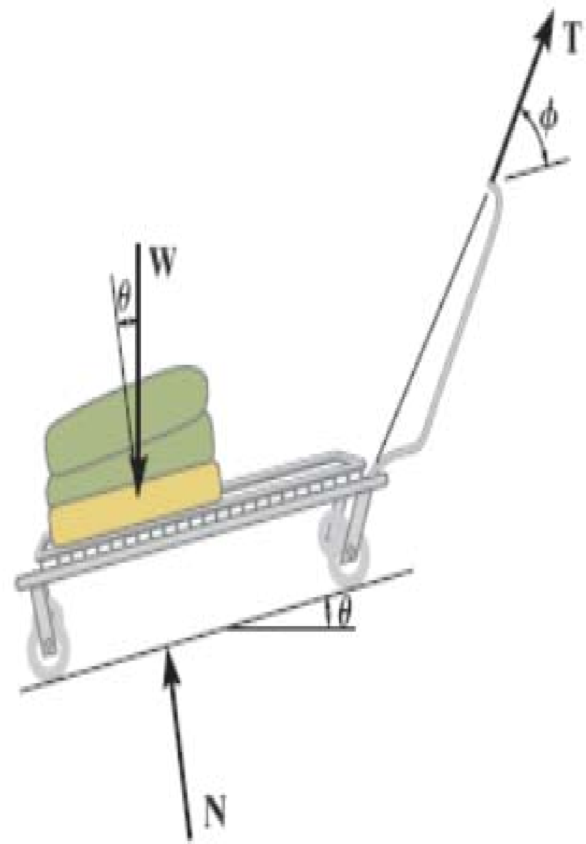
$$U_{1-2} = \int_{s_1}^{s_2} F_s\, ds = \int_{s_1}^{s_2} -ks\, ds$$

$$\boxed{U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)}$$

This work represents the trapezoidal area under the line Fs = k.s, Fig.5b.

Note: **A mistake in sign can be avoided when applying this equation if one simply notes the direction of the spring force acting on the particle and compares it with the sense of direction of displacement of the particle if both are in the same sense, positive work results; if they are opposite to one another, the work is negative.**

The forces acting on the cart, as it is pulled a distance up the incline, are shown on its free-body diagram. The constant towing force **T** does positive work of **UT = (T cos Ø).s**, the weight does negative work of **UW = -(W sin θ).s**, and the normal force **N** does no work since there is no displacement of this force along its line of action



Ex.1: The l0 kg block shown in Fig. 6 rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force P = 400 N pushes the block up the plane s = 2 m.

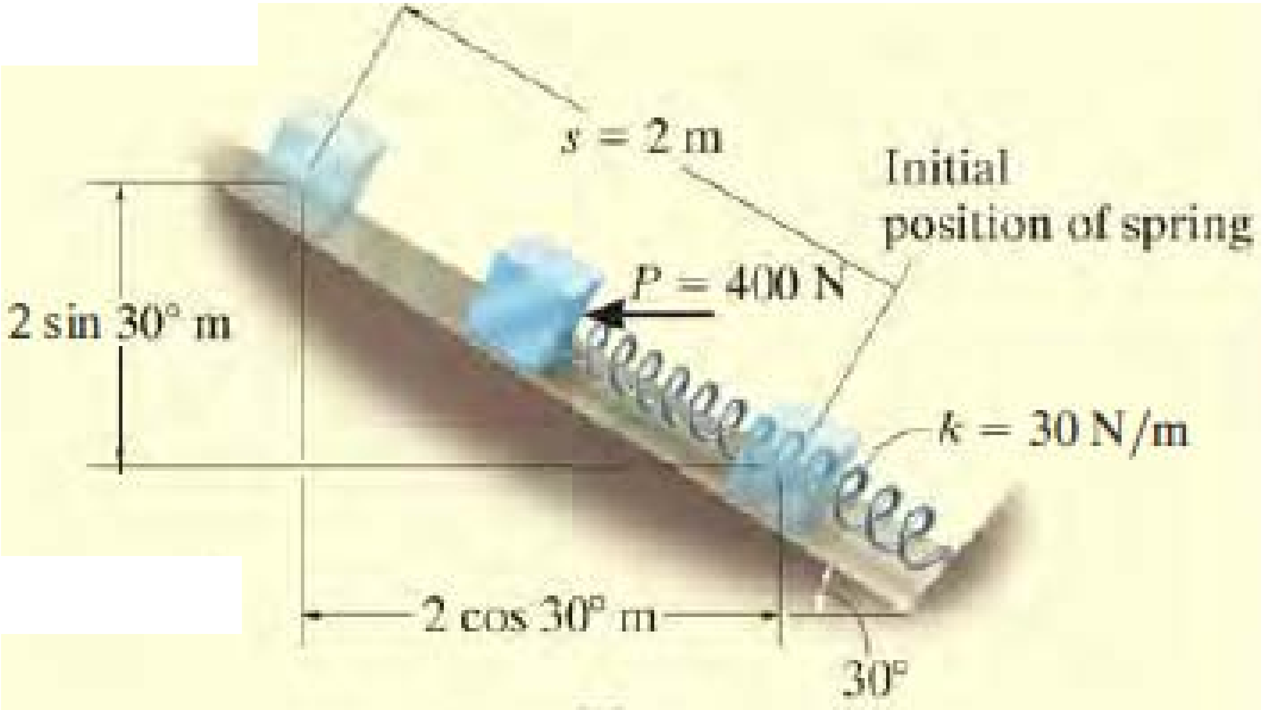
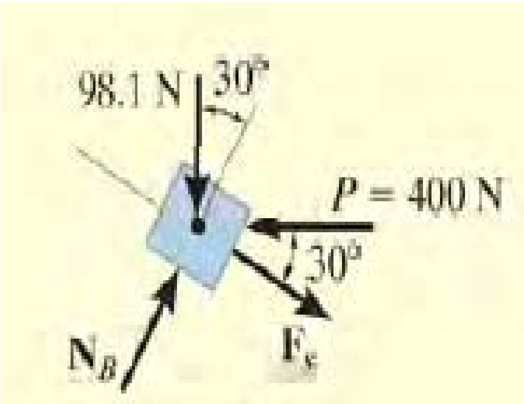
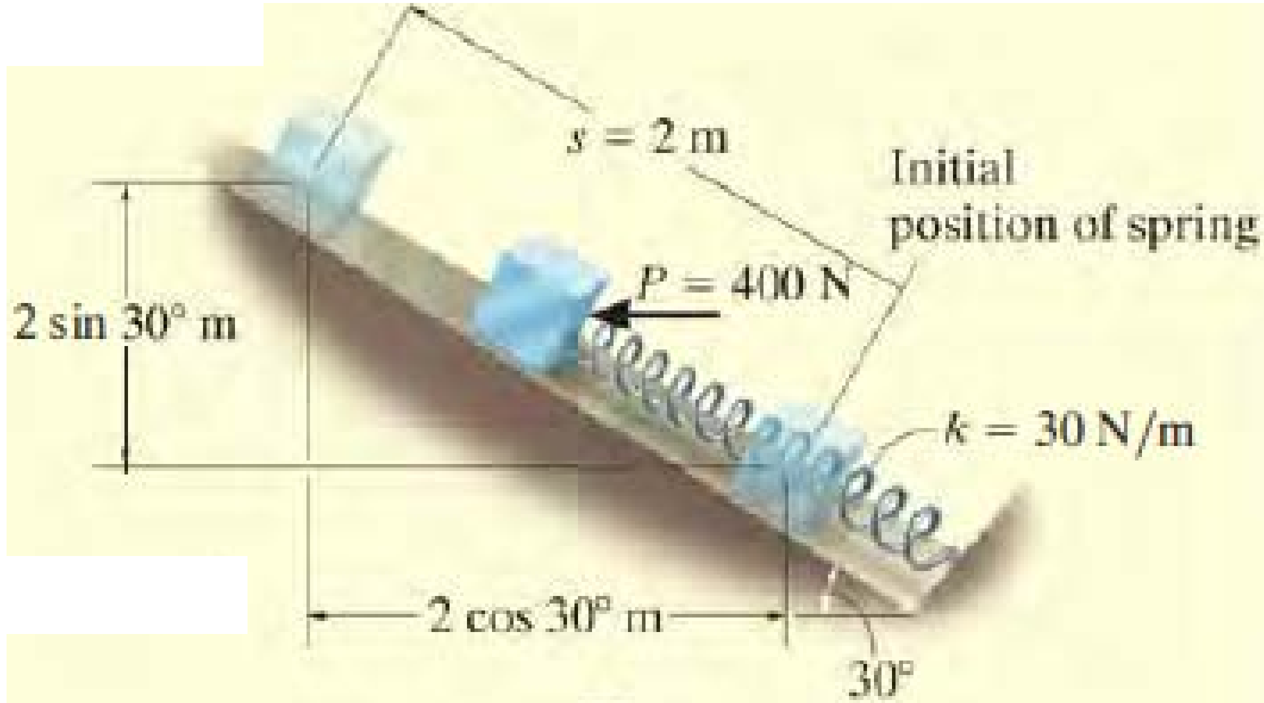


Figure 5

Solution:





Horizontal Force P:

Horizontal Force P. Since this force is constant, the work is determined using Eq. 2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e., $U_{1\text{-}2} = F_c \cos\theta\,(s_2 - s_1)$

$$U_P = 400\text{ N}\,(2\text{ m}\cos 30^\circ) = 692.8\text{ J}$$

Or the displacement times the component of force in the direction of displacement, i.e.,

$$U_P = 400\text{ N}\cos 30^\circ(2\text{ m}) = 692.8\text{ J}$$

Spring Force Fs:

In the initial position the spring is stretched $S_1 = 0.5$ m and in the final position it is stretched $S_2 = 0.5$ m + 2 m = 2.5 m. We require the work to be negative since the force and displacement are opposite to each other. The work of Fs is thus

$$U_s = -\left[\tfrac{1}{2}(30\text{ N/m})(2.5\text{ m})^2 - \tfrac{1}{2}(30\text{ N/m})(0.5\text{ m})^2\right] = -90\text{ J}$$

Weight W:

Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1\text{ N})\,(2\text{ m}\sin 30^\circ) = -98.1\text{ J}$$

Or it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1\sin 30^\circ\text{ N})\,(2\text{ m}) = -98.1\text{ J}$$

Normal Force N_B :

This force does no work since it is always perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced **2 m** is therefore

$$U_T = 692.8\text{ J} - 90\text{ J} - 98.1\text{ J} = 505\text{ J} \qquad \textit{Ans.}$$

2. **Principle of Work and Energy**

If the particle has a mass m (Fig.7) and is subjected to a system of external forces represented by the resultant $F_R = \sum F$, then the equation of motion for the particle in the tangential direction is $\sum F_t = m.\ a_t$. Applying the kinematic equation $a_t = v\ dv/ds$ and integrating both sides, assuming initially that the particle has a position $s = s_1$ and a speed $v = v_1$, and later at $s = s_2$, $v = v_2$, we have

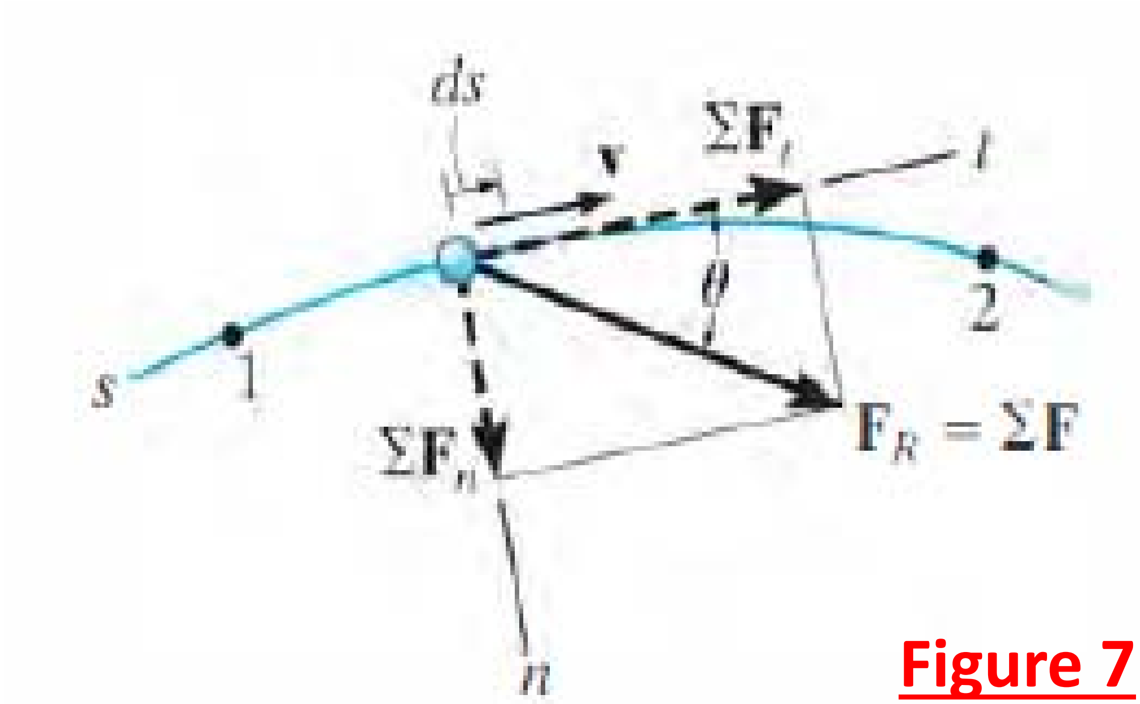


Figure 7

$$\sum \int_{s_1}^{s_2} F_t\, ds = \int_{v_1}^{v_2} mv\, dv \quad \Rightarrow \quad \sum \int_{s_1}^{s_2} F_t\, ds = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \qquad \mathbf{5}$$

From Fig. 7, note that $\sum F_t = \sum F \cos\theta$, and since work is defined from Eq. 1, the final result can be written as:

$$\sum U_{1-2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \qquad \mathbf{6}$$

This equation represents the principle of work and energy for the Particle:

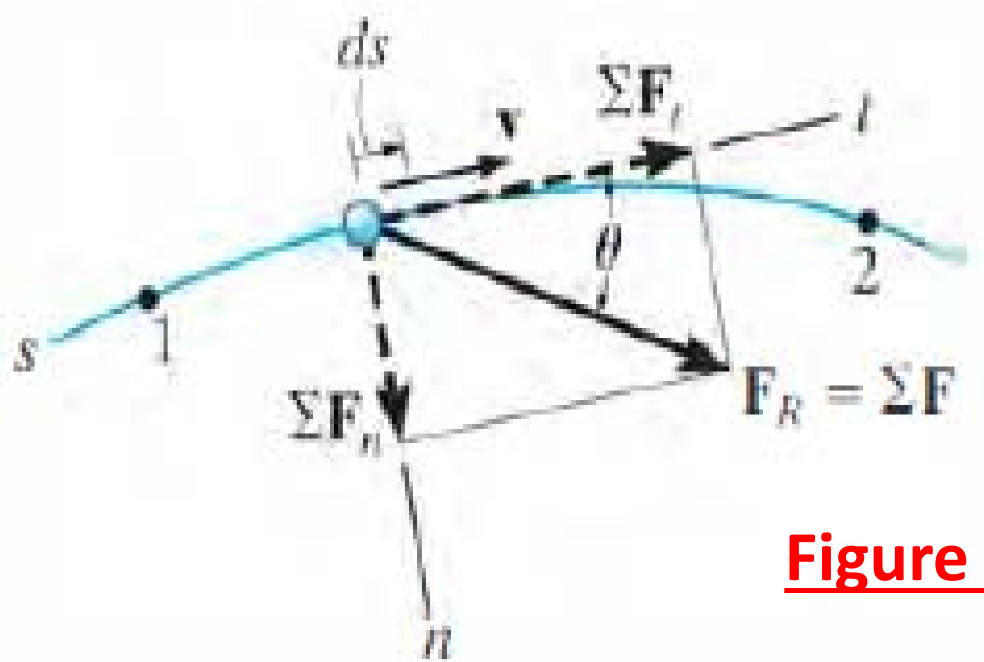


Figure 7

$$\Sigma U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \qquad 6$$

Left side = **the sum of the work done by all the forces acting on the particle as the particle moves from point 1 to point 2**

Right side : $T = \frac{1}{2}mv^2$ **; the particle's final and initial kinetic energy**

Like work, kinetic energy is a scalar and has units of joules (J);

Unlike work, which can be either positive or negative, the kinetic energy is always positive, regardless of the direction of motion of the particle.

$$\boxed{T_1 + \Sigma U_{1-2} = T_2} \qquad 6$$

which states that the particle's initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to its final position is equal to the particle's final kinetic energy.

