Optimization
Fourth Class
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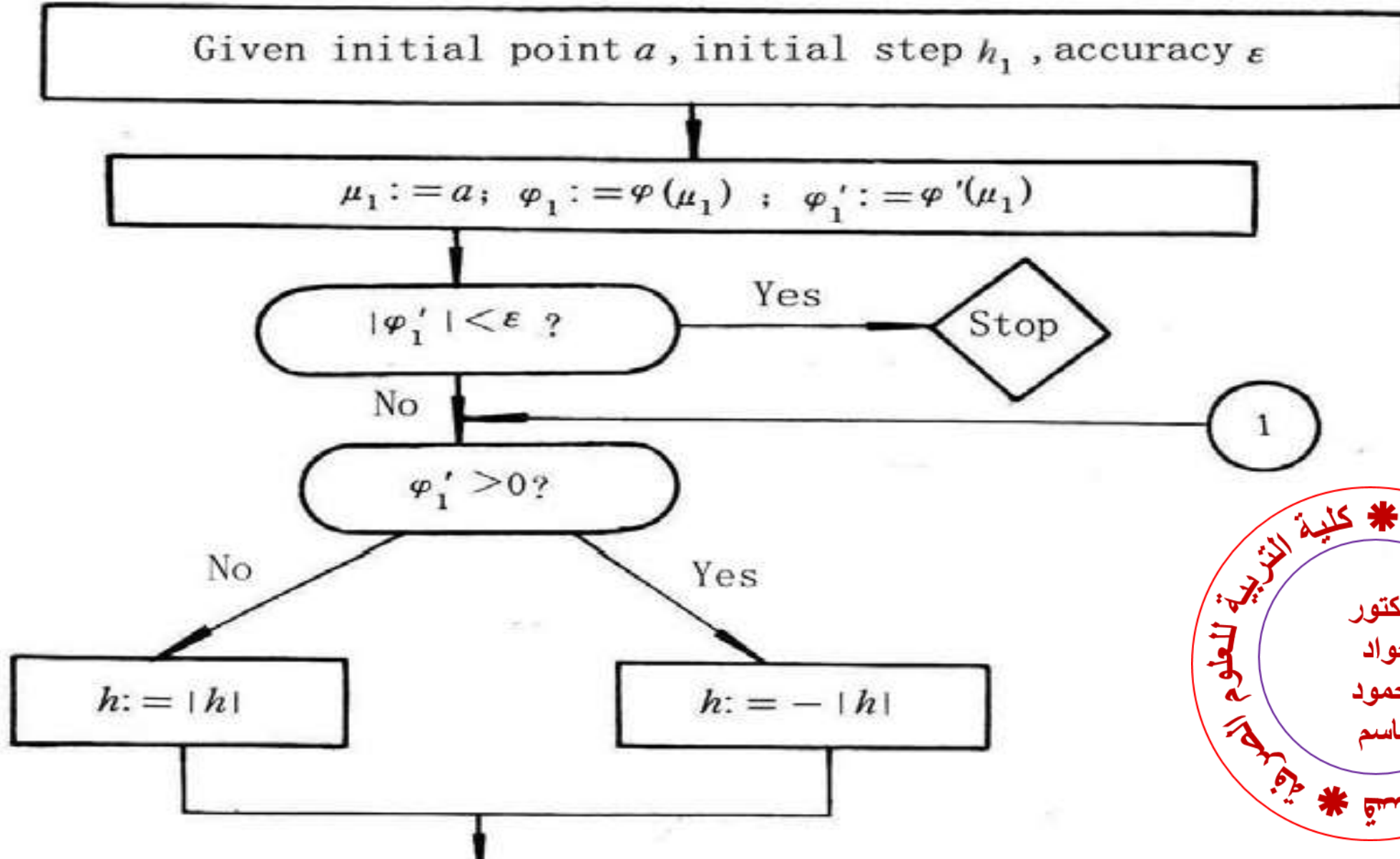### University of Basrah
