جامعة البصرة كلية الهندسة

قسم هندسة النفط

المرحلة الثانية – موائع ساكنة (PeE211)

اسم الموضوع: موائع ساكنة

رقم الفصل: الاول

رقم المحاضرة:

اسم التدريسي: د. عمار علي

- Dimensions & units, dimensional analysis.
- Process variables: physical state, overall mass balance, overall energy balance, overall momentum balance.
- Concept of fluid behavior, Newtonian and non- Newtonian fluids, laminar and turbulent flow in circular tube.
- Flow measurement.
- Pitot tube, venturi menter, orifice meter, rota meter.

References: -

1. "Fluid mechanics" by V.L. Streeter, 9th Edition.
2. "Fluid Mechanics" Frank. M. White, 5th edition.
3. "Fundamentals of Fluid Mechanics" 5th edition B. R. Munson et al - John Wiley and Sons.
4. "Fluid Mechanics and Hydraulic Machines" 5th edition Er.R.K. RAJPUT - . CHAND & COMPANY LTD. RAM NAGAR, NEW DELHI-110 055.
5. "ميكانيك الموائع" د. كامل الشماع طبعة دار الكتب في جامعة البصرة
6. Lectures of other instructors in the collage & department.
7. Any other references in this field.

1. These lectures were prepared and used by me to conduct lectures for 2st year B. Tech. students.
2. Theories, Figures, Problems, Concepts used in the lectures to fulfill the course requirements are taken from the general fluids references
3. I take responsibility for any mistakes in solving the problems. Readers are requested to rectify when using the same.
4. I thank the following authors for making their books & lectures available for reference

A. Ali

Chapter 1

Fluids Mechanics

What is fluid mechanics?

Fluid mechanics may be defined as that branch of Engineering-science which deals with the behavior of fluid under the conditions of ***rest and motion.***

The fluid mechanics may be divided into three parts: ***Statics, kinematics and dynamics***

Statics. The study of ***incompressible fluids*** under ***static conditions*** is called hydrostatics and that dealing with the ***compressible static gases is termed as aerostatics.***

Kinematics. It deals with the ***velocities, accelerations and the patterns of flow only***. ***Forces or energy*** causing velocity and acceleration are ***not*** dealt under this heading.

Dynamics. It deals with the relations between ***velocities, accelerations*** of fluid with the ***forces or energy*** causing them.

Properties of Fluids:

The matter can be classified on the basis of the ***spacing between the molecules*** of the matter as follows:

- **Solids**, the molecules are *very closely* spaced whereas
- **Liquids** (Liquid state, Gaseous state) the spacing between the different molecules is *relatively large* and in gases the spacing between the molecules is still large.

1 Fluid

A fluid may be defined as follows:

"A fluid is a substance which is capable of flowing."

or

"A fluid is a substance which ***deforms continuously when subjected to external shearing force****."*

Ideal fluids. An ideal fluid is one which ***has no viscosity and surface tension and is incompressible***. In true sense no such fluid exists in nature. However fluids which have ***low viscosities such as water and air*** can be treated as ideal fluids under certain conditions. The assumption of ideal fluids ***helps in simplifying the mathematical analysis***.

Real fluids. A real practical fluid is one which ***has*** viscosity, surface tension and compressibility in addition to the density. The real fluids are actually available in nature.

Continuum. A continuous and homogeneous medium is called continuum. From the continuum view point, the overall properties and behavior of fluids can be studied ***without regard for its atomic and molecular structure***.

Dimensions:

Mass	Length	Time	Force
M	L	T	F

Types of Systems:

i: M-L-T

ii: F-L-T

Units:

System/Quantity	Mass	Length	Time	Force
Standard International (S.I)	kg	m	sec	N
British System (English)	slug	ft	sec	lb
French system (c.g.s)	gm	cm	sec	dyne
Kilogram weight system	kg	m	sec	kgw

Length			**Mass**	
1 ft =	12 inches or 12”		1 slug =	14.59 kg
1 inch =	2.54 cm		1 ton =	1000 kg
1 ft =	m		1 kg =	1000 g
mile =	1609 m			
Volume			**Gravitational acceleration**	
1 m^3 =	1000 liters = 10^6 cm^3		g = 9.81 m/sec^2 =	32.2 ft/sec^2
1 gallon	3.785 liters			
Force				
1 N =	1 kg.m/sec^2			
1 N =	10^5 dyne			
1 N =	(1/4.44) lb			
1 N =	(1/9.81) kgw			
1 kgw =	2.20462 lb			

2 Liquids and Their Properties

The properties of water are of much importance because the subject of hydraulics is mainly concerned with it. Some important properties of water which will be considered are:

(i) Density,
(ii) Specific gravity,
(iii) Viscosity,
(iv) Vapor pressure,
(v) Cohesion,
(vi) Adhesion,
(vii) Surface tension,
(viii) Capillarity, and
(ix) Compressibility

2.1 DENSITY

- ***Mass Density***

Its units are kg/m^3 , i.e., $\rho = \frac{\text{mass}}{\text{Volum}} = \frac{\text{m}}{\text{V}}$

- ***Weight Density***

also known as ***specific weight***, weight per unit volume at the standard temperature and pressure. It is usually denoted by w.,

i.e., $w = \text{g}$

- ***Specific volume***

i.e., $v = \frac{\text{V}}{\text{m}}$

2.2 SPECIFIC GRAVITY

For liquids, the standard fluid is pure water at 4°C,

i.e., $Specific\ gravity = \frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}} = \frac{w_{liquid}}{w_{water}}$

Relative density or specific gravity (S of S.g): the ratio of mass density of a substance to a standard mass density. Generally, the standard mass density is taken of water at 4 ^{o}C. ρ_{wtare} at 4^{o}C = 1000 kg/m^3 .

Specific Volume (v): is the reciprocal of the density; that is, the volume occupied by unity mass of fluid

$$v = \frac{1}{\rho}\ , m^3/kg$$

Specific weight (γ or w): is the weight per unit volume.

$$\gamma = w = \frac{weight}{Volume} = \frac{mg}{V} = \rho g$$

Units: N/m^3

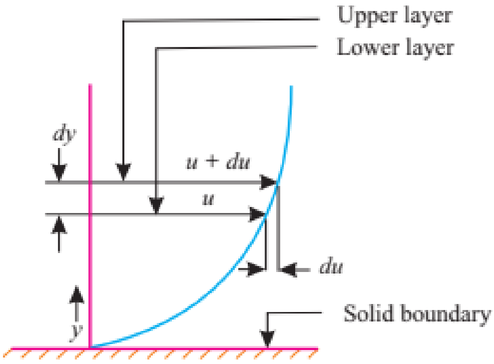
2.3 Viscosity

Property of a fluid which determines its ***resistance to shearing stresses***. It is a measure of the ***internal fluid friction which causes resistance*** to flow. It is primarily due to ***cohesion and molecular momentum exchange between fluid layers,*** and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid.

An ideal fluid has no viscosity.

The ***viscosity*** together with relative ***velocity*** causes a shear stress acting between the fluid layers. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by τ (called Tau)

$$\tau = \mu \frac{du}{dy}$$



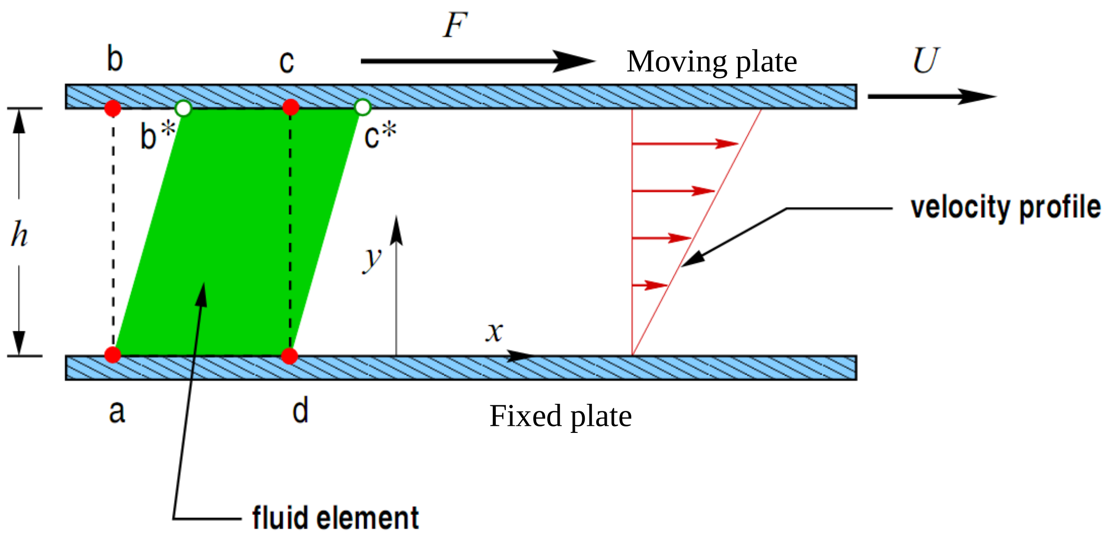
Now, μ is called the viscosity (or dynamic viscosity) of fluid, and the relation above is the ***Newton's law of viscosity.***

A **fluid** is a substance that ***deforms continuously*** when subjected to a shear stress.

Shear force, the force component tangent to a surface, and this force divided by the area of the surface is the ***average shear stress over the area***.

Note:

The fluid in immediate contact with a solid boundary has the same velocity as the boundary i.e. there is no slip at the boundary.



The fluid in the area a-b-c-d flows to the new position a-b*-c*-d, each fluid particle ***varying uniformly from zero at the fixed plate to U*** at the moving plate.

$$F \alpha \frac{A\,U}{h}$$

Where *A* is the surface area of the moving plate. The proportionality constant depends on fluid type, it is generally termed as μ.

$$Let \quad \tau = \frac{F}{A} : is\ the\ shear\ stress$$

$$\therefore \ \tau = \mu \frac{U}{h} : is\ the\ shear\ stress$$

$\frac{U}{h}$: is the ratio of angular velocity of line ab or it is **the *rate of deformation*** of fluid

$\frac{U}{h} = \frac{du}{dy}$ $\ for\ linear\ velocity\ distribution\ only$

$$\rightarrow \tau = \mu \frac{du}{dy} = \mu \frac{U}{h}$$

Units of viscosity:

μ =τ/(U/h) : (N/m^2)/(m/sec/m) = $\frac{N}{m^2}.sec$ = Pa. sec (in SI) or $\mu = \frac{gm}{cm.sec} = \frac{dyne\ sec.}{cm^2} = poise$

1 poise = 0.1 Pa. sec.

Kinematic viscosity (ν): is the dynamic viscosity μ divided by the density ρ

$$\nu = \frac{\mu}{\rho}$$

Units:

$$\frac{N.sec/m^2}{kg/m^3} = \frac{m^2}{sec}$$

$1 cm^2/sec$ = St (Stokes)

1 St = 10^{-4} m^2/sec

1 cSt = 10^{-6} m^2/sec

Newton's Law of Viscosity

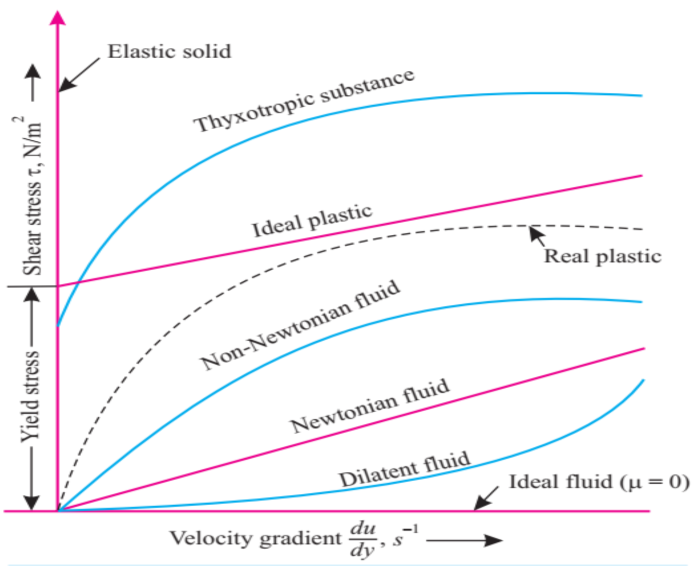
This law states that the ***shear stress(τ)on a fluid element layer is directly proportional to the rate of shear strain***. The constant of proportionality is called the co-efficient of viscosity.

$$Mathematically, \rightarrow \qquad \tau = \mu \frac{du}{dy}$$

Types of Fluids

The fluids may be of the following types: Refer to Fig.

1.Newtonian fluids: These fluids ***follow Newton's viscosity equation.*** For such fluids μ does not change with rate of deformation. Examples. Water, kerosene, air etc.



Variation of shear stess with velocity gradient.

2. Non-Newtonian fluids: Fluids which ***do not follow*** the linear relationship between shear stress and rate of deformation. Such fluids are relatively uncommon. Examples. Solutions or suspensions (slurries), mud flows, polymer solutions, blood etc.

3 .Plastic fluids. In the case of a plastic substance which is ***non-Newtonian fluid*** an initial yield stress is to be exceeded to cause a continuous deformation. These substances are represented by straight line intersecting the vertical axis at the “yield stress”.

- ***An ideal plastic*** (or Binigham plastic) has a definite yield stress and a constant linear relation between shear stress and the rate of angular deformation. Examples: Sewage sludge, drilling muds etc.
- ***A thyxotropic substance***, which is non-Newtonian fluid, has a non-linear relationship between the shear stress and the rate of angular deformation, beyond an initial yield stress. The printer’s ink is an example of thyxotropic substance.

4. Ideal fluid. An ideal fluid is one which is ***incompressible and has zero viscosity*** Thus an ideal fluid is represented by the horizontal axis ($\tau = 0$)

A true elastic solid may be represented by the vertical axis of the diagram.

Summary of relations between shear stress (τ) and rate of angular deformation for various types of fluids:

(i) *Ideal fluids*: $\tau = 0$, (ii) *Newtonian fluids*: $\tau = \mu \cdot \frac{du}{dy}$,

(iii) *Ideal plastics*: $\tau = const. + \mu \cdot \frac{du}{dy}$, (iv) *Thyxotropic fluids*: $\tau = const. + \mu . \left(\frac{du}{dy}\right)^n$, *and*

(v) *Non-Newtonian fluids*: $\tau = \left(\frac{du}{dy}\right)^n$.

Effect of Temperature on Viscosity

Viscosity is effected by temperature. The viscosity of liquids ***decreases*** but that of gases ***increases*** with increase in temperature. ***Why?***

For liquids: $\mu_T = Ae^{\beta/T}$

For gases: $\mu T = \dfrac{bT^{1/2}}{1 + a/T}$

where, μ_T = Dynamic viscosity at absolute temperature T,

A, β = Constants (for a given liquid), and

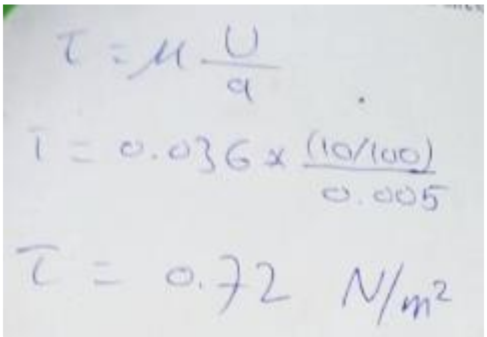
a, b = Constants (for a given gas).

Effect of Pressure on Viscosity

The viscosity under ordinary conditions is ***not appreciably affected*** by the changes in pressure. However, the viscosity of some oils has been found to increase with increase in pressure.

Read Examples (1.6-1.20)

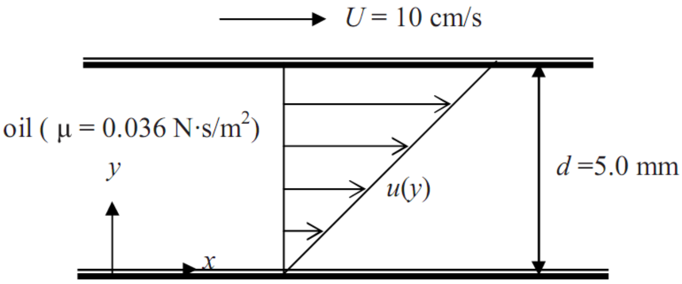
Example 1.1: Determine the shear stress exerted on the bottom fixed surface shown in figure.



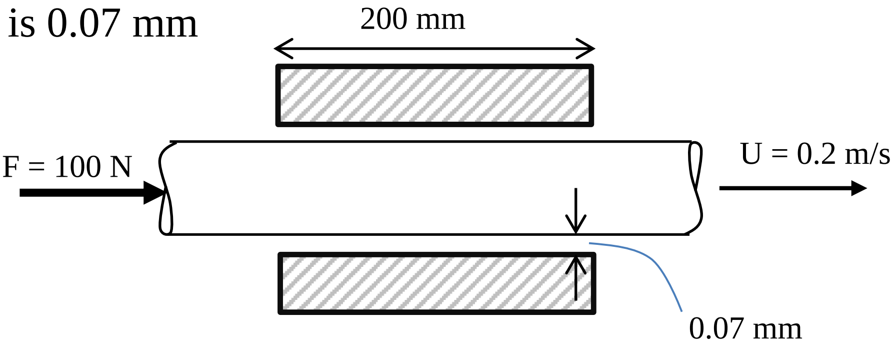
$\tau = \mu \frac{U}{a}$

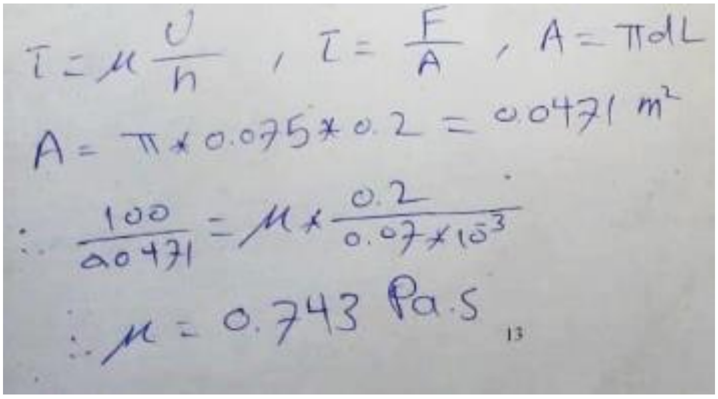
$\tau = 0.036 \times \frac{(10/100)}{0.005}$

$\tau = 0.72 \ N/m^2$



Example 1.2: Determine the dynamic viscosity of fluid between the 75 mm-diameter shaft and sleeve shown in figure. The clearance between the shaft and sleeve is 0.07 mm





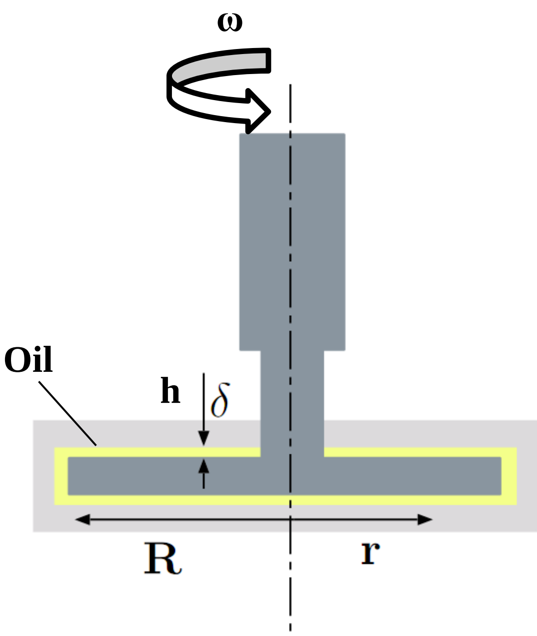
$\tau = \mu \frac{U}{h}$, $\tau = \frac{F}{A}$, $A = \pi d L$

$A = \pi * 0.075 * 0.2 = 0.0471 \ m^2$

$\therefore \frac{100}{0.0471} = \mu * \frac{0.2}{0.07 * 10^{-3}}$

$\therefore \mu = 0.743 \ Pa.s$

Example 1.3: A disk of radius R rotates at angular velocity ω inside an oil bath of viscosity μ as shown in figure. Derive an expression for the viscous torque on the disk. Neglect shear stress on the outer disk end.