Procedure for Analysis

Work (Free-Body Diagram).

Establish the inertial coordinate system and draw a free-body diagram of the particle in order to account for all the forces that do work on the particle as it moves along its path.

Principle of Work and Energy.

1. Apply the principle of work and energy, $T_1 + \sum U_{1\text{-}2} = T_2$.

2. The kinetic energy at the initial and final points is always positive, since it involves the speed squared ($T = 0.5\ mv^2$).

3. A force does work when it moves through a displacement in the direction of the force.

4. Work is positive when the force component is in the same sense of direction as its displacement, otherwise it is negative.

4. Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force-displacement curve.

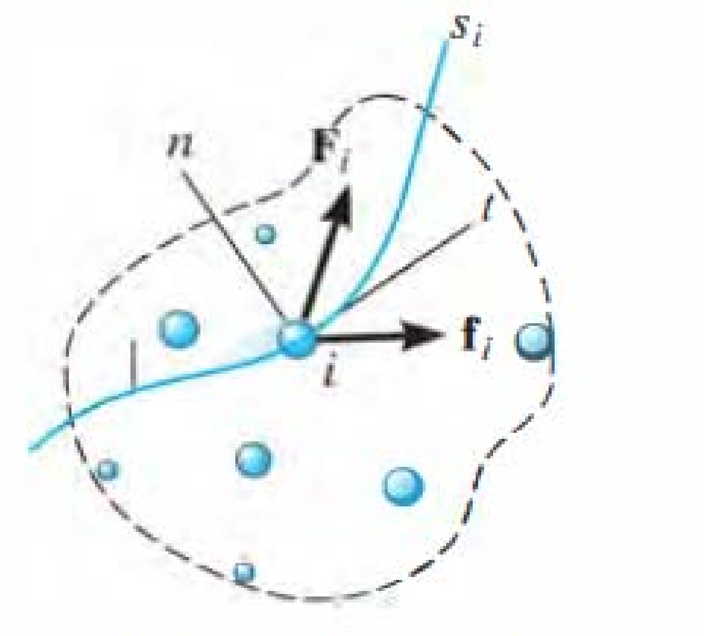
5. The work of a weight is the product of the weight magnitude and the vertical displacement, $U_w = \pm W.y$. It is positive when the weight moves downwards.

6. The work of a spring is of the form $U_s = 0.5\ k\ s^2$, where k is the spring stiffness and s is the stretch or compression of the spring.

3. Principle of Work a n d Energy for a System of Particles.

The principle of work and energy can be extended to include a system of particles isolated within an enclosed region of space as shown in Fig.8.

The arbitrary *ith* particle, having a mass m_i, is subjected to a resultant external force F_i and a resultant internal force f_i which all the other particles exert on the *ith* particle.



Inertial coordinate system

Figure 8

Apply the principle of work and energy to this and each of the other particles in the system, then since work and energy are scalar quantities, the equations can be summed algebraically:

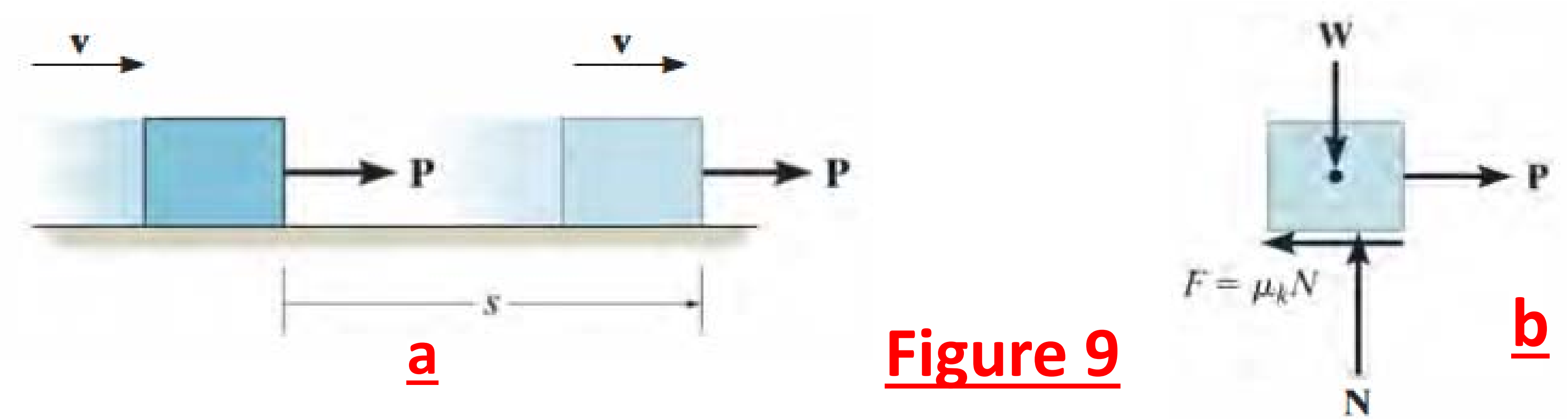
$$\Sigma T_1 + \Sigma U_{1\text{-}2} = \Sigma T_2 \qquad \text{...... (8)}$$

$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$ …… (8)

The initial kinetic energy of the system plus the work done by all the external and internal forces acting on the system is equal to the final kinetic energy of the system.

Work of Friction Caused by Sliding. These problems involve cases where a body slides over the surface of another body in the presence of friction. Consider, for example, a block which is translating a distance **s** over a rough surface as shown in **Fig.9a**. If the applied force **P** just balances the resultant frictional force $\mu_k N$, **Fig.9b**, then due to equilibrium a constant velocity v is maintained, and one would expect **Eq. 8** to be applied as follows:

$$\tfrac{1}{2}mv^2 + Ps - \mu_k N s = \tfrac{1}{2}mv^2$$



a

b

Figure 9

Ex.2: The 3500-lb automobile shown in Fig. 10a travels down the 10° inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far s the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.

Solution

This problem can b e solved using the principle of work and energy, since it involves force, velocity, and displacement.

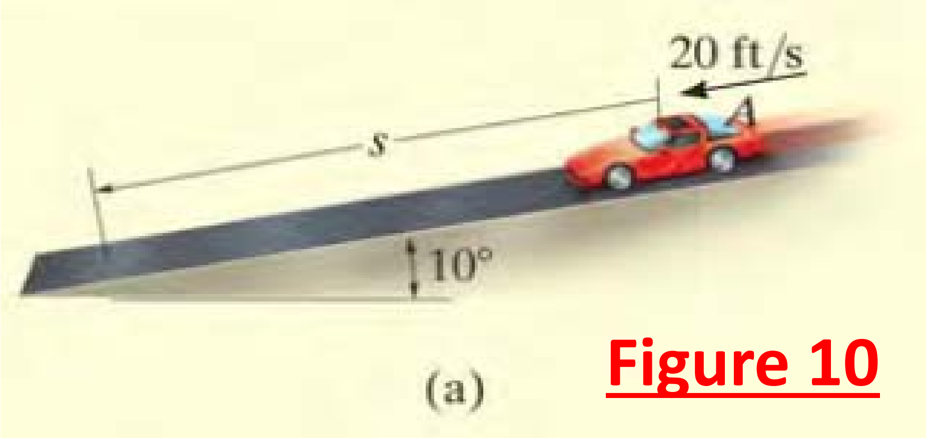


Figure 10

Work (Free-Body Diagram). As shown in Fig. 10b, the normal force N_A does no work since it never undergoes displacement along its line of action. The weight, 3500 lb, is displaced s.sin10° and does positive work. Why? The frictional force F_A does both external and internal work when it undergoes a displacement s. This work is negative since it is in the opposite sense of direction to the displacement.

Applying the equation of equilibrium normal to the road, we have:

$$+\nwarrow \Sigma F_n = 0;$$

$$N_A - 3500 \cos 10° \text{ lb} = 0$$

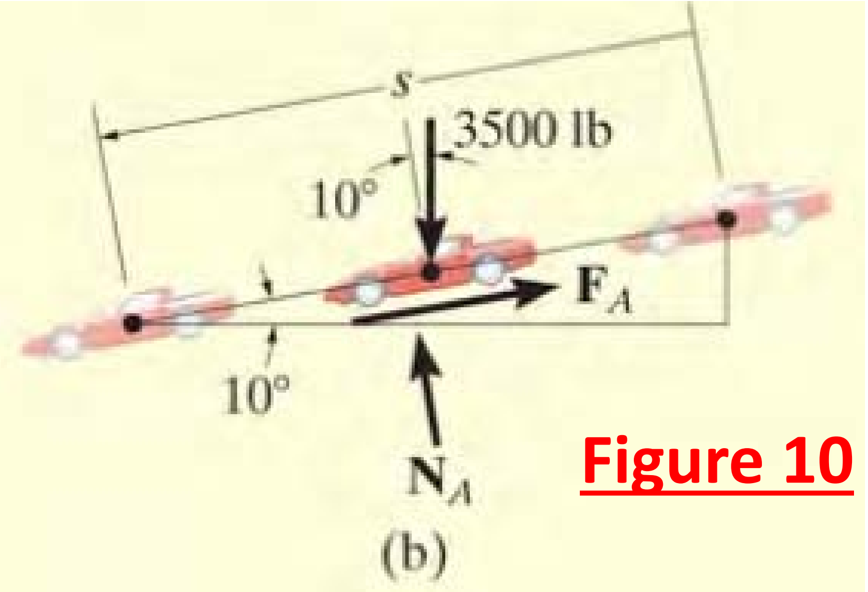


Figure 10

$$N_A = 3446.8 \text{ lb}$$ $$F_A = \mu_k N_A = 0.5\,(3446.8 \text{ lb}) = 1723.4 \text{ lb}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

$$\frac{1}{2}\left(\frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(20 \text{ ft/s})^2 + 3500 \text{ lb}(s \sin 10°) - (1723.4 \text{ lb})s = 0$$

$$s = 19.5 \text{ ft}$$ *Ans.*

H.W. Solve using equation of motion (force and acceleration0

Ex.3: For a short time the crane in Fig. 11 lifts the 2.50 Mg beam with a force of $F = (28 + 3\ s^2)$ kN. Determine the speed of the beam when it has risen $s = 3$ m. Also, how much time does it take to attain this height starting from rest?

Solution

Note that at $s = 0$,

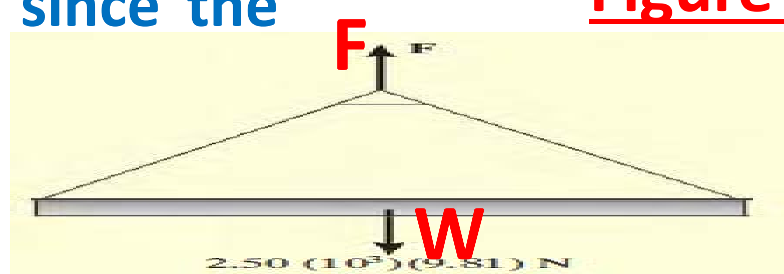
$F = 28(103)N > W = 2.50(10^3)(9.81)N$,

so motion will occur.

Work (Free-Body Diagram).

As shown on the free-body diagram, the lifting force F does positive work, which must be determined by integration since this force is a variable. Also, the weight is constant and will do negative work since the displacement is upwards.

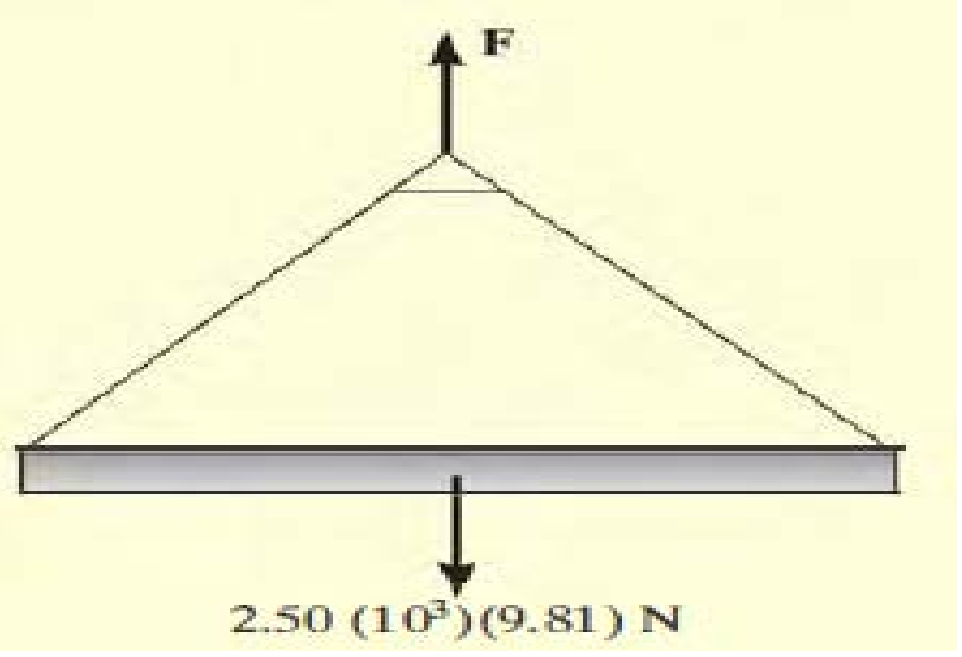
Figure 11



F.B.D

Principles of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$



$$0 + \int_0^s (28 + 3s^2)(10^3)\,ds - (2.50)(10^3)(9.81)s = \tfrac{1}{2}(2.50)(10^3)v^2$$

$$28(10^3)s + (10^3)s^3 - 24.525(10^3)s = 1.25(10^3)v^2$$

$$v = (2.78s + 0.8s^3)^{\frac{1}{2}}$$

When $s = 3$ m, $\Rightarrow$ $v = 5.47$ m/s

Kinematics.

Since we were able to express the velocity as a function of displacement, the time can be determined using v = *dsldt*. In this case,

$$v = (2.78s + 0.8s^3)^{\frac{1}{2}}$$

$$\frac{ds}{dt} = (2.78s + 0.8s^3)^{\frac{1}{2}} \quad \Rightarrow \quad t = \int_0^3 \frac{ds}{(2.78s + 0.8s^3)^{\frac{1}{2}}}$$

$$t = 1.79 \text{ s}$$

H.W. Find the acceleration :

1. By Applying the Kinematics;

2. By Using Force and acceleration.

Ex.4: The platform P, shown in Fig. 12a, has negligible mass and is tied down so that the 0.4 m long cords keep a 1 m long spring compressed 0.6 m when nothing is on the platform. If a 2 kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, Fig. 112b, determine the maximum height h the block rises in the air, measured from the ground .