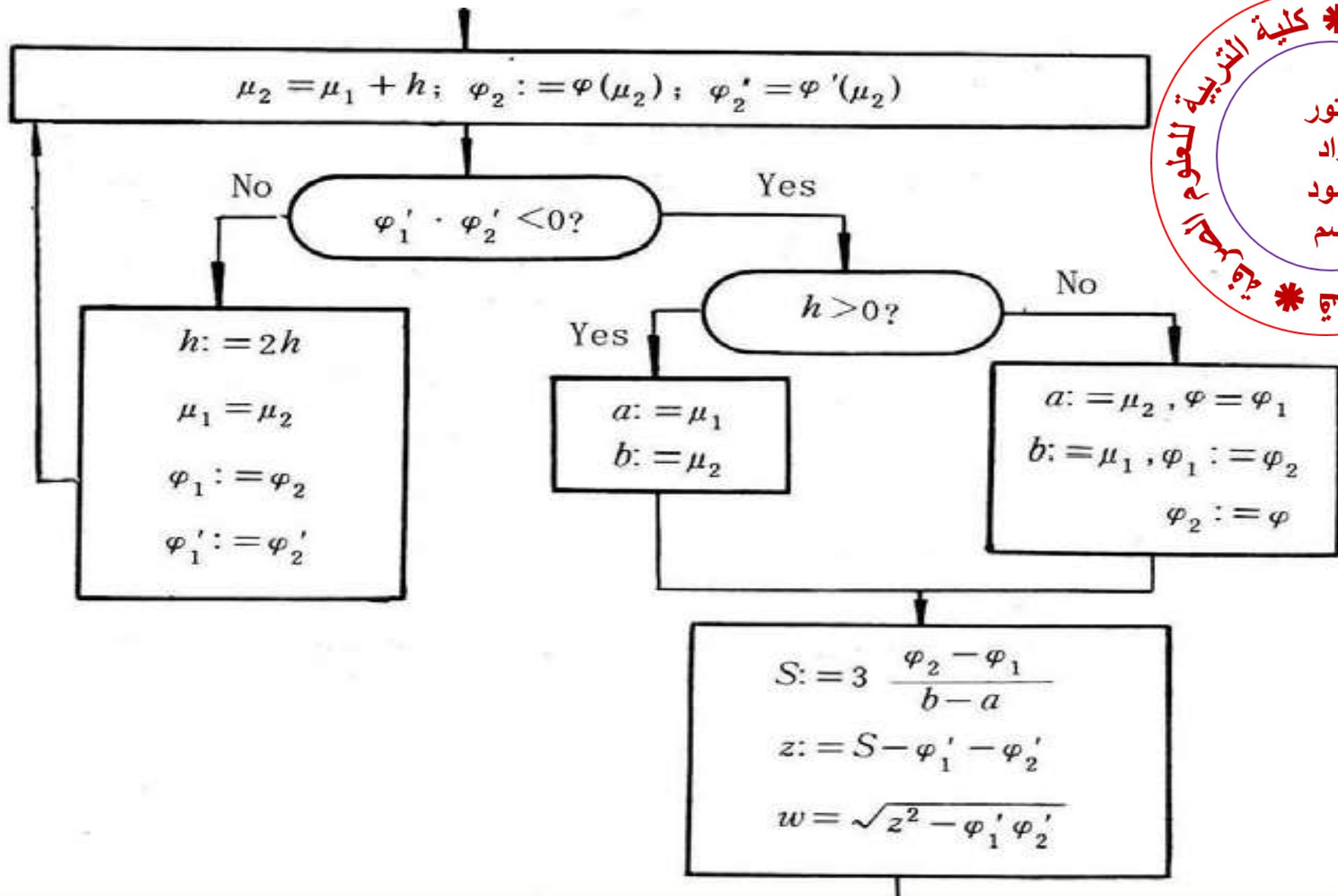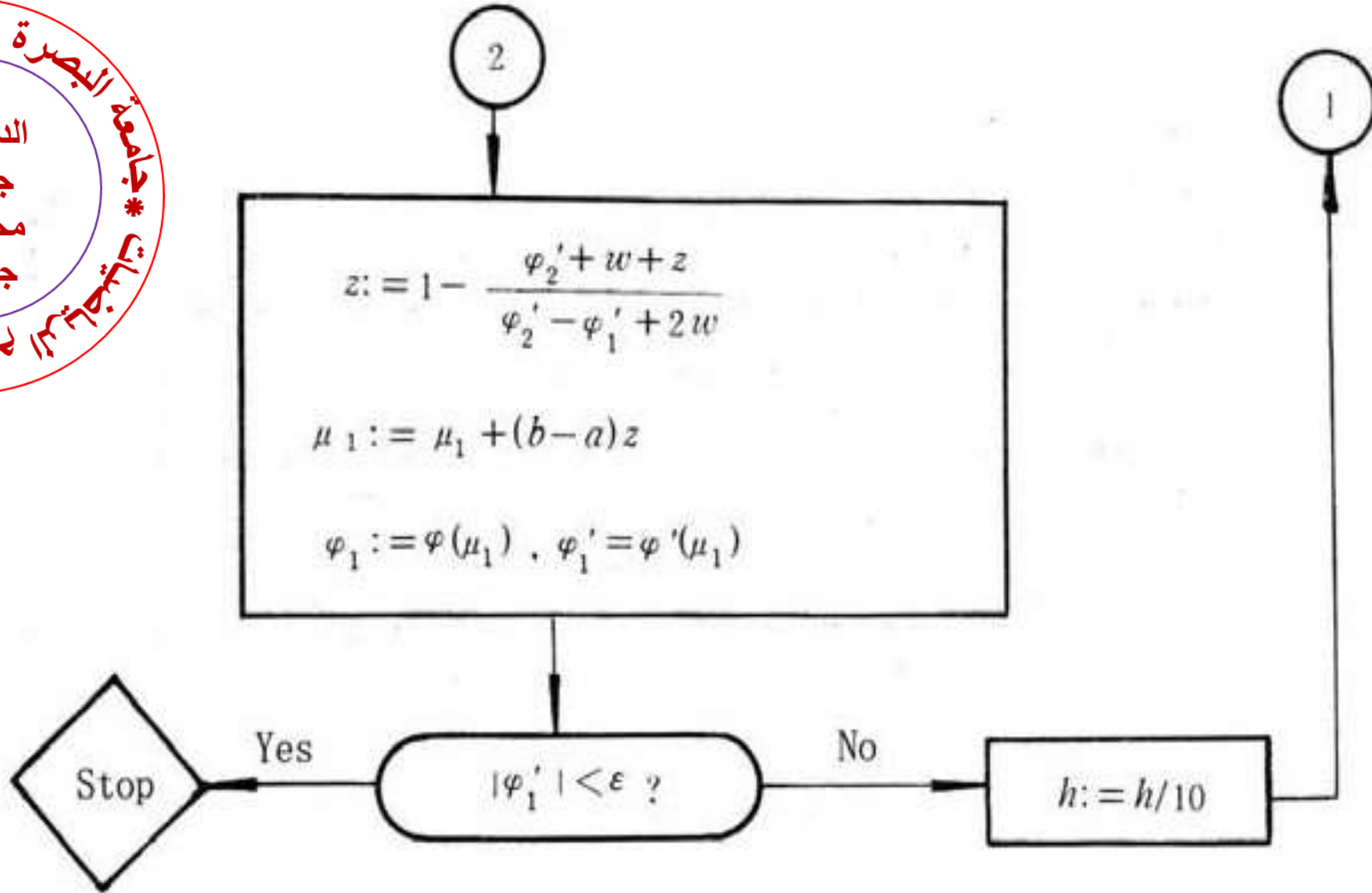## Iraq