Sol: $T = \frac{\mu\omega\pi R^4}{h}$

$$\tau = \frac{F}{A} = \mu \frac{U}{h}$$

$$T = F * r \quad , \quad dT = dF * r$$

$$\frac{dF}{dA} = \frac{dT}{r\,dA} = \mu \frac{U}{h} = \mu \frac{\omega r}{h}$$

$$dT = \frac{\mu\omega}{h} r^2 dA$$

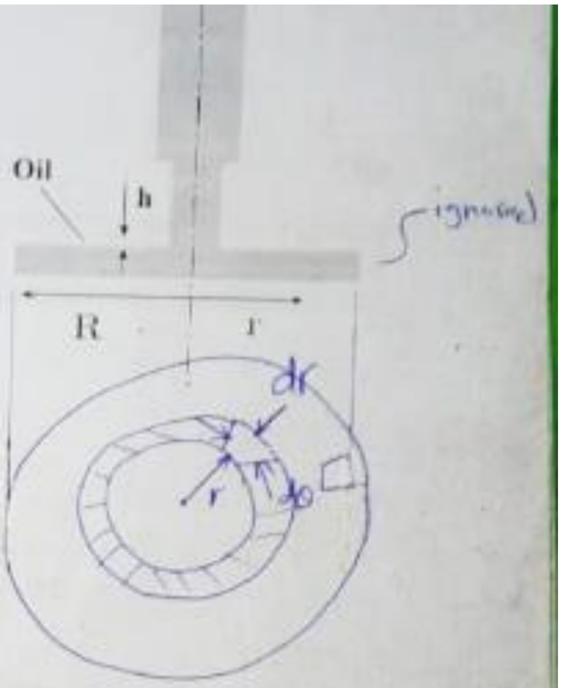
$$\because dA = r\,dr\,d\theta$$

$$dT = \frac{\mu\omega}{h} r^3 dr\,d\theta$$

$$T = \frac{2\mu\omega}{h} \int_0^R \int_0^{2\pi} r^3 dr\,d\theta$$

$$= \frac{2\mu\omega}{h} (2\pi) \int_0^R r^3 dr = \frac{4\mu\omega\pi}{h} \frac{r^4}{4} \Big|_0^R$$

$$= \frac{\mu\omega\pi R^4}{h}$$



ملاحظة: تم ضرب العزم * 2 لأن الزيت موجود على طرفي القرص

2.4 SURFACE TENSION AND CAPILLARITY

2.4.1 Surface Tension

Cohesion means intermolecular attraction between molecules of the same liquid. It enables a liquid to resist small amount of tensile stresses. Cohesion is a tendency of the liquid to remain as one assemblage of particles. "Surface tension" is due to ***cohesion between particles*** at the free surface.

Adhesion. Means attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a ***liquid to stick to another body***.

Capillary action is due to both cohesion and adhesion.

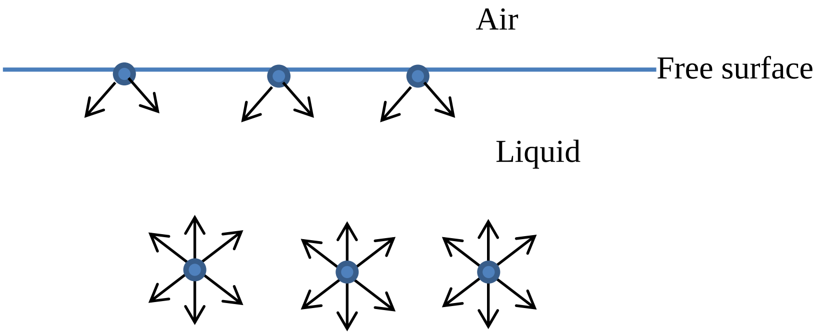
Surface tension (σ) is caused by the force of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by other molecules all around and is in equilibrium.

Surface tension of liquid is due to the ***force of attraction*** between similar molecules, called cohesion, and those between different molecules, called adhesion.

- The interior molecules are in balance.
- Near a free surface, the cohesion force ***between liquid molecules*** is much greater than that between an ***air molecule and a liquid*** molecule, *hence*, there is ***a resultant force on a liquid molecule*** acting toward the interior of the liquid. This force called surface tension.
- It is the force that *holds a water droplet or mercury globule together*.
- It is the force that ***form a film*** at the interface between a liquid and gas or two immiscible liquids

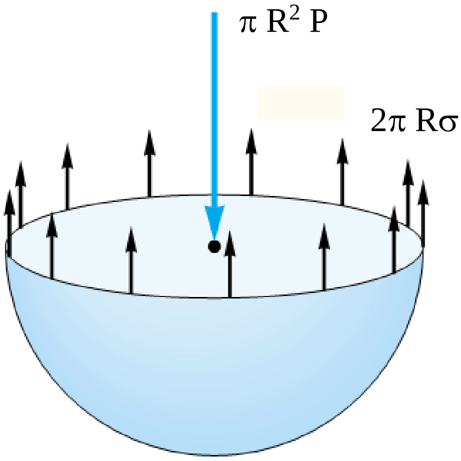
- This force is ***proportional*** to the product of ***the surface tension coefficient σ and the length of the free surface***.

Surface tension force = $\sigma *$ length of the free surface



Pressure Inside a Water Droplet, Soap Bubble and a Liquid Jet

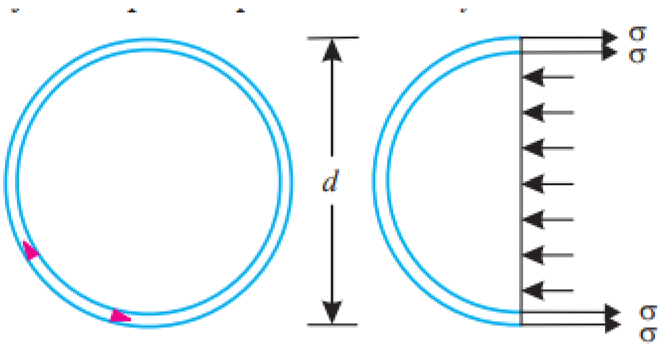
For a ***spherical droplet***: radius *R*, internal pressure *P*, the force balance on a hemispherical free body gives:



$$\pi R^2 P = 2\pi R \sigma$$

$$\therefore P = \frac{2\sigma}{R}$$

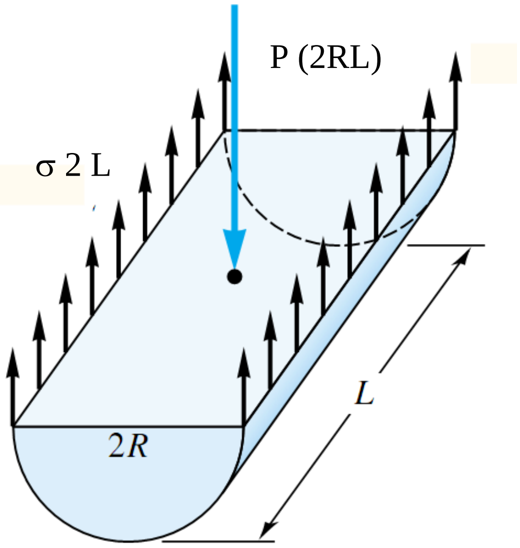
For a ***Soap(or hollow)bubble***: Soap bubbles have two surfaces on which surface tension σ acts. free body gives:



$$\frac{\pi}{4} d^2 P = 2(\pi d \sigma)$$

$$\therefore \quad P = \frac{8}{d}\sigma$$

Fore a **cylindrical liquid jet** of radius R, the force balance gives:



$$2RL\,P = \sigma\,2L$$

$$\therefore P = \frac{\sigma}{R}$$

Hence, the action of ***surface tension*** is to increase the pressure within a ***drople***t of liquid or ***Soap bubble*** or within a small ***liquid jet***.

See Example 1.22-1.26

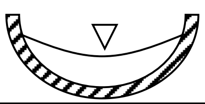
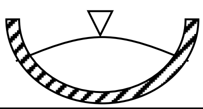
2.4.2 Capillarity: Capillarity is a phenomenon by which a liquid (depending upon its specific gravity) rises into a thin glass tube above or below its general level. This phenomenon is ***due to*** the combined effect of cohesion and adhesion of liquid particles.

It is useful to re-mention here the attraction force types.

Attraction force

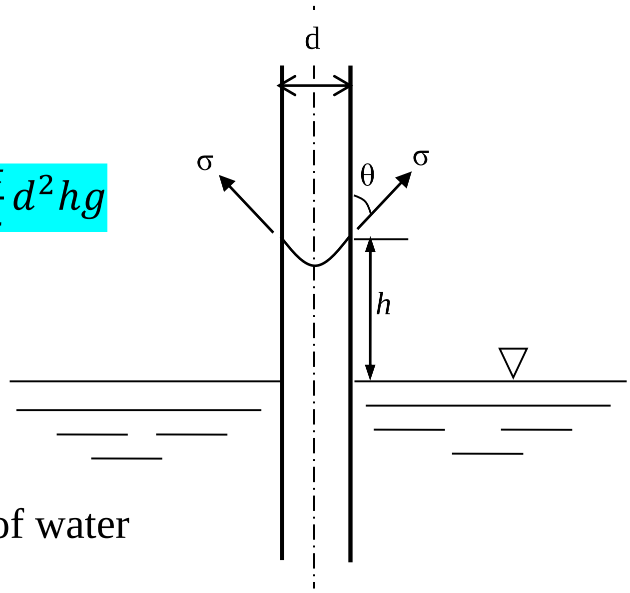
Adhesion: between the fluid and the boundary

Cohesion: between the fluid particles

Adhesion > Cohesion Meniscus concave	Cohesion > Adhesion Meniscus convex
	

Capillarity in a tube:

Balancing forces in y direction:



1- Surface tension force = $\pi d\sigma\ cos\theta$

2- Force due to weight = $mg = \rho V g = \rho \frac{\pi}{4} d^2 hg$

$$\pi d\sigma\ cos\theta = \rho \frac{\pi}{4} d^2 hg$$

$$\therefore h = \frac{4\sigma \cos\theta}{\rho g d}$$

For example: the surface tension coefficient of water

Equals to 0.074 N/m at 20 ^{o}C.

For water and glass :θ≈0. (θ=Angle of contact of the water surface)

Hence the capillary rise of water in the glass tube

$$\therefore h = \frac{4\sigma}{\rho g d}$$

Bulk Modulus of compression (E)

$$E = \frac{Change\ in\ pressure}{volumetric\ strain} = \frac{dP}{\left|\frac{dV}{V}\right|}$$

Units: N/m^2

Perfect Gas

The perfect gas is defined as a substance that satisfies the perfect gas law.

$$P v = R T, \qquad or \qquad P = \rho R T$$

T: must be absolute (in Kelvin)

R is the gas constant (J/kg.K)

P: absolute pressure (N/m^2)

***Vapor pressure* (Pv)**

The pressure value at which the ***liquid molecules escaping from the liquid surface.*** The vapor pressure of a given fluid ***increases with temperature***.

For example, table below, displays some values of water vapor pressures at different temperatures,

Temperature (^oC)	Pv (Pa)
0	588.3
5	882.54
10	1176.36

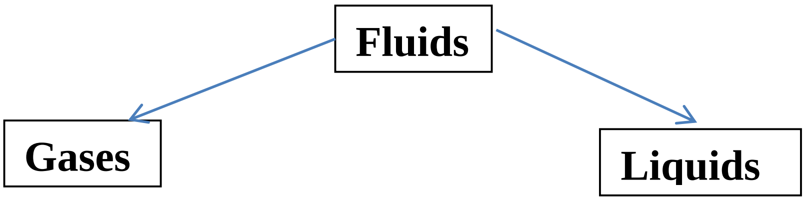
See Example 1.27-1.40

HW.01 SOLVE PROBLEM PAGE 41-42

Chapter Two

FLUID STATICS

PRESSURE MEASUREMENT



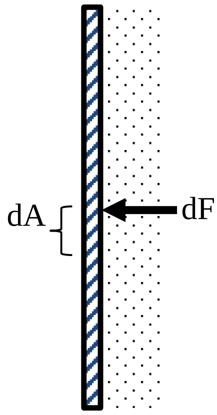
Gases: occupy the whole volume of container. The viscosity increases with increasing temperature, ***due to the increase of momentum change between layers.***

Liquids: form a free surface. The viscosity ***decreases*** with increasing temperature. Because in liquids, the molecules are so ***much closer*** than in gases, so with temperature increase, the ***cohesive*** forces hold the molecules may decrease.

2.1. Pressure of A Liquid

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The force per unit area is called pressure.

It is the normal force pushing against a plane area divided by the area. It ***results from the continuous movement of molecules***.



$$P = \frac{dF}{dA}$$

Units:

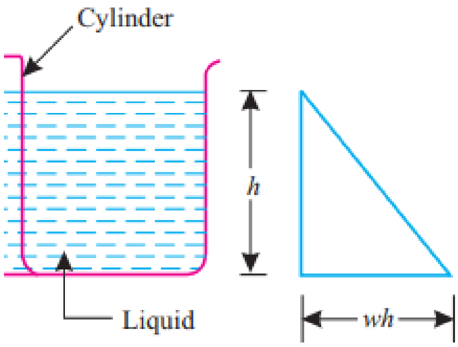
- N/m^2 (Pascal), lb/ft^2 (psf), lb/in^2 (psi), or bar
- 1 bar = 10^5 Pascal
- Sometimes, the pressure is expressed as a pressure head, m-fluid

- In industrial, they may used $\frac{kg}{cm^2} = \frac{kgf}{cm^2} = \frac{1 \times 9.81\ N}{cm^2} = 98.1\ kPa$

2.2. Pressure Head of a Liquid

Consider a vessel containing liquid,

Now, Total pressure on the base of the cylinder = Weight of liquid in the cylinder



$$F.A = \gamma.A.h$$

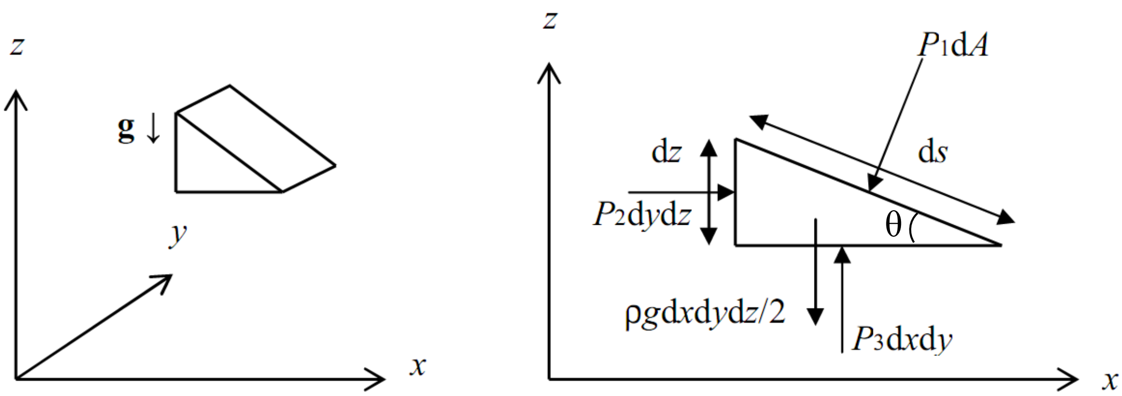
$$\therefore\ F = \gamma.h$$

$$\therefore h = \frac{F}{\gamma}$$

$$where\ \gamma = w = (\rho g)$$

2.3. Pascal's Law (Pressure Acting on A Point)

It can be proven that the pressures acting on a point at rest, has the same value in all directions. Let us assume a particle of a fluid at rest, with free body diagram shown in figure.



