Optimization

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Chapter Two

Line Search

Lecture 7

Algorithm (4): (Quadratic Interpolation with Three Points)

Step 1:

$Given\ a\ tolerance\ \varepsilon > 0$, initial point α_1 and step length h.

$Take\ \alpha_2 = \alpha_1 + h$. Evaluate $\Phi(\alpha_1) = \Phi_1\ and\ \Phi(\alpha_2) = \Phi_2$.

Step 2:

If $\Phi_1 < \Phi_2$ take the third point $\alpha_3\ as : \alpha_3 = \alpha_1 - h$. Otherwise take the third point $\alpha_3\ \ as$: $\alpha_3 = \alpha_1 + 2h$. Evaluate $\Phi(\alpha_3) = \Phi_3$.

Step 3:

Use Equation (56) $to\ find\ \ \bar{\alpha}$ and evaluate $\Phi(\bar{\alpha})$.

Step 4:

If the difference between the two lowest function values is less than or equal the given tolerance stop. Otherwise go to next step.

Step 5:

Discard the point with the highest function value and go to step 3.

<u>Theorem (8):</u>

1: Let $\Phi(\alpha)$ have continuous forth order derivatives.

2: Let α^* be such that $\Phi'(\alpha^*) = 0$ *and* $\Phi''(\alpha^*) \neq 0$.

Then the sequence $\{\alpha_k\}$ generated by Equation (56) converges to α^* with order 1.32 of converge rate.

<u>Example:</u>

Use quadratic interpolation with three points method to find the position of the *minimizer of* $f(x) = 2x^2 - e^x$. Given the *initial point* 1, length step 0.5 and tolerance 0.0001.

Solution:

Let $\alpha_1 = 1 \rightarrow \alpha_2 = \alpha_1 + h = 1 + 0.5 = 1.5.$

$\therefore f(\alpha_1) = f(1) = -0.71828 \quad and\; f(\alpha_2) = f(1.5) = 0.01831.$

Since $f(\alpha_1) < f(\alpha_2)$, we take the third point

$\alpha_3 = \alpha_1 - h = 1 - 0.5 = 0.5.$

$\therefore f(\alpha_3) = f(0.5) = -1.14872.$

Now, we have $\alpha_1 = 1\,, \alpha_2 = 1.5\; and\; \alpha_3 = 0.5.$

And $\Phi_1 = f(\alpha_1) = -0.71828\,, \Phi_2 = f(\alpha_2) = 0.01831,$

$\Phi_3 = f(\alpha_3) = -1.14872.$

Now, we find the value of $\bar{\alpha}$ as follows:

$$\therefore \quad \bar{\alpha} = \frac{\Phi_1\left({\alpha_2}^2 - {\alpha_3}^2\right) + \Phi_2\left({\alpha_3}^2 - {\alpha_1}^2\right) + \Phi_3\left({\alpha_1}^2 - {\alpha_2}^2\right)}{2\left[\Phi_1\,(\alpha_2 - \alpha_3) + \Phi_2\,(\alpha_3 - \alpha_1) + \Phi_3(\alpha_1 - \alpha_2)\right]} = 0.37459.$$

$\therefore f(\bar{\alpha}) = -1.17375.$

Now, we put the function values in descent order:

$$\Phi_2 = f(\alpha_2) = f(1.5) = \quad 0.01831$$

$$\Phi_1 = f(\alpha_1) = f(1) = \quad -0.71828$$

$$\Phi_3 = f(\alpha_3) = f(0.5) = -1.14872$$

$$f(\bar{\alpha}) = f(0.37459) = \quad -1.17375$$

Now, we find the difference between Φ_3 *and* $f(\bar{\alpha})$.

$$\therefore \Phi_3 - f(\bar{\alpha}) = -1.14872 + 1.17375 = 0.02503 \ > 0.0001 \,.$$

Then, we discard the highest function value $\Phi_2 = f(\alpha_2) = 0.01831$ and set $\alpha_1 = 1\,, \alpha_2 = 0.5$ *and* $\alpha_3 = 0.37459$ *and*

$$\Phi_1 = -0.71828\,, \Phi_2 \text{ and } \Phi_3 = -1.17375\,.$$

Now, we find the value of $\bar{\alpha}$ as follows:

$$\therefore \quad \bar{\alpha} \ = \frac{\Phi_1({\alpha_2}^2 - {\alpha_3}^2) + \Phi_2({\alpha_3}^2 - {\alpha_1}^2) + \Phi_3({\alpha_1}^2 - {\alpha_2}^2)}{2[\Phi_1\,(\alpha_2 - \alpha_3) + \Phi_2\,(\alpha_3 - \alpha_1) + \Phi_3(\alpha_1 - \alpha_2)]} = 0.3615 \ \text{ and}$$

$$f(\bar{\alpha}) = f(0.3615) = -1.17411.$$

Now, we put the function values in descent order:

$\boldsymbol{\Phi_1 = f(\alpha_1) = f(1) = -0.71828}$

$\boldsymbol{\Phi_2 = f(\alpha_2) = f(0.5) = -1.14872}$

$\boldsymbol{\Phi_3 = f(\alpha_3) = f(0.37459) = -1.17375}$

$\boldsymbol{f(\overline{\alpha}) = f(0.3615) = -1.17411}$

Now, we find the difference between $\boldsymbol{\Phi_3 \; and \; f(\overline{\alpha})}$.

$\boldsymbol{\therefore \Phi_3 - f(\overline{\alpha}) = -1.17375 + 1.17411 = 0.00036 > 0.0001}$.

$\therefore$**We discard the highest value** $\boldsymbol{\Phi_1 = f(\alpha_1) = f(1)}$.

Now, we set $\boldsymbol{\alpha_1 = 0.5, \alpha_2 = 0.37459 \; and \; \alpha_3 = 0.3615}$.

And $\boldsymbol{\Phi_1 = f(\alpha_1) = f(0.5) = -1.14872}$,

$\boldsymbol{\Phi_2 = f(\alpha_2) = f(0.37459) = -1.17375}$

$\boldsymbol{\Phi_3 = f(0.3615) = -1.17411}$.

Now, we find the value of $\boldsymbol{\overline{\alpha}}$ **as follows:**

$$\therefore \; \overline{\alpha} = \frac{\Phi_1({\alpha_2}^2 - {\alpha_3}^2) + \Phi_2({\alpha_3}^2 - {\alpha_1}^2) + \Phi_3({\alpha_1}^2 - {\alpha_2}^2)}{2[\Phi_1 \, (\alpha_2 - \alpha_3) + \Phi_2 \, (\alpha_3 - \alpha_1) + \Phi_3(\alpha_1 - \alpha_2)]} = 0.35709 \, .$$

$\boldsymbol{\therefore f(\overline{\alpha}) = f(0.35709) = -1.17413}$.

Now, we put the function values in descent order:

$$\boldsymbol{\Phi_1 = f(\alpha_1) = f(0.5) = -1.14872}$$

$$\boldsymbol{\Phi_2 = f(\alpha_2) = f(0.37459) = -1.17375}$$

$$\boldsymbol{\Phi_3 = f(0.3615) = -1.17411}$$

$$\boldsymbol{f(\overline{\alpha}) = f(0.35709) = -1.17413}$$

Now, we find the difference between $\boldsymbol{\Phi_3}$ $\boldsymbol{and}$ $\boldsymbol{f(\overline{\alpha})}$.

$$\boldsymbol{\therefore \Phi_3 - f(\overline{\alpha}) = -1.17411 + 1.17413 = 0.00002 < 0.0001.}$$

$\therefore$The required minimizer is $\boldsymbol{0.35709}$.