Subject :Ph 101

Classical Mechanics

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Chapter one
Vectors

- Vector and scalar
- Coordinate system
- Properties of vector
- The unit vector
- Components of a vector
- Product of a vector

Mathematical Addition of Vectors

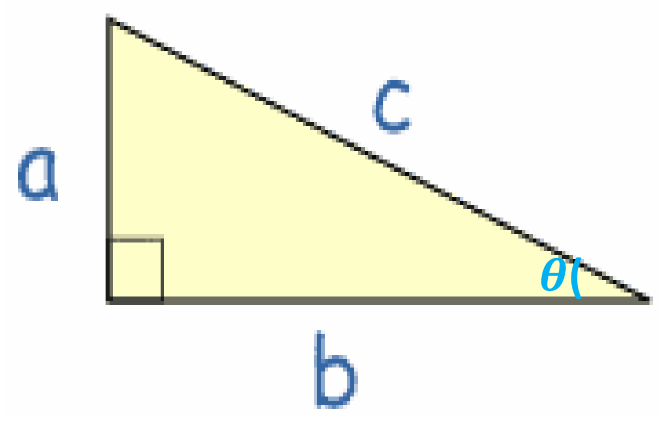
Pythagoras' Theorem:- in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

$a^2 + b^2 = c^2$



Types of Quantities

- The magnitude of a quantity tells how large the quantity is.
- :There are two types of quantities
- 1.Scalar quantities : have magnitude only.
- 2. Vector quantities : have both magnitude and direction.
- Can you give some examples of each?

Scalars

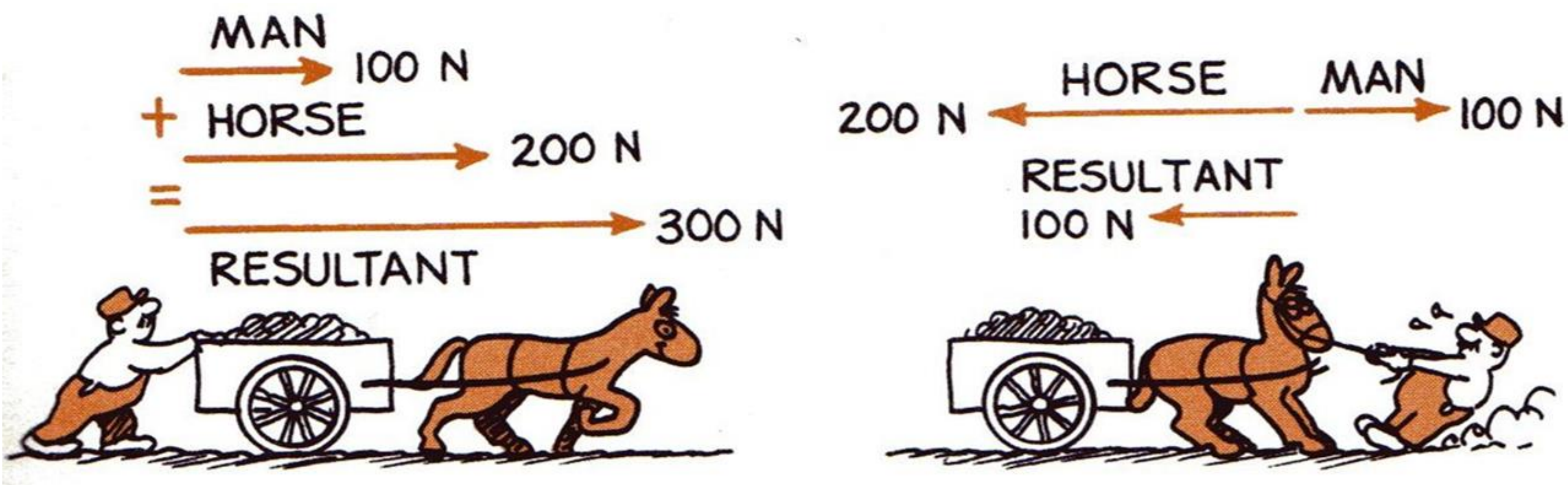
- Mass
- Distance
- Speed
- Time
- Temperature