$dA = ds \cdot dy = dy \cdot dz/\sin\theta$

$$\sum \mathbf{F} = 0$$

$$F_x = P_2 \mathrm{d}y\mathrm{d}z - P_1 \mathrm{d}A \sin\theta = 0$$

$$P_2 \mathrm{d}y\mathrm{d}z = P_1 \mathrm{d}y \frac{\mathrm{d}z}{\sin\theta} \sin\theta$$

$$P_2 = P_1$$

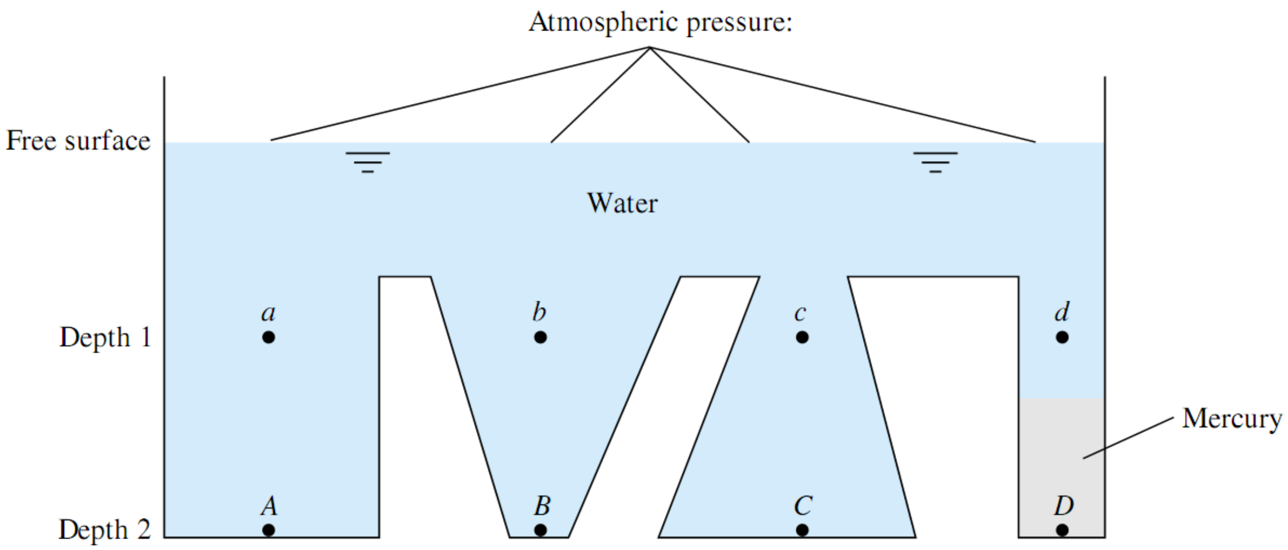
$$F_z = P_3 \mathrm{d}y\mathrm{d}x = \frac{1}{2}\rho g \mathrm{d}x\mathrm{d}y\mathrm{d}z + P_1 \mathrm{d}y \frac{\mathrm{d}x}{\cos\theta} \cos\theta$$

$$P_3 = P_1 + \frac{1}{2}\rho g \mathrm{d}z$$

$$\mathrm{d}z \rightarrow 0,\ P_3 = P_1$$

$$\therefore P_1 = P_2 = P_3$$

Note: Pressure ***doesn't vary horizontally***, provided that the fluid is connected. To illustrate this statement, we may refer to the figure below.



Points *a*, *b*, *c*, and *d* are at equal depths in water and therefore have identical pressures. Points *A*, *B*, and *C* are also at equal depths in water and have identical pressures higher than *a*, *b*, *c*, and *d*. Point ***D* has a different** pressure from *A*, *B*, and *C* because it is not connected to them by a water path.

2.4. Absolute and Gauge Pressures

Atmospheric pressure:

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as ***atmospheric pressure***. The atmospheric pressure is also known as '***Barometric pressure'***.

Gauge pressure:

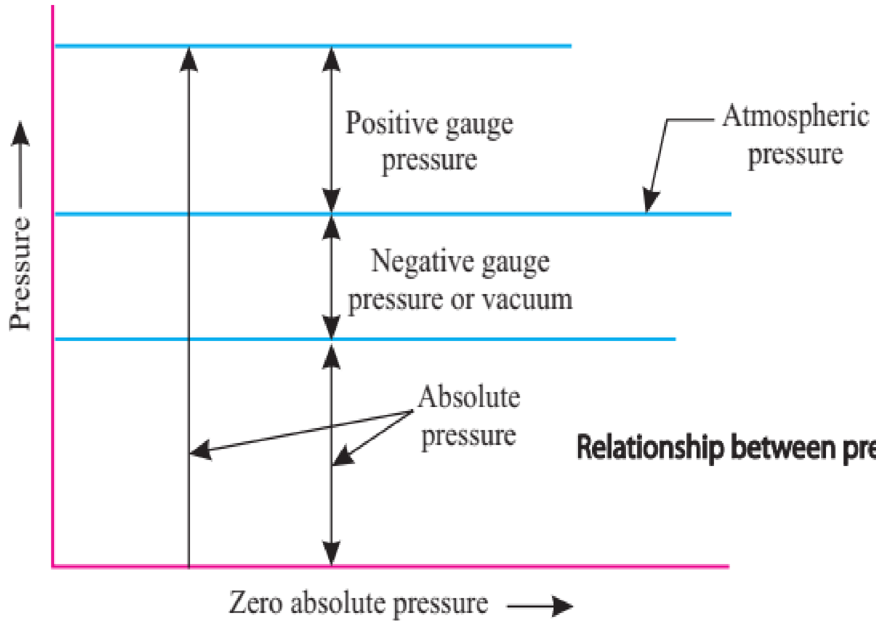
It is the pressure, measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken as ***datum***.

Gauges record pressure above or below the local atmospheric pressure, since they measure the ***difference in pressure*** of the liquid to which they are connected and that of surrounding air.

Absolute pressure:

It is necessary to establish an absolute pressure scale which is ***independent of the changes in atmospheric pressure***. A pressure of absolute zero can exist only in complete vacuum.

A schematic diagram showing the gauge pressure, vacuum pressure and the absolute pressure is gi



Relationship between pressures.

Mathematically:

i.e., $P_{abs} = P_{atm} + P_g$ & $P_{vac} = P_{atm} - P_{abs}$

See Example 2.1-2.10

2.5. Measurement of Pressure

The pressure of a fluid may be measured by the following devices:

1- Manometers:

devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid. These are classified as follows:

(a) Simple manometers:

(i) Piezometer,

(ii)U-tube manometer, and

(iii)Single column manometer.

(b)Differential manometers.

2- Mechanical gauges:

in which the pressure is measured by balancing the fluid column by spring (elastic element) or dead weight. Generally, these gauges are used for measuring **high pressure** and where high precision is not required. Some commonly used mechanical gauges are:

(i)Bourdon tube pressure gauge,

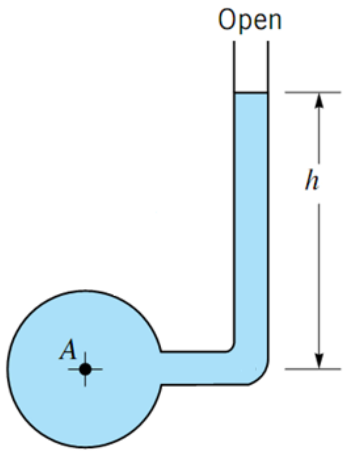
(ii)Diaphragm pressure gauge,

(iii)Bellow pressure gauge, and

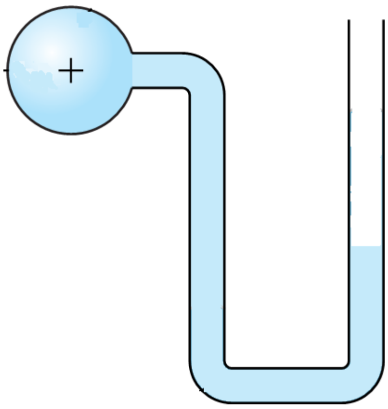
(iv)Dead-weight pressure gauge.

Manometers: devices that employ liquid columns for determining differences in pressure:

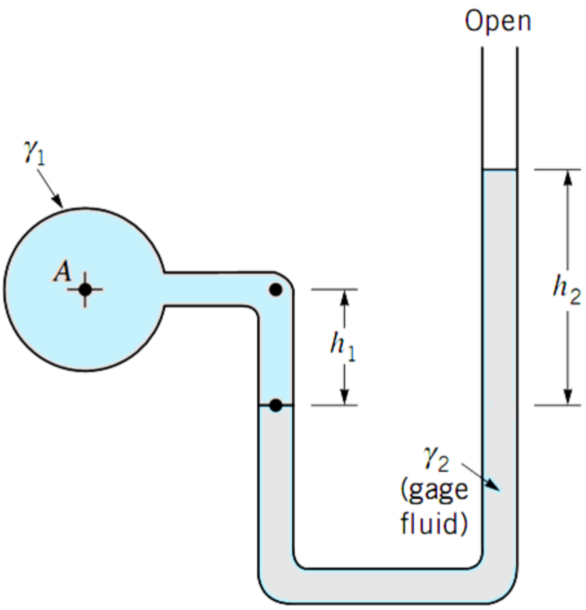
1- **Piezometer Manometer**: The simplest type of manometer consists of a vertical tube, open at the top, and attached to the container in which the pressure is required, it is used for small positive pressures.



2- **U-Tube Manometer**: This type of manometer consists of a tube formed into the shape of a U filled with the ***same fluid*** to be measured. It is used for small positive and negative pressures.



3- **U-Tube Manometer with Multi-Liquids**: It is U tube with using another liquid(s) of greater gravity. It is used for greater positive and negative pressure.



General Procedure in Working with Manometers Problems.

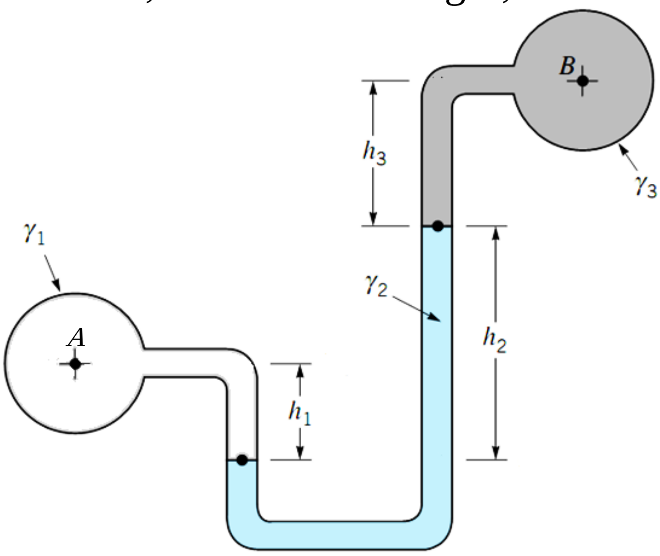
1- Start at one end and write the pressure there.
2- Add to the started pressure the change in pressure in the same unit from one meniscus (liquid surface) to the next (plus for lower meniscus and minus for higher)
3- Continue until the other end of the gage, and equate the expression to the pressure at that point.

$$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

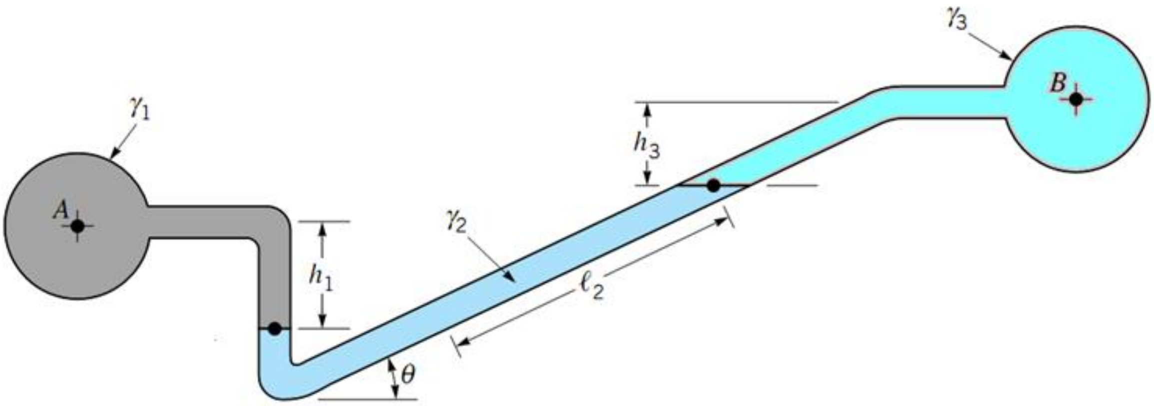
Or, $P_A - P_B = -\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3$

Note: If any tube section is filled with gas, then the elevation in this section can be ignored because the specific weight (γ) of gases is much less than liquids. For example, in the figure shown, if fluid 1 is a gas, then the manometer relation will be:

$$P_A - P_B = \gamma_2 h_3 + \gamma_3 h_3$$

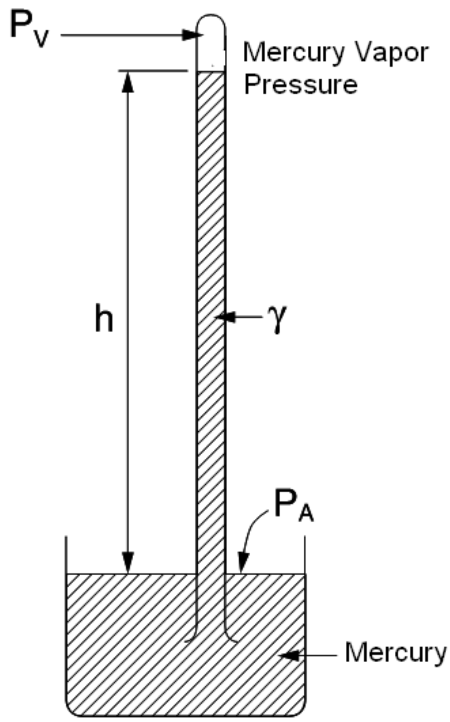


Inclined Tube Manometer: this type of manometer is designed to increase the accuracy of pressure measurements.



$$P_A + \gamma_1 h_1 - \gamma_2 l \sin\theta - \gamma_3 h_3 = P_B$$

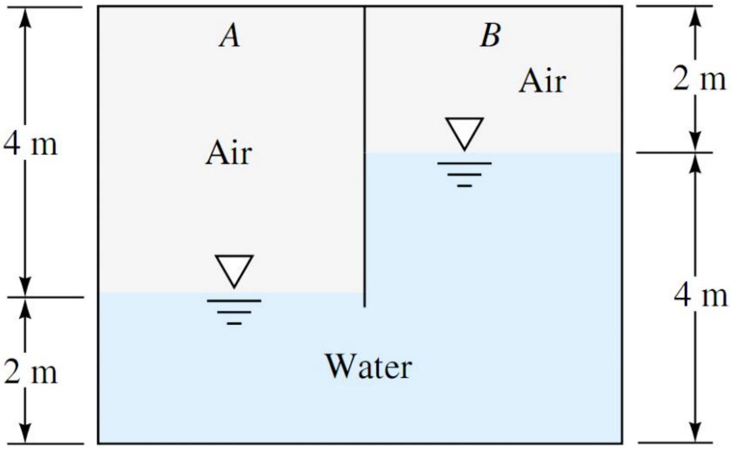
Mercury Barometer: it consists of a glass tube closed at one end and filled with mercury, and inverted so that the open end is submerged in mercury. It is used to measure the atmospheric pressure, P_A



$$P_A = \gamma_{Hg} h + P_V$$

P_V: is the pressure of mercury vapor

Example 2.3: For the closed tank shown in figure, the pressure at point *A* is 95 kPa absolute, what is the absolute pressure at point *B*?



Solution: First compute $\rho_A = p_A/RT = (95000)/[287(293)] \approx 1.13\ kg/m^3$, hence $\gamma_A \approx (1.13)(9.81) \approx 11.1\ N/m^3$. Then proceed around hydrostatically from point A to point B:

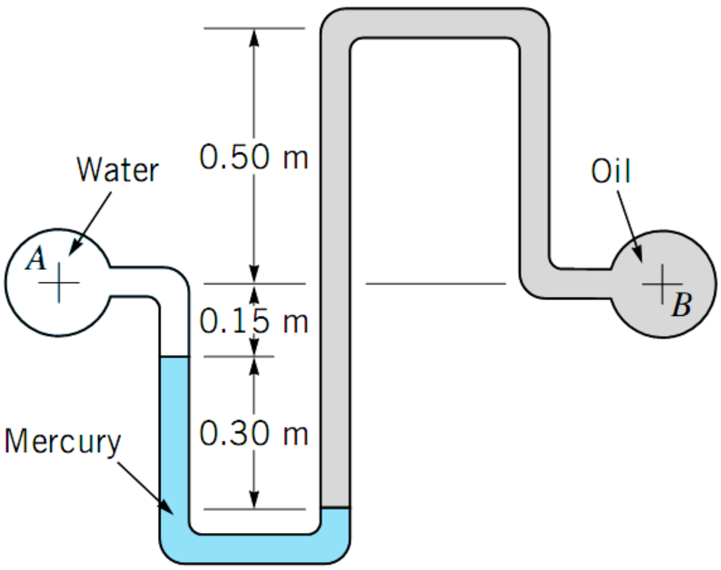
$$95000\ Pa + (11.1\ N/m^3)(4.0\ m) + 9790(2.0) - 9790(4.0) - \left(\frac{p_B}{RT}\right)(9.81)(2.0) = p_B$$

Solve for $p_B \approx$ **75450 Pa** *Accurate answer.*

If we neglect the air effects, we get a much simpler relation with comparable accuracy:

$$95000 + 9790(2.0) - 9790(4.0) \approx p_B \approx \mathbf{75420\ Pa}$$ *Approximate answer.*

Example 2.4: The mercury manometer shown indicates a differential reading of 0.30 m. Determine the differential pressure between pipe *A* and pipe *B*. What is the pressure value in pipe *B* when the pressure in pipe *A* is 30-mm Hg vacuum? (S_{oil}=0.83, S_{Hg}=13.6)



vacuum.

$P_A + \gamma_w \times 0.15 + \gamma_{Hg} \times 0.3 - \gamma_{oil} \times 0.45 = P_B$

$P_A - P_B = -\gamma_w (0.15 + S_{Hg} \times 0.3 - S_{oil} \times 0.45)$

$= -9810 (0.15 + 13.6 \times 0.3 - 0.83 \times 0.45)$

$= -37832.265\ Pa$

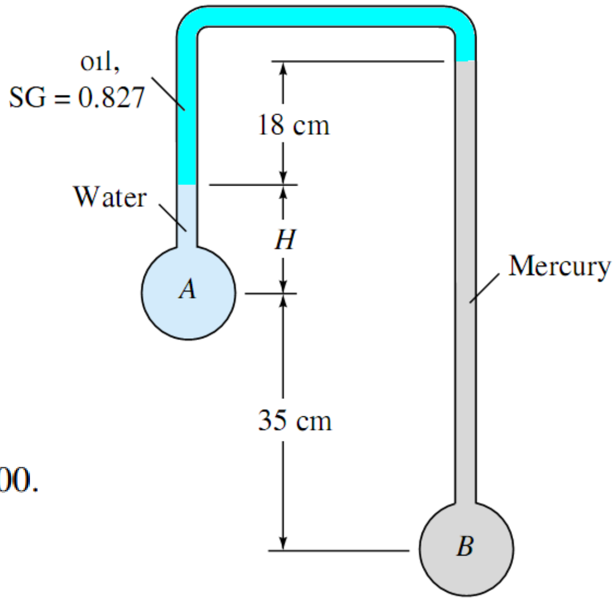
$\therefore P_A - P_B = -37.832\ kPa$

$P_A = 30\ mm\ Hg$

$P_A = 30 \times 10^{-3} \times \gamma_{Hg} = 0.03 \times 13.6 \times 9810 = 4002.5\ Pa$

$\therefore P_B = P_A + 37.832 \times 10^3 = 41.834 \times 10^3\ Pa$

~~Example~~ 2.5: For the inverted manometer shown in figure, if P_B-P_A= 90 kPa, what must the height *H* be?