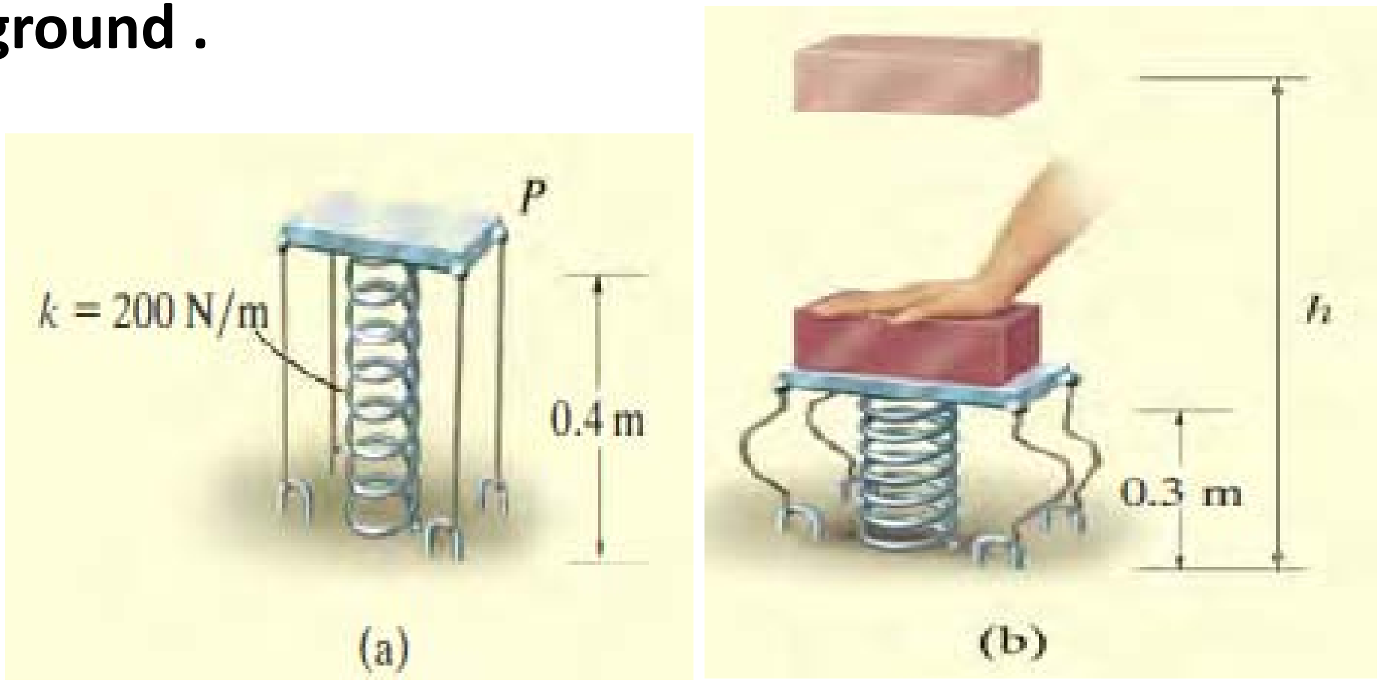


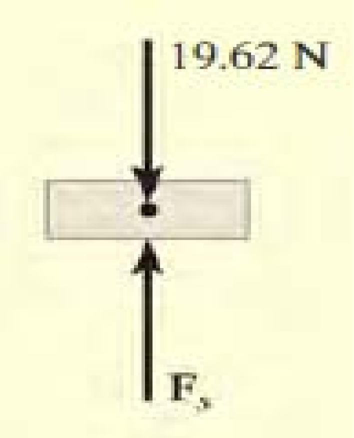
Figure 12

Solution:

Work (Free-Body Diagram).

Since the block is released from rest and later reaches its maximum height, the initial and final velocities are zero.

The free-body diagram of the block when it is still in contact with the platform is shown below



Note that the weight does negative work and the spring force does positive work.

The initial compression in the spring is

$$s_1 = 0.6\text{ m} + 0.1\text{ m} = 0.7\text{ m}.$$

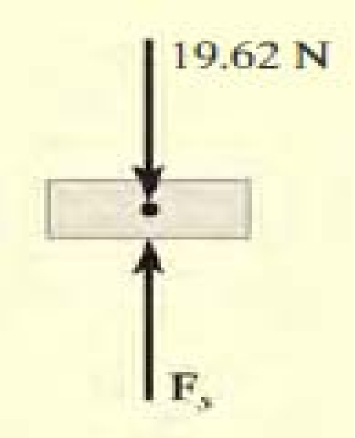
Due to the cords, the spring's final compression is

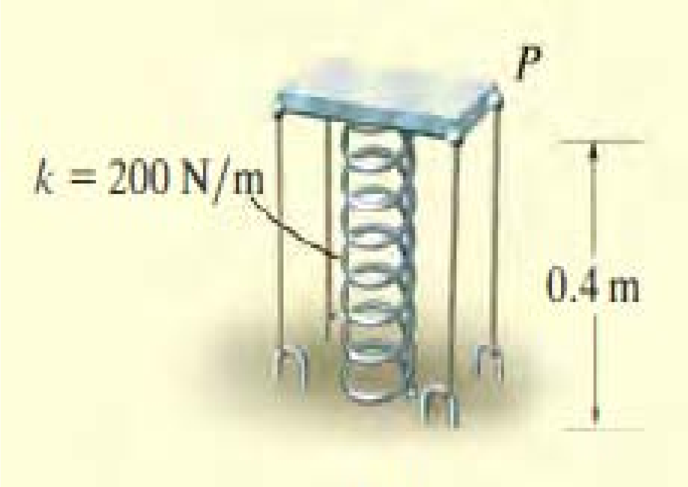
$$s_2 = 0.6\text{ m}$$ (after the block leaves the platform)

The bottom of the block rises from a height of :

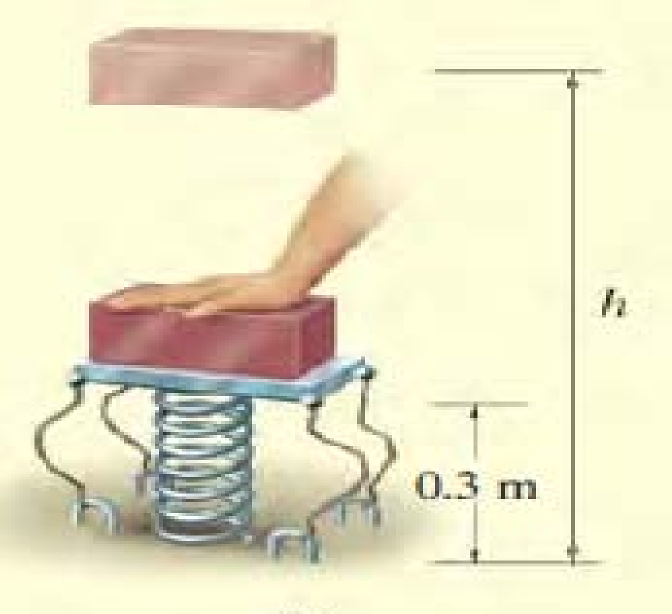
$$(0.4\text{ m} - 0.1\text{ m}) = 0.3\text{ m to a final height } h.$$

Principle of Work and Energy:





(a)



(b)

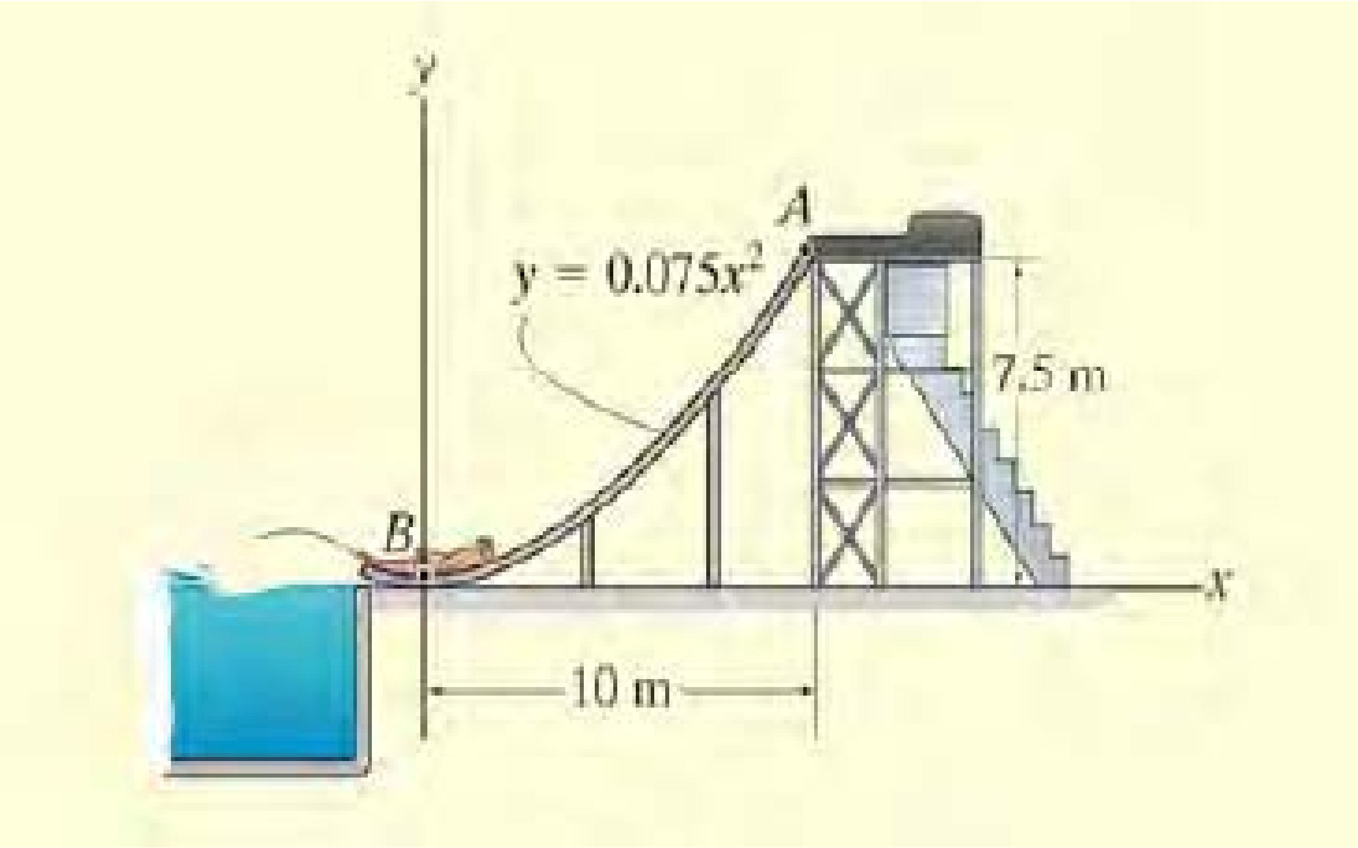
$$T_1 + \Sigma U_{1\text{-}2} = T_2$$

$$\tfrac{1}{2}mv_1^2 + \left\{-\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right) - W\,\Delta y\right\} = \tfrac{1}{2}mv_2^2$$

$$0 + \left\{-\left[\tfrac{1}{2}(200\text{ N/m})(0.6\text{ m})^2 - \tfrac{1}{2}(200\text{ N/m})(0.7\text{ m})^2\right] - (19.62\text{ N})[h - (0.3\text{ m})]\right\} = 0$$

$$h = 0.963\text{ m} \qquad \textit{Ans.}$$

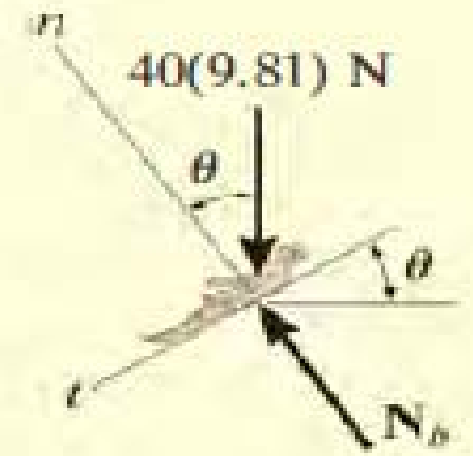
Ex.5: The **40 kg** boy in **Fig. 13** slides down the smooth water slide. If he starts from rest at **A**, determine his speed when he reaches **B** and the normal reaction the slide exerts on the boy at this position.



Solution:

Work (Free-Body Diagram).

A s shown o n the free-body diagram, there are two forces acting on the boy as he goes down the slide. Note that the normal force does no work.



Principle of Work and Energy.

$$T_A + \Sigma U_{A-B} = T_B$$

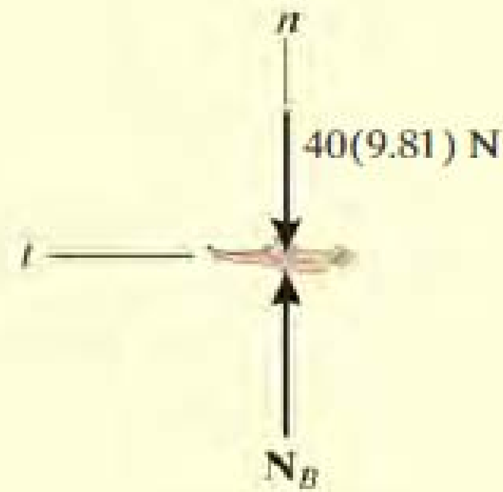
$$0 + (40(9.81)\,\text{N})(7.5\,\text{m}) = \tfrac{1}{2}(40\,\text{kg})v_B^2$$

$$v_B = 12.13\ \text{m/s} = 12.1\ \text{m/s} \qquad \textit{Ans.}$$

Equation of Motion.