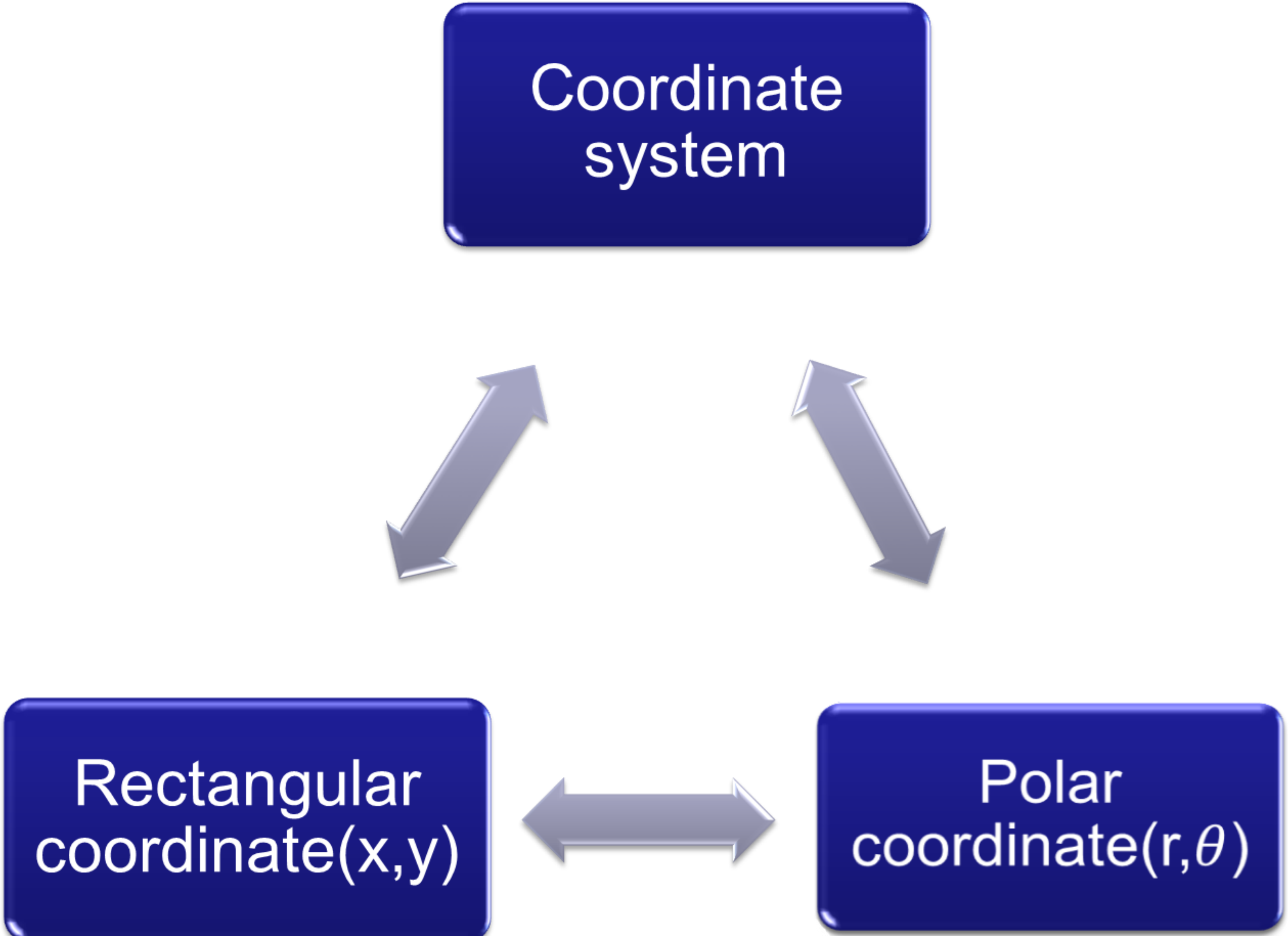
Vectors

- Displacement
- Velocity
- Acceleration
- Force
- Weight

- Velocity is a vector quantity that includes both speed and direction. A vector is represented by an arrowhead line
- magnitude
- With direction

- Any vector is uniquely specified by its three components x, y, z




Coordinate system
Rectangular coordinate(x,y)
Polar coordinate(r,θ)

Rectangular &Polar coordinate

A Cartesian coordinate system in two dimensions (also called a rectangular coordinate system or an orthogonal coordinate system) is defined by an ordered pair of perpendicular lines (axes), a single unit of length for both axes, and an orientation for each axis.

