Fluid Mechanics II (2^{nd} Grade / 2^{nd} Semester)

Syllabus:

- Fluid Dynamics: Types of fluid & Flow; Continuity Eq. & Equation of Fluid Motion (Energy & Bernoulli Eqs).

- Measurement of flow Rate: Measurement of flow rate in pipe lines; Tanks, & open channels.

- Flow in pipes: Laminar & Turbulent flows; Reynolds No.; Major & Minor losses; Equivalent Pipe, Parllel pipes; Series pipes, Branching pipes, & pipes network.

- Flow in open channels: Types of flow, Best hydraulic Section, Specific Energy & Critical depth

References

1- Fluid Mechanics by Streeter & Wylie.

2- Fluid Mechanics for Engineers by Albertson, Barton, & Simons.

3- Fluid Mechanics & Hydraulics (Schaum's Series) by Giles.

4- ميكانيك الموائع - د. نزار علي سبتي

5- مبادئ ميكانيك الموائع - د. جميل الملائكة

6- Fluid Mechanics with Engineering Applications by Daugherty, Franzini, & Finnemore.

7- Elementary Fluid Mechanics by Vennard & Street.

Fluid Dynamics

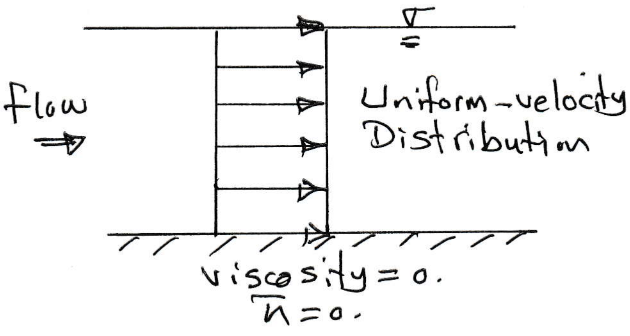
Fluid dynamics is a study of fluids in motion, the parts of which move at different velocities are subjected to various & changing accelerations - both +ve & -ve. These accelerations occur both in the direction of motion & normal to the direction of motion.

Types of Fluid

There are two types of fluid
- Perfect (Ideal) (Non-Viscous) Fluid
- Real (viscous) fluid

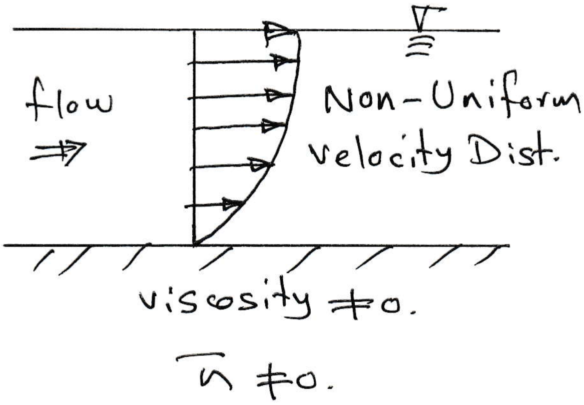
- Perfect Fluid ⇒ Viscosity = 0.

  * In perfect fluid there is a slipped Boundary condition



- Real fluid ⇒ Viscosity ≠ 0.

  * In Real fluid, there is no slip Boundary condition due to viscous



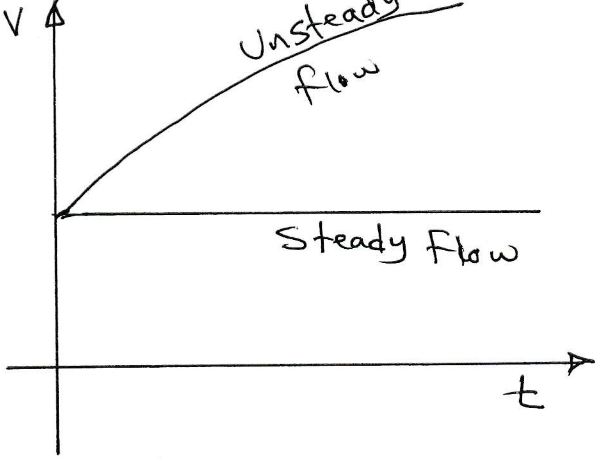
Types of flow

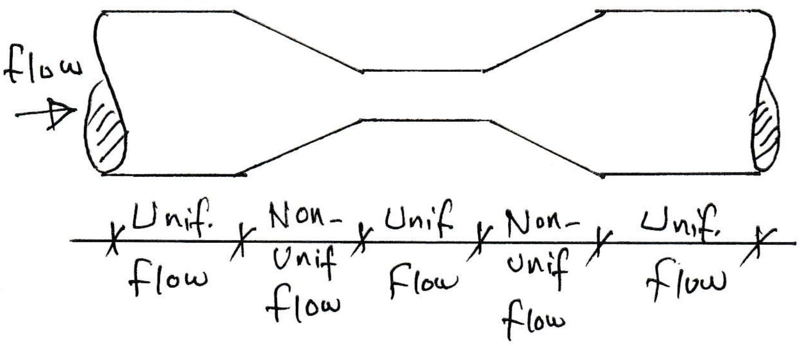
1- Steady Flow: exists if the velocity at a point, for example, remains constant with respect to time ($\frac{\partial V}{\partial t} = 0.$).

2- Unsteady Flow: exists if the velocity at a point, for example, changes either in magnitude or in direction with respect to time ($\frac{\partial V}{\partial t} \neq 0.$).

3- Uniform Flow: exists if the velocity, for example, remains constant with respect to distance ($\frac{\partial V}{\partial s} = 0.$).

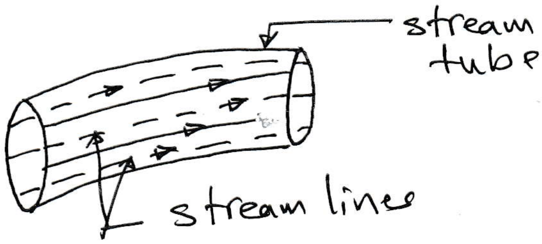
4- Non-Uniform Flow: exists if the velocity, for example, changes either in magnitude or in direction with respect to distance ($\frac{\partial V}{\partial s} \neq 0.$).





Stream Line : Is an imaginary line within the flow for which the tangent at any point is the time average of the direction of motion at that point.