Referring to the free-body diagram of the boy when he is at B, Fig. below, the normal reaction N_B can now be obtained by applying the equation of motion along the n axis. Here the radius of curvature of the path is

$$\rho_B = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{|d^2y/dx^2|} = \frac{\left[1 + (0.15x)^2\right]^{3/2}}{|0.15|}\Bigg|_{x=0} = 6.667\text{ m}$$

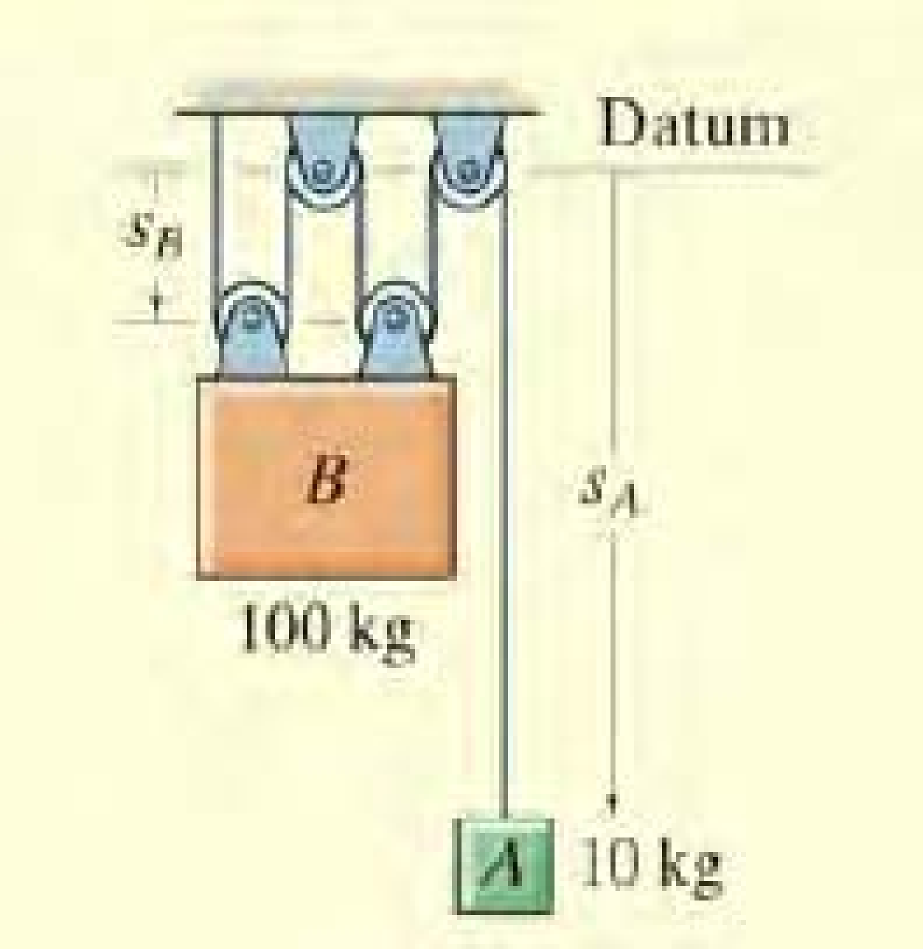


$$+\uparrow \Sigma F_n = ma_n;$$

$$N_B - 40(9.81)\text{ N} = 40\text{ kg}\left(\frac{(12.13\text{ m/s})^2}{6.667\text{ m}}\right)$$

$$N_B = 1275.3\text{ N} = 1.28\text{ kN} \qquad \textit{Ans.}$$

Ex.6: Blocks **A** and **B** shown in **Fig. 14** have a mass of **10 kg** and **100 kg**, respectively. Determine the distance B travels when it is released from rest to the point where its speed becomes **2 m/s**.



Solution:
Work (Free-Body Diagram).

As shown on the free-body diagram of the system, the cable force T and reactions R_1 and R_2 do no work, since these forces represent the reactions at the supports and consequently they do not move while the blocks are displaced. The weights both do positive work if we assume both move downward, in the positive sense of direction of S_A and S_B.

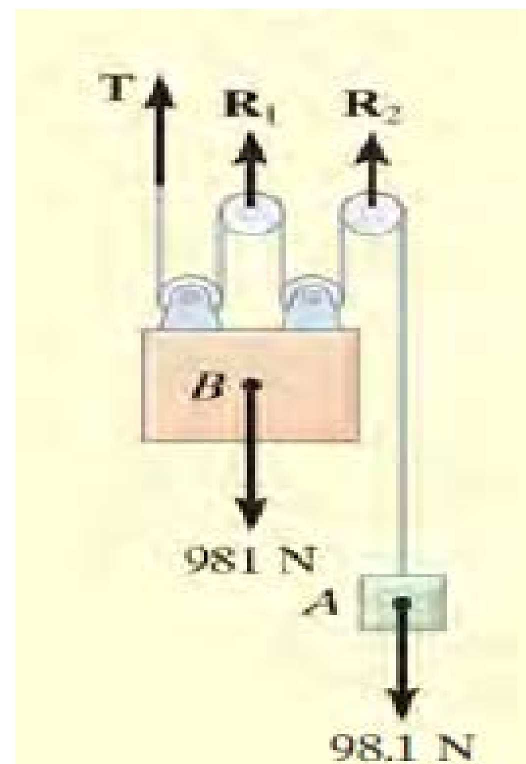
Principle of Work and Energy.

Realizing the blocks are released from rest, we have

$$\Sigma T_1 + \Sigma U_{1\text{-}2} = \Sigma T_2$$

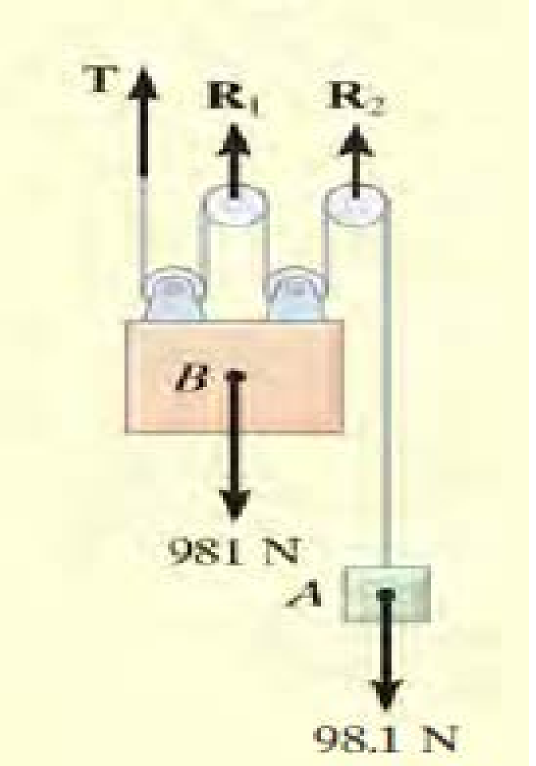
$$\left\{\tfrac{1}{2}m_A(v_A)_1^2 + \tfrac{1}{2}m_B(v_B)_1^2\right\} + \{W_A\,\Delta s_A + W_B\,\Delta s_B\} =$$

$$\left\{\tfrac{1}{2}m_A(v_A)_2^2 + \tfrac{1}{2}m_B(v_B)_2^2\right\}$$



$$\{0 + 0\} + \{98.1\ \text{N}\ (\Delta s_A) + 981\ \text{N}\ (\Delta s_B)\} =$$

$$\left\{\tfrac{1}{2}(10\ \text{kg})(v_A)_2^2 + \tfrac{1}{2}(100\ \text{kg})(2\ \text{m/s})^2\right\} \qquad (1)$$



Kinematics.

Using the methods of kinematics it may be seen from Fig. that the total length L of all the vertical segments of cable may be expressed in terms of the position coordinates S_A and S_B as

$$s_A + 4s_B = l$$

$$v_A = -4v_B$$

$$\Delta s_A + 4\,\Delta s_B = 0$$

$$\Delta s_A = -4\,\Delta s_B$$

$$\Delta s_B = 0.883\ \text{m} \downarrow$$

$$v_A = -4v_B = -4(2\ \text{m/s}) = -8\ \text{m/s}$$

4. Power and Efficiency

Power. **The term "power" provides a useful basis for choosing the type of motor or machine which is required to do a certain amount of work in a given time.**

For example, two pumps may each be able to empty a reservoir if given enough time; however, the pump having the larger power will complete the job sooner.

The power generated by a machine or engine that performs an amount of work dU within the time interval dt is therefore:

$$\boxed{P = \frac{dU}{dt}}$$

If the work dU is expressed as $dU = \mathbf{F} \cdot d\mathbf{r}$, then

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} \quad \Rightarrow \quad \boxed{P = \mathbf{F} \cdot \mathbf{v}}$$

Hence, power is a scalar, where in this formulation **v** represents the velocity of the particle which is acted upon by the force **F**.

The basic units of power used in the SI and FPS systems are the watt (W) and horsepower (hp), respectively. These units are defined as

$$1\ \mathrm{W} = 1\ \mathrm{J/s} = 1\ \mathrm{N \cdot m/s} \qquad 1\ \mathrm{hp} = 550\ \mathrm{ft \cdot lb/s}$$

For conversion between the two systems of units, 1 hp = 746 W.

Efficiency. The mechanical efficiency of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\boxed{\epsilon = \frac{\text{power output}}{\text{power input}}}$$

If energy supplied to the machine occurs during the same time interval at which it is drawn, then the efficiency may also be expressed in terms of the ratio:

$$\boxed{\epsilon = \frac{\text{energy output}}{\text{energy input}}}$$

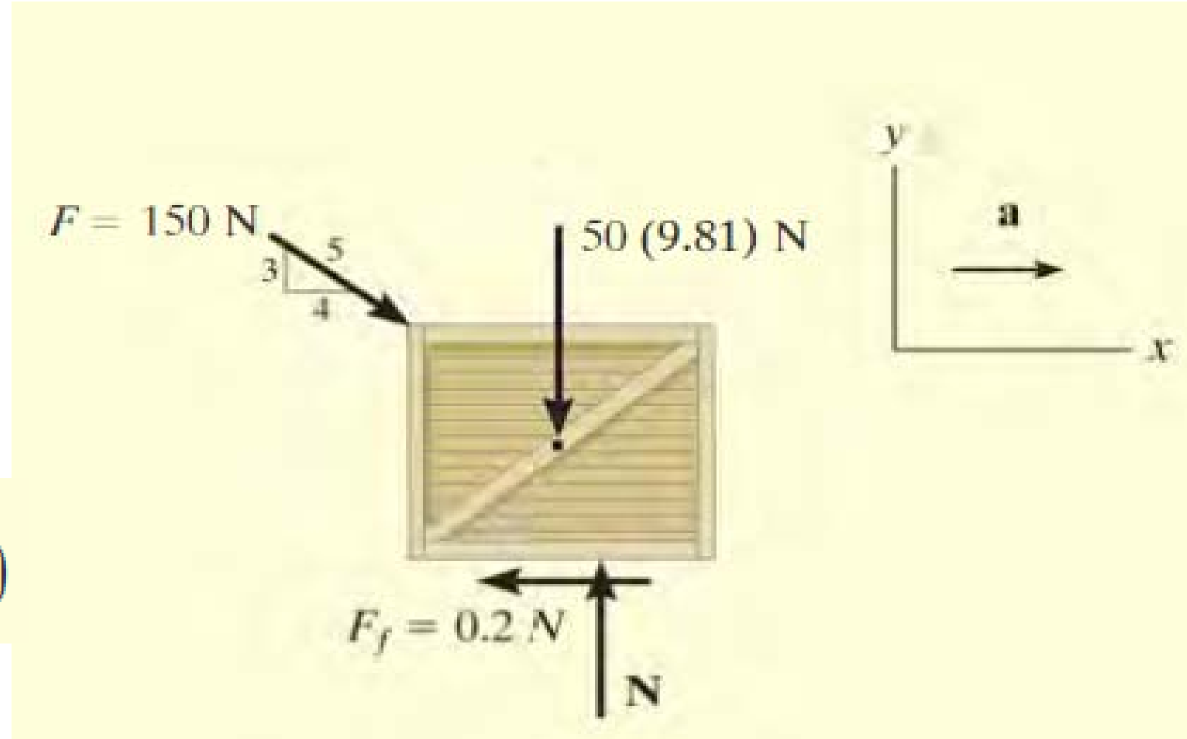
Note: Since machines consist of a series of moving parts, frictional forces will always be developed within the machine, and as a result, extra energy or power is needed to overcome these forces. Consequently, power output will be less than power input and so the efficiency of a machine is always less than 1 .

Procedure for Analysis

- First determine the external force $\mathbf{F}$ acting on the body which causes the motion. This force is usually developed by a machine or engine placed either within or external to the body.
- If the body is accelerating, it may be necessary to draw its free-body diagram and apply the equation of motion $(\Sigma\mathbf{F} = m\mathbf{a})$ to determine $\mathbf{F}$.
- Once $\mathbf{F}$ and the velocity $\mathbf{v}$ of the particle where $\mathbf{F}$ is applied have been found, the power is determined by multiplying the force magnitude with the component of velocity acting in the direction of $\mathbf{F}$, (i.e., $P = \mathbf{F}\cdot\mathbf{v} = Fv\cos\theta$).
- In some problems the power may be found by calculating the work done by $\mathbf{F}$ per unit of time ($P_{avg} = \Delta U/\Delta t$,).

Ex.7: The man in Fig. below pushes on the 50 kg crate with a force of F = 150 N. Determine the power supplied by the man when t = 4 s. The coefficient of kinetic friction between the floor and the crate is μ_k = 0.2. Initially the create is at rest .

Solution: The free-body diagram of the crate is



Applying the equation of motion,

$$+\uparrow \Sigma F_y = ma_y;$$

$$N - \left(\tfrac{3}{5}\right)150\text{ N} - 50(9.81)\text{ N} = 0$$

$$N = 580.5\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \quad \Rightarrow \quad \left(\tfrac{4}{5}\right)150\text{ N} - 0.2(580.5\text{ N}) = (50\text{ kg})a \quad \Rightarrow \quad a = 0.078\text{ m/s}^2$$

The velocity of the crate when t = 4 s is therefore

$$v = v_0 + a_c t \quad \Rightarrow \quad v = 0 + (0.078\text{ m/s}^2)(4\text{ s}) = 0.312\text{ m/s}$$

The power supplied to the crate by the man when t = 4 s is therefore

$$P = \mathbf{F} \cdot \mathbf{v} = F_x v = \left(\frac{4}{5}\right)(150\ \text{N})(0.312\ \text{m/s}) = 37.4\ \text{W}$$

Chapter Six

Impulse and Momentum

6.1 Principle of Linear Impulse and Momentum

Using kinematics, the equation of motion for a article of mass m can be written as:

$$\Sigma \mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

$$\Sigma \int_{t_1}^{t_2} \mathbf{F}dt = m\int_{\mathbf{v}_1}^{\mathbf{v}_2} d\mathbf{v} \qquad \text{or} \qquad \Sigma \int_{t_1}^{t_2} \mathbf{F}dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

This equation is referred to as the ***principle of linear impulse and momentum.***

Linear Momentum Unit

Since *m is a* positive scalar, the linear-momentum vector has the same direction as v, and its magnitude *mv has units of mass times velocity, e.g.:*

kg . m/s

Linear Impulse

The integral of force in time domain is writen:

$$\mathbf{I} = \int_{t_1}^{t_2} \mathbf{F}_c dt = \mathbf{F}_c(t_2 - t_1)$$

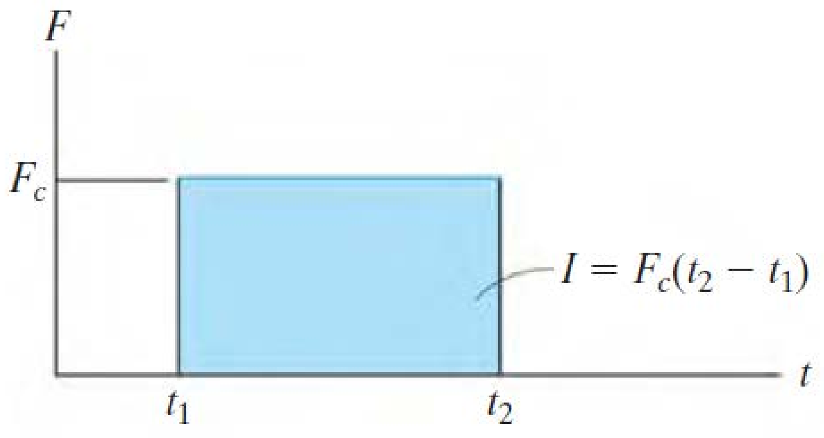
and referred to as the linear impulse. This term is a vector quantity which measures the effect of a force during the time the force acts. Since time is a positive scalar, the impulse acts in the same direction as the force, and its magnitude has units of force times time, e.g.:

N.s

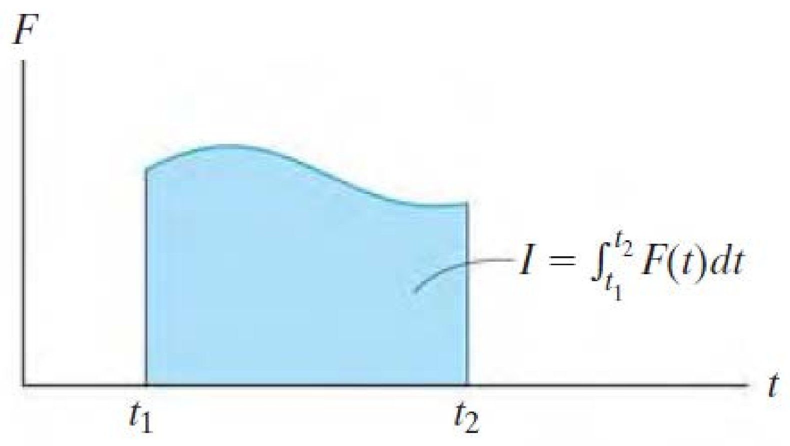
Note:

Although the units for impulse and momentum are defined differently, it can be shown is dimensionally homogeneous.