لتحديد موقع النقطة p(x,y)او p(r,θ) يجب ان تحدد بالنسبة للنقطة المرجعية.(0,0)

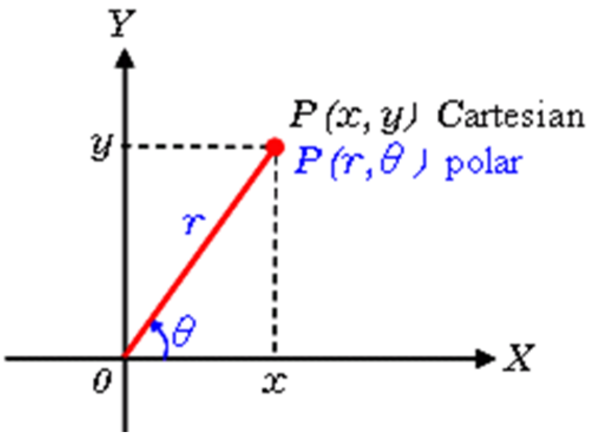
What is the relationship of Cartesian coordinate (x,y) with polar coordinate (r,θ)?

$r^2 = x^2 + y^2$

$X = r\cos\theta$

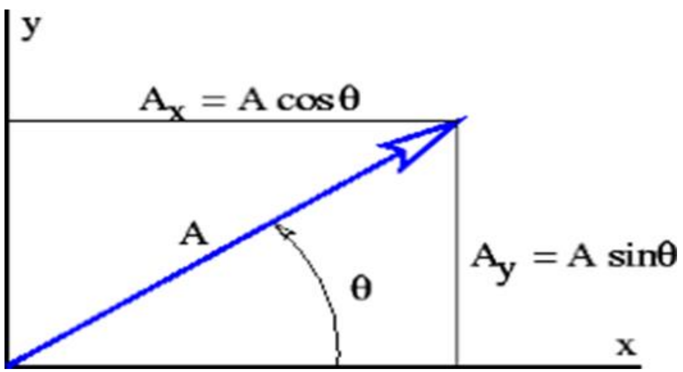
$Y = r\sin\theta$

$\theta = \tan^{-1}(\frac{y}{x})$

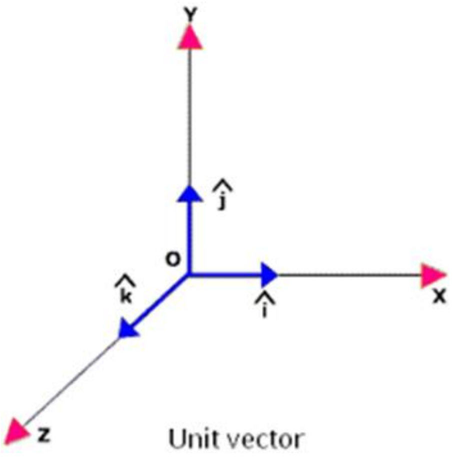


Vector Components

- A vector quantity has both magnitude and direction.
- $|\vec{A}| = \sqrt{A_x{}^2 \quad + A_y{}^2}$
- $\theta = \tan^{-1}(\frac{A_y}{A_x})$
- $\vec{A} = \vec{a}|\vec{A}|$
- *Where*
- $|\vec{A}|$ *magnitude of vector*
- $\vec{a}$ *is the direction of vector (unit vector)*
- $\hat{\imath}$ ≡ a unit vector along the x-axis
- $\hat{\jmath}$ ≡ a unit vector along the y-axis
- $\hat{k}$ ≡ a unit vector along the z-axis
- $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$
-



A_x=Acos θ



Example(1)

Given the vectors: A = 3i + 2j – k and B = 5i +5j.

Determine:

1- Their magnitude.

2- The direction of B.

3- A + B

4- A -2 B

5- A unit vector parallel to A.

solution

Vectors A and B are written using the unit vector notation.