Chapter Two

Line Search

Lecture 9

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Flow chart for the cubic interpolation method with two points

Given initial point a, initial step h_1, accuracy ε

$\mu_1 := a$; $\varphi_1 := \varphi(\mu_1)$; $\varphi_1' := \varphi'(\mu_1)$

$|\varphi_1'| < \varepsilon$? — Yes → Stop

No

$\varphi_1' > 0$?

No → $h := |h|$

Yes → $h := -|h|$

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$$\mu_2 = \mu_1 + h; \quad \varphi_2 := \varphi(\mu_2); \quad \varphi_2' = \varphi'(\mu_2)$$

No $\qquad$ $\varphi_1' \cdot \varphi_2' < 0?$ $\qquad$ Yes

$$h := 2h$$
$$\mu_1 = \mu_2$$
$$\varphi_1 := \varphi_2$$
$$\varphi_1' := \varphi_2'$$

Yes $\qquad$ $h > 0?$ $\qquad$ No

$$a := \mu_1$$
$$b := \mu_2$$

$$a := \mu_2, \quad \varphi = \varphi_1$$
$$b := \mu_1, \quad \varphi_1 := \varphi_2$$
$$\varphi_2 := \varphi$$

$$S := 3\,\frac{\varphi_2 - \varphi_1}{b - a}$$
$$z := S - \varphi_1' - \varphi_2'$$
$$w = \sqrt{z^2 - \varphi_1'\varphi_2'}$$

Algorithm (5): (Cubic Interpolation Method with Two Points)

Step 0:

Given initial point a, initial step h and accuracy $\varepsilon > 0$.

Step 1:

Set $\mu_1 = a$. Evaluate $\Phi_1 = \Phi(\mu_1)$ and $\Phi'_1 = \Phi'(\mu_1)$.

Step 2:

If $|\Phi'_1| < \varepsilon$, stop. Otherwise go to step 3.

Step 3:

If $\Phi'_1 > 0$, set $h = -|h|$ and go to step 5. Otherwise go to step 4.

Step 4:

Set $h = |h|$ and go to step 5.

Step 5:

Set $\mu_2 = \mu_1 + h$ and evaluate $\Phi_2 = \Phi(\mu_2)$ and $\Phi'_2 = \Phi'(\mu_2)$.

Step 6:

If $\Phi'_1 \Phi'_2 \geq 0$, set $h = 2h, \mu_1 = \mu_2, \Phi_1 = \Phi_2, \Phi'_1 = \Phi'_2$ and go to step 5. Otherwise go to step 7.

Step 7:

If $h > 0$, set $a = \mu_1, b = \mu_2$ and go to step 9. Otherwise go to step 8.

Step 8:

Set $a = \mu_2, \Phi = \Phi_1, b = \mu_1, \Phi_1 = \Phi_2, \Phi_2 = \Phi$.

Step 9: Compute

$$s = 3 \left[\frac{\Phi_2 - \Phi_1}{b - a} \right], z = s - \Phi'_1 - \Phi'_2 ,$$

$$w = \sqrt{z^2 - \Phi'_1 \Phi'_2}, z = 1 - \frac{\Phi'_2 + w + z}{\Phi'_2 - \Phi'_1 + 2w} ,$$

$$\mu_1 = \mu_1 + (b - a)z, \Phi_1 = \Phi(\mu_1), \Phi'_1 = \Phi'(\mu_1).$$

Step 10:

If $|\Phi'_1| < \varepsilon$, stop. Otherwise set $h = \frac{h}{10}$ and go to step 3.

Example:

Use cubic interpolation method to find the minimizer of the function $f(x) = \dfrac{x}{\ln x}$, $x > 1$.

Given initial point $a = 2$, $length\ step\ h = 0.1$ and the accuracy 0.03.