Graphical of Linear Impulse



Constant Force



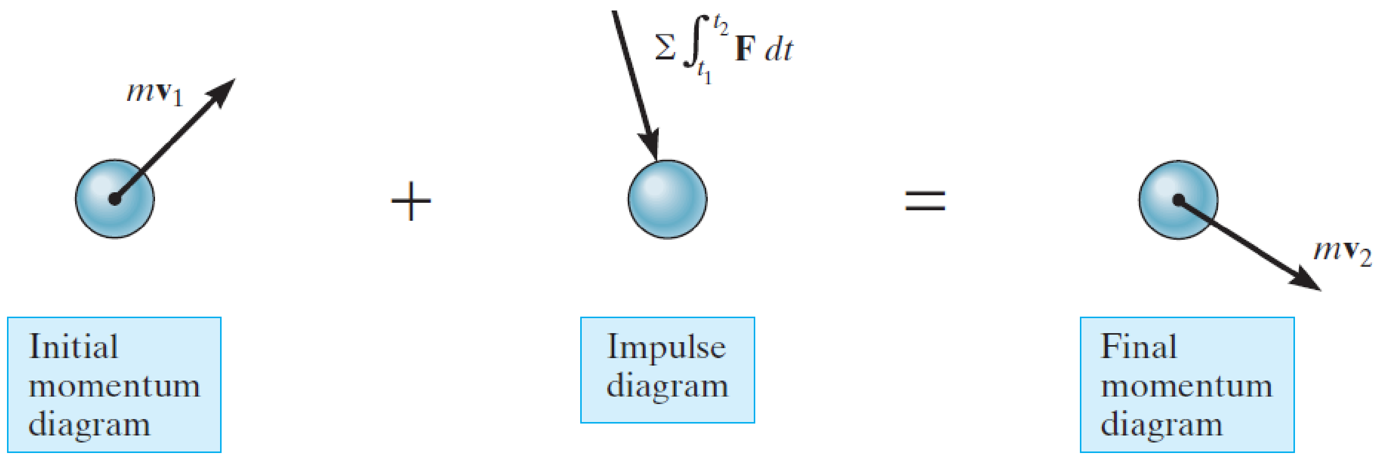
Variable Force

Principle of Linear Impulse and Momentum

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

$$m(v_z)_1 + \Sigma \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$



Principle of Linear Impulse and Momentum for a System of Particles

States that the initial linear momentum of the system plus the impulses of all the external forces acting on the system from t1 to t2 is equal to the system's final linear momentum.

$$\boxed{\Sigma m_i(\mathbf{v}_i)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}_i\,dt = \Sigma m_i(\mathbf{v}_i)_2}$$

Procedure for Analysis (1-2)

Free-Body Diagram.

- Establish the x, y, z inertial frame of reference and draw the particle's free-body diagram in order to account for all the forces that produce impulses on the particle.
- The direction and sense of the particle's initial and final velocities should be established.
- If a vector is unknown, assume that the sense of its components is in the direction of the positive inertial coordinate(s).
- As an alternative procedure, draw the impulse and momentum diagrams for the particle

Procedure for Analysis (2-2)

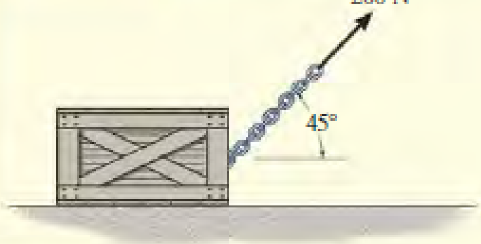
Principle of Impulse and Momentum.

- In accordance with the established coordinate system, apply the principle of linear impulse and momentum, $m\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\, dt = m\mathbf{v}_2$. If motion occurs in the x–y plane, the two scalar component equations can be formulated by either resolving the vector components of $\mathbf{F}$ from the free-body diagram, or by using the data on the impulse and momentum diagrams.

- Realize that every force acting on the particle's free-body diagram will create an impulse, even though some of these forces will do no work.

- Forces that are functions of time must be integrated to obtain the impulse. Graphically, the impulse is equal to the area under the force–time curve.

EXAMPLE 15.1



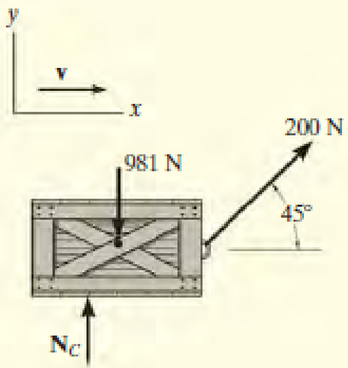
(a)

The 100-kg crate shown in Fig. 15–5*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied for 10 s, determine the final velocity and the normal force which the surface exerts on the crate during this time interval.

SOLUTION

This problem can be solved using the principle of impulse and momentum since it involves force, velocity, and time.

Free-Body Diagram. See Fig. 15–5*b*. Since all the forces acting are *constant*, the impulses are simply the product of the force magnitude and 10 s $[\mathbf{I} = \mathbf{F}_c(t_2 - t_1)]$



(b)

Fig.15–5

Principle of Impulse and Momentum.

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$0 + 200\text{ N}\cos 45°(10\text{ s}) = (100\text{ kg})v_2$$

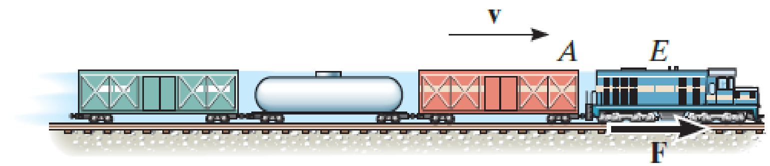
$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N}\sin 45°(10\text{ s}) = 0$$

$$N_C = 840\text{ N} \qquad \textit{Ans.}$$

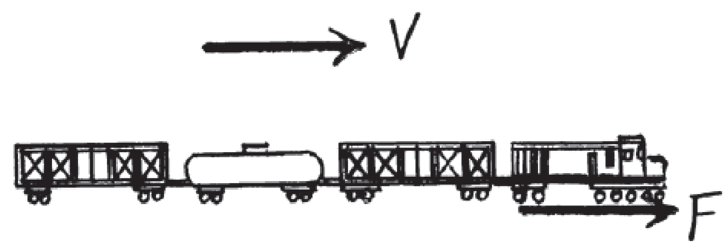
15–6.



A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.

SOLUTION

$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$



Entire train:

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

$$F = 19.4 \text{ kN}$$ **Ans.**

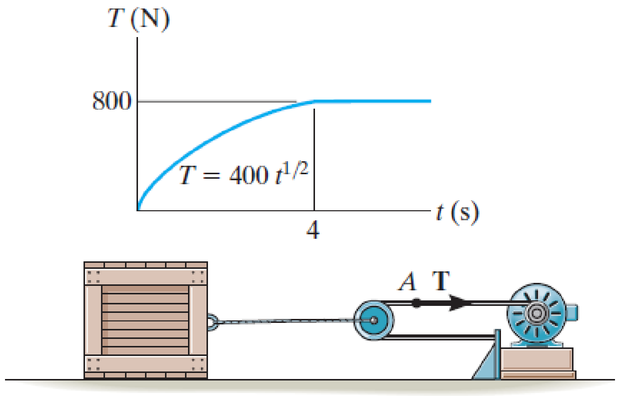
Three cars:

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN}$$ **Ans.**

15–9.

The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force $\boldsymbol{T}$ to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the crate.



SOLUTION

Equilibrium. The time required to move the crate can be determined by considering the equilibrium of the crate. Since the crate is required to be on the verge of sliding, $F_f = \mu_s N = 0.5\ N$. Referring to the FBD of the crate, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \quad N - 200(9.81) = 0 \quad N = 1962\ \text{N}$$

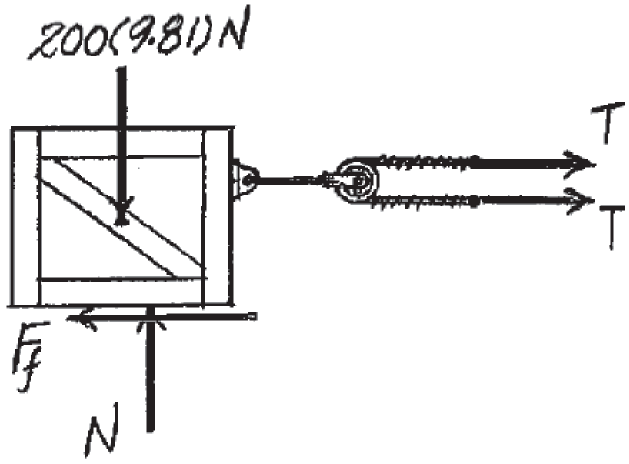
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 2\left(400t^{\frac{1}{2}}\right) - 0.5(1962) = 0 \quad t = 1.5037\ \text{s}$$

Principle of Impulse and Momentum. Since the crate is sliding, $F_f = \mu_k N = 0.4(1962) = 784.8$ N. Referring to the FBD of the crate, Fig. *a*

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$0 + 2\int_{1.5037\ \text{s}}^{4\ \text{s}} 400t^{\frac{1}{2}}\, dt - 784.8(4 - 1.5037) = 200v$$

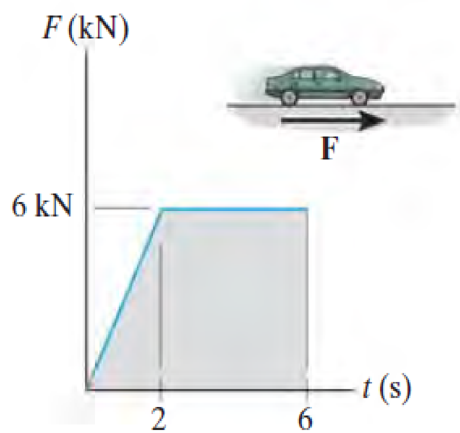
$$v = 6.621\ \text{m/s} = 6.62\ \text{m/s} \qquad \textbf{Ans.}$$



(a)

Problems

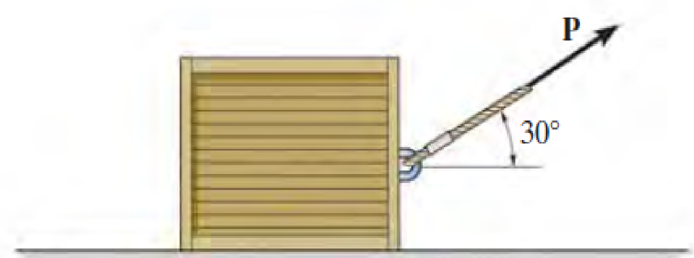
F15–4. The wheels of the 1.5-Mg car generate the traction force **F** described by the graph. If the car starts from rest, determine its speed when $t = 6$ s.



Prob. F15–4

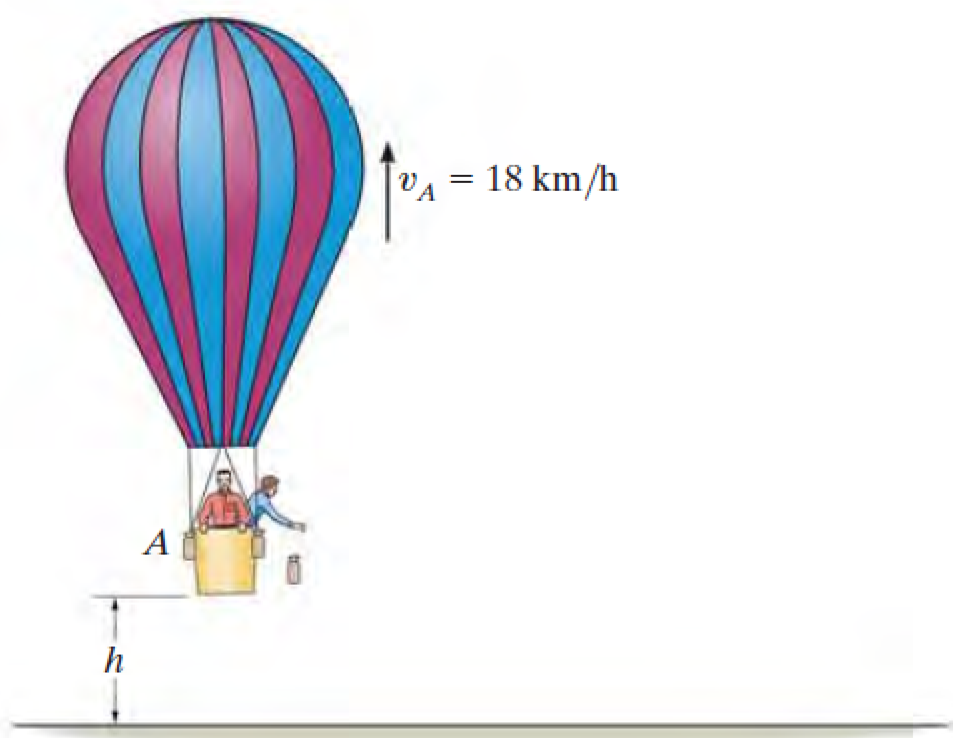
Prob. F15–1

15–10. The 50-kg crate is pulled by the constant force **P**. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of **P**. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



Prob. 15–10

15–25. The balloon has a total mass of 400 kg includ the passengers and ballast. The balloon is rising at a const velocity of 18 km/h when $h = 10$ m. If the man drops 40-kg sand bag, determine the velocity of the balloon wh the bag strikes the ground. Neglect air resistance.