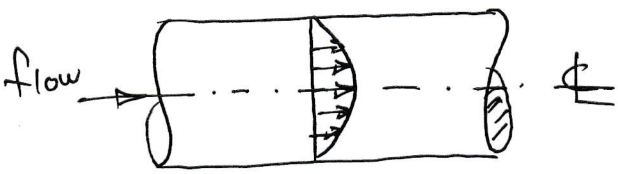
Stream Tube : Is an element of fluid bounded by a special group of stream lines which enclose or confine the flow.



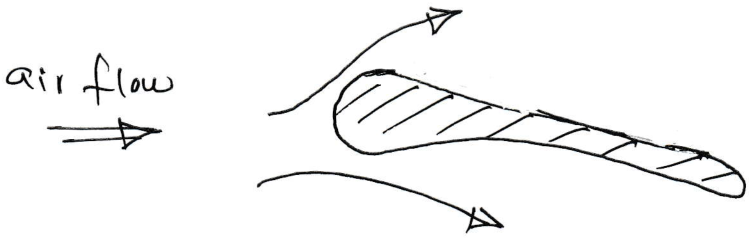
One, Two & Three Dimensional Flow

- 1D-Flow : Such as flow in pipe

1D & Axisymmetry flow



- 2D-Flow: such as flow around the wing of aircraft.



Velocity & Acceleration

Motion of fluid is specified by velocity components expressed as functions of space & time;

$u = F(x, y, z, t)$ – velocity component in X-dir.

$v = F(x, y, z, t)$ – " " " Y-dir.

$w = F(x, y, z, t)$ – " " " Z-dir.

Acceleration: Rate of change of velocity.

for example;

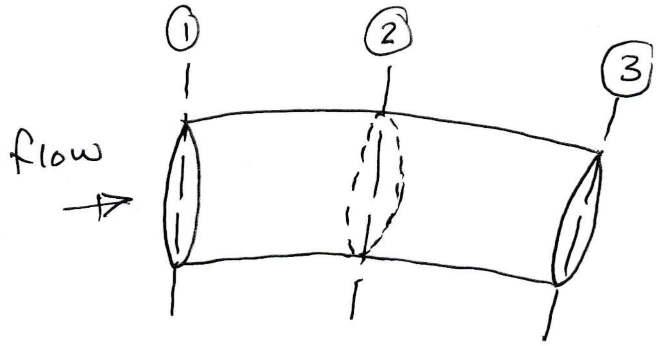
$$a_x = \frac{Du}{Dt} = \underbrace{\frac{\partial u}{\partial t}}_{\text{local acc.}} + \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{convective acc.}}$$

The Continuity Eq.

By conservation of mass:

Inflow – Outflow = Rate of change of accumulating materials inside the control volume.

For 1D, steady flow & incompressible fluid,



$$\boxed{\begin{aligned} Q &= A \cdot V \\ Q_1 &= Q_2 = Q_3 \end{aligned}}$$

where : Q = flowrate (discharge) in (L^3/T) L = length unit, T = time "

A = cross-sectional area in (L^2)

V = average velocity in (L/T)

$Q_1, Q_2, \& Q_3$: flow rate at sections (1), (2), & (3), respectively.

<u>Ex.1</u>: (3 KN) of water per second flow through pipeline reducer. Calculate the flowrate in (m^3/s.) & the mean velocities in the (300 mm) & (200 mm) pipe diameters.

<u>Solution</u>:

Since, $W = \gamma \forall$

$\therefore \forall = \frac{W}{\gamma} = \frac{3000}{9810}$

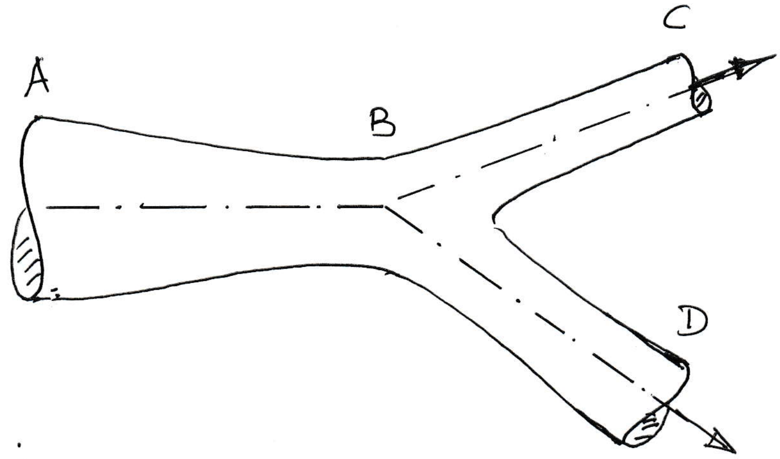
$= 0.306 \; m^3$

$Q = \frac{\forall}{t}$; for $t = 1$ s. $\Rightarrow Q = \frac{0.306}{1} = 0.306 \; m^3/s.$

$\because Q = A.V \Rightarrow V_1 = \frac{Q}{A_1} = \frac{0.306}{\frac{\pi}{4}(0.3)^2} = 4.33$ m/s

Similarily; $V_2 = \frac{Q}{A_2} = 9.74$ m/s

Ex.2 : As shown in figure below, if $D_A = 450$ mm, $D_B = 300$ mm, $D_c = 150$ mm, $D_D = 225$ mm, $V_A = 1.8$ m/s., & $V_D = 3.6$ m/s., determine V_B & V_c.



Solution : By continuity;

$$Q_A = Q_B = Q_c + Q_D$$

$$\therefore \quad A_A \cdot V_A = A_B \cdot V_B = A_c \cdot V_c + A_D \cdot V_D \quad — \quad (1)$$

from eq. (1) : $A_A \cdot V_A = A_B \cdot V_B \Rightarrow \frac{\pi}{4}(0.45)^2(1.8) = \frac{\pi}{4}(0.3)^2 V_B$

$$\therefore V_B = 4.05 \text{ m/s.}$$

from eq. (1) : $A_A \cdot V_A = A_c \cdot V_c + A_D \cdot V_D$

$$\frac{\pi}{4}(0.45)^2(1.8) = \frac{\pi}{4}(0.15)^2 V_c + \frac{\pi}{4}(0.225)^2(3.6)$$