Solution:

First, we find the derivative of $f(x)$.

$$f'(x) = \frac{\ln x - 1}{(\ln x)^2} .$$

1: Set $\mu_1 = a = 2$.

2: Compute $\Phi_1 = f(\mu_1)$ and $\Phi'_1 = f'(\mu_1)$ as follows:

$\Phi_1 = f(2) = 2.88539$.

$\Phi'_1 = f'(2) = -0.63867$.

Since $|\Phi'_1| = 0.63867 > \epsilon = 0.03$ and $\Phi'_1 < 0$.

Set $h = |h| = 0.1$.

3: Compute μ_2 , Φ_2 and Φ'_2 as follows:

$\mu_2 = \mu_1 + h = 2 + 0.1 = 2.1$.

$\Phi_2 = f(2.1) = 2.83043$.

$\Phi'_2 = f'(2.1) = -0.53712$.

Since $\Phi'_1 \Phi'_2 > 0$, set

$h = 2h = 0.2$.

$\mu_1 = \mu_2 = 2.1, \Phi_1 = \Phi_2 = 2.83043,$

$$\Phi'_1 = \Phi'_2 = -0.53712.$$

4: Compute μ_2 , Φ_2 **and** Φ'_2 **as follows:**

$$\mu_2 = \mu_1 + h = 2.1 + 0.2 = 2.3 \ .$$

$$\Phi_2 = f(2.3) = 2.76141 \ .$$

$$\Phi'_2 = f'(2.3) = -0.24086.$$

Since $\Phi'_1 \Phi'_2 > 0$, set

$$h = 2h = 0.4 \ .$$

$$\mu_1 = \mu_2 = 2.3, \boldsymbol{\Phi}_1 = \Phi_2 = 2.76141,$$
$$\Phi'_1 = \Phi'_2 = -0.24086.$$

5: Compute μ_2 , Φ_2 **and** Φ'_2 **as follows:**

$\mu_2 = \mu_1 + h = 2.3 + 0.4 = 2.7$.

$\Phi_2 = f(2.7) = 2.71834$.

$\Phi'_2 = f'(2.7) = -0.00684$.

Since $\Phi'_1 \Phi'_2 > 0$, set

$h = 2h = 0.8$.

$\mu_1 = \mu_2 = 2.7, \Phi_1 = \Phi_2 = 2.71834,$

$\Phi'_1 = \Phi'_2 = -0.00684$.

6: Compute μ_2 , Φ_2 **and** Φ'_2 **as follows:**

$$\mu_2 = \mu_1 + h = 2.7 + 0.8 = 3.5.$$

$$\Phi_2 = f(3.5) = 2.79382.$$

$$\Phi'_2 = f'(3.5) = 0.16106.$$

Since $\Phi'_1 \Phi'_2 < 0$, **and** $h > 0$,

Set $a = \mu_1 = 2.7$ *and* $b = \mu_2 = 3.5.$

7: Compute s , z *and* w *as follows*:

$$s = 3\left[\frac{\Phi_2 - \Phi_1}{b - a}\right] = 0.2832.$$

$$z = s - \Phi'_1 - \Phi'_2 = 0.12898.$$

$$w = \sqrt{z^2 - \Phi'_1 \Phi'_2} = 0.13318. \left((w > 0)\right).$$

$$z = 1 - \frac{\Phi'_2 + w + z}{\Phi'_2 - \Phi'_1 + 2w} = 0.02681.$$

8: Compute μ_1 , Φ_1 **and** Φ'_1 **as follows:**

$$\mu_1 = \mu_1 + (b - a)z = 3.52681 .$$

$$\Phi_1 = f(3.52681) = 2.79818 .$$

$$\Phi'_1 = f'(3.52681) = 0.1639 .$$

Since $|\Phi'_1| = 0.1639 > \epsilon = 0.03$.