1-The magnitude of A is given by:

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{A}| = \sqrt{3^2 + 2^2 + (-1)^2} = 3.74$$

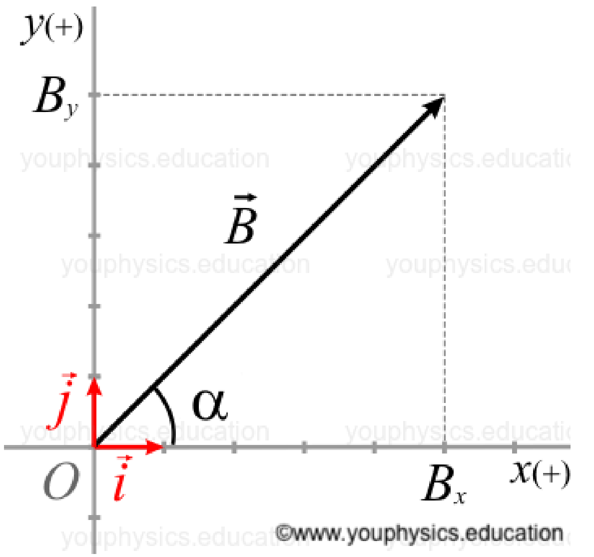
Similarly, the magnitude of B is

$$|\vec{B}| = \sqrt{B_x{}^2 \quad + B_y{}^2 + B_z{}^2}$$

$$|\vec{B}| = \sqrt{(5)^2 \quad +(5)^2 + (0)^2} \qquad = \sqrt{50}$$

The magnitude of a vector is always a positive number.

2- In the figure below vector B is shown, as well as the standard unit vectors (in red).



$$\alpha = \arctan\frac{B_y}{B_x} = \arctan\frac{5}{5} = 45^0$$

3- The vector sum of A and B is given by:

$$\vec{A}+\vec{B} = (A_x+B_x)\vec{i} + (A_y+B_y)\vec{j} + (A_z+B_z)\vec{k}$$
$$\vec{A}+\vec{B} = (3+5)\vec{i} + (2+5)\vec{j} + (-1+0)\vec{k}$$
$$\vec{A}+\vec{B} = 8\vec{i} + 7\vec{j} - \vec{k}$$

4- we multiply B by -2 and then we add A

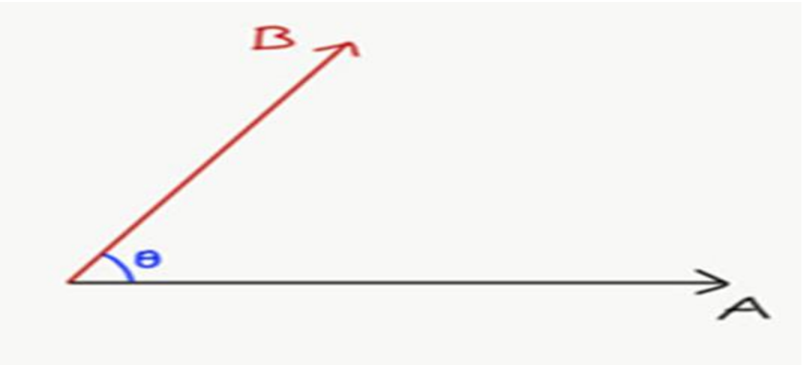
$$\vec{A}-2\vec{B} = (A_x-2B_x)\vec{i} + (A_y-2B_y)\vec{j} + (A_z-2B_z)\vec{k}$$
$$\vec{A}-2\vec{B} = (3-2\cdot5)\vec{i} + (2-2\cdot5)\vec{j} + (-1-2\cdot0)\vec{k}$$
$$\vec{A}-2\vec{B} = -7\vec{i} + 8\vec{j} - \vec{k}$$

5-

$$\vec{u}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\vec{i} + A_y\vec{j} + A_z\vec{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$
$$\vec{u}_A = \frac{3\vec{i} + 2\vec{j} - \vec{k}}{\sqrt{3^2 + 2^2 + (-1)^2}} = \frac{1}{3.74}(3\vec{i} + 2\vec{j} - \vec{k})$$

Product of vector

- There are two kinds of vector product the first one is called scalar product or dot product because the result of the product is a scalar quantity . The second is called vector product or cross product because the result is a vector perpendicular to the plane of the two vectors .



The scalar product

يعرف الضرب القياسي scalar product بالضرب النقطي dot product وتكون نتيجة الضرب القياسي لمتجهين كمية قياسية .