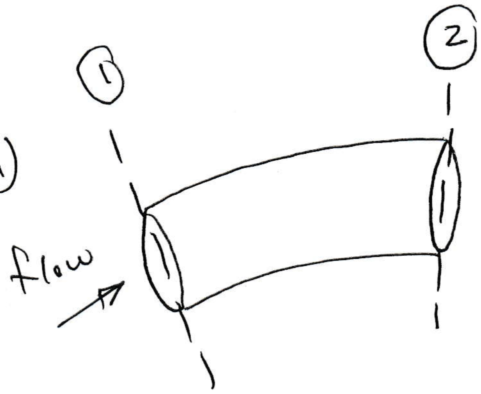
$$\therefore \quad V_c = 8.09 \text{ m/s.}$$

Equations of Fluid Motion

- Based on Newton's second law ($\sum \vec{F}$ = mass * Acceleration)

& for 1D, steady flow,

& incompressible fluid;



$$\boxed{\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} + h_{L_{1-2}}}$$ Energy Equation

- & for Ideal fluid (viscosity $= 0. \Rightarrow \eta = 0. \Rightarrow h_{L_{1-2}} = 0.$)

$$\boxed{\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} = \text{Constant}}$$ Bernoulli's Equation

where;

$\frac{P}{\gamma}$ = pressure head (L).

Z = elevation head (L).

$\frac{V^2}{2g}$ = velocity head (L)

$h_{L_{1-2}}$ = head loss between section 1 & 2 (L).

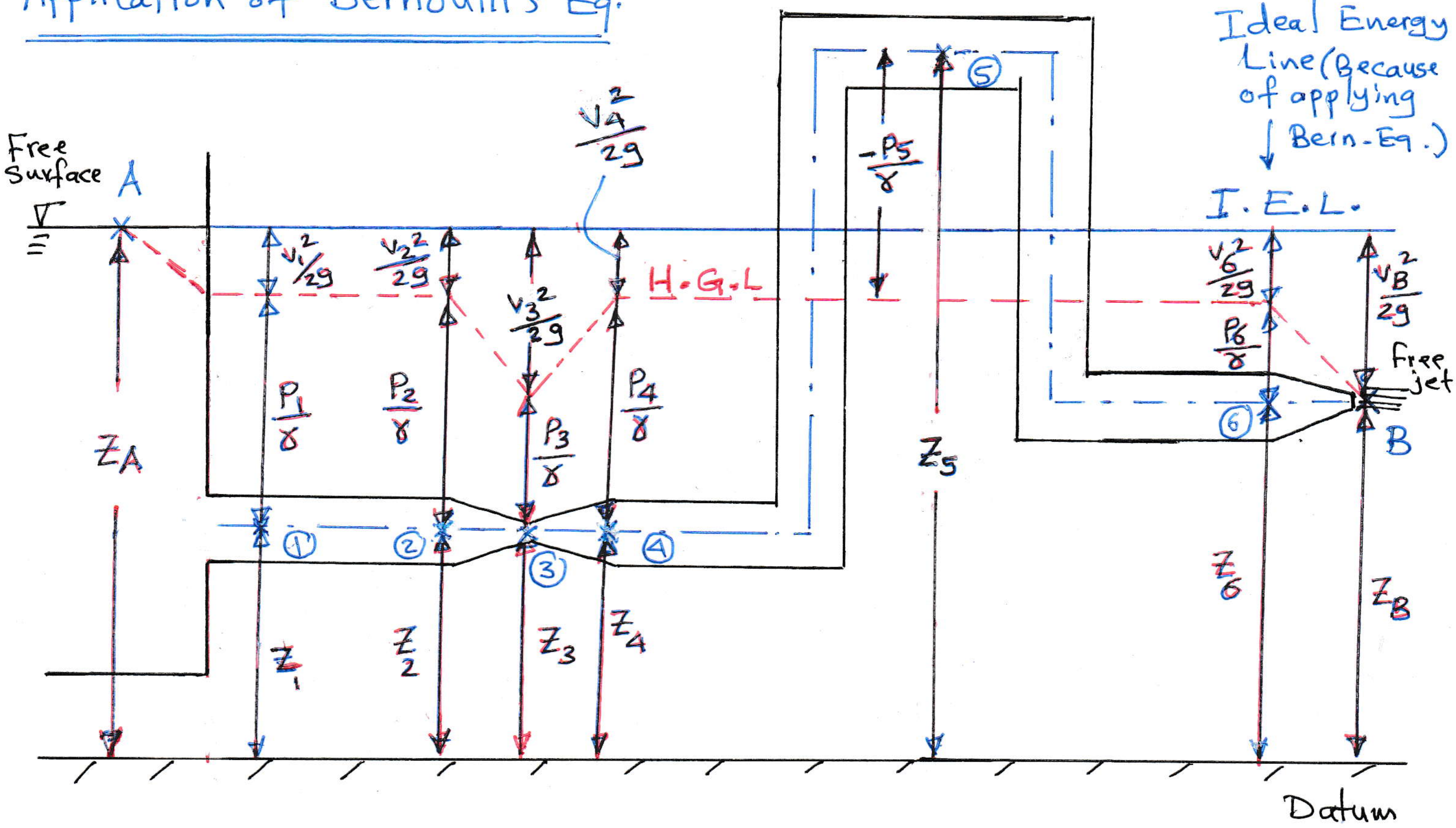
1 & 2: sections 1 & 2, respectively

Energy Line (E.L.) & Hydraulic Grade Line (H.G.L.)

$$E.L. = \text{Total Energy} = \frac{P}{\gamma} + Z + \frac{V^2}{2g}$$

$$H.G.L = \text{Potential Energy} = \frac{P}{\gamma} + Z$$

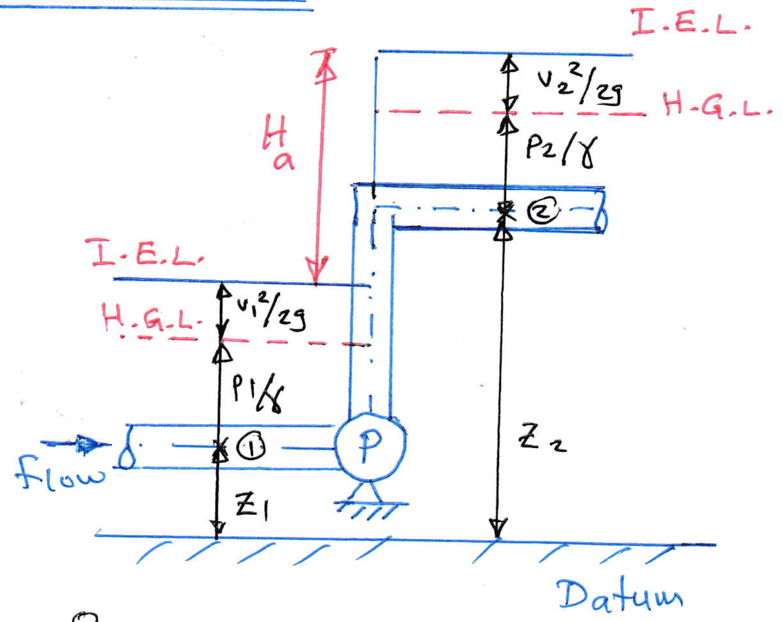
Application of Bernoulli's Eq.