6.2 Conservation of Linear Momentum

When the sum of the external impulses acting on a system of particles is zero, Linear Momentum equation reduces to a simplified form:

$$\boxed{\Sigma m_i(\mathbf{v}_i)_1 = \Sigma m_i(\mathbf{v}_i)_2}$$

This equation is referred to as the conservation of linear momentum. It states that:

The total linear momentum for a system of particles remains constant during the time period t1 to t2

Substituting $m\mathbf{v}_G = \Sigma m_i \mathbf{v}_i$ to previous Eq. gives

$$(\mathbf{v}_G)_1 = (\mathbf{v}_G)_2$$

which indicates that the velocity $(\mathbf{v}_G)$ of the mass center for the system of particles does not change if no external impulses are applied to the system

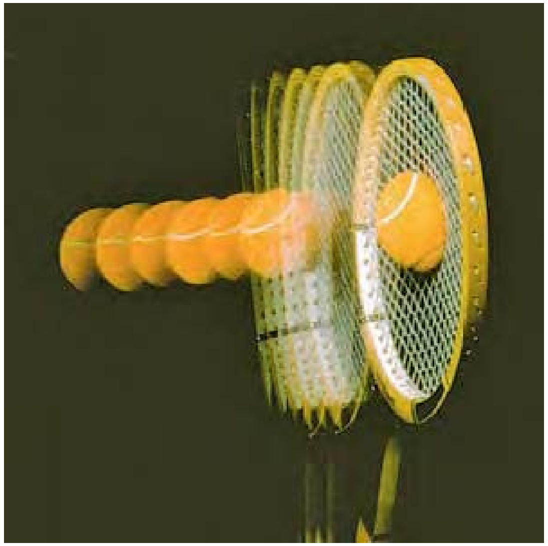
Application

The conservation of linear momentum is often applied when particles **collide or interact**. For application, a careful study of the free-body diagram for the entire system of particles should be made in order to identify the forces which create either **external or internal impulses** and thereby determine in what direction(s) linear momentum is conserved.

Non-impulsive forces are forces causing negligible impulses if the time period over which the motion is studied is very short, these external impulses may be neglected or considered approximately equal to zero such as the weight of body

Impulsive forces are forces which are very large and act for a very short period of time produce a significant change in momentum and these normally occur due to an explosion or the striking of one body against another

Example



The effect of striking a tennis ball with a racket as shown in the photo.
During the very short time of interaction, the force of the racket on the ball is impulsive since it changes the ball's momentum drastically. By comparison, the ball's weight will have a negligible effect on the change in momentum, and therefore it is nonimpulsive. Consequently, it can be neglected from an impulse–momentum analysis during this time. If an impulse–momentum analysis is considered during the much longer time of flight after the racket–ball interaction, then the impulse of the ball's weight is important since it, along with air resistance, causes the change in the momentum of the ball.

Procedure for Analysis (1-2)

Free-Body Diagram.

- Establish the x, y, z inertial frame of reference and draw the free-body diagram for each particle of the system in order to identify the internal and external forces.
- The conservation of linear momentum applies to the system in a direction which either has no external forces or the forces can be considered nonimpulsive.
- Establish the direction and sense of the particles' initial and final velocities. If the sense is unknown, assume it is along a positive inertial coordinate axis.
- As an alternative procedure, draw the impulse and momentum diagrams for each particle of the system.

Procedure for Analysis (2-2)

Momentum Equations.

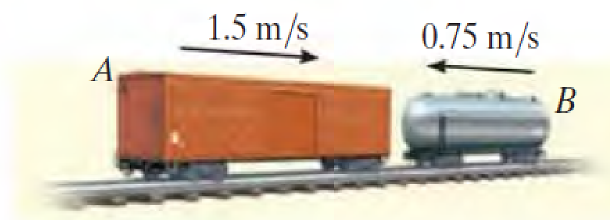
- Apply the principle of linear impulse and momentum or the conservation of linear momentum in the appropriate directions.

- If it is necessary to determine the *internal impulse* $\int F\,dt$ acting on only one particle of a system, then the particle must be *isolated* (free-body diagram), and the principle of linear impulse and momentum must be applied *to this particle*.

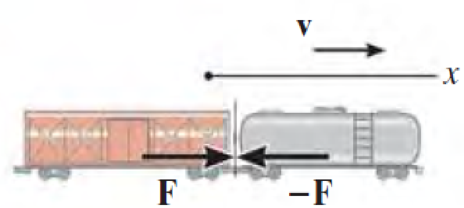
- After the impulse is calculated, and provided the time Δt for which the impulse acts is known, then the *average impulsive force* F_{avg} can be determined from $F_{avg} = \int F\,dt/\Delta t$.

Problem

The 15-Mg boxcar A is coasting at 1.5 m/s on the horizontal track when it encounters a 12-Mg tank car B coasting at 0.75 m/s toward it as shown in Fig. 15–8*a*. If the cars collide and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in 0.8 s.



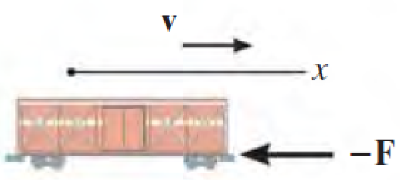
(a)



(b)

SOLUTION

Part (a) Free-Body Diagram.* Here we have considered *both* cars as a single system, Fig. 15–8*b*. By inspection, momentum is conserved in the x direction since the coupling force $\mathbf{F}$ is *internal* to the system and will therefore cancel out. It is assumed both cars, when coupled, move at $\mathbf{v}_2$ in the positive x direction.



(c)

Fig. 15–8

Conservation of Linear Momentum.

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$(15\,000 \text{ kg})(1.5 \text{ m/s}) - 12\,000 \text{ kg}(0.75 \text{ m/s}) = (27\,000 \text{ kg})v_2$$

$$v_2 = 0.5 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

Part (b). The average (impulsive) coupling force, $\mathbf{F}_{avg}$, can be determined by applying the principle of linear momentum to *either one* of the cars.

Free-Body Diagram. As shown in Fig. 15–8*c*, by isolating the boxcar the coupling force is *external* to the car.

Principle of Impulse and Momentum. Since $\int F\,dt = F_{avg}\,\Delta t = F_{avg}(0.8 \text{ s})$, we have

$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + \Sigma \int F\,dt = m_A v_2$$

$$(15\,000 \text{ kg})(1.5 \text{ m/s}) - F_{avg}(0.8 \text{ s}) = (15\,000 \text{ kg})(0.5 \text{ m/s})$$

$$F_{avg} = 18.8 \text{ kN} \qquad \textit{Ans.}$$

EXAMPLE 15.7

The 80-kg man can throw the 20-kg box horizontally at 4 m/s when standing on the ground. If instead he firmly stands in the 120-kg boat and throws the box, as shown in the photo, determine how far the boat will move in three seconds. Neglect water resistance.

(© R.C. Hibbeler)

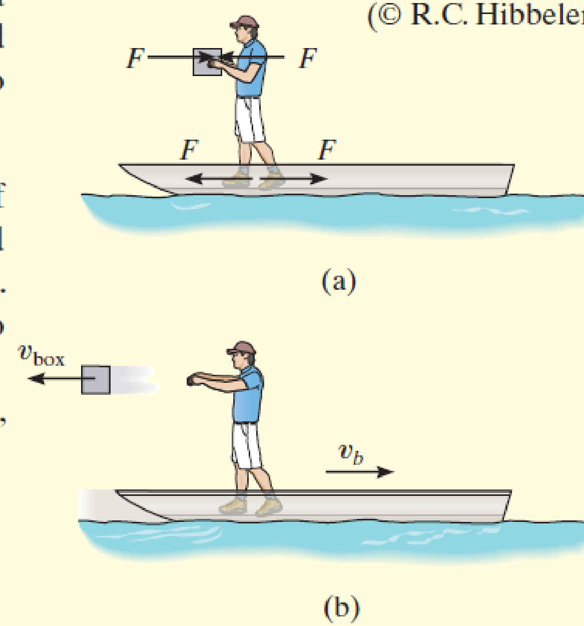


Fig. 15–11

SOLUTION

Free-Body Diagram. If the man, boat, and box are considered as a single system, the horizontal forces between the man and the boat and the man and the box become internal to the system, Fig. 15–11*a*, and so linear momentum will be conserved along the x axis.

Conservation of Momentum. When writing the conservation of momentum equation, it is *important* that the velocities be measured from the same inertial coordinate system, assumed here to be fixed. From this coordinate system, we will assume that the boat and man go to the right while the box goes to the left, as shown in Fig. 15–11*b*.

Applying the conservation of linear momentum to the man, boat, box system,

$$(\xrightarrow{+}) \qquad 0 + 0 + 0 = (m_m + m_b)\, v_b - m_{box}\, v_{box}$$

$$0 = (80 \text{ kg} + 120 \text{ kg})\, v_b - (20 \text{ kg})\, v_{box}$$

$$v_{box} = 10\, v_b \qquad (1)$$

Kinematics. Since the velocity of the box *relative* to the man (and boat), $v_{box/b}$, is known, then v_b can also be related to v_{box} using the relative velocity equation.

$$(\overset{+}{\rightarrow}) \qquad v_{box} = v_b + v_{box/b}$$

$$-v_{box} = v_b - 4 \text{ m/s} \tag{2}$$

Solving Eqs. (1) and (2),

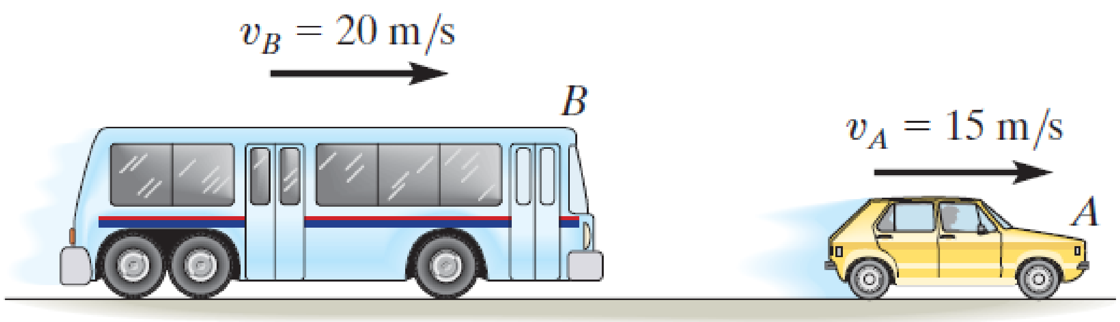
$$v_{box} = 3.64 \text{ m/s} \leftarrow$$

$$v_b = 0.3636 \text{ m/s} \rightarrow$$

The displacement of the boat in three seconds is therefore

$$s_b = v_b t = (0.3636 \text{ m/s})(3 \text{ s}) = 1.09 \text{ m} \qquad \textit{Ans.}$$

15–35. The 5-Mg bus B is traveling to the right at 20 m/s. Meanwhile a 2-Mg car A is traveling at 15 m/s to the right. If the vehicles crash and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.



SOLUTION

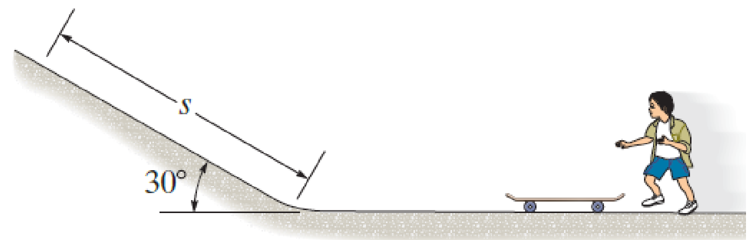
Conservation of Linear Momentum.

$$(\xrightarrow{+}) \qquad m_A v_A + m_B v_B = (m_A + m_B)v$$

$$\left[5(10^3)\right](20) + \left[2(10^3)\right](15) = \left[5(10^3) + 2(10^3)\right]v$$

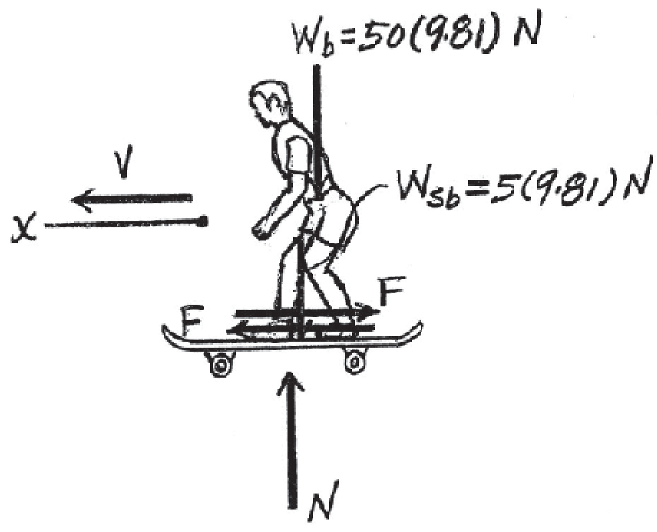
$$v = 18.57 \text{ m/s} = 18.6 \text{ m/s} \rightarrow$$

***15–36.** The 50-kg boy jumps on the 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance s the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard's rolling resistance.

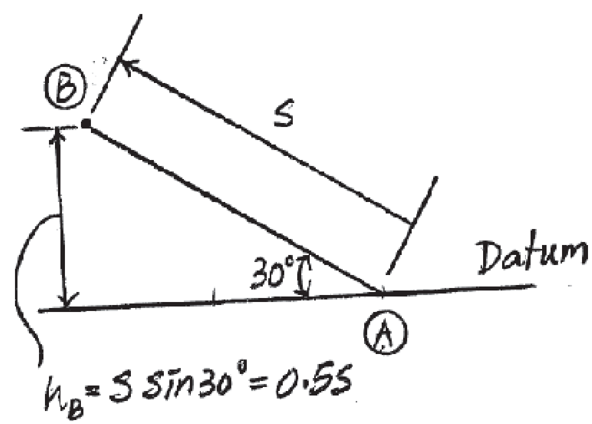


SOLUTION

Free-Body Diagram:



(a)



(b)

Conservation of Linear Momentum: Since the resultant of the impulsive force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m_b(v_b)_1 + m_{sb}(v_{sb})_1 = (m_b + m_{sb})v$$

$$50(5) + 5(0) = (50 + 5)v$$

$$v = 4.545 \text{ m/s}$$

Conservation of Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the boy and skateboard at positions A and B are $\left(V_g\right)_A = (m_b + m_{sb})gh_A = 0$ and $\left(V_g\right)_B = (m_b + m_{sb})gh_B = (50 + 5)(9.81)(s \sin 30^\circ) = 269.775s$.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(m_b + m_{sb}){v_A}^2 + \left(V_g\right)_A = \frac{1}{2}(m_b + m_{sb}){v_B}^2 + \left(V_g\right)_B$$

$$\frac{1}{2}(50 + 5)\left(4.545^2\right) + 0 = 0 + 269.775s$$

$$s = 2.11 \text{ m} \qquad \textbf{Ans.}$$

15–38. A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

SOLUTION

$(\overset{+}{\rightarrow})\quad \Sigma mv_1 = \Sigma mv_2 \qquad 15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$

$$v_2 = 0.5 \text{ m/s}$$

Ans.

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

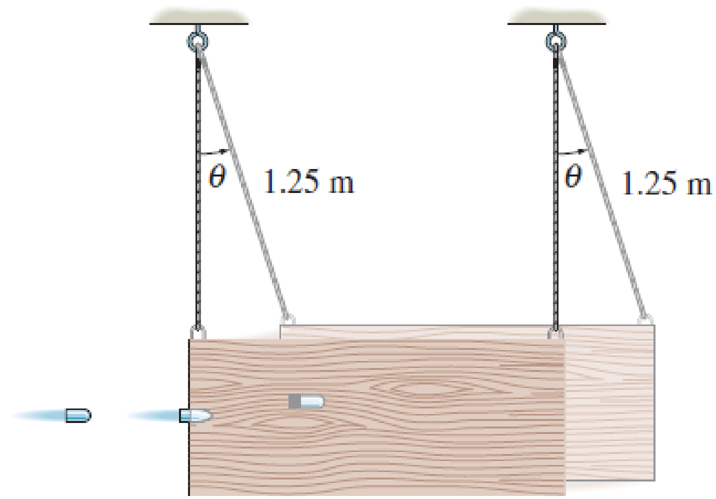
Ans.

$$\Delta T = T_2 - T_1 \quad = 3.375 - 20.25 = -16.9 \text{ kJ}$$

This energy is dissipated as noise, shock, and heat during the coupling.