تكون هذه القيمة موجبة إذا كانت الزاوية المحصورة بين المتجهين بين 0 و 90 درجة وتكون النتيجة سالبة إذا كانت الزاوية المحصورة بين المتجهين بين 90 و 180 درجة وتساوي 0 إذا كانت الزاوية 90

يعرف الضرب القياسي للمتجهين بحاصل ضرب مقدار المتجه الاول في مقدار المتجه الثاني في جيب تمام الزاوية المحصورة بينهما

يمكن ايجاد قيمه الضرب القياسي للمتجهين باستخدام مركبات كل متجه كمما يلي:

$$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$A.B = +\text{ve when } 0 \le \theta < 90^o$$

$$A.B = -\text{ve when } 90^o < \theta \le 180^o$$

$$A.B = \text{zero when } \theta = 90$$

$$\vec{A}.\vec{B}= (A_x \mathrm{i}.B_x \mathrm{i}+A_x \mathrm{i}.B_y \mathrm{j}+A_x \mathrm{i}.B_z \mathrm{k}$$

$$+A_y j.B_y j+A_y\, j.B_z k+$$

$$A_z k.B_x i+A_z k.B_y j+A_z k.B_z k)$$

$$\vec{A}.\vec{B}= A_x B_x+A_y B_y+A_z B_z$$

$$\vec{A}.\vec{B}= |A||B| \cos\theta$$

$$\cos\theta=\frac{\vec{A}.\vec{B}}{|A||B|}=\frac{A_x B_x+A_y B_y+A_z B_z}{|A||B|}$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}=1$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}=0$$

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}=\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}=0$$

$$\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}=\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}=0$$

Example (2)

Find the angle between the two vector

$\vec{A}$=2i+3j+4k $\vec{B}$=i-2j+3k

SOLUTION

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$$

$\vec{A}.\vec{B}$= $A_x B_x + A_y B_y + A_z B_z$=(2)(1)+(3)(-2)+(4)(3)=8

$$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$|B| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$\cos\theta = \frac{8}{\sqrt{29}\sqrt{14}} = 0.397$$

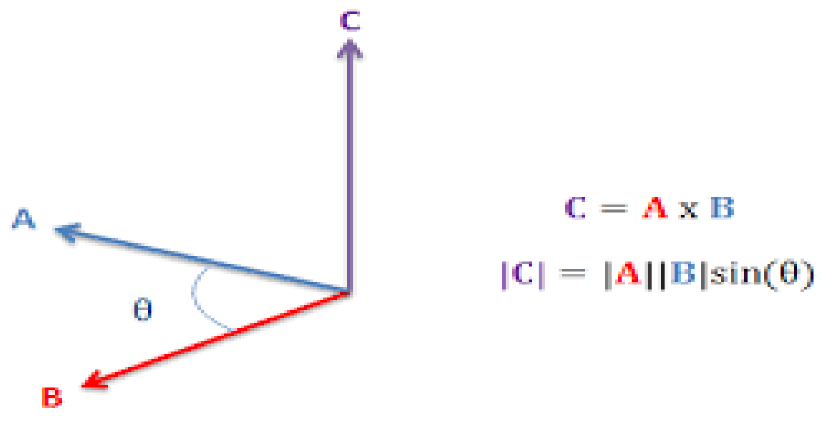
$$\theta = 66.6°$$

The vector product(cross product)

The cross product $\vec{A} \times \vec{B}$ is defined as a vector $\vec{C}$ that is perpendicular (orthogonal) to both A and B, with a direction given by the right-hand rule and a magnitude equal to the area of the parallelogram that the vectors span

يكون ناتج الضرب الاتجاهي لمتجهين AوB كمية متجه هي المتجه C, واتجاهه عمودي على كل من المتجهين A وBكما في الشكل. وتستخدم قاعدة البريمة لتحديد اتجاه المتجه الناتج عن الضرب الاتجاهي (ادا كان الدوران مع عكس عقارب الساعة يكون المتجه المحصل عمودي الى الخارج من الصفحة وادا كان الدوران مع عقارب الساعة يكون اتجاه المتجه المحصل عمودي الى الداخل على الصفحة.