System of Pipes with Pump or Turbine

with pump:

By applying Bern. Eq. between points 1 & 2:



$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} + H_a = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

where;

H_a = pump head (m)

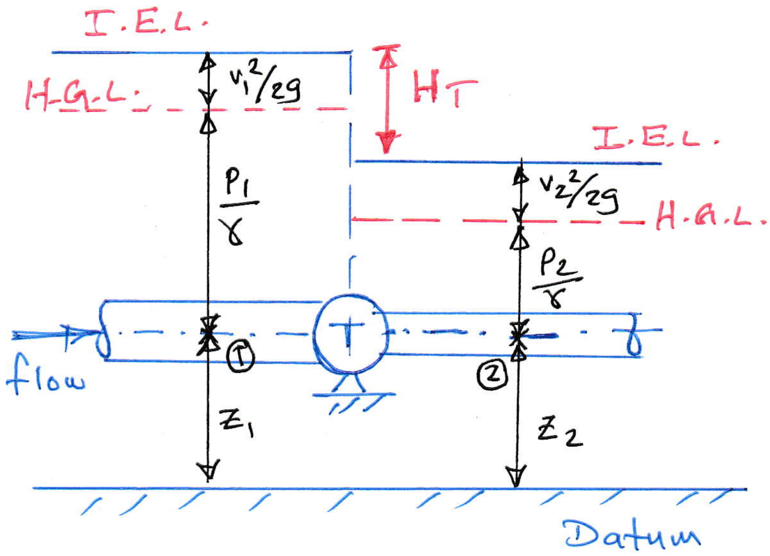
with Turbine:

By applying Bern. Eq. between points ① & ②:

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} - H_T = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

where;

H_T = turbine head (m).



Power: The power is the Energy per unit time.

$$\boxed{\text{power} = \gamma Q H}$$

where; Power measured Watt or horsepower (hp) or J./s.

Note: 1 hp = 746 Watt)

Q = flowrate ($m^3/s.$)

H = head (m).

γ : specific weight of the liquid (N/m^3)

So; Power of pressure $= \gamma Q \frac{P}{\gamma} = QP$

" " elevation $= \gamma Q z$

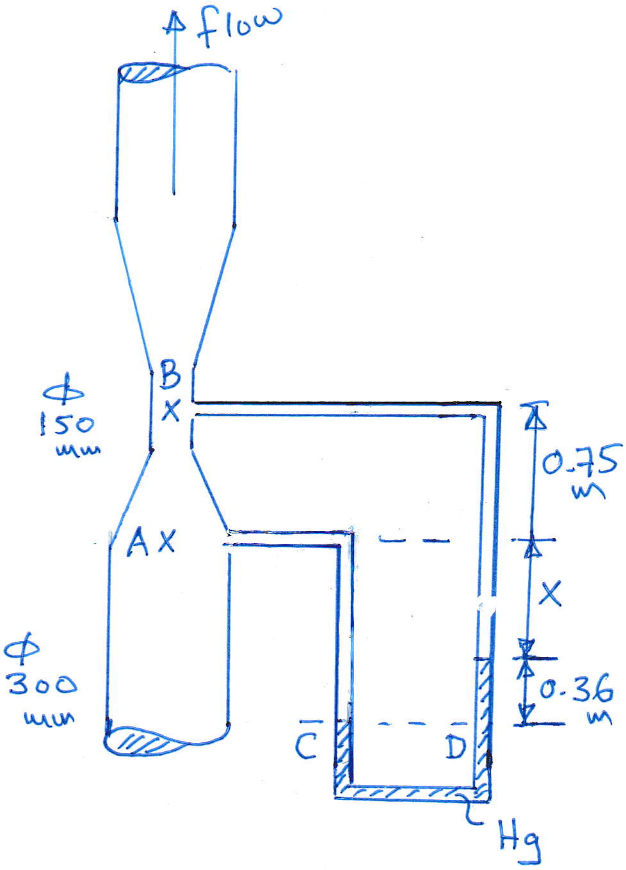
" " velocity $= \gamma Q \frac{V^2}{2g} = \frac{\rho Q v^2}{2}$

" " Pump $= \gamma Q H_a$

" " turbine $= \gamma Q H_T$

dissipation power due to friction $= \gamma Q h_L$

Ex.1: For the Venturi meter shown in figur below, the deflection of the mercury in the differential gauge is 0.36 m. Determine the flow of water through the meter if no energy is lost between A & B.



Sol.: Since there is no energy loss between A & B, by applying Bern. Eq. between points A & B:

$$\frac{P_A}{\gamma_w} + Z_A + \frac{V_A^2}{2g} = \frac{P_B}{\gamma} + Z_B + \frac{V_B^2}{2g}$$

Take datum at A ⇒ $Z_A = 0$.
$Z_B = 0.75$

So, Bern. Eq. becomes

$$\frac{P_A}{\gamma_w} + \frac{V_A^2}{2g} = \frac{P_B}{\gamma_w} + 0.75 + \frac{V_B^2}{2g} \quad \text{—— (1)}$$

From the differential gauge; $P_C = P_D$

$$P_A + \cancel{\gamma_w X} + \gamma_w (0.36) = P_B + \gamma_w (0.75) + \cancel{\gamma_w \cdot X} + 13.6\gamma_w (0.36) \quad \text{—— (2)}$$

Dividing Eq. (2) by γ_w:

$$\therefore \frac{P_A}{\gamma_w} = \frac{P_B}{\gamma_w} + 5.286 \quad \text{—— (3)}$$

