Bays law for calculating conditional probabilities قانون بيز لحساب الاحتمالات المشروطة

ان حساب الاحتمالات المشروطة يغطيه قانون يعرف بقانون بيز والذي يحمل اسم مكتشفه توماس بيز في القرن 18

Suppose $A_1, A_2, A_3, \cdots, A_m$ are mutually exclusive events such that $A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_m = S, \quad and, \quad P(A_i) > 0 \quad where \quad i = 1,2,3,\cdots,m$ then for any event B we have
$P(B) = P(A_1) * P(B / A_1) + P(A_2) * P(B / A_2) + \cdots + P(A_m) * P(B / A_m)$

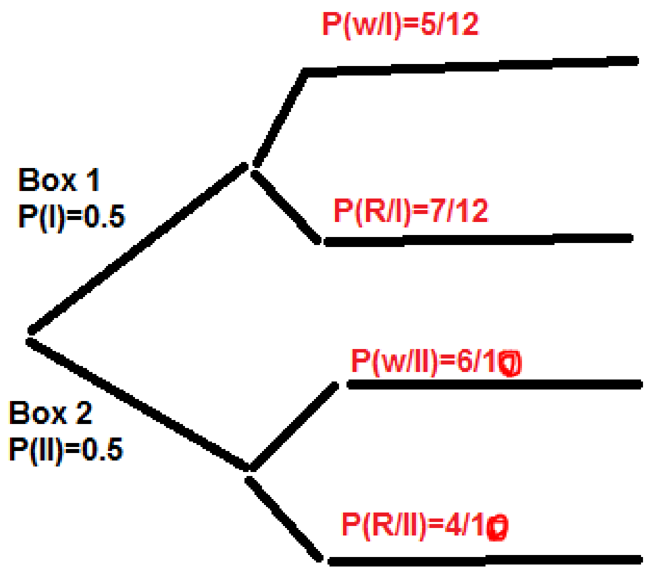
$$\therefore \quad P(B) = \sum_{i=1}^{m} P(A_i) * P(B / A_i)$$

Example: Consider two boxes, box1 contains 5 white and 7 red balls, box2 contains 6 white and 4 red balls. One of boxes is selected at random, and a ball is drawn from it. Find the probability that the ball drawn will be white.

th solution:

$$P(W) = P(W \cap I) + P(W \cap II)$$

$$P(W) = P(I) * P(W / I) + P(II) * P(W / II)$$
$$= 1/2 * 5/12 + 1/2 * 6/10 = 0.51$$



Bayes theory نظرية بيز

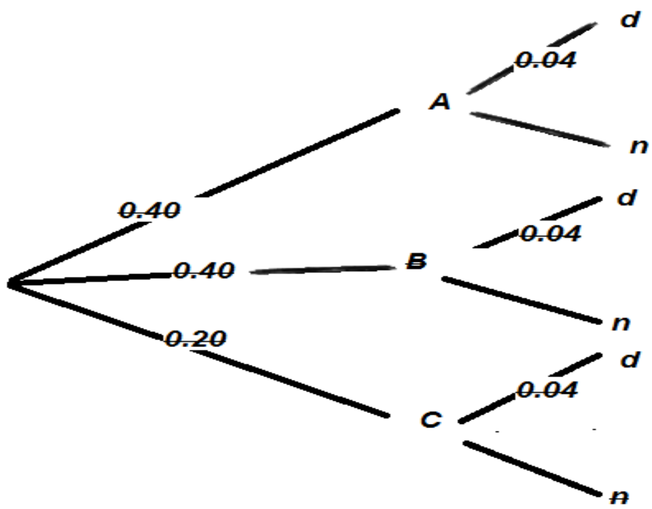
هي أحدى أهم أدوات الذكاء الصناعي, والتي تدخل في تطبيقات لا تعد ولاتحصى مثل فرز رسائل البريد الألكتروني المزعجة, وصولا الى مجال الكيمياء البيولوجية مرورا بعالم المال والادارة. كذلك أعتمدت في فك ترميز الرسائل في الحرب العالمية الثانية.