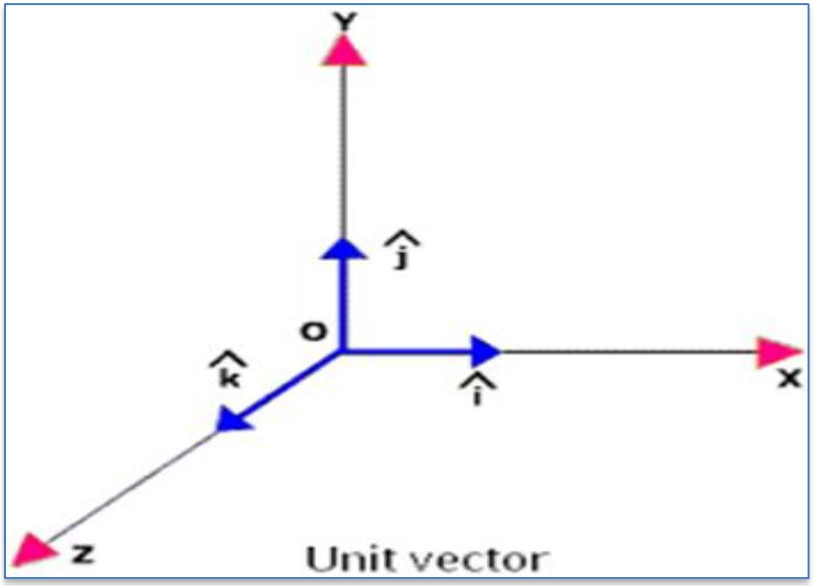
$\vec{A} \times \vec{B}$=ABsin θ



$i \times i = 0$ $\quad$ $i \times j = k$ $\quad$ $j \times i = -k$

$j \times j = 0$ $\quad$ $j \times k = i$ $\quad$ $k \times j = -i$

$k \times k = 0$ $\quad$ $k \times i = j$ $\quad$ $i \times k = -j$

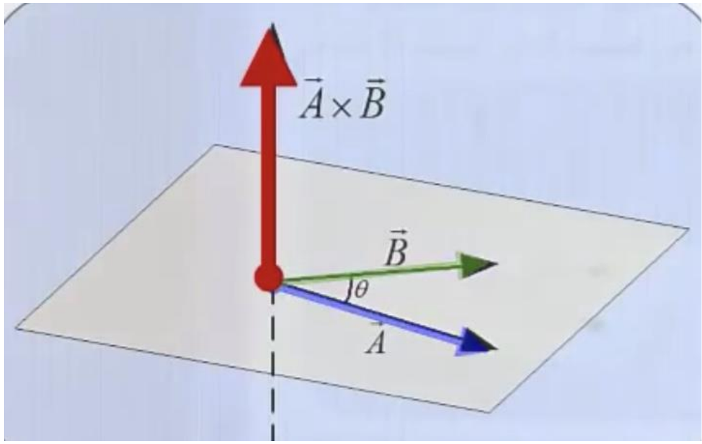


$\vec{A} \times \vec{B} = |A||B| \sin\theta$

$\vec{A} \times \vec{B} = (A_x\hat{i}+A_y\hat{j}+A_z\hat{k}) \times (B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$

$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)i + (A_zB_x - A_xB_z)j + (A_xB_y - A_yB_x)k$

If $a = a_1 i + a_2 j + a_3 k$ and $b = b_1 i + b_2 j + b_3 k$



$$a \times b = \begin{matrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$= (a_2 \times b_3 - a_3 \times b_2)\, i - (a_1 \times b_3 - a_3 \times b_1)\, j + (a_1 \times b_2 - a_2 \times b_1)\, k$$

$$\vec{C} = \vec{A} \times \vec{B}$$

$$C_x = (A_y B_{z} - A_z B_x)$$

$$C_y = (A_z B_{x} - A_x B_z)$$

$$C_z = (A_x B_{y} - A_y B_x)$$

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\therefore \quad \vec{C} = C_x \text{ i} + C_y \text{ j} + C_z \text{ k}$$

Example (3)

- If $\vec{C} = \vec{A} \times \vec{B}, where\ \vec{A} = 3i - 4j, and\ \vec{B} = -2i + 3k\ , what\ is\ C?$
- Solution
- $\vec{C} = \vec{A} \times \vec{B}$ = $(3i - 4j) \times (-2i + 3k)$
- Which by distributive law , becomes
- $\vec{C} = (3i \times -2i) + (3i \times 3k) + (-4j \times -2i) + (-4j \times 3k)$
- Using equation $\vec{A} \times \vec{B} = AB \sin\theta\ to\ evaluate$
- Each term in the equation above we got
- $\vec{C} = 0 - 9j - 8k - 12i = -12i - 9j - 8k$
- the vector C is perpendicular to both vectors A and B