From continuity $\Rightarrow Q_A = Q_B$

$$\therefore A_A \cdot V_A = A_B V_B$$

$$\therefore V_A = \frac{A_B}{A_A} V_B$$

$$\therefore V_A = 0.25 V_B \quad \text{—} \quad (4)$$

Subs. eqs. (3) & (4) into eq. (1):

$$\cancel{\frac{P_B}{\gamma_w}} + 5.286 + \frac{(0.25 V_B)^2}{2g} = \cancel{\frac{P_B}{\gamma_w}} + 0.75 + \frac{V_B^2}{2g}$$

$$\therefore V_B = 9.74 \text{ m/s.}$$

$$\& \; Q_B = 0.17 \text{ m}^3/\text{s.}$$

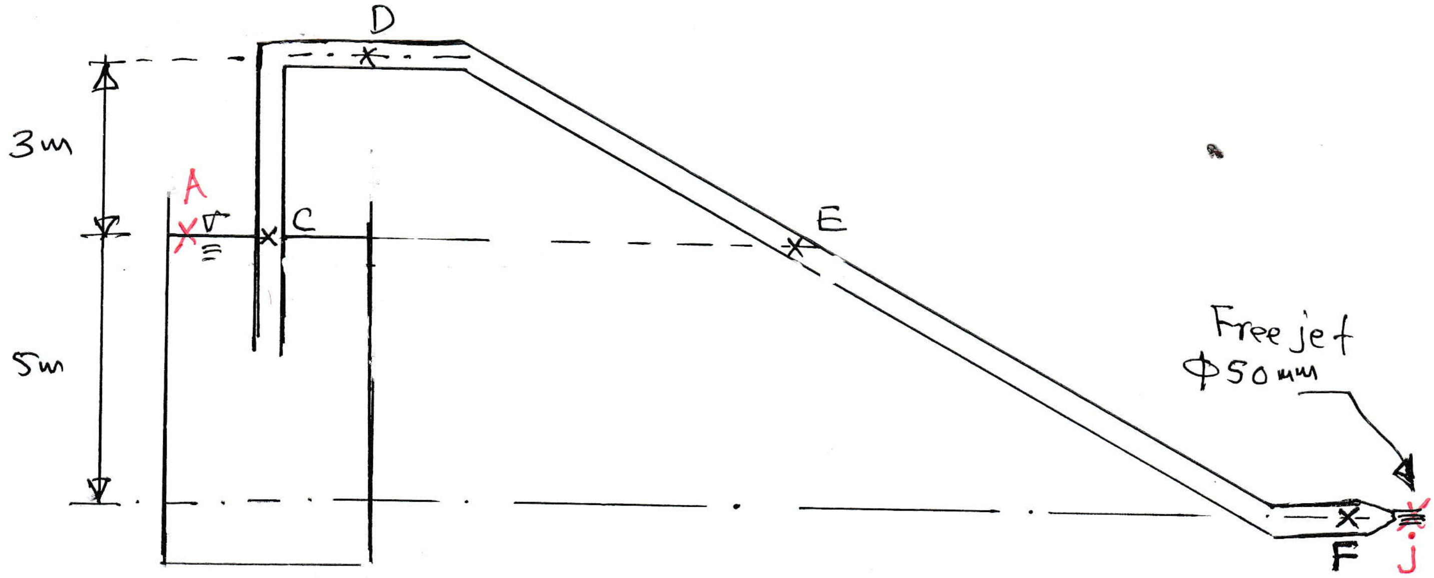
Ex.2: For the siphone shown in figure below, if its diameter is 100m, determine:

i- the outlet flow.

ii- the pressures at points C, D, E, & F.

iii- Plot the E.L. & H.G.L.

Note: Assume no energy loss.



Sol.: i- Since there is no energy loss,

by applying Bern. Eq. between points

A & j :

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_j}{\gamma} + Z_j + \frac{V_j^2}{2g}$$

Take datum at point j :

Bern. Eq. becomes;

$$0 + 5 + 0 = 0 + 0 + \frac{V_j^2}{2g}$$

$\therefore V_j = 9.9$ m/s.

$Q = 0.019\ m^3/s.$

ii- By continuity $\Rightarrow Q_c = Q_D = Q_E = Q_F = Q = 0.019\ m^3/s$

Since, $A_C = A_D = A_E = A_F$

$$\therefore V_c = V_D = V_E = V_F = \frac{0.019}{\frac{\pi}{4}(0.1)^2} = 2.42\ m/s$$

By applying Bern. Eq. between A & C : Datum at A

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_c}{\gamma} + Z_c + \frac{V_c^2}{2g}$$

Datum at A $\Rightarrow$ $0 + 0 + 0 = \frac{P_c}{\gamma} + 0 + \frac{V_c^2}{2g}$

$$\therefore \frac{P_c}{\gamma} = \frac{V_c^2}{2g} \Rightarrow P_c = -2928.2\ Pa.$$