Set $h = \dfrac{h}{10} = \dfrac{0.8}{10} = 0.08$.

Since $\Phi'_1 > 0$, $set\ h = -|h| = -0.08$.

9: Compute μ_2 , Φ_2 **and** Φ'_2 **as follows:**

$$\mu_2 = \mu_1 + h = 3.52681 - 0.08 = 3.44681 .$$

$$\Phi_2 = f(3.44681) = 2.78542 .$$

$$\Phi'_2 = f'(3.44681) = 0.15535.$$

Since $\Phi'_1 \Phi'_2 > 0$, set

$h = 2h = -0.16$.

$\mu_1 = \mu_2 = 3.44681, \Phi_1 = \Phi_2 = 2.78542$

$\Phi'_1 = \Phi'_2 = 0.15535.$

10: Compute μ_2 , Φ_2 and Φ'_2 as follows:

$\mu_2 = \mu_1 + h = 3.44681 - 0.16 = 3.28681.$

$\Phi_2 = f(3.28681) = 2.77553$.

$\Phi'_2 = f'(3.28681) = 0.13413$.

Since $\Phi'_1 \Phi'_2 > 0$, set $h = 2h = -0.32$,

$\mu_1 = \mu_2 = 3.28681, \Phi_1 = \Phi_2 = 2.77553.$

$\Phi'_1 = \Phi'_2 = 0.13413.$

11: Compute μ_2 , Φ_2 **and** Φ'_2 **as follows:**

$\mu_2 = \mu_1 + h = 3.28681 - 0.32 = 2.96681$.

$\Phi_2 = f(2.96681) = 2.72813$.

$\Phi'_2 = f'(2.96681) = 0.07392$.

Since $\Phi'_1 \Phi'_2 > 0$, set $h = 2h = -0.64$,

$\mu_1 = \mu_2 = 2.96681, \boldsymbol{\Phi_1} = \Phi_2 = 2.72813.$

$\Phi'_1 = \Phi'_2 = 0.07392.$

12: Compute μ_2 , Φ_2 **and** Φ'_2 **as follows:**

$\mu_2 = \mu_1 + h = 2.96681 - 0.64 = 2.32681$.

$\Phi_2 = f(2.32681) = 2.75526$.

$\Phi'_2 = f'(2.32681) = -0.21804.$

13: Since $\Phi'_1 \Phi'_2 < 0$ *and* $h < 0$, *set*

$$a = \ \mu_2 = 2.32681$$
$$b = \mu_1 = 2.96681$$
$$\Phi = \boldsymbol{\Phi_1} = 2.72813$$
$$\Phi_1 = \Phi_2 = 2.75526$$
$$\Phi_2 = \Phi = 2.72813.$$

14: Compute s, z and w as $follows$:

$$s = 3\left[\frac{\Phi_2 - \Phi_1}{b - a}\right] = -0.12717 .$$
$$z = s - \Phi'_1 - \Phi'_2 = 0.27129 .$$
$$w = \sqrt{z^2 - \Phi'_1 \Phi'_2} = 0.27425 . \big((w > 0)\big).$$
$$z = 1 - \frac{\Phi'_2 + w + z}{\Phi'_2 - \Phi'_1 + 2w} = -0.2766 .$$

15: Compute μ_1 , Φ_1 **and** Φ'_1 **as follows:**

$\mu_1 = \mu_1 + (b - a)z = 2.78978.$

$\Phi_1 = f(2.78978) = 2.71918 .$

$\Phi'_1 = f'(2.78978) = 0.02467 .$

Since $|\Phi'_1| = 0.02467 < \epsilon = 0.03 .$

$\therefore$ **The minimizer of the function is**

$\mu_1 = 2.78978.$

H.W.

Use cubic interpolation method to find the minimizer of the function $f(x) = x^5 - 5x^3 - 20x + 5.$

Take $a = 0, h = 0.4$ *and the accuracy* 0.2 .