15–39.

A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0°$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6°$. Estimate the speed of the bullet.



SOLUTION

Just after impact:

Datum at lowest point.

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(4 + 0.002)(v_B)_2^2 + 0 = 0 + (4 + 0.002)(9.81)(1.25)(1 - \cos 6°)$$

$$(v_B)_2 = 0.3665 \text{ m/s}$$

For the system of bullet and block:

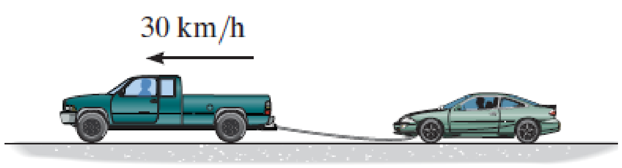
$$(\overset{+}{\rightarrow}) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$0.002(v_B)_1 = (4 + 0.002)(0.3665) \qquad (v_B)_1 = 733 \text{ m/s}$$ **Ans.**

Problems

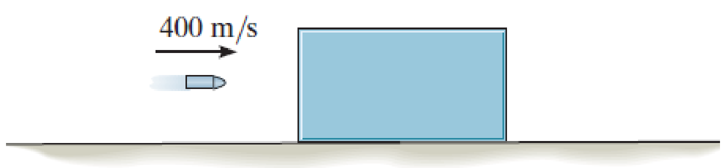
5–37.

Γhe 2.5-Mg pickup truck is towing the 1.5-Mg car using a :able as shown. If the car is initially at rest and the truck is :oasting with a velocity of 30 km/h when the cable is slack, letermine the common velocity of the truck and the car just ıfter the cable becomes taut. Also, find the loss of energy.



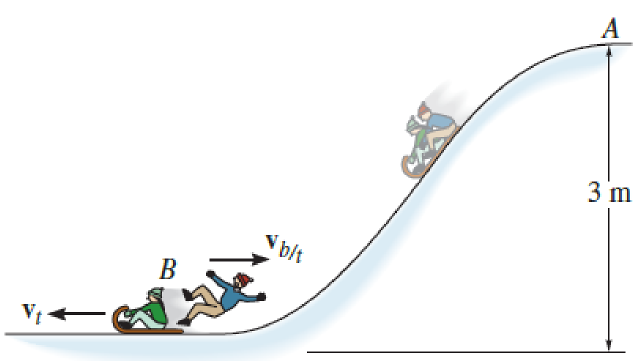
15–43.

The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



***15–44.**

A toboggan having a mass of 10 kg starts from rest at A and carries a girl and boy having a mass of 40 kg and 45 kg, respectively. When the toboggan reaches the bottom of the slope at B, the boy is pushed off from the back with a horizontal velocity of $v_{b/t} = 2$ m/s, measured relative to the toboggan. Determine the velocity of the toboggan afterwards. Neglect friction in the calculation.



6.3 Impact

Impact occurs when two bodies collide with each other during a very short period of time, causing relatively large (impulsive) forces to be exerted between the bodies. The striking of a hammer on a nail, or a golf club on a ball, are common examples of impact loadings.

line of impact

A line passing through the mass centers of the particles which is perpendicular to the plane of contact.

Types of Impact

Central impact and oblique impact

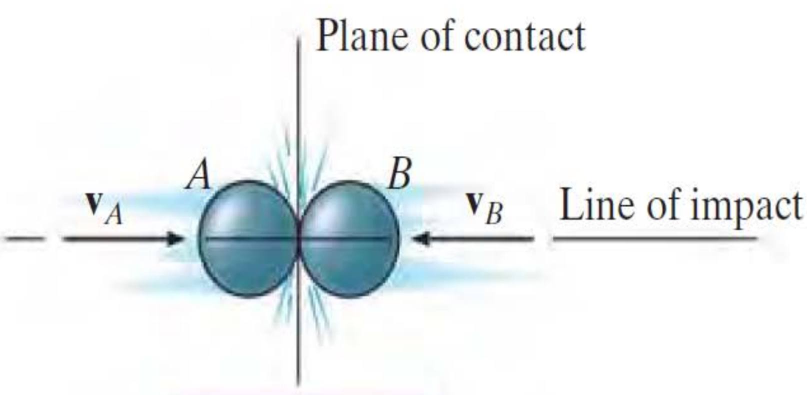
Types of Impact

1-Central impact

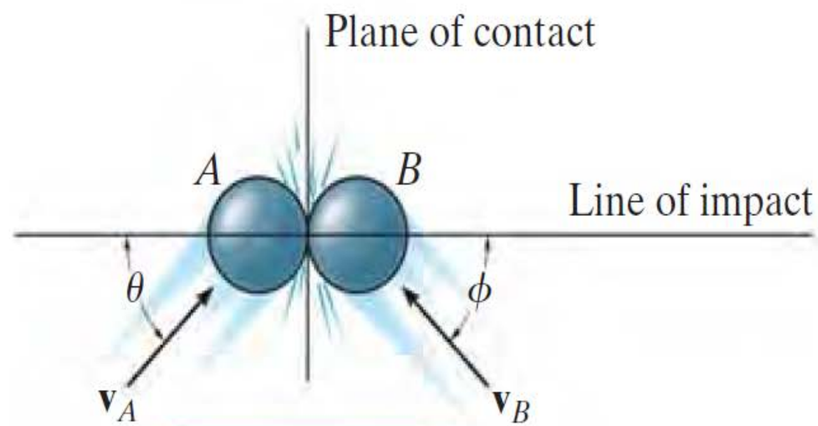
Occurs when the direction of motion of the mass centers of the two colliding particles is along a line passing through the mass centers of the particles.

2-oblique impact

Occurs when the motion of one or both of the particles make an angle with the line of impact.

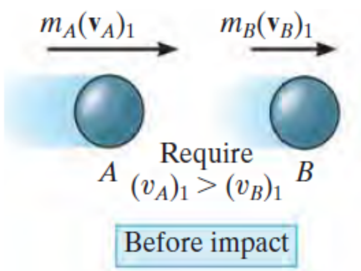


Central impact



Oblique impact

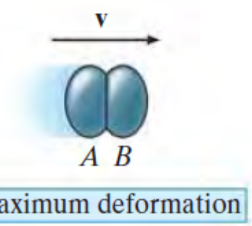
Central impact Analysis 1-3



(a)

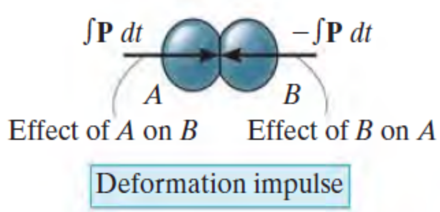
The particles have the initial momen

Provided $(v_A)_1 > (v_B)_1$



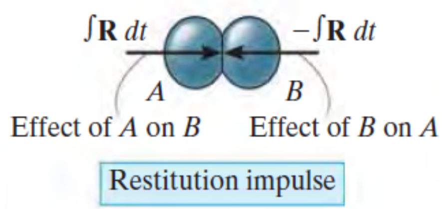
(c)

Only at the instant of *maximum deformation* will both particles move with a common velocity $\mathbf{v}$, since their relative motion is zero.



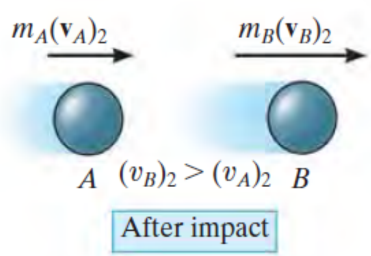
(b)

During the collision the particles must be thought of as *deformable* or nonrigid. that they exert an equal but opposite deformation impulse $\int \mathbf{P}\, dt$ on each other.



(d)

Afterward a *period of restitution* occurs, in which case the particles will either return to their original shape or remain permanently deformed. The equal but opposite *restitution impulse* $\int \mathbf{R}\, dt$ pushes the particles apart from one another



(e)

Just after separation the particles will have the final momenta where $(v_B)_2 > (v_A)_2$.

Central impact Analyses 2-3

1- momentum for the system of particles is conserved before and after impact:

$$(\xrightarrow{+}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

2- Apply the principle of impulse and momentum to each particle such that during the deformation phase for particle A and for the restitution phase, the following two equation are obtained:

$$(\xrightarrow{+}) \qquad m_A(v_A)_1 - \int P\,dt = m_A v$$

$$(\xrightarrow{+}) \qquad m_A v - \int R\,dt = m_A(v_A)_2$$

3- The ratio of the restitution impulse to the deformation impulse is called the coefficient of restitution, e. From the above equations, this value for particle A is:

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{v - (v_A)_2}{(v_A)_1 - v}$$

Central impact Analyses 3-3

4- In a similar manner, we can establish e by considering particle B:

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{(v_B)_2 - v}{v - (v_B)_1}$$

5- If the unknown v is eliminated from the above two equations, the coefficient of restitution can be expressed in terms of the particles' initial and final velocities as

$$(\xrightarrow{+}) \qquad \boxed{e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}}$$

Coefficient of Restitution

is equal to the ratio of the relative velocity of the particles' separation just after impact, (vB) 2 - (vA) 2 , to the relative velocity of the particles' approach just before impact, (vA)1 - (vB)1 .

Elastic Impact ($e = 1$). If the collision between the two particles is *perfectly elastic*, the deformation impulse $\left(\int \mathbf{P}\, dt\right)$ is equal and opposite to the restitution impulse $\left(\int \mathbf{R}\, dt\right)$. Although in reality this can never be achieved, $e = 1$ for an elastic collision.

Plastic Impact ($e = 0$). The impact is said to be *inelastic or plastic* when $e = 0$. In this case there is no restitution impulse $\left(\int \mathbf{R}\, dt = \mathbf{0}\right)$, so that after collision both particles couple or stick *together* and move with a common velocity.

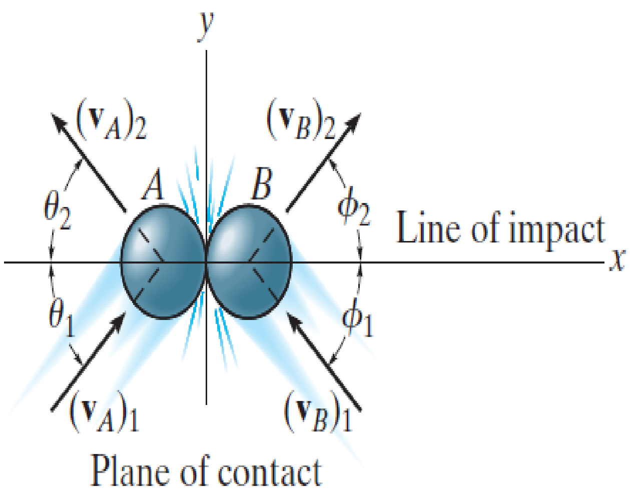
Procedure for Analysis (Central Impact)

In most cases the *final velocities* of two smooth particles are to be determined *just after* they are subjected to direct central impact. Provided the coefficient of restitution, the mass of each particle, and each particle's initial velocity *just before* impact are known, the solution to this problem can be obtained using the following two equations:

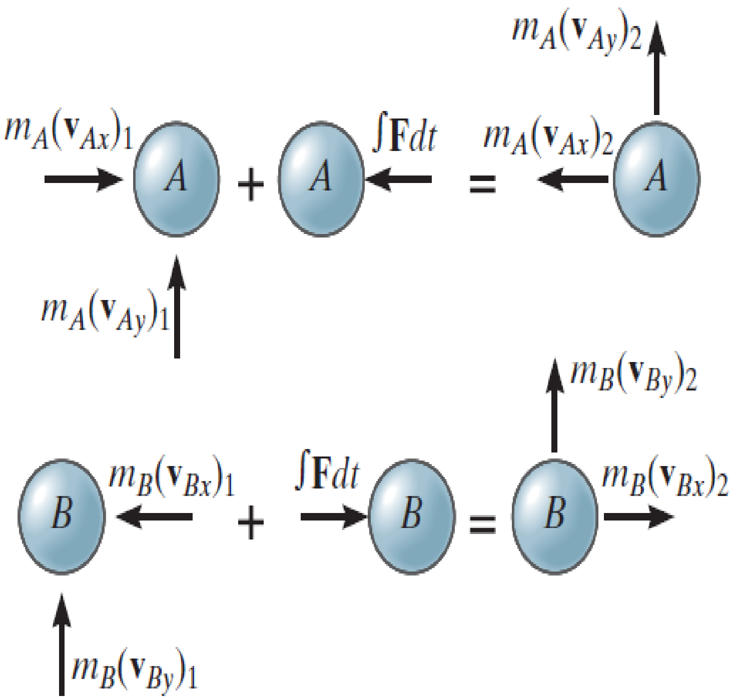
- The conservation of momentum applies to the system of particles, $\Sigma m v_1 = \Sigma m v_2$.
- The coefficient of restitution, $e = [(v_B)_2 - (v_A)_2]/[(v_A)_1 - (v_B)_1]$, relates the relative velocities of the particles along the line of impact, just before and just after collision.

When applying these two equations, the sense of an unknown velocity can be assumed. If the solution yields a negative magnitude, the velocity acts in the opposite sense.

Oblique impact Analysis 1-2



(a)



(b)

Oblique impact Analysis 2-2

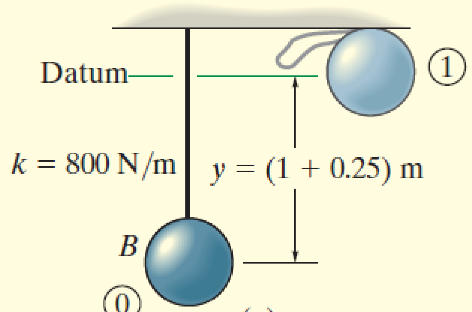
Procedure for Analysis (Oblique Impact)

If the y axis is established within the plane of contact and the x axis along the line of impact, the impulsive forces of deformation and restitution act *only in the x direction*, Fig. b, . By resolving the velocity or momentum vectors into components along the x and y axes, Fig. b, it is then possible to write four independent scalar equations in order to determine $(v_{Ax})_2$, $(v_{Ay})_2$, $(v_{Bx})_2$, and $(v_{By})_2$.

- Momentum of the system is conserved *along the line of impact, x axis*, so that $\Sigma m(v_x)_1 = \Sigma m(v_x)_2$.
- The coefficient of restitution, $e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$, relates the relative-velocity *components* of the particles *along the line of impact* (x axis).
- If these two equations are solved simultaneously, we obtain $(v_{Ax})_2$ and $(v_{Bx})_2$.
- Momentum of particle A is conserved along the y axis, perpendicular to the line of impact, since no impulse acts on particle A in this direction. As a result $m_A(v_{Ay})_1 = m_A(v_{Ay})_2$ or $(v_{Ay})_1 = (v_{Ay})_2$
- Momentum of particle B is conserved along the y axis, perpendicular to the line of impact, since no impulse acts on particle B in this direction. Consequently $(v_{By})_1 = (v_{By})_2$.

EXAMPLE 15.10

Ball B shown in Fig. 15–17a has a mass of 1.5 kg and is suspended from the ceiling by a 1-m-long elastic cord. If the cord is *stretched* downward 0.25 m and the ball is released from rest, determine how far the cord stretches after the ball rebounds from the ceiling. The stiffness of the cord is $k = 800$ N/m, and the coefficient of restitution between the ball and ceiling is $e = 0.8$. The ball makes a central impact with the ceiling.