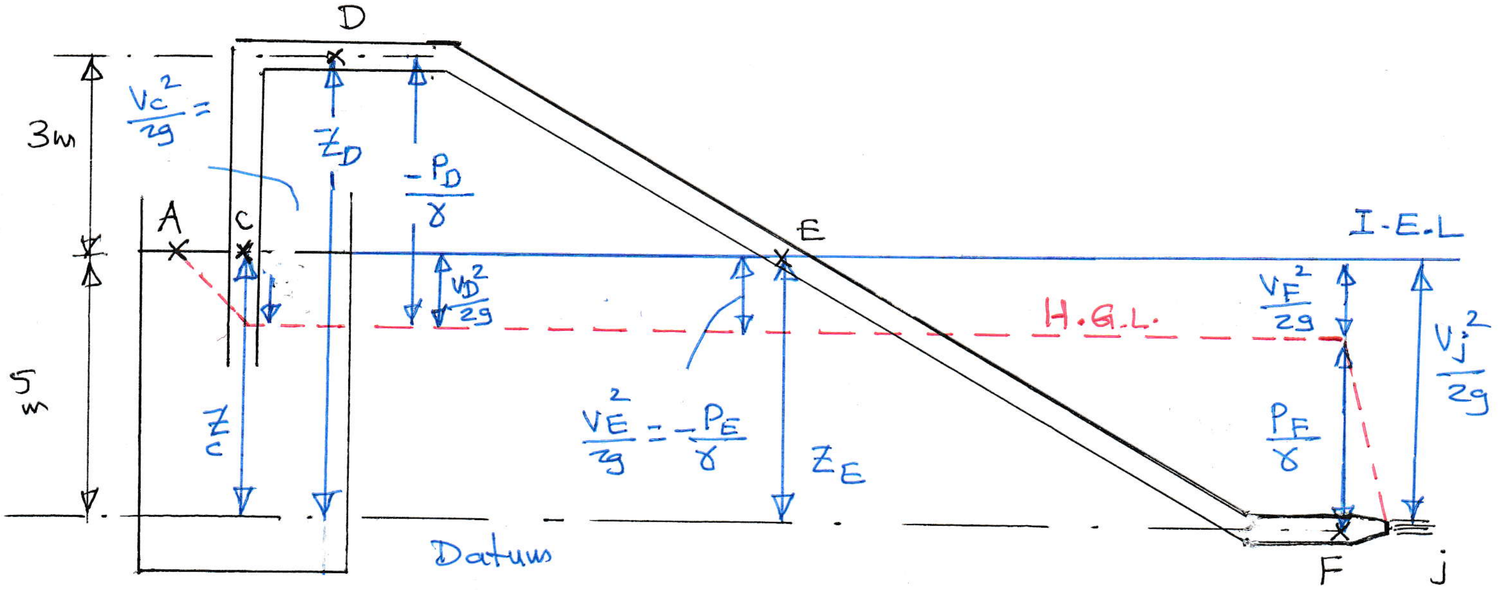
H.W.: Find P_D, P_E, & P_F

Ans. : $P_D = -32358$ Pa.

$P_E = P_c$

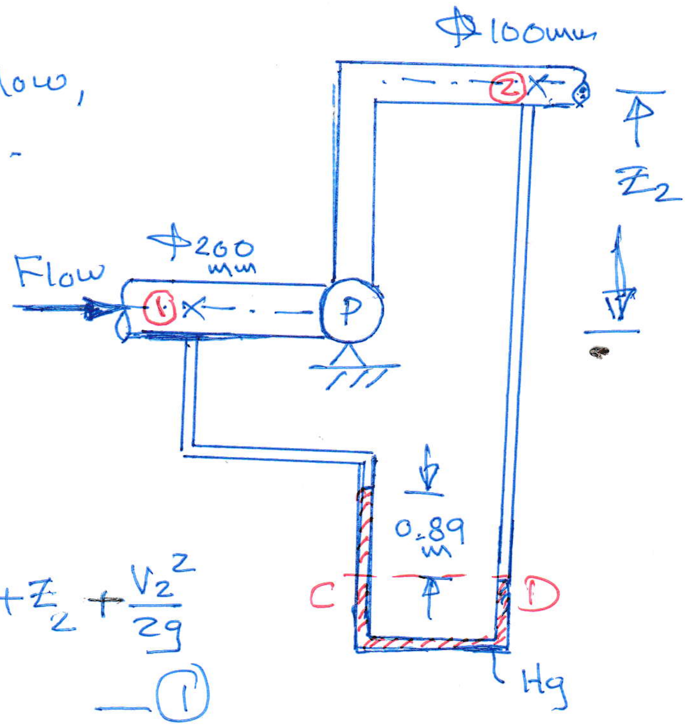
$P_F = 46122$ Pa.

iii-



EX-3: For the figure shown below, calculate the flowrate of water delivered by a pump which added 12 hp. Assume frictionless flow.

Sol.: For frictionless flow, applying Bern. Eq. between points ① & ②: (Take datum at point ①):



$$\frac{P_1}{\gamma_w}+0+\frac{V_1^2}{2g}+H_a=\frac{P_2}{\gamma_w}+Z_2+\frac{V_2^2}{2g} \quad \text{——(1)}$$

Since; $\text{Power} = \gamma_w Q H_a$

$\therefore\ 12 * 746 = 9810\ Q * H_a$

$$\therefore\ H_a = \frac{0.912}{Q} \quad \text{—— (2)}$$

since $Q_1 = Q_2 = Q \Rightarrow V_1 = \dfrac{Q}{A_1} = 31.83Q$ —— (3)

$$V_2 = \frac{Q}{A_2} = 127.32Q \quad \text{—— (4)}$$

From the manometer; $P_C = P_D$

$$\therefore \left[P_1 + \gamma_{Hg}(0.89) = P_2 + \gamma_w Z_2 + \gamma_w (0.89)\right] \div \gamma_w$$

$$\therefore\ \frac{P_2}{\gamma_w} + Z_2 = \frac{P_1}{\gamma_w} + 11.214 \quad \text{—— (5)}$$

Subs. eqs. (2), (3), (4), & (5) into eq. (1):

$$\cancel{\frac{P_1}{\gamma_w}} + \frac{(31.83Q)^2}{2g} + \frac{0.912}{Q} = \cancel{\frac{P_1}{\gamma_w}} + 11.214 + \frac{(127.32Q)^2}{2g}$$

$$\therefore\ 774.577\,Q^3 + 11.214\,Q - 0.912 = 0.$$

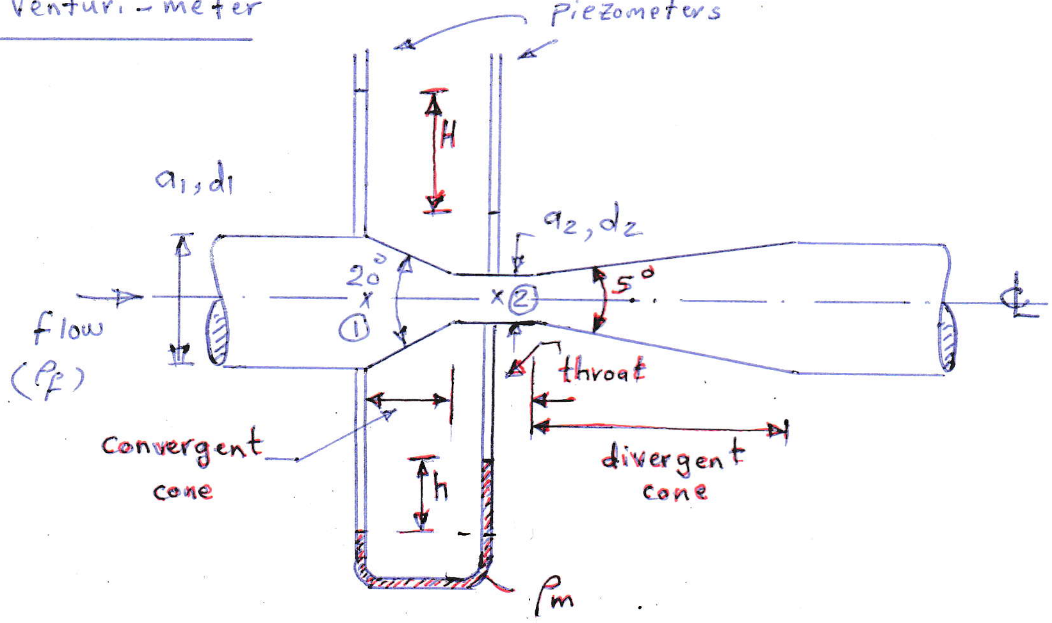
By trial & error $\Rightarrow Q = 0.0814\ m^3/s.$