Solution: Gamma = 9790 N/m^3 for water and 133100 N/m^3 for mercury and (0.827)(9790) = 8096 N/m^3 for Meriam red oil. Work your way around from point A to point B:

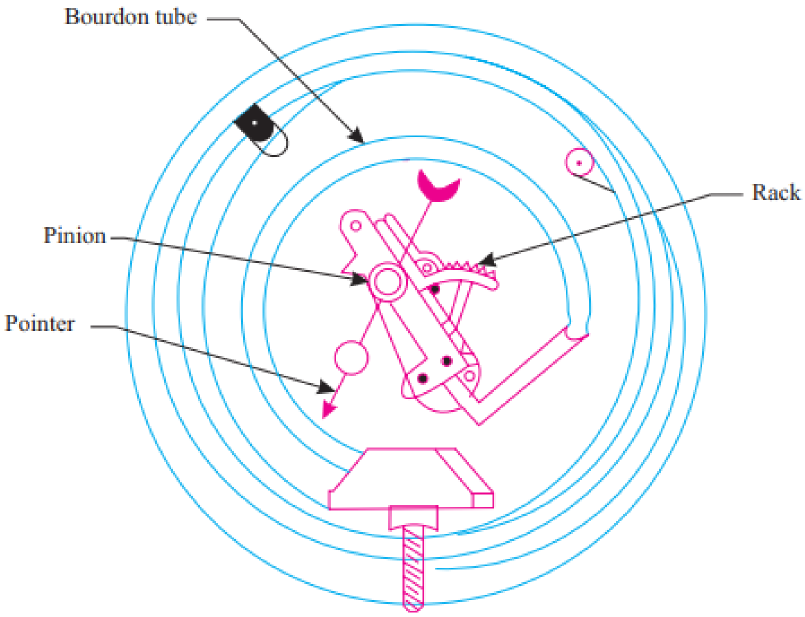
$$p_A - (9790\ N/m^3)(H\ meters) - 8096(0.18)$$

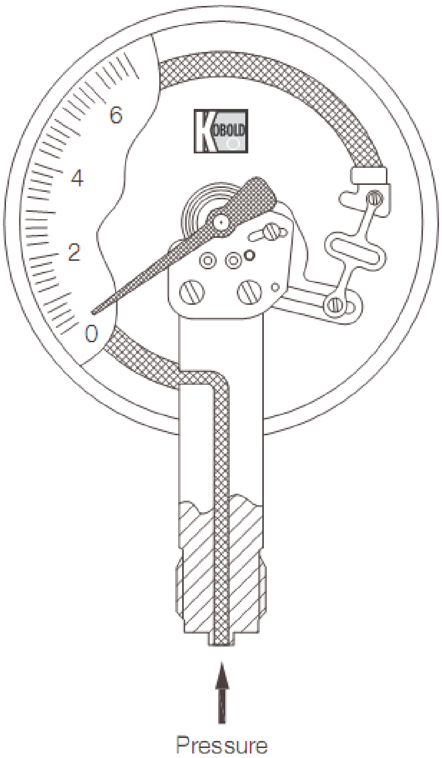
$$+133100(0.18 + H + 0.35) = p_B = p_A + 97000.$$

Solve for $H \approx 0.226\ m = \mathbf{22.6\ cm}$ *Ans.*

Bourdon Gauge

Bourdon gauge is a typical device used for measuring (high as well as low pressures) gauge pressure. It consists of a hollow, curved, flat metallic tube closed at one end; the other end in connected to the pressure to be measured. A scaled plate and pointer are needed for indication.





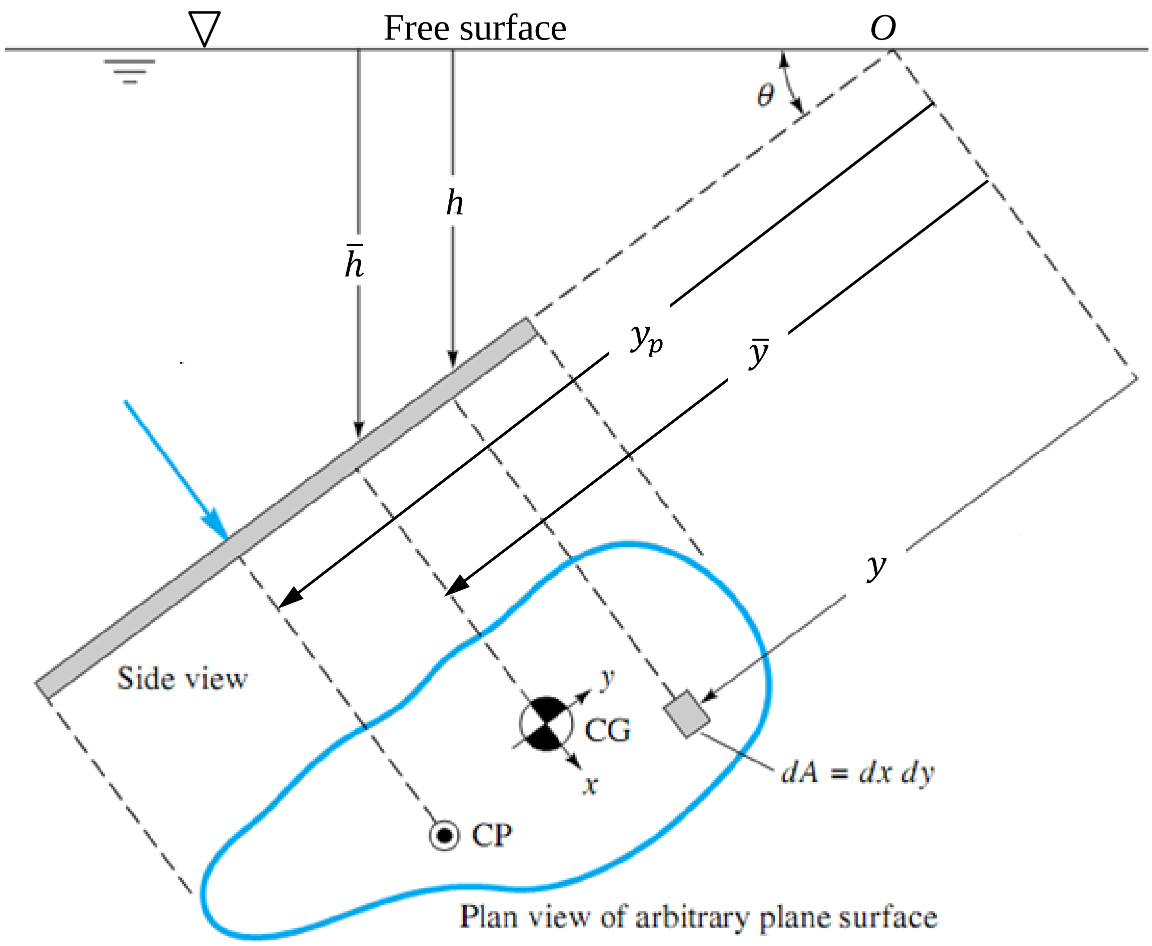
HW.02 SOLVE PROBLEM PAGE 94-96

Chapter 3

Forces on Immersed Surfaces

In the design of submerged devices and objects, surfaces, dams, surfaces on ships, and holding tanks, it is necessary to calculate the ***magnitude and location of forces*** that act on both plane and curved surfaces. This subject will be divided into two titles; ***plane*** and ***curved surfaces***.

1- Plane Surfaces (Horizontal; Vertical and Inclined Surface)



Cp: Center of pressure force
CG: center of geometry

The total force of the liquid on the plane surface is found by integrating the pressure over area.

$$F = \int_A P\, dA$$

Using gauge pressure, the local pressure is

$$P = \gamma h = \gamma\, y\, sin\theta$$

$\therefore F = \int_A \gamma\, y\, sin\theta\, dA$

$$\therefore F = \gamma\, sin\theta \int_A y\, dA$$

h is measured vertically down from the free surface and *y* is measured from point *O* on the free surface.

We know that the distance to a centroid is defined as:

$$\bar{y} = \frac{1}{A}\int_A y\, dA$$

$$\therefore F = \gamma\, sin\theta\, \bar{y}A = \gamma\, \bar{h}\, A = P_c A$$

Where Pc is the pressure at the centroid.

How to find the location of the resultant force F?

Generally, we termed to the location of the resultant force by y_p. Firstly, we should defined the well-known rule that says: *the sum of the moments of all the infinitesimal forces acting on the area A must equal the moment of the resultant force.*

$$y_p F = \int_A y\, P dA = \int_A y\, \gamma\, sin\theta\, y\, dA = \gamma\, sin\theta \int_A y^2\, dA$$

But, $I_x = \int_A y^2\, dA$ is the second moment of area about x-axis

$$y_p F = \gamma \sin\theta \, I_x$$

$$\therefore y_p = \frac{\gamma \sin\theta \, I_x}{\gamma \, \bar{y} \, A \sin\theta}$$

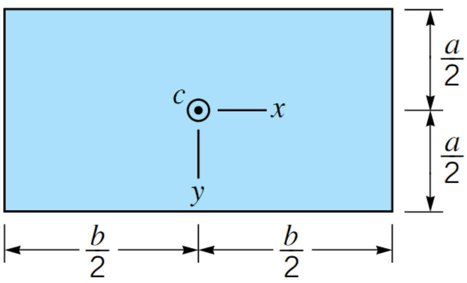
$I_x = I_{xc} + A\bar{y}^2$ (Parallel axis theorem),

Where I_{xc} is the second moment of area about the centroid axis.

$$\therefore y_p = \frac{I_{xc} + A\bar{y}^2}{\bar{y} \, A}$$

$$\therefore y_p = \bar{y} + \frac{I_{xc}}{\bar{y} \, A}$$

This equation clearly shows that the resultant force F doesn't pass through the centroid but it always below it.

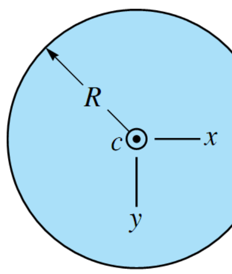


$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

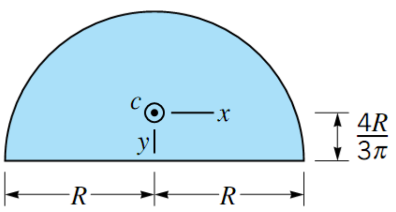
$I_{xyc} = 0$



$A = \pi R^2$

$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$

$I_{xyc} = 0$

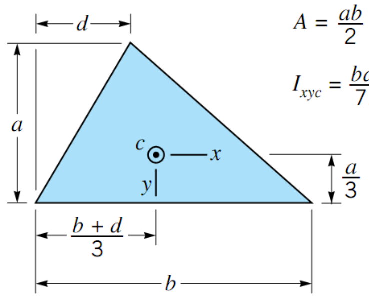


$A = \frac{\pi R^2}{2}$

$I_{xc} = 0.1098 R^4$

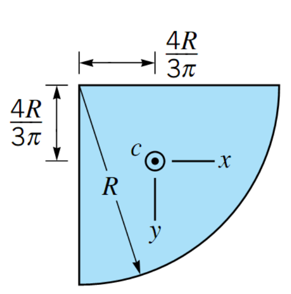
$I_{yc} = 0.3927 R^4$

$I_{xyc} = 0$



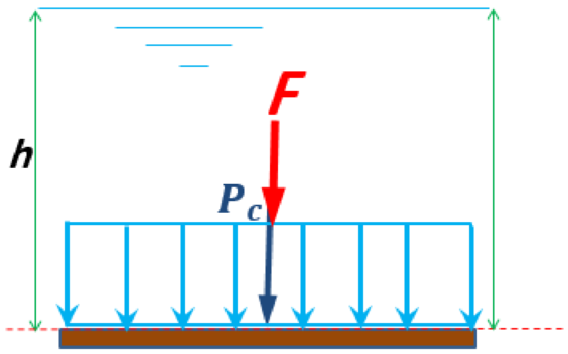
$A = \frac{ab}{2}$ $I_{xc} = \frac{ba^3}{36}$

$I_{xyc} = \frac{ba^2}{72}(b - 2d)$

 $A = \frac{\pi R^2}{4}$ $I_{xc} = I_{yc} = 0.05488R^4$ $I_{xyc} = -0.01647R^4$	

Note for horizontal and vertical surfaces:

❖ Horizontal surface



$$P_c = \gamma . h$$

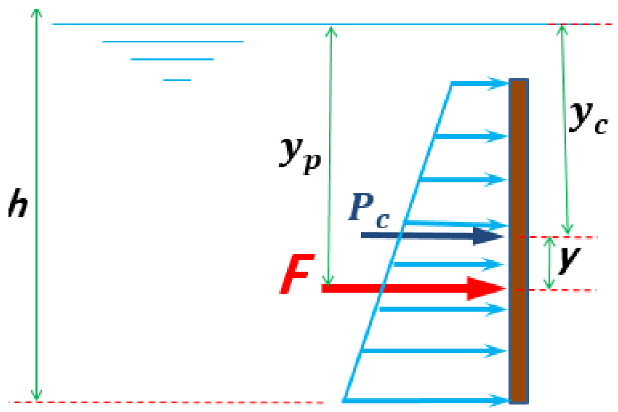
$$F = P_c . A$$

$y_p = h$ [redacted] $= y_c$

$where: P_c = pressure\ at\ the\ gate\ centroid = \gamma . h_p\ \ for\ same\ fluid$

$A = Area\ of\ the\ gate$

❖ Vertical surface



$$P_c = \gamma . y_c$$

$$F = P_c . A$$

$$y_p = y_c + y = y_c + \frac{I_{xc}}{h_c . A}$$

$where : I_{xc} = moment\ of\ inertia\ for\ the\ gate\ about\ the\ centroid$

Example .1. *Fig. 3.7 shows a circular plate of diameter 1.2 m placed vertically in water in such a way that the centre of the place is 2.5 m below the free surface of water. Determine: (i) Total pressure on the plate. (ii) Position of centre of pressure.*

Solution. Diameter of the plate, $d = 1.2$ m

Area,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 1.2^2 = 1.13 \text{ m}^2$$

$$\bar{x} = 2.5 \text{ m}$$

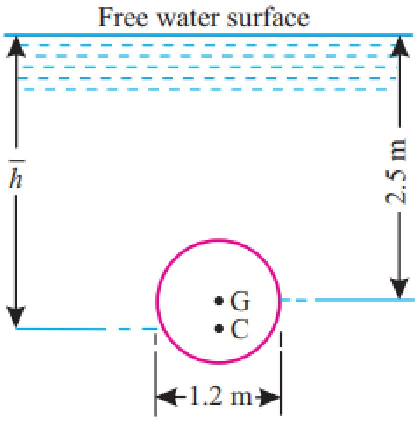


Fig. 3.7

(*i*) Total pressure, P:

Using the relation:

$$P = wA\bar{x} = 9.81 \times 1.13 \times 2.5$$

$$= \mathbf{27.7\ kN}\ \textbf{(Ans.)}$$

(*ii*) Position of centre of pressure, $\bar{h}$:

Using the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where,

$$I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 1.2^4 = 0.1018 \text{ m}^4$$

$$\bar{h} = \frac{0.1018}{1.13 \times 2.5} + 2.5 = 2.536 \text{ m}$$

i.e.

$$\bar{h} = \mathbf{2.536\ m}\ \textbf{(Ans.)}$$

Example .2. *A rectangular plate 3 metres long and 1 metre wide is immersed vertically in water in such a way that its 3 metres side is parallel to the water surface and is 1 metre below it. Find: (i) Total pressure on the plate, and (ii) Position of centre of pressure.*

Solution. Width of the plane surface, $b = 3$ m

Depth of the plane surface, $d = 1$ m

Area of the plane surface,

$$A = b \times d = 3 \times 1 = 3\ \text{m}^2$$

$$\bar{x} = 1 + \frac{1}{2} = 1.5\ \text{m}$$

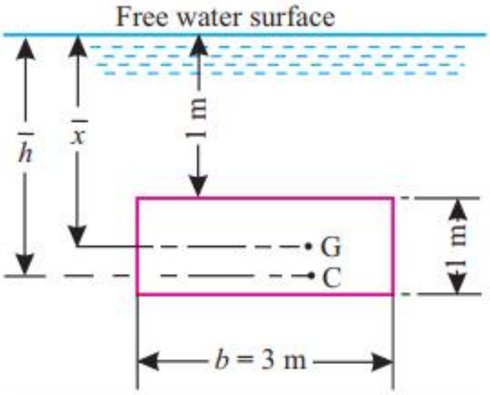


Fig. 3.8

(i) Total pressure P:

Using the relation:

$$P = wA\bar{x} = 9.81 \times 3 \times 1.5$$

$$= \textbf{44.14 kN (Ans.)}$$

(ii) Centre of pressure, $\bar{h}$:

Using the relation: $$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

But, $$I_G = \frac{bd^3}{12} = \frac{3 \times 1^3}{12} = 0.25\text{m}^4$$

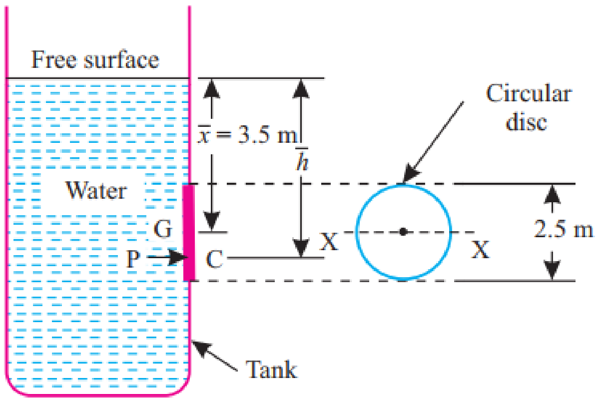
∴ $$\bar{h} = \frac{0.25}{3 \times 1.5} + 1.5 = 1.556\ \text{m}$$

i.e. $$\bar{h} = \textbf{1.556 m (Ans.)}$$

Example 3. *A circular opening, 2.5 m diameter, in a vertical side of tank is closed by a disc of 2.5 m diameter which can rotate about a horizontal diameter. Determine:*

(i) *The force on the disc;*

(ii) *The torque required to maintain the disc in equilibrium in vertical position when the head of water above horizontal diameter is 3.5 m.*



∴ Area of the opening,

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 2.5^2 = 4.91 \text{ m}^2$$

Depth of C.G., $\bar{x} = 3.5$ m

(*i*) Force on the disc, P:

Using the relation:

$$P = wA\bar{x} = 9.81 \times 4.91 \times 3.5$$
$$= \mathbf{168.6\ kN}\ \textbf{(Ans.)}$$

(*ii*) Torque required, *T*:

In order to determine the torque (T) required to maintain the disc in equilibrium, let us first calculate the point of application of force acting on the disc, *i.e.* centre of pressure of the force P. The depth of centre of pressure ($\bar{h}$) is given by the relation:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(\pi/64 \times d^4)}{(\pi/4 \times d^2)\bar{x}} + \bar{x} \qquad \left[\because I_G = \frac{\pi}{64} \times d^4\right]$$

$$= \frac{(\pi/64 \times 2.5^4)}{(\pi/4 \times 2.5^2) \times 3.5} + 3.5 = 3.61 \text{ m}$$

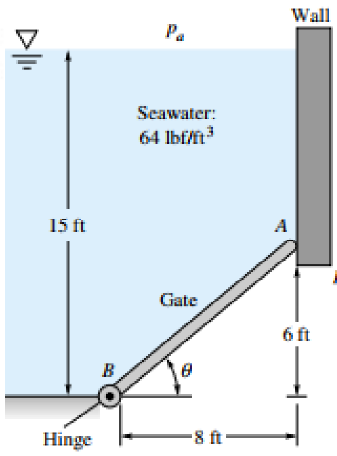
i.e., the force P is acting at a distance of 3.61 m from the free surface. Moment of this force about horizontal diameter $X - X$

$$= P(\bar{h} - \bar{x}) = 168.6\,(3.61 - 3.5)$$
$$= 18.55 \text{ kNm.} \qquad (anticlockwise)$$

Hence a torque (T) of **18.55 kNm** must be applied on the disc in the **clockwise direction** to maintain the disc in equilibrium position. **(Ans.)**

EXAMPLE:

The gate in Fig. below is 5 ft wide, is hinged at point *B*, and rests against a smooth wall at point *A*. Compute (*a*) the force on the gate due to seawater pressure, (*b*) the horizontal force *P* exerted by the wall at point *A*, and (*c*) the reactions at the hinge *B*.