(a)

SOLUTION

First we must obtain the velocity of the ball *just before* it strikes the ceiling using energy methods, then consider the impulse and momentum between the ball and ceiling, and finally again use energy methods to determine the stretch in the cord.

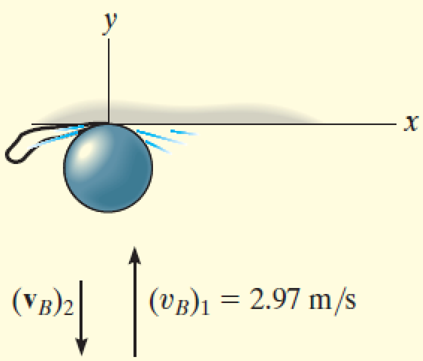
Conservation of Energy. With the datum located as shown in Fig. 15–17a, realizing that initially $y = y_0 = (1 + 0.25)$ m $= 1.25$ m, we have

$$T_0 + V_0 = T_1 + V_1$$

$$\tfrac{1}{2}m(v_B)_0^2 - W_B y_0 + \tfrac{1}{2}ks^2 = \tfrac{1}{2}m(v_B)_1^2 + 0$$

$$0 - 1.5(9.81)\text{N}(1.25\text{ m}) + \tfrac{1}{2}(800\text{ N/m})(0.25\text{ m})^2 = \tfrac{1}{2}(1.5\text{ kg})(v_B)_1^2$$

$$(v_B)_1 = 2.968\text{ m/s}\uparrow$$



(b)

The interaction of the ball with the ceiling will now be considered using the principles of impact.* Since an unknown portion of the mass of the ceiling is involved in the impact, the conservation of momentum for the ball–ceiling system will not be written. The "velocity" of this portion of ceiling is zero since it (or the earth) are assumed to remain at rest *both* before and after impact.

Coefficient of Restitution. Fig. 15–17*b*.

$$(+\uparrow) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.8 = \frac{(v_B)_2 - 0}{0 - 2.968 \text{ m/s}}$$

$$(v_B)_2 = -2.374 \text{ m/s} = 2.374 \text{ m/s} \downarrow$$

Conservation of Energy. The maximum stretch s_3 in the cord can be determined by again applying the conservation of energy equation to the ball just after collision. Assuming that $y = y_3 = (1 + s_3)$ m, Fig. 15–17*c*, then

$$T_2 + V_2 = T_3 + V_3$$

$$\tfrac{1}{2}m(v_B)_2^2 + 0 = \tfrac{1}{2}m(v_B)_3^2 - W_B y_3 + \tfrac{1}{2}ks_3^2$$

$$\tfrac{1}{2}(1.5 \text{ kg})(2.37 \text{ m/s})^2 = 0 - 9.81(1.5) \text{ N}(1 \text{ m} + s_3) + \tfrac{1}{2}(800 \text{ N/m})s_3^2$$

$$400s_3^2 - 14.715s_3 - 18.94 = 0$$

Solving this quadratic equation for the positive root yields

$$s_3 = 0.237 \text{ m} = 237 \text{ mm} \qquad \textit{Ans.}$$

*The weight of the ball is considered a nonimpulsive force.

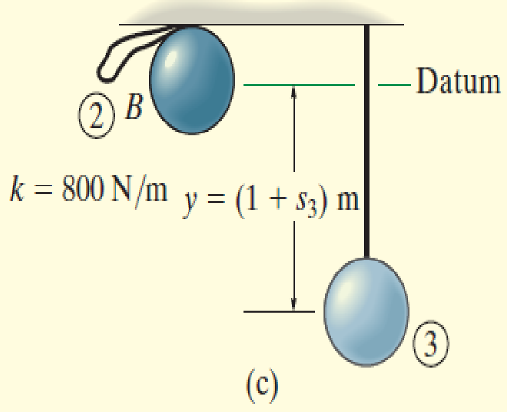
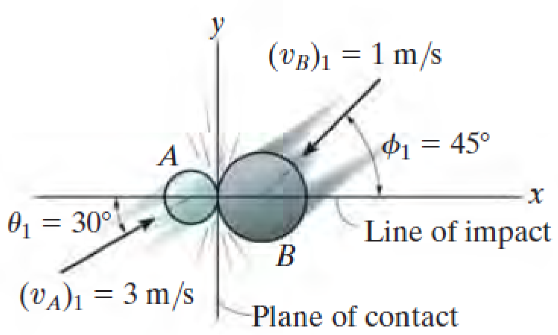
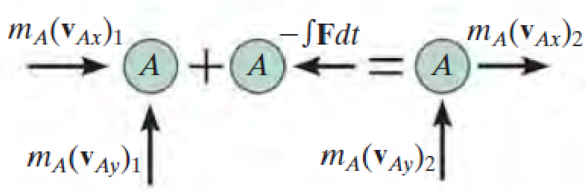


Fig. 15–17

Example 15.11



(a)



Two smooth disks A and B, having a mass of 1 kg and 2 kg, respectively, collide with the velocities shown in Fig. 15–18*a*. If the coefficient of restitution for the disks is $e = 0.75$, determine the x and y components of the final velocity of each disk just after collision.

SOLUTION

This problem involves *oblique impact*. Why? In order to solve it, we have established the x and y axes along the line of impact and the plane of contact, respectively, Fig. 15–18*a*.

Resolving each of the initial velocities into x and y components, we have

$$(v_{Ax})_1 = 3 \cos 30° = 2.598 \text{ m/s} \qquad (v_{Ay})_1 = 3 \sin 30° = 1.50 \text{ m/s}$$

$$(v_{Bx})_1 = -1 \cos 45° = -0.7071 \text{ m/s} \qquad (v_{By})_1 = -1 \sin 45° = -0.7071 \text{ m/s}$$

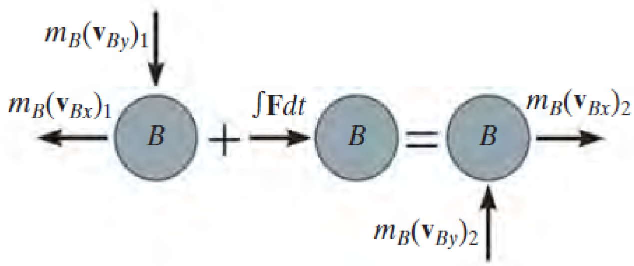
The four unknown velocity components after collision are *assumed to act in the positive directions*, Fig. 15–18*b*. Since the impact occurs in the x direction (line of impact), the conservation of momentum for *both* disks can be applied in this direction. Why?

Conservation of "x" Momentum. In reference to the momentum diagrams, we have

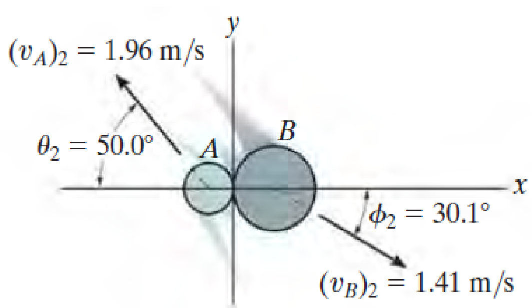
$$(\overset{+}{\rightarrow}) \qquad m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$

$$1 \text{ kg}(2.598 \text{ m/s}) + 2 \text{ kg}(-0.707 \text{ m/s}) = 1 \text{ kg}(v_{Ax})_2 + 2 \text{ kg}(v_{Bx})_2$$

$$(v_{Ax})_2 + 2(v_{Bx})_2 = 1.184 \qquad (1)$$



(b)



(c)

Fig. 15–18

Coefficient of Restitution (*x*).

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1}; \; 0.75 = \frac{(v_{Bx})_2 - (v_{Ax})_2}{2.598 \text{ m/s} - (-0.7071 \text{ m/s})}$$

$$(v_{Bx})_2 - (v_{Ax})_2 = 2.482 \qquad (2)$$

Solving Eqs. 1 and 2 for $(v_{Ax})_2$ and $(v_{Bx})_2$ yields

$(v_{Ax})_2 = -1.26 \text{ m/s} = 1.26 \text{ m/s} \leftarrow \quad (v_{Bx})_2 = 1.22 \text{ m/s} \rightarrow$ *Ans.*

Conservation of "*y*" Momentum. The momentum of *each disk* is *conserved* in the *y* direction (plane of contact), since the disks are smooth and therefore *no* external impulse acts in this direction. From Fig. 15–18*b*,

$(+\uparrow)\; m_A(v_{Ay})_1 = m_A(v_{Ay})_2; \quad (v_{Ay})_2 = 1.50 \text{ m/s} \uparrow$ *Ans.*

$(+\uparrow)\; m_B(v_{By})_1 = m_B(v_{By})_2; \quad (v_{By})_2 = -0.707 \text{ m/s} = 0.707 \text{ m/s} \downarrow$ *Ans.*

NOTE: Show that when the velocity components are summed vectorially, one obtains the results shown in Fig. 15–18*c*.

15–58. Disk A has a mass of 250 g and is sliding on a *smooth* horizontal surface with an initial velocity $(v_A)_1 = 2\text{ m/s}$. It makes a direct collision with disk B, which has a mass of 175 g and is originally at rest. If both disks are of the same size and the collision is perfectly elastic ($e = 1$), determine the velocity of each disk just after collision. Show that the kinetic energy of the disks before and after collision is the same.

SOLUTION

$$(\overset{+}{\rightarrow}) \qquad (0.250)(2) + 0 = (0.250)(v_A)_2 + (0.175)(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad e = 1 = \frac{(v_B)_2 - (v_A)_2}{2 - 0} \qquad \text{Solving}$$

$$(v_A)_2 = 0.353\text{ m/s} \qquad (v_B)_2 = 2.35\text{ m/s}$$ **Ans.**

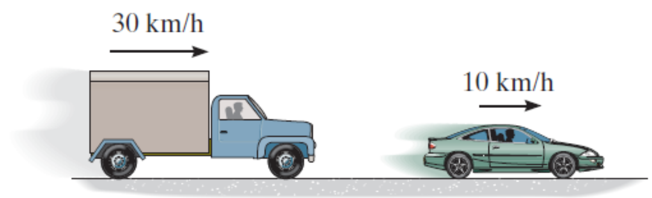
$$T_1 = \frac{1}{2}(0.25)(2)^2 = 0.5\text{ J}$$ **Ans.**

$$T_2 = \frac{1}{2}(0.25)(0.353)^2 + \frac{1}{2}(0.175)(2.35)^2 = 0.5\text{ J}$$ **Ans.**

$$T_1 = T_2$$ **QED**

15–59.

The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.

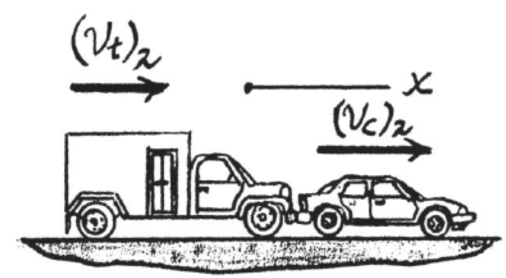


SOLUTION

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are $(v_t)_1 = \left[30\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 8.333\text{ m/s}$ and $(v_c)_1 = \left[10\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\ h}{3600\text{ s}}\right) = 2.778\text{ m/s}$.

By referring to Fig. a,



$$(\overset{+}{\rightarrow}) \qquad m_t(v_t)_1 + m_c(v_c)_1 = m_t(v_t)_2 + m_c(v_c)_2$$

$$5000(8.333) + 2000(2.778) = 5000(v_t)_2 + 2000(v_c)_2$$

$$5(v_t)_2 + 2(v_c)_2 = 47.22 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_{c/t}) = \left[15\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 4.167\text{ m/s} \rightarrow$.

Applying the relative velocity equation,

$$(\mathbf{v}_c)_2 = (\mathbf{v}_t)_2 + (\mathbf{v}_{c/t})_2$$

$$(\overset{+}{\rightarrow}) \qquad (v_c)_2 = (v_t)_2 + 4.167$$

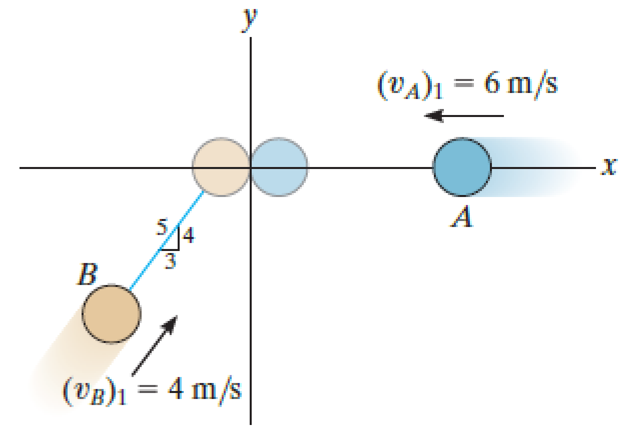
$$(v_c)_2 - (v_t)_2 = 4.167 \qquad \textbf{(2)}$$

Applying the coefficient of restitution equation,

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_1 - (v_c)_1}$$

$$e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \qquad \textbf{(3)}$$

15–73. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.



SOLUTION

$(\overset{+}{\rightarrow})$ $$\Sigma m v_1 = \Sigma m v_2$$

$$0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$(\overset{+}{\rightarrow})$ $$e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1} \qquad 0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4\left(\frac{3}{5}\right) - (-6)}$$

$$(v_A)_{2x} = 1.35 \text{ m/s} \rightarrow \qquad (v_B)_{2x} = 4.95 \text{ m/s} \leftarrow$$

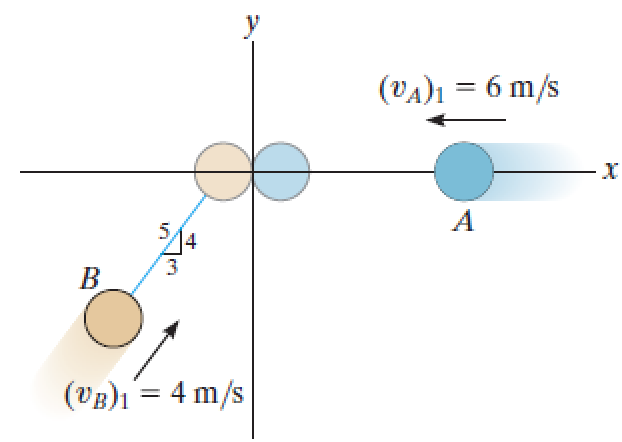
$(+\uparrow)$ $$m v_1 = m v_2$$

$$0.5\left(\frac{4}{5}\right)(4) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow \qquad v_A = 1.35 \text{ m/s} \rightarrow$$

$$v_B = \sqrt{(4.59)^2 + (3.20)^2} = 5.89 \text{ m/s} \qquad \theta = \tan^{-1}\frac{3.20}{4.95} = 32.9$$

15–74. Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



SOLUTION

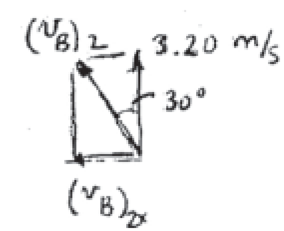
$$\Sigma m v_1 = \Sigma m v_2$$

$(\xrightarrow{+})$ $$0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$(+\uparrow)$ $$0.5(4)\left(\frac{4}{5}\right) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$



$$(v_B)_{2x} = 3.20 \tan 30° = 1.8475 \text{ m/s} \leftarrow$$

$$(v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s} \leftarrow$$

$(\xrightarrow{+})$ $$e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$e = \frac{-1.752-(-1.8475)}{4(\frac{3}{5})-(-6)} = 0.0113$$