Measurement of Flow Rate

A- Measurement of Flow Rate in Pipeline

1- Venturi - meter.
2- Nozzle meter.
3- Orifice "

1- Venturi-meter



For any inclination of Venturi-meter

$$\boxed{Q_{act.} = c_d \cdot a_1 \sqrt{\frac{2gH}{m^2 - 1}}}$$

where;

$Q_{act.}$ = actual discharge ($m^3/sec.$)

c_d = coefficient of discharge < 1.0 , (0.95 – 0.99)

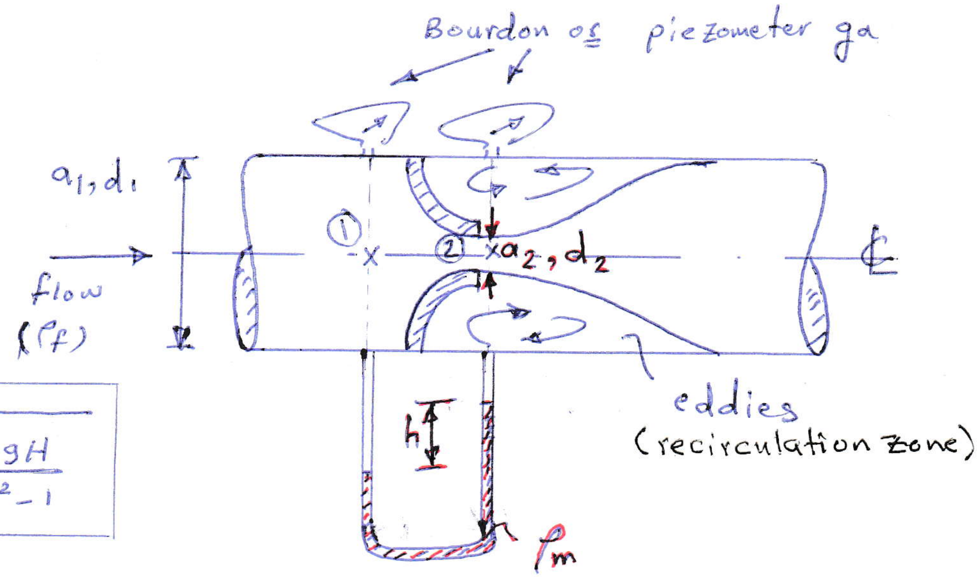
$$H = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right) = h\left(\frac{\rho_m}{\rho_f} - 1\right)$$

$$m = \frac{a_1}{a_2}$$

a_1 = Cross-sectional area of the pipe (m^2) $= \frac{\pi}{4} d_1^2$

a_2 = " " " " " throat (m^2) $= \frac{\pi}{4} d_2^2$

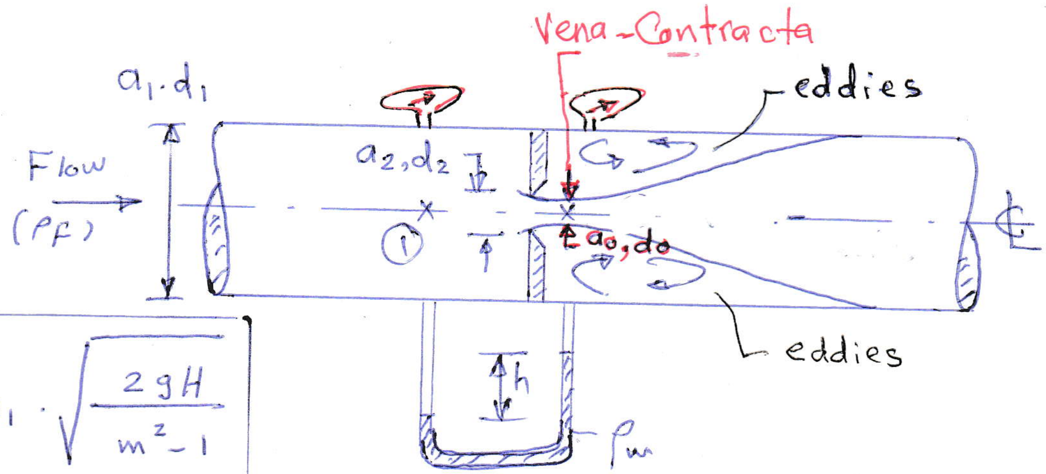
2- Nozzle meter



$$\boxed{Q_{act.} = C_d \cdot a_1 \cdot \sqrt{\frac{2gH}{m^2-1}}}$$

C_d - varies between (0.9 - 0.99)

3- Orifice meter



$$\boxed{Q_{act.} = C_d \cdot a_1 \cdot \sqrt{\frac{2gH}{m^2-1}}}$$

a_2 = cross-sectional area of the orifice opening (m^2).

C_c = coefficient of contraction $= \frac{a_o}{a_2}$

a_o = cross-sectional area of the flow at Vena-Contracta.

C_d - varies from 0.55 to 0.7.

$m = \frac{a_1}{a_2}$

a_2 : cross-sectional of the orifice opening $= \frac{\pi}{4} d_2^2$

B - Measurement of Flow in Tanks & Open channel

1. Orifices.
2. Weirs
   a- Rectangular Weir.
   b- Triangular (V-notch) Weir.
   c- Trapezoidal Weir.
3. Sluice Gate.
4. Pitot Tube.