Let the events $A_1, A_2, A_3, \cdots, A_m$ Partition the sample space $P(A_i) > 0$ for i = 1, 2, 3, ..., m

$$P(A_i / B) = \frac{P(A_i) * P(B / A_i)}{\sum_{i=1}^{m} P(A_i) * P(B / A_i)}$$

بنيت نظرية بيز على أساس الاحتمالات القبلية prior والاحتمالات البعدية posterior , فالاحتمال البعدي يعتمد على الاحتمال القبلي.

Q) The machines A, B and C produce respectively 40%, 40% and 20% of the total number of items of a factory. The percentages of defective output are 4% for each machine. If an item is selected at random was defective, find the probability that the item is produced by machine B.



$$P(B / d) = \frac{P(d / B) * P(B)}{P(d / B) * P(B) + P(d / A) * P(A) + P(d / c) * P(c)}$$

$$P(B / d) = \frac{0.04 * 0.40}{0.04 * 0.40 + 0.04 * 0.40 + 0.04 * 0.20} = 0.40$$

Q- From a previous information, we know that 49% of the populations are female. Of the female patients, 8% are high risk for heart attack, while 12% of the male patients are high risk. A single person is selected at random and found to be high risk. What is the probability that it is a male?

solu. Define H: high risk F: female M: male

We know:	
P(F) =	.49
P(M) =	.51
P(H|F) =	.08
P(H|M) =	.12

$$P(M \mid H) = \frac{P(M)P(H \mid M)}{P(M)P(H \mid M) + P(F)P(H \mid F)}$$

$$= \frac{.51\,(.12)}{.51\,(.12) + .49\,(.08)} = .61$$

****

Q- The chances that a doctor will diagnose a cancer disease correctly are 80%. The chance that patient will die by his treatment after correct diagnosis is 25%, and the chance of death by wrong diagnosis is 65%. One of this doctor patients, who had a cancer died, what is the chance that his disease was diagnosed correctly?

The solution:

Let E_1 = the event that the cancer is diagnosed correctly.

E_2 = the event that a patient suffered from cancer dies.

$P(E_1) = 0.80, \quad P(E_1') = 0.20, \quad P(E_2 / E_1) = 0.25, \quad P(E_2 / E_1') = 0.65, \quad P(E_1 / E_2)?$

$$P(E_1 / E_2) = \frac{P(E_2 / E_1) * P(E_1)}{P(E_2 / E_1) * P(E_1) + P(E_2 / E_1') * P(E_1')}$$

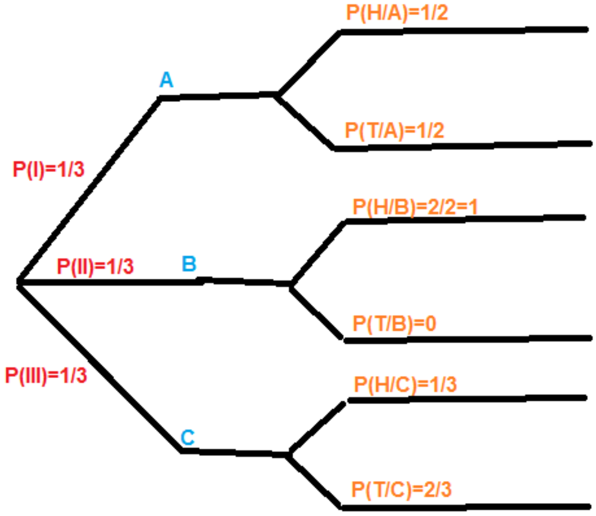
$$P(E_1 / E_2) = \frac{0.25 * 0.80}{0.25 * 0.80 + 0.65 * 0.20} = 20/33$$

****

Q- A box contains three coins, one coin is fair, one coin is two headed, and one coin is weighted, so that the probability of head appearing is 1/3. A coin is selected at random and tossed once. If we are given that the result is tail, find the probability that the weighted coin was chosen.

The solution:

Let A = fair coin, B = two headed coin, C = weighted coin



$$P(C/T)=\frac{P(T/C)*P(C)}{P(T/A)*P(A)+P(T/B)*P(B)+P(T/C)*P(C)}$$

$$P(C/T)=\frac{1/3*2/3}{1/3*1/2+1/3*0+1/3*2/3}=0.57$$