Solution

Part (a) By geometry the gate is 10 ft long from A to B, and its centroid is halfway between, or at elevation 3 ft above point B. The depth h_{CG} is thus $15 - 3 = 12$ ft. The gate area is $5(10) = 50$ ft². Neglect p_a as acting on both sides of the gate. From Eq. (2.38) the hydrostatic force on the gate is

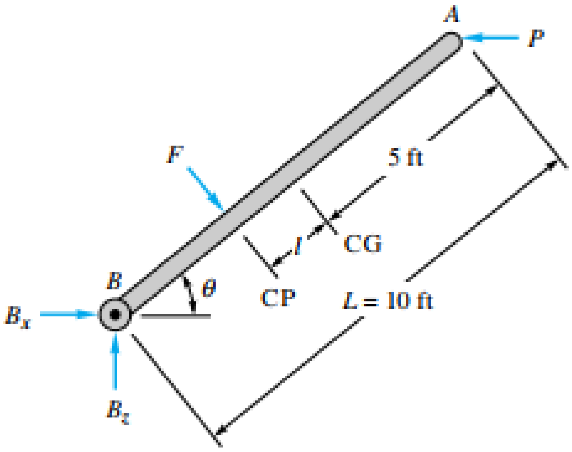
$$F = p_{CG}A = \gamma h_{CG}A = (64 \text{ lbf/ft}^3)(12 \text{ ft})(50 \text{ ft}^2) = 38{,}400 \text{ lbf} \qquad \textit{Ans. (a)}$$

Part (b) First we must find the center of pressure of F. A free-body diagram of the gate is shown in Fig. E2.5*b*. The gate is a rectangle, hence

$$I_{xy} = 0 \quad \text{and} \quad I_{xx} = \frac{bL^3}{12} = \frac{(5 \text{ ft})(10 \text{ ft})^3}{12} = 417 \text{ ft}^4$$

The distance l from the CG to the CP is given by Eq. (2.44) since p_a is neglected.

$$l = -y_{CP} = +\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{(417 \text{ ft}^4)(\frac{6}{10})}{(12 \text{ ft})(50 \text{ ft}^2)} = 0.417 \text{ ft}$$



The distance from point B to force F is thus $10 - l - 5 = 4.583$ ft. Summing moments counterclockwise about B gives

$$PL \sin \theta - F(5 - l) = P(6 \text{ ft}) - (38{,}400 \text{ lbf})(4.583 \text{ ft}) = 0$$

or $$P = 29{,}300 \text{ lbf}$$ *Ans. (b)*

Part (c) With F and P known, the reactions B_x and B_z are found by summing forces on the gate

$$\sum F_x = 0 = B_x + F \sin \theta - P = B_x + 38{,}400(0.6) - 29{,}300$$

or $$B_x = 6300 \text{ lbf}$$

$$\sum F_z = 0 = B_z - F \cos \theta = B_z - 38{,}400(0.8)$$

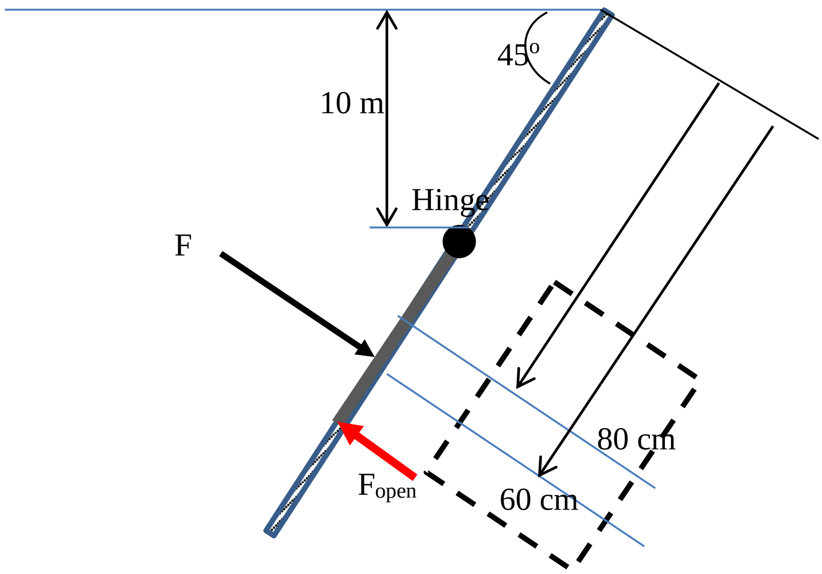
or $$B_z = 30{,}700 \text{ lbf}$$ *Ans. (c)*

This example should have reviewed your knowledge of statics.

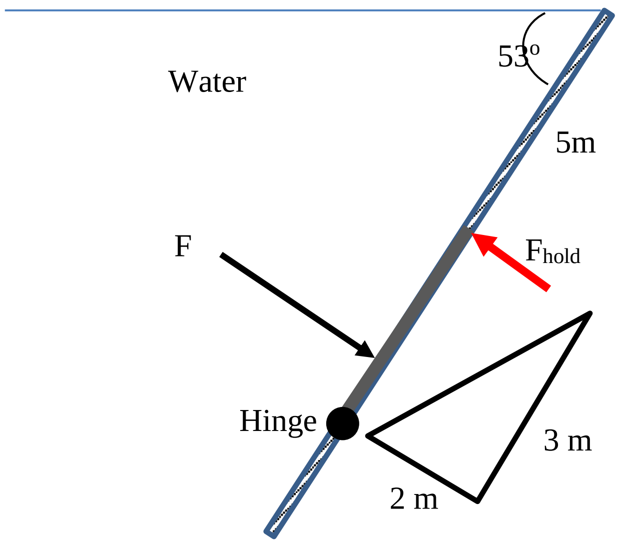
See Examples (3.1-3.31) Ref. 4.

<u>*Example:*</u> A 60 x 80 cm window on a submersible lake. If it is on a 45° angle with horizontal, what force applied normal to the window at the bottom edge in needed to just open the window, if is hinged at the top edge when the top edge is 10 m below the surface?

F=24.445 KN ans.

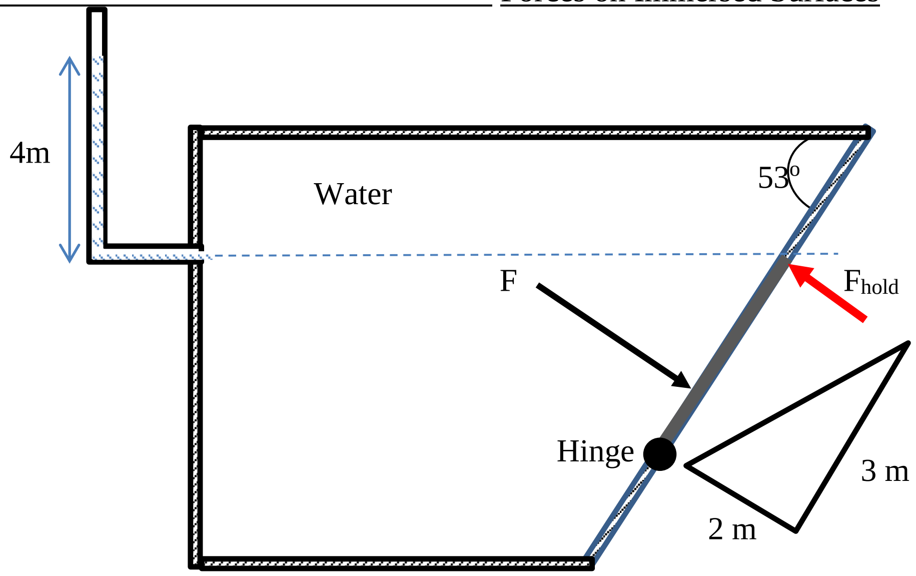


Example: find the force necessary to hold the gate in the position shown in figure.

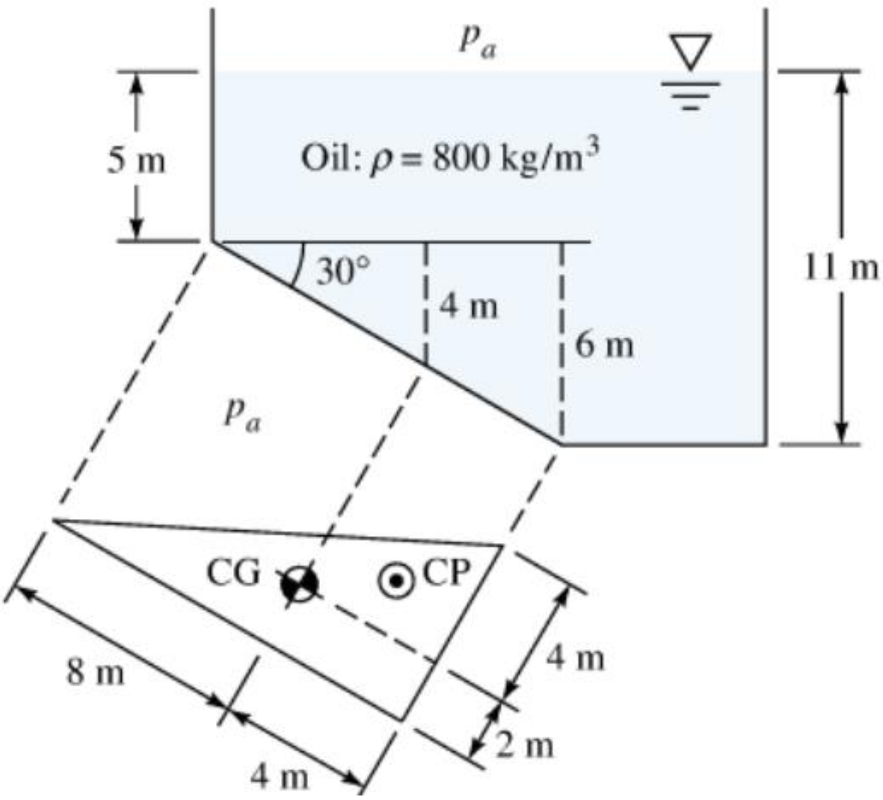


F=50.95 KN ans.

Example: find the force necessary to hold the gate in the position shown in figure.



Example 3: A tank of oil has a right-triangular panel near the bottom, as in ,Figure. Omitting Pa, find: a) hydrostatic force 2) CP on the panel



Solution

Part (a) The triangle has properties given in Fig. 2.13*c*. The centroid is one-third up (4 m) and one-third over (2 m) from the lower left corner, as shown. The area is

$$\tfrac{1}{2}(6\text{ m})(12\text{ m}) = 36\text{ m}^2$$

The moments of inertia are

$$I_{xx} = \frac{bL^3}{36} = \frac{(6\text{ m})(12\text{ m})^3}{36} = 288\text{ m}^4$$

and

$$I_{xy} = \frac{b(b-2s)L^2}{72} = \frac{(6\text{ m})[6\text{ m} - 2(6\text{ m})](12\text{ m})^2}{72} = -72\text{ m}^4$$

The depth to the centroid is $h_{CG} = 5 + 4 = 9$ m; thus the hydrostatic force from Eq. (2.44) is

$$F = \rho g h_{CG} A = (800\text{ kg/m}^3)(9.807\text{ m/s}^2)(9\text{ m})(36\text{ m}^2)$$

$$= 2.54 \times 10^6\ (\text{kg}\cdot\text{m})/\text{s}^2 = 2.54 \times 10^6\text{ N} = 2.54\text{ MN} \qquad \textit{Ans. (a)}$$

Part (b) The CP position is given by Eqs. (2.44):

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{(288\text{ m}^4)(\sin 30°)}{(9\text{ m})(36\text{ m}^2)} = -0.444\text{ m}$$

$$x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} = -\frac{(-72\text{ m}^4)(\sin 30°)}{(9\text{ m})(36\text{ m}^2)} = +0.111\text{ m} \qquad \textit{Ans. (b)}$$

The resultant force $F = 2.54$ MN acts through this point, which is down and to the right of the centroid, as shown in Fig. E2.6.

2- Curved Surfaces

We know that the pressure force is normal on each element of the surface. For curved surfaces, we calculate the *components* (horizontal and vertical) rather than the resultant, this for simplicity.

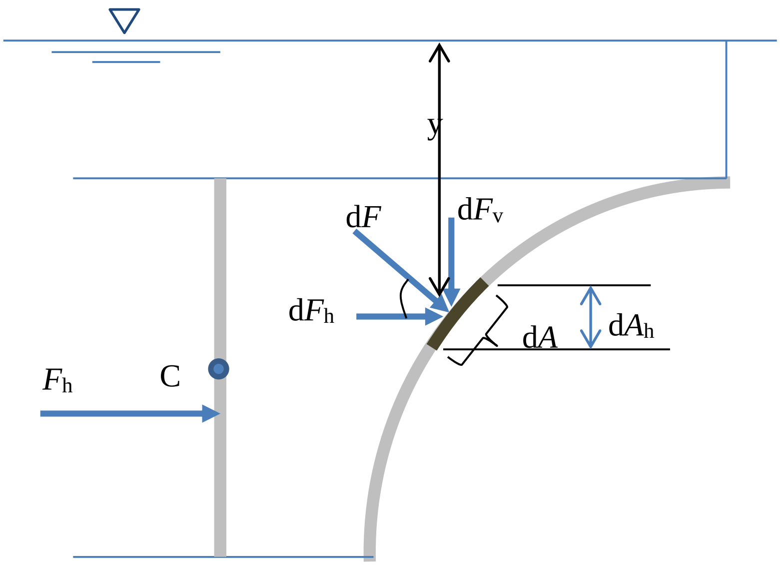
2-1 The horizontal component

$$F_h = \int_A dF_h = \int_A P\, dA\, cos\theta = \int_A \gamma\, y\, dA\, cos\theta$$

$$F_h = \gamma \int_A y\, dA_h$$

$$\therefore F_h = \gamma\, \bar{y}\, A_h$$

Where A_h is the vertical projection of the curved area and $\bar{y}$ is the centroid of the projected area.



2-2 The vertical component

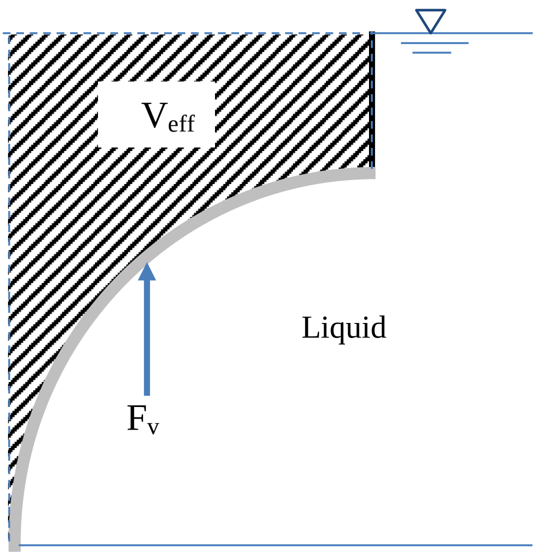
$$F_V = \int dF_V = \int P\, dA\, sin\theta = \int \gamma\, y\, dA\, sin\theta$$

$$F_V = \gamma \int y\, dA_V$$

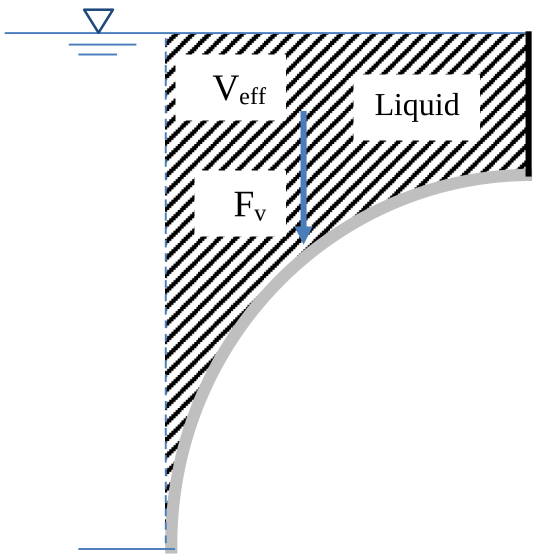
The last integral represents the **fluid volume** over the curved surface until the free surface (at which the pressure is atmospheric), hence we can say that the vertical component is the fluid weight over the curved surface.

$$F_V = \gamma\, V$$

- The fluid volume V is found by extending the curved surface to the free surface level ($P = P_{atm} = 0$).
- When the liquid is below the curved surface, an imaginary or equivalent free surface can be constructed. The weight of the imaginary volume of liquid vertically above the curved surface is then the vertical component of pressure force on the curved surface.
- The imaginary liquid must be of the same specific weight as the liquid in contact with curved surface.



Liquid below the curved surface



Liquid above the curved surface

Note: the location of the vertical component action must pass through the centroid of the effective volume.

See Examples (3.32-3.42) Ref. 4.

Example: Gate AB is a quarter-circle 10 ft wide and hinged at B. Find the force F just sufficient to keep the gate from opening. The gate is uniform and weighs 3000 lbf.

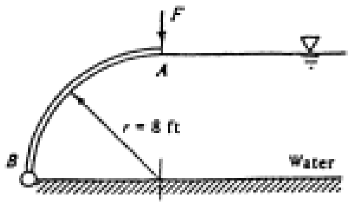
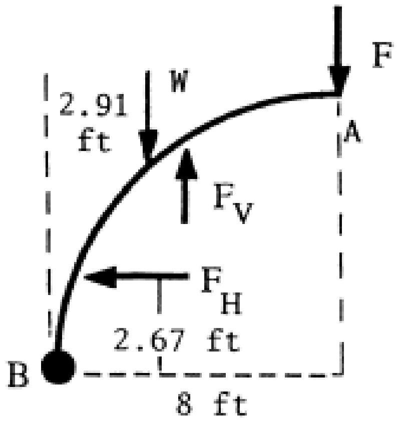


Fig. P2.83

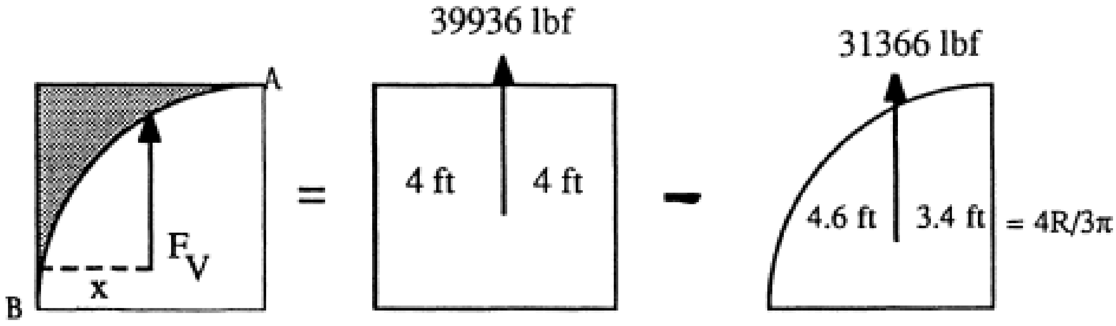
Solution: The horizontal force is computed as if AB were vertical:

$$F_H = \gamma h_{CG} A_{vert} = (62.4)(4\text{ ft})(8 \times 10\text{ ft}^2)$$
$$= 19968\text{ lbf} \quad \text{acting 5.33 ft below A}$$



The vertical force equals the weight of the missing piece of water above the gate, as shown below.

$$F_V = (62.4)(8)(8 \times 10) - (62.4)(\pi/4)(8)^2(10)$$
$$= 39936 - 31366 = 8570\text{ lbf}$$



The line of action x for this 8570-lbf force is found by summing moments from above:

$$\sum M_B(\text{of } F_V) = 8570x = 39936(4.0) - 31366(4.605), \quad \text{or} \quad x = 1.787\text{ ft}$$

Finally, there is the 3000-lbf gate weight W, whose centroid is $2R/\pi = 5.093$ ft from force F, or 8.0 – 5.093 = 2.907 ft from point B. Then we may sum moments about hinge B to find the force F, using the freebody of the gate as sketched at the top-right of this page:

$$\sum M_B(\text{clockwise}) = 0 = F(8.0) + (3000)(2.907) - (8570)(1.787) - (19968)(2.667),$$

$$\text{or} \quad F = \frac{59840}{8.0} = \mathbf{7480\ lbf} \quad \textit{Ans.}$$

Example: The quarter circle gate BC in Fig. P2.86 is hinged at C. Find the horizontal force *P* required to hold the gate stationary. The width *b* into the paper is 3 m.

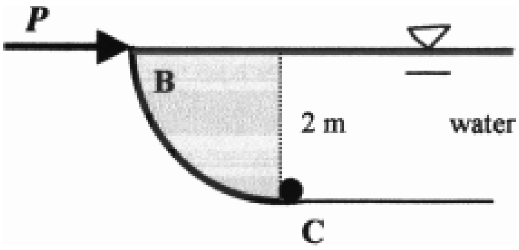


Fig. P2.86

Solution: The horizontal component of water force is

$$F_H = \gamma h_{CG} A = (9790\ N/m^3)(1\ m)[(2\ m)(3\ m)] = 58{,}740\ N$$

This force acts 2/3 of the way down or 1.333 m down from the surface (0.667 m up from C). The vertical force is the weight of the quarter-circle of water above gate BC:

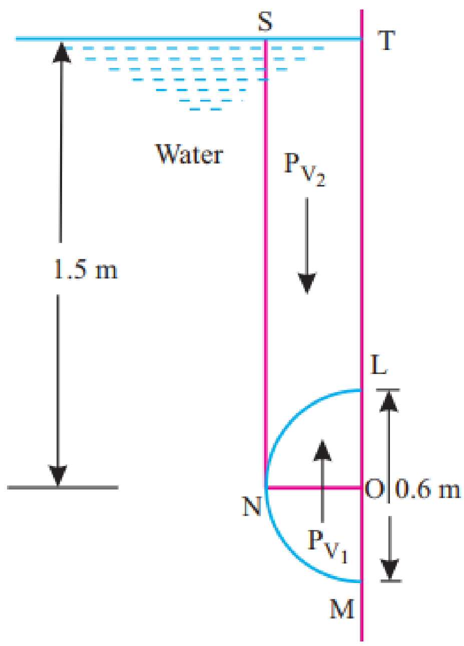
$$F_V = \gamma(Vol)_{water} = (9790\ N/m^3)[(\pi/4)(2\ m)^2(3\ m)] = 92{,}270\ N$$

F_V acts down at $(4R/3\pi) = 0.849$ m to the left of C. Sum moments clockwise about point C:

$$\sum M_C = 0 = (2\ m)P - (58740\ N)(0.667\ m) - (92270\ N)(0.849\ m) = 2P - 117480$$

Solve for $P = 58{,}700\ N = \mathbf{58.7\ kN}$ *Ans.*

Example .A hemisphere projection of diameter0.6 m exists on one of the vertical sides of a tank. If the tank contains water to an elevation of 1.5 m above the centre of the hemisphere, calculate the vertical and horizontal forces acting on the projection.



Solution

Vertical force, $F_V = F_{V_1} - F_{V_2}$

= Weight volume of water MNST – weight of volume of water LNST

= Weight of water contained by the hemisphere LNM

$$= w \times \frac{1}{2}\left(\frac{4}{3}\pi R^3\right)$$

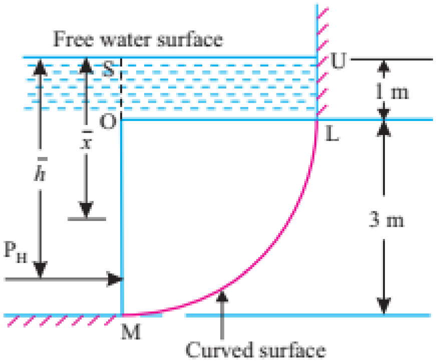
$$= 9.81 \times \frac{1}{2} \times \frac{4}{3} \times \pi \times (0.3)^3$$

= **0.555 kN (Ans.)**

Horizontal force, $P_H = wA\bar{x}$

$$= 9.81 \times \pi \times (0.3)^2 \times 1.5 = \mathbf{4.16\ kN}$$ **(Ans.)**

Example. shows a curved surface LM, which is in the form of a quadrant of a circle of radius 3 m, immersed in the water. If the width of the gate is unity, calculate the horizontal and vertical components of the total force acting on the curved surface.



Solution.

Radius of the gate=3 m
Width of the gate=1 m
LO=OM =3 m
Horizontal component of total force,
F_H: *Horizontal force exerted by water on gate is given by,*
F_h=Total pressure force on the projected area of curved surface LM on vertical plane = Total pressure force on OM (projected area of curved surface on vertical plane)

$$= OM \times 1) = wA\bar{x}$$

But, $A = OM \times 1 = 3 \times 1 = 3\text{m}^2$ and $\bar{x} = 1 + \frac{3}{2} = 2.5$ m

$F_H = 9.81 \times (3 \times 1) \times 2.5 = \mathbf{73.57\ kN}$ **(Ans.)**

The point of application of P_H is given by:

$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x}$$

where, I_G = M.O.I. of OM about its c.g. $= \frac{bd^3}{12} = \frac{1 \times 3^3}{12} = 2.25\ \text{m}^4$

$\therefore$ $\bar{h} = \frac{2.25}{(3 \times 1) \times 2.5} + 2.5 =$ **2.8 m from water surface (Ans.)**

Vertical force (F_V) exerted by water is given by:

F_V = Weight of water supported by LM up to free surface

= weight of portion ULMOS

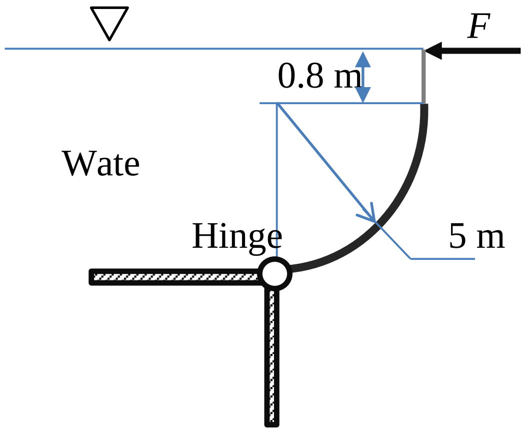
= weight of ULOS + weight of water in LOM

= w (volume of ULOS + volume of LOM)

$$= 9.81\left[UL \times LO + \frac{\pi \times (LO)^2}{4} \times 1\right] = 9.81\left[1 \times 3 + \frac{\pi \times 3^2}{4} \times 1\right]$$

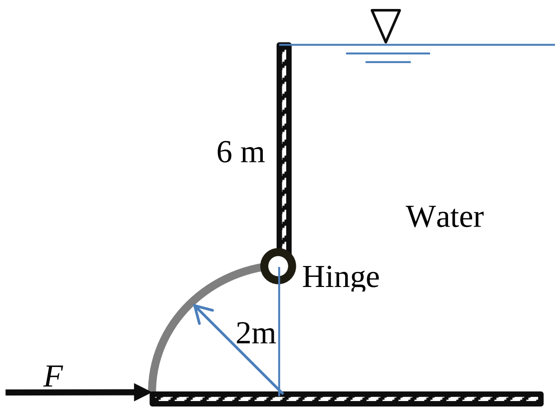
= 9.81 (3 + 7.068) kN = **98.77 kN (Ans.)**

Example: Find the force *F* required to hold the gate in the position shown in figure. The gate is 5 m wide.



F=437210.7N *ans*

Example: Find the force F needed to just open the gate shown. The gate is 4 m wide.

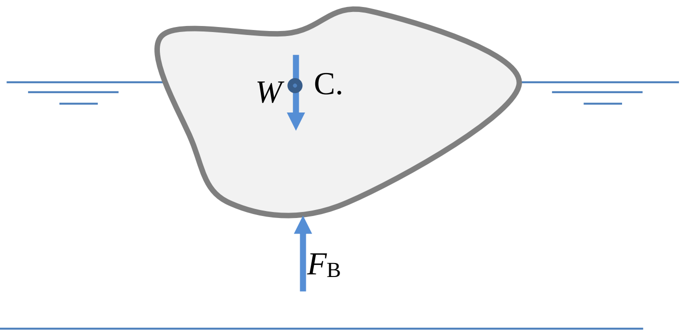


F=549228.7422 N *ans*

Chapter 4

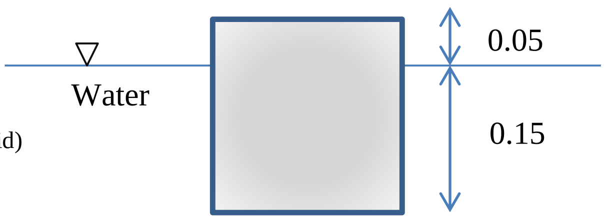
Buoyancy and Stability

Buoyancy: resultant force exerted on a body by static fluid which is submerged or floating. It always acts vertically upward.



- The buoyancy force acts through the centroid of the displaced liquid volume.
- It can be proven that the Buoyancy force equals the weight of the displaced (المزاح) liquid.
- For equilibrium, $F_B = W$, $F_B = \gamma V_{\text{displaced liquid}}$

<u>*Example:*</u> A 0.2 m cube is floating as shown, find the density of the cube material.



Solution:

Volume of the cube $= 0.2 \times 0.2 \times 0.2 = 0.008\ m^3$

Weight of cube , $w = \gamma V_{\text{displaced liquid}}$

Weight of cube displaced in water$(= \gamma V_{\text{displaced liquid}})$

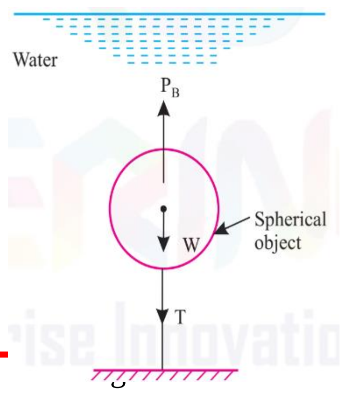
$= 9810 \times 0.2 \times 0.2 \times 0.15 = 58.86\ N$

Weight of cube , $w = \gamma V$

$58.86 = \gamma\ (0.008)$

$\therefore\ \gamma = \rho \times 9.81 = 7357.5;\ \therefore\ \rho = 750\ m^3/Kg$

<u>*Example*</u>: A spherical object of 1.45m diameter is completely immersed in a water reservoir and chained to the bottom. If the chain has a tension of 5.20 kN, find the weight of the object when it is taken out of the reservoir into the air.



Solution. Given: d= 1.45 m;T= 5.20 kN.

Buoyant force, F_B=W (weight of the object) +T(tension in the chain)

$\therefore W = F_B - T$

$$= \frac{4}{3}\pi \times \left(\frac{1.45}{2}\right)^3 \times 9.81 - 5.20$$

$= \mathbf{10.46\ kN}$ **(Ans.)**

Hydrometer: an instrument used to measure the specific gravity of liquids. It consists of bulb and constant area stem. When placed in pure water the specific gravity is marked to read 1.0. The force balance is

$$W = \gamma_{water}\, V_{displaced}$$

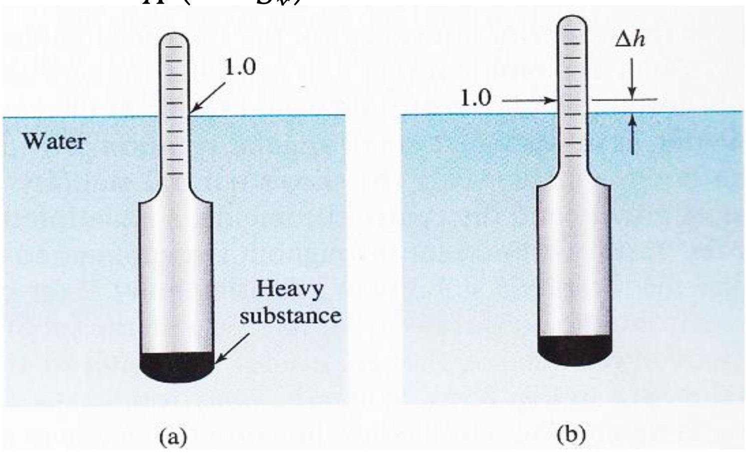
Where W is the weight of the hydrometer and V is the submerged volume below the S=1.0 line. In an unknown liquid of specific gravity, γ_x, a force balance would be:

$$W = \gamma_x (V - A\,\Delta h)$$

Where A is the cross-sectional area of the stem. Equating the two equations above gives

$$\Delta h = \frac{V}{A}\left(1 - \frac{1}{S_x}\right)$$

Where Sx $=\gamma_x / \gamma_{water}$

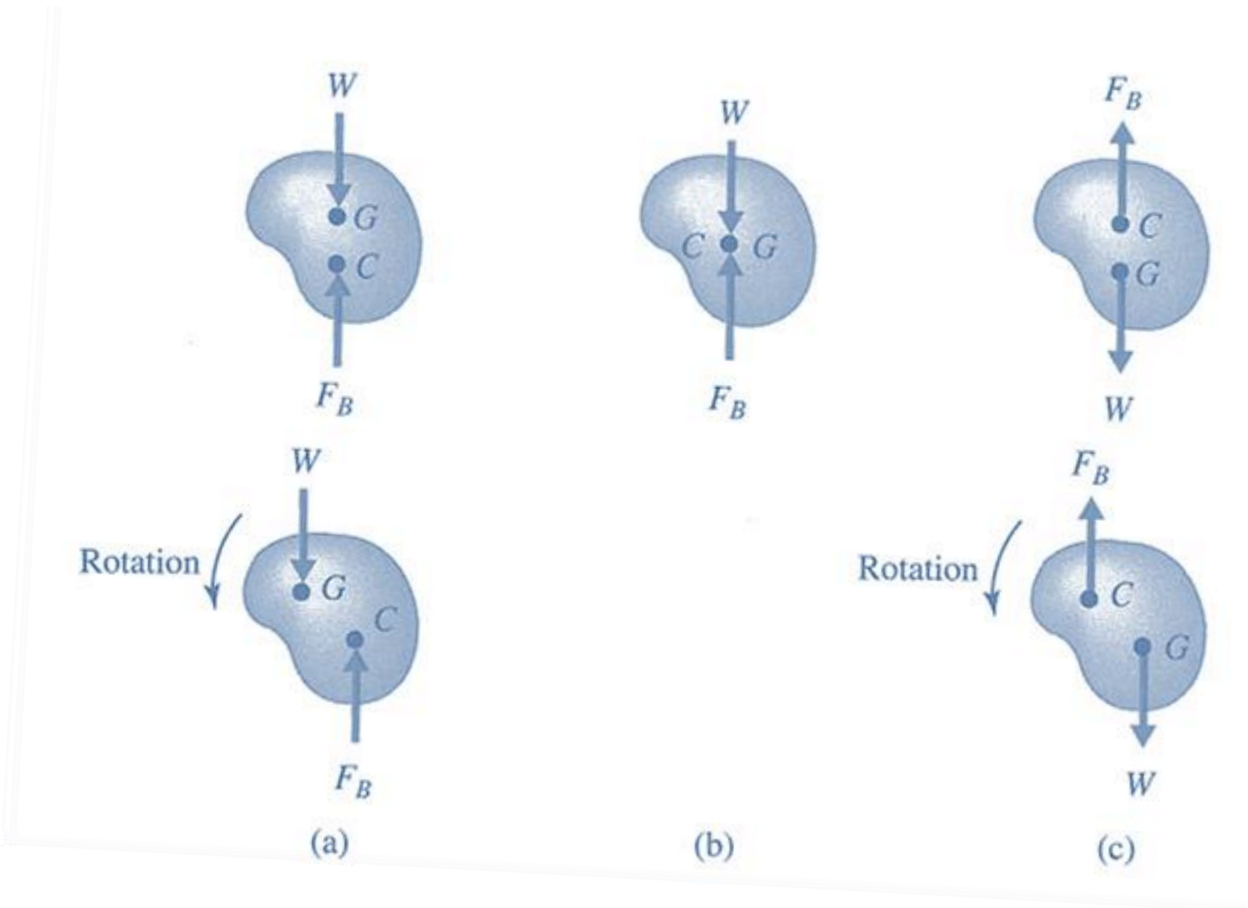


Hydrometer: (a) in water, (b) in unknown liquid

Stability

Stability becomes an important consideration when floating bodies such as a boat or ferry is designed. It is an obvious requirement that a floating body such as a boat does not topple when slightly disturbed. We say that a body is in stable equilibrium if it is able to return to its position when slightly disturbed. Failure to do so denotes unstable equilibrium

Stability of submerged bodies



Unstable Neutral Stable

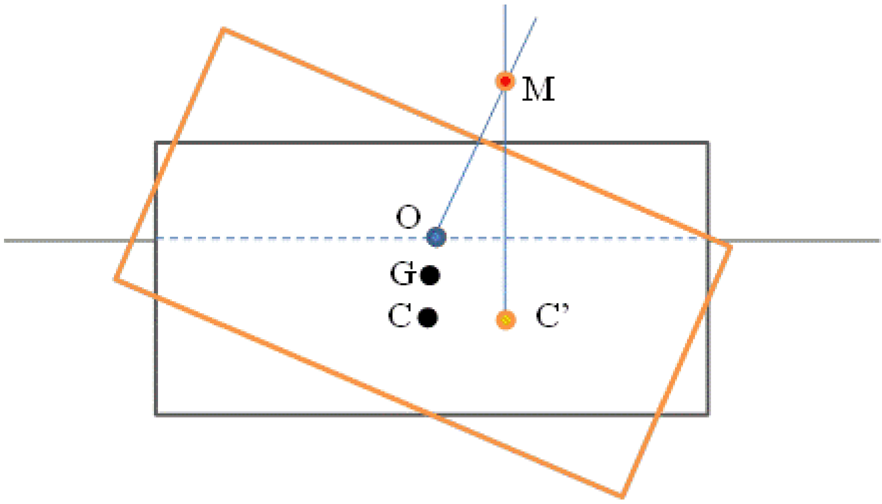
Stability of floating bodies: in this case, the stability is more complicated to deal with.

When the body is slightly rotated about O,

1- The center of gravity remains unchanged.
2- The center of buoyancy is changed to C’

The center of the buoyancy (C, the centroid of the displaced volume of fluid) of a floating body depends on the shape of the body and on the position in which it is floating.

Extending a line from C' vertically. It will intercept with a line extended from the point O (axis of rotation) at a point M. This point M is called the **Metacenter**.

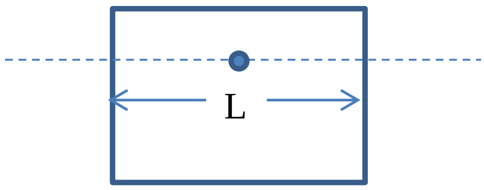


Now:

If M is above G, the body is **stable**, otherwise, it is **unstable** and according to the following relation:

$$\overline{GM} = \frac{I_o}{V_{displac}} \mp \overline{CG}$$

Where $\overline{GM}$: distance between G and M (**metacentric height**).



$$I_o = \frac{bL^3}{12}$$

b: length into the paper

+ ve sign : when G is **lower** *than C*

– ve sign : when G is **higher** *than C*

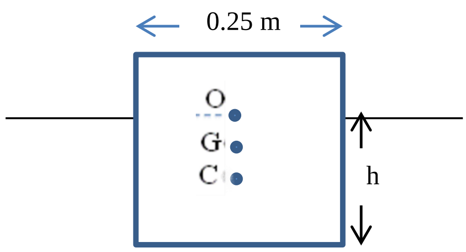
$\overline{CG}$ distance between C and G

I_o: second moment area of the waterline area about an axis passing through the Origin O.

$V_{displac}$: Volume of displaced liquid or (submerged body)

Example: A 0.25 long cylinder with 0.25 m diameter composed of material with density of ρ = 815 kg/m^3. Will it float on water on its base?

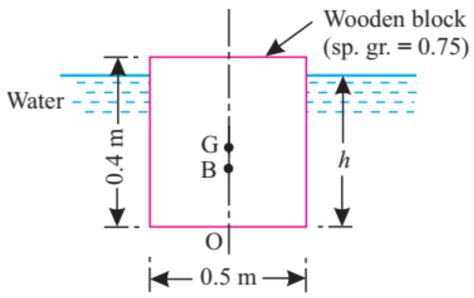
Solution:



$\overline{GM}$ positive? If yes that lead to the body is stable.

$\overline{GM}$ = negative the body is unstable.

Example: .A wooden block of specific gravity 0.75 floats in water. If the size of the block is 1m×0.5m×0.4m, find its metacentric height.



Note in this example: B ≡ C

Solution.

Size of the block = 1m× 0.5m× 0.4m= 0.2 m^3

Specific gravity of wood = 0.75

γ_{wood} =0.75 × 9.81 = 7.36 kN/m3

Weight of wooden block = γ_{wood} × volume

= 7.36 × 1× 0.5 × 0.4 = 1.472 kN

Let depth of immersion =h metres.

Weight of water displaced = γ_{water} × volume of the wood submerged in water

$$= 9.81 \times 1 \times 0.5 \times h = 4.9\,h \text{ kN}$$

Now, for equilibrium:

Weight of wooden block = Weight of water displaced *i.e.,* 1.472 = 4.9 *h*

or, $$h = \frac{1.472}{4.9} = 0.3 \text{ m}$$

∴ Distance of centre of buoyancy from bottom *i.e.,*

$$OB = \frac{h}{2} = \frac{0.3}{2} = 0.15 \text{ m}$$

and, $$OG = \frac{0.4}{2} = 0.2 \text{ m}$$

∴ $$BG = OG - OB = 0.2 - 0.15 = 0.05 \text{ m}$$

Also, $$\text{BM} = \frac{I}{V}$$

Where, I = Moment of inertia of a rectangular section

$$= \frac{1 \times 0.5^3}{12} = 0.014 \text{ m}^4$$

and, V = Volume of water displaced (or volume of wood in water)

$$= 1 \times 0.5 \times h = 1 \times 0.5 \times 0.3 = 0.15 \text{ m}^3$$

$$\text{BM} = \frac{1}{V} = \frac{0.0104}{0.15} = 0.069 \text{ m}$$

We know that the metacentric height,

$$GM = BM - BG \qquad (\because G \text{ is above } B)$$

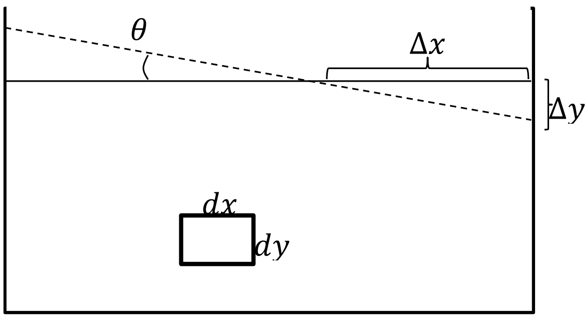
$$= 0.069 - 0.05 = \mathbf{0.019\ m}$$ **(Ans.)**

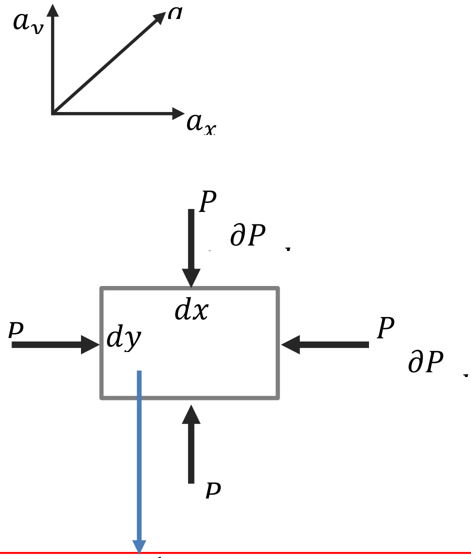
Chapter-5 Accelerated Fluid (Forced Vortex Flow)

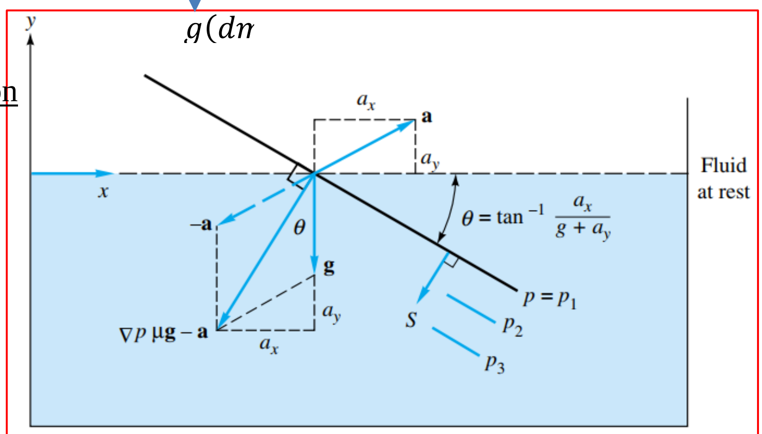
When a fluid mass is moving with constant acceleration, we assume no relative motion between the fluid layers, i.e. no shear stress.

1- Linear motion with constant acceleration.

Assume a fluid in a vessel (of unit width), the vessel is moving with constant acceleration.







Equation of Newton 2nd law in x-direction

$$m a_x = \sum F_x$$

$$d_m a_x = P d_y - \left(P + \frac{\partial P}{\partial x} dx\right) dy$$

$$d_m a_x = -\frac{\partial P}{\partial x} dx dy$$

$$\therefore a_x = -\frac{1}{\rho} \frac{\partial P}{\partial x} \qquad (1)$$

$$\frac{dp}{ds} = \rho G \qquad \text{where } G = [a_x^2 + (g + a_y)^2]^{1/2}$$

These results are independent of the size or shape of the container as long as the fluid is continuously connected throughout the container.

Equation of Newton 2nd law in y-direction

$ma_y = \sum F_y$

$d_m a_y = Pd_x - \left(P + \frac{\partial P}{\partial y}dy\right)dx - gdm$

$$d_m a_y = -\frac{\partial P}{\partial y}dxdy - gdm$$

$dm = \rho(dx * dy * 1)$

$$\therefore a_y = -g - \frac{1}{\rho}\frac{\partial P}{\partial y} \qquad (2)$$

Note: if $a_y = 0$, the pressure along y direction will vary hydrostatically i.e. P = γh.

But, $dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy$

Hence, from equations (1) and (2),

$$\boldsymbol{dP = -\rho a_x d_x - (\rho g + \rho a_y)dy} \qquad (3)$$

The line of constant pressure, can be found from the above equation,

$\boldsymbol{Presure\ at\ any\ point\ a}$ $P_a = \rho.G.\Delta s$ $where \quad \Delta s = h.cos(\theta)$

by setting $dP = 0$

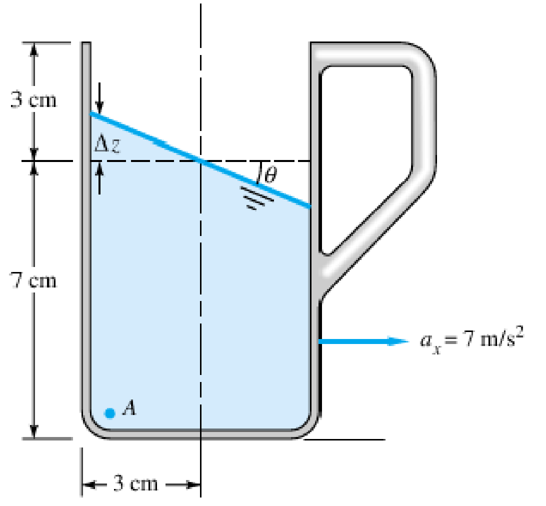
⇨ $\rho a_x d_x = -\rho(g + a_y)dy$

$$\therefore \boldsymbol{\frac{dy}{dx} = -\frac{a_x}{g+a_y} = \tan\theta} \quad \text{(negative slope).}$$

The line of constant pressure is free surface itself.

for free surface $\boldsymbol{\frac{y}{x} = -\frac{a_x}{g+a_y}}$

Example 01: A drag racer rests her coffee mug on a horizontal tray while she accelerates at 7 m/s^2. The mug is 10 cm deep and 6 cm in diameter and contains coffee 7 cm deep at rest. (a) Assuming rigid-body acceleration of the coffee, determine whether it will spill out of the mug. (b) Calculate the gage pressure in the corner at point A if the density of coffee is 1010 kg/m^3.



Solution

a)) The free surface tilts at the angle $\boldsymbol{\theta}$ given by above Eq. regardless of the shape of the mug. With $a_z = 0$ and standard gravity,

$$\theta = \tan^{-1}\frac{a_x}{g} = \tan^{-1}\frac{7.0}{9.81} = 35.5°$$

If the mug is symmetric about its central axis, the volume of coffee is conserved if the tilted surface intersects the original rest surface exactly at the centerline, as shown in Fig.

Thus the deflection at the left side of the mug is

z = (3 cm)(tan $\boldsymbol{\theta}$) = 2.14 cm **Ans. (a)**

This is less than the 3-cm clearance available, so the coffee will not spill unless it was sloshed during the start-up of acceleration.

b)) When at rest, the gage pressure at point A is given

$$p_A = \rho g(z_{surf} - z_A) = (1010\ kg/m^3)(9.81\ m/s^2)(0.07\ m) = 694\ N/m^2 = 694\ Pa$$

During acceleration, applies, with $G= [(7.0)^2\ (9.81)^2]^{1/2} = 12.05\ m/s^2$.
The distance Δs down the normal from the tilted surface to point A is

$$\Delta s= (7.0+ 2.14)\ (\cos\theta) = 7.44\ cm$$

Thus the pressure at point A becomes

$$p_A= \rho\ G\ \Delta s=1010(12.05)(0.0744)= 906\ Pa$$

Ans. (b)

which is an increase of 31 percent over the pressure when at rest.

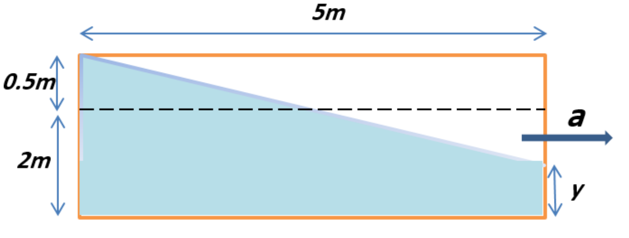
Example:02

An open rectangular tank mounted on a truck is 5 m long, 2 m wide and 2.5 m high is filled with water to a depth of 2 m. Determine the following:

1.1 Maximum horizontal acceleration that can be imposed on the tank without spilling any water.

1.2 The accelerating force on the liquid mass.

1.3 If the acceleration is increased to 6 m/s^2, how much water is spilled out?



Solution

$$1.1: at\ \max(a)\ without\ spill\ V1 = V2 \rightarrow 2x5x2 = (\frac{2.5+y}{2})x5x2 \rightarrow y = 1.5$$

$$tan\theta = \frac{a}{g}\ \ and\ Max\ a\ when\ tan\theta = \frac{2.5-1.5}{5} = \frac{a}{9.81} \rightarrow a = 1.96\ m/s^2$$

$$1.2: F = mxa = \rho.V.a = 1000*2*5*2*1.962 = 39240N \approx 39.24KN$$

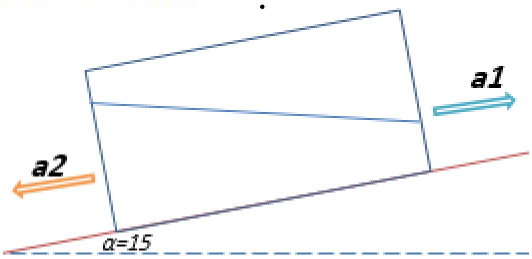
$$1.3: a = 6 \rightarrow tan\theta = \frac{6}{9.81} = \frac{2.5}{x} \rightarrow x = 4.08m$$

$$\therefore V3 = 0.5x4.08x2.5x2 = 10.21\ m^3\ ,\ \rightarrow Vspill = V1 - V3 = (2x5x2) - 10.21 = 9.78\ m^3$$

Example: 03

A vessel containing oil is accelerated on a plane incline 15^0 with the horizontal at 1.2 m/s^2. Determine the following:

1. The inclination of the oil surface when the motion is upwards.
2. The inclination of the oil surface when the motion is downwards.



Solution:

$$1:\ ax = a_1.cos\alpha = 1.2\cos(15) = 1.16\ m/s^2$$

$$az = a_1.sin\alpha = 1.2\sin(15) = 0.31\ m/s^2$$

$$\theta = tan^{-1}\left(\frac{-ax}{g+az}\right) = \left(\frac{-1.16}{9.81+0.31}\right) = -6.533^o$$

$$2:\ ax = -a_1.cos\alpha = -1.2\cos(15) = -1.16\ m/s^2$$

$$az = -a_1.sin\alpha = -1.2\sin(15) = -0.31\ m/s^2$$

$$\theta = tan^{-1}\left(\frac{-ax}{g+az}\right) = \left(\frac{+1.16}{9.81-0.31}\right) = 6.96^o$$

Example H.W.1

The tank shown in Fig. 1a is accelerated to the right. Calculate the acceleration a_x needed to cause the free surface shown in Fig. 1b to touch point A. Calculate also the pressure at point B.

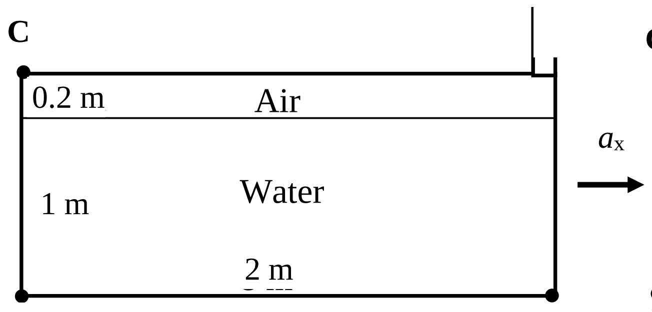


Fig. 1a

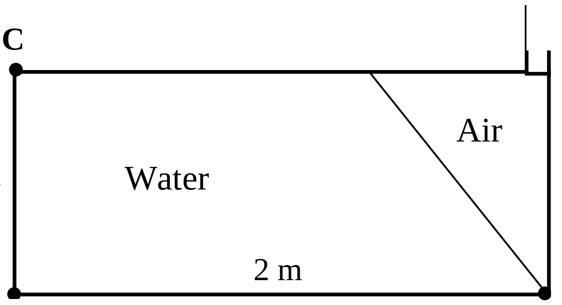
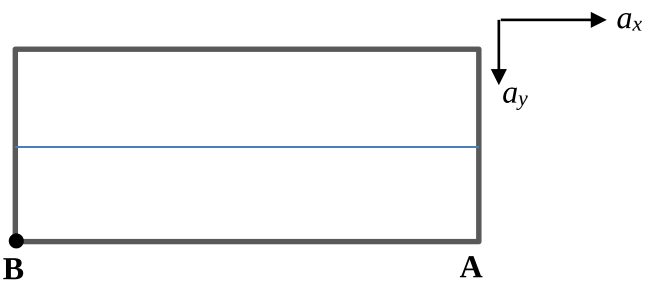


Fig. 1b

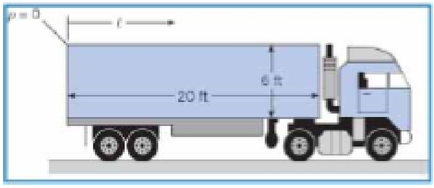
Example HW.2

A closed box with horizontal base of 6x6 m and height of 2 m is half filled with water. It is given a_x =g/2 and a_y=-g/4. Find the pressure at point b as shown.



Example: H.W

Example : Pressure in A Decelerating Tank of Liquid

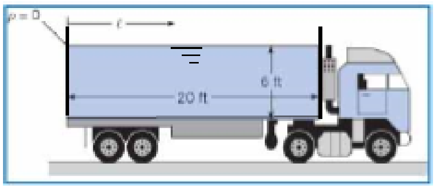


The tank on a trailer truck is filled completely with gasoline, which has a specific weight of 42 lbf/ft^3 (6.60 kN/m^3). The truck is decelerating at a rate of 10 ft/s^2 (3.05 m/s^2).

(a) If the tank on the trailer is 20 ft (6.1 m) long and if the pressure at the top rear end of the tank is atmospheric, what is the pressure at the top front?

(b) If the tank is 6 ft (1.83 m) high, what is the maximum pressure in the tank?

Now if the tank is assumed to be opened



find the following:

If the tank on the trailer is 20 ft (6.1 m) long and 6 ft (1.83 m) high, what is the pressure at the top front and bottom rear? and what is the maximum pressure in the tank? The width of truck is 2 m.

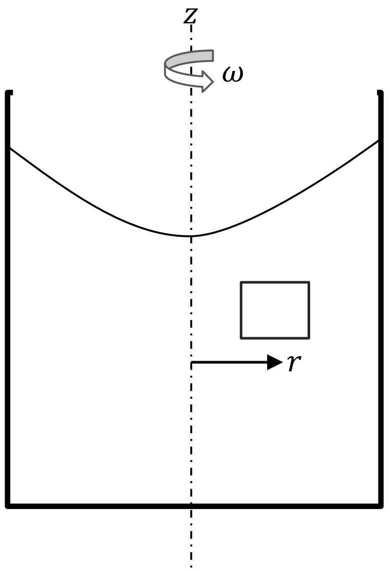
2- Rotation with constant acceleration

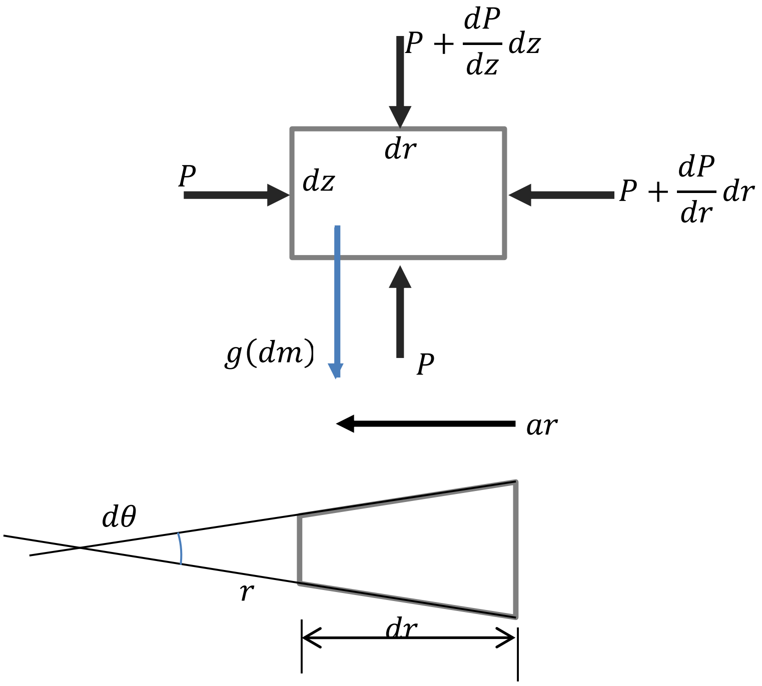
Assumptions:

- No pressure variation with θ direction
- The horizontal rotation will not alter the pressure distribution in the vertical direction (i.e. the pressure equals to $P = \gamma h$).

Applying Newton's 2nd low in r-direction:

$$-ma_r = \sum F_r$$





$$-dm\, a_r = \sum F_r$$

$$-\rho r d\theta dr dz\, a_r = P r d\theta dz - \left(P + \frac{\partial P}{\partial r} dr\right) dz\, r\, d\theta$$

$$\therefore \frac{\partial P}{\partial r} = \rho\, a_r$$

$$a_r = r\omega^2$$

$$\therefore \frac{\partial P}{\partial r} = \rho\, r\omega^2 \qquad (1)$$

$$-ma_z = \sum F_z = 0\,, \quad a_z = 0$$

$$P\, r d\theta\, dr - \left(P + \frac{\partial P}{\partial z} dz\right) r\, dr\, d\theta - \rho\, r\, dr\, d\theta\, dz\, g = 0$$

$$\therefore \frac{\partial P}{\partial z} = -\rho\, g \qquad (2)$$

But, $dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$

$$\boldsymbol{dP = \rho\, r\, \omega^2 dr - \rho g\, dz} \qquad (3)$$

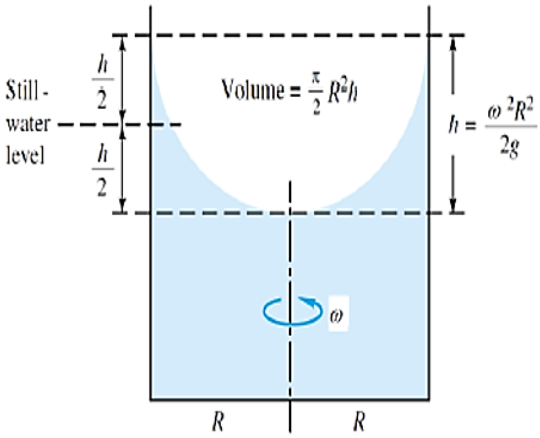
→Pressure at any Point P(r,z): $\boldsymbol{P = \frac{1}{2} \rho\, r^2\, \omega^2 - \rho g z}$

on the free surface, dP = 0 .

$$\omega^2 \left(\frac{r_2^2}{2} - \frac{r_1^2}{2} \right) = g(z_2 - z_1)$$

If we put point 1 at the z-axis so that $r_1 = 0$

$$\omega^2 \frac{r_2^2}{2} = g(z_2 - z_1) \text{ Equation of Parabola.}$$

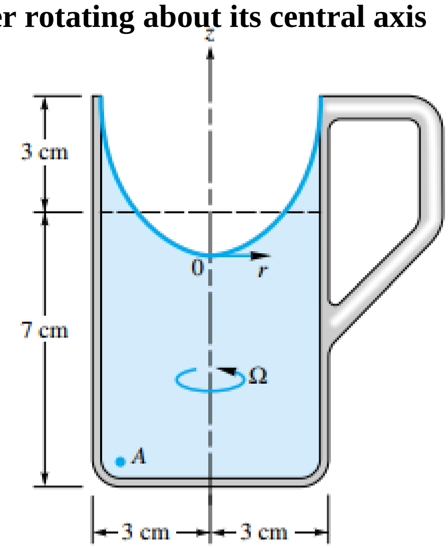


$$h = \frac{\omega^2 r^2}{2g}$$

For a cylinder rotating about its central axis

Example 04

The coffee cup in Example above is removed from the drag racer, placed on a turntable, and rotated about its central axis until a rigid-body mode occurs. Find (a) the angular velocity which will cause the coffee to just reach the lip of the cup and (b) the gage pressure at point A for this condition.



Solution

a) The cup contains 7 cm of coffee. The remaining distance of 3 cm up to the lip must equal the distance *h*/2 in Fig. Thus

$$\frac{h}{2} = 0.03 \text{ m} = \frac{\Omega^2 R^2}{4g} = \frac{\Omega^2 (0.03 \text{ m})^2}{4(9.81 \text{ m/s}^2)}$$

Solving, we obtain

$$\Omega^2 = 1308 \quad \text{or} \quad \Omega = 36.2 \text{ rad/s} = 345 \text{ r/min}$$

b) To compute the pressure, it is convenient to put the origin of coordinates *r* and *z* at the bottom of the free-surface depression, as shown in Fig. The gage pressure here is $p_0 = 0$, and point *A* is at (*r*, *z*) = (3 cm, -4 cm). Equation (3) can then be evaluated

$$\boldsymbol{P - P_0 = \frac{1}{2} \rho\, r^2\, \omega^2 - \rho g z}$$

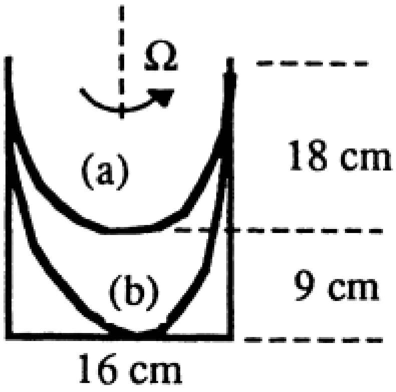
$p_A = 0 + \frac{1}{2}(1010 \text{ kg/m}^3)(0.03 \text{ m})^2(1308 \text{ rad}^2/\text{s})^2 - (1010 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(-0.04 \text{ m})$
$= 396 \text{ N/m}^2 + 594 \text{ N/m}^2 = 990 \text{ Pa}$

This is about 43 percent greater than the still-water pressure p_A = 694 Pa

***E*xample 05**

A 16-cm-diameter open cylinder 27 cm high is full of water. Compute the rigid-body rotation rate about its central axis, in r/min, (a) for which one-third of the water will spill out and (b) for which the bottom will be barely exposed.

Solution:



(a) One-third will spill out if the resulting paraboloid surface is 18 cm deep:

$$h = 0.18\ \text{m} = \frac{\Omega^2 R^2}{2g} = \frac{\Omega^2 (0.08\ \text{m})^2}{2(9.81)}, \quad \text{solve for} \quad \Omega^2 = 552,$$

$$\Omega = 23.5\ \text{rad/s} = \mathbf{224\ r/min} \quad \textit{Ans.}\ (a)$$

b) The bottom is barely exposed if the paraboloid surface is 27 cm deep:

$$h = 0.27\ \text{m} = \frac{\Omega^2 (0.08\ \text{m})^2}{2(9.81)}, \quad \text{solve for} \quad \Omega = 28.8\ \text{rad/s} = \mathbf{275\ r/min} \quad \textit{Ans.}\ (b)$$

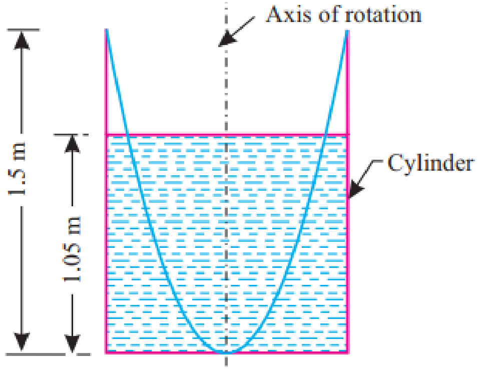
Example 06

A 0.225mdiameter cylinder is1.5mlong and contains water up to a height of 1.05m. Estimate the speed at which the cylinder may be rotated about its vertical axis so that the axial depth becomes zero.

Also, find the difference in total pressure force due to rotation:

(i) At the bottom of cylinder, and

(ii) On the sides of the cylinder



Solution.

$$z = \frac{\omega^2 R^2}{2g}, \text{ we get:}$$

$$1.5 = \frac{\omega^2 \times 0.1125^2}{2 \times 9.81}$$

$$\omega^2 = \frac{1.5 \times 2 \times 9.81}{0.1125^2} = 2325.33$$

$$\omega = 48.22\ \text{rad/s}$$

$$\omega = \frac{2\pi N}{60}$$

$$48.22 = \frac{2\pi N}{60}$$

$$N = \frac{48.22 \times 60}{2\pi} = \mathbf{460.46\ r.p.m}\ \textbf{(Ans.)}$$

(*i*) Difference in total pressure force at the bottom of the cylinder:

Total pressure force at the bottom **before rotation,**

$$F_{\text{before rot.}} = wA\bar{h} \qquad = w\,V$$

where, $w = 9810\ \text{N/m}^3$,

A = Area of bottom $= \pi R^2 = \pi \times 0{\cdot}1125^2 = 0.03976\ \text{m}^2$

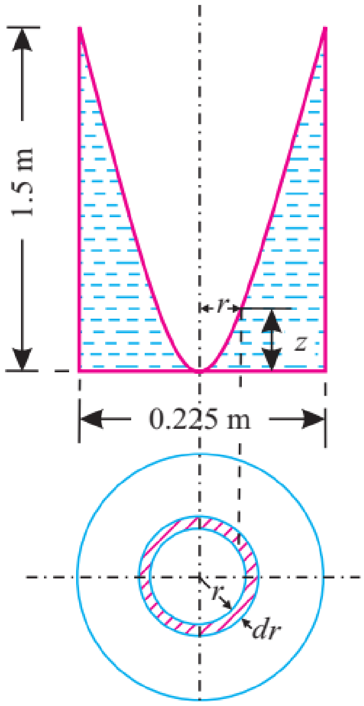
$\bar{h}$ = 1.05 m.

$\therefore \quad F_{\text{before rot.}} = 9810 \times 0.03976 \times 1.05 = 409.55\ \text{N}$

After rotation, the depth of water at the bottom is not constant and hence the pressure force due to the height of water *will not be constant.*

Consider an elementary ring of radius r and width dr as shown in Fig. Let $z\left(= \dfrac{\omega^2 r^2}{2g}\right)$ be the height of water from the bottom of the tank up to free suface of water at a radius r.

Hydrostatic force on the ring at the bottom,



$$dF = w \times \text{area of ring} \times z$$

$$= 9810 \times 2\pi r\ dr \times \frac{\omega^2 r^2}{2g}$$

$$= 9810 \times 2\pi r \times \frac{\omega^2}{2g} \times r^3 dr$$

$$= 3141.6\ \omega^2 r^3 dr$$

Total pressure force at the bottom,

$$F_{\text{after rot.}} = \int dF = \int_0^{0.1125} 3141.6\omega^2 r^3 dr$$

$$= 3141.6\omega^2 \left| \frac{r^4}{4} \right|_0^{0.1125}$$

[$\because$ ω = 48.22 rad/s, example 6.68]

$$= 3141.6 \times 48.22^2 \times \frac{0{\cdot}1125^4}{4}$$

$$= 292.52\ \text{N}$$

$\therefore$ Difference in pressure force at the bottom

$$= F_{\text{before rot.}} - F_{\text{after rot.}}$$

$= 409.55 - 292.52 =$ **117.03 N (Ans.)**

(*ii*) Difference in total pressure force on the sides of the cylinder:

Total pressure force on the sides of the cylinder **before rotation,**

$$F_{\text{before rot.}} = wA\bar{h}$$

where, $\omega = 9810\ \text{N/m}^3$,

A = Surface area of the sides of the cylinder up to height of water

$= \pi D \times$ height of water

$= \pi \times 0.225 \times 1.05 = 0{\cdot}7422\ \text{m}^2$, and

$\bar{h}$ = *c.g.* of the wetted area of the sides

$$= \frac{1}{2} \times 1.05 = 0.525\,\text{m}.$$

$$\therefore \quad F_{\text{before rot.}} = 9810 \times 0.7422 \times 0.525 = 3822.5\ \text{N}$$

After rotation, the water is up to the top of the cylinder and force on the sides,

$$F_{\text{after rot.}} = w \times A \times \bar{h}$$

where, $w = 9810\ \text{N/m}^3$

A = Wetted area of sides

$= \pi D \times$ height of water $= \pi \times 0.225 \times 1.5 = 1.06\ \text{m}^2$, and

$$\bar{h} = \frac{1}{2} \times \text{height of water} = \frac{1}{2} \times 1.5 = 0.75\ \text{m}.$$

$$\therefore \quad F_{\text{after rot.}} = 9810 \times 1.06 \times 0.75 = 7798.95\ \text{N}$$

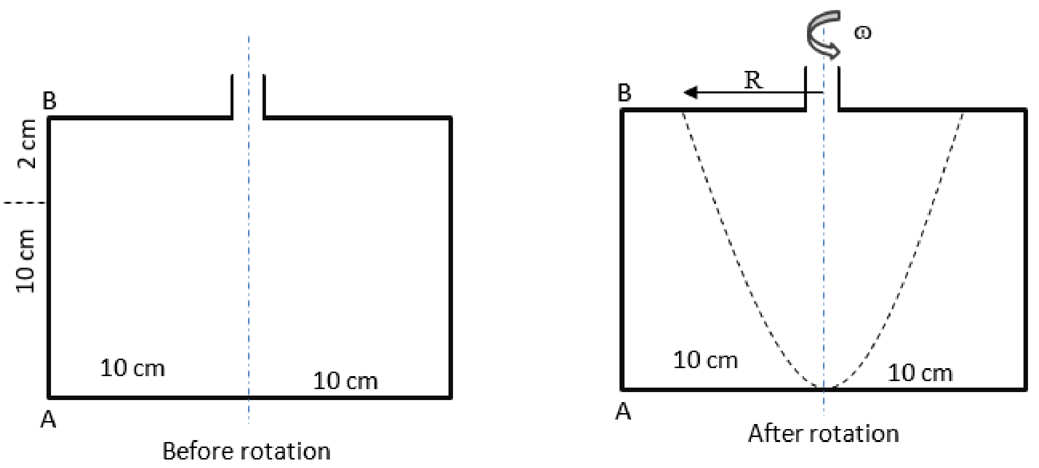
$\therefore$ Difference in pressure force on the sides

$= F_{\text{after rot,}} - F_{\text{before rot.}}$

$= 7798.95 - 3822.5 =$ **3976·45 N (Ans.)**

Example hw

A water-filled cylinder is rotating about its center line. Calculate the rotational speed that is necessary for the water to just touch the origin and the pressures at A and B.



Solved examples