1- Orifice

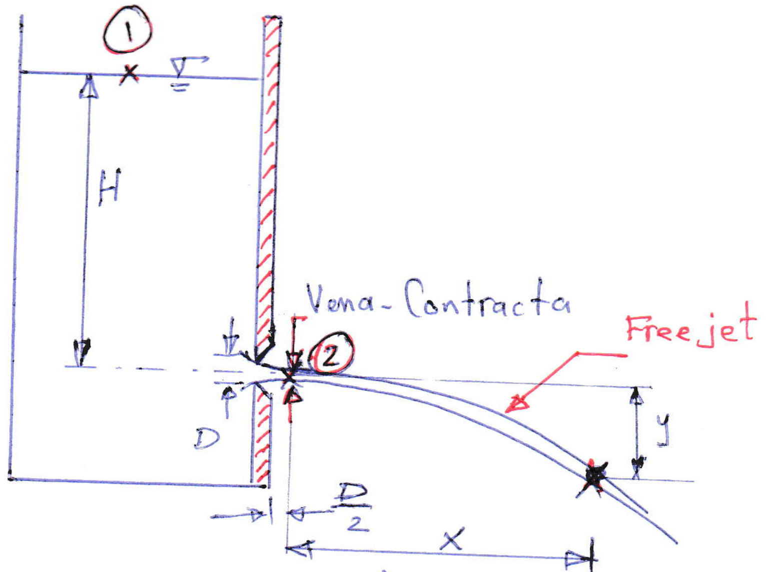
a- Small Orifice Under Constant Head ($H \geq 5D$) (Steady Flow)

$$Q_{act.} = C_d \cdot Q_{theo.}$$

$$Q_{theo.} = A_{theo.} * V_{theo.}$$

$$\therefore Q_{theo.} = a * \sqrt{2gH}$$

$$\therefore \boxed{Q_{act.} = C_d \cdot a \cdot \sqrt{2gH}}$$



sharp-orifice
(wall thickness < D (orifice size))

where;

$Q_{act.}$ = actual discharge (m^3/sec.)

$Q_{theo.}$ = theoritical discharge (m^3/sec.)

C_d = coeff. of discharge (< 1.0)

a = theoritical area (area of the orifice)

$V_{theo.}$ = theoritical velocity (Torricelli Theorem) = $\sqrt{2gH}$ (By applying Bern. Eq. between points 1 & 2

H = Constant head above the centerline of the orifice (m).

$$V_{act.} = C_v * V_{theo.}$$

$$\therefore \boxed{C_v = \frac{V_{act.}}{V_{theo.}}}$$

$$\boxed{V_{act.} = \sqrt{\frac{g x^2}{2y}}} \quad ; \quad V_{theo.} = \sqrt{2gH}$$

$$\therefore C_v = \frac{\sqrt{\frac{g x^2}{2y}}}{\sqrt{2gH}} \Rightarrow \boxed{C_v = \frac{x}{2\sqrt{yH}}}$$

where; C_v = coefficient of velocity (<1.0).

$V_{theo.}$ = theoritical velocity.

$V_{act.}$ = actual " .

$$C_c = \frac{A_{act.}}{A_{theo.}} = \frac{\text{Area of jet at vena-Contracta}}{\text{area of the orifice}}$$

$$= \frac{\frac{Q_{act.}}{V_{act.}}}{\frac{Q_{theo.}}{V_{theo.}}} = \frac{Q_{act.}}{V_{act.}} * \frac{V_{theo.}}{Q_{theo.}} = \frac{C_d}{C_v}$$

$$\therefore \boxed{C_c = \frac{C_d}{C_v}}$$

where; C_c = coefficient of contraction (<1.0).

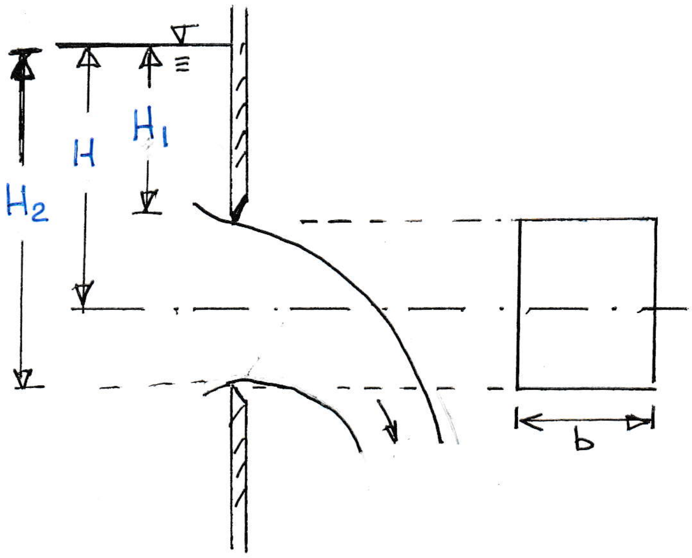
Losses in the Orifice Flow

By applying Energy Eq. between points 1 & 2 (vena-contracta).

$$\boxed{h_f = H(1 - C_v^2)}$$

2- Large Sharp Orifice Under Constant Head "H<5D"

$$\boxed{Q_{act.} = \frac{2}{3} C_d \cdot b\sqrt{2g}\left[H_2^{3/2} - H_1^{3/2}\right]}$$



Discharge of an orifice Under Falling Head (Unsteady Flow)

let;

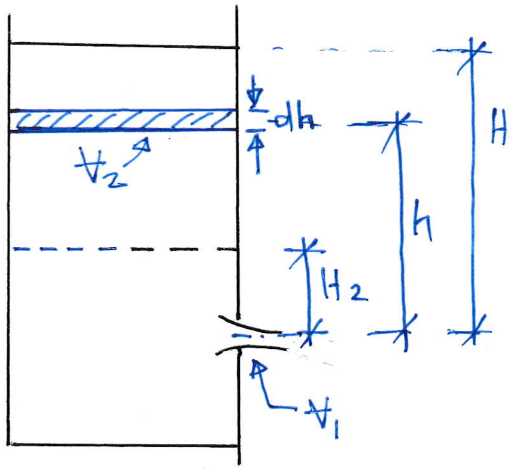
A: cross-sectional area of the tank

a: " " " " orifice

H_1 = orifice head at time t.

H_2 = " " " " t

h = orifice " " any time (0-t).



By continuity requirements

$$\forall_1 = -\forall_2$$

(-ve sign because the flow is under falling head)

$$\forall_1 = dQ.dt \quad ; \quad \forall_2 = A.dh$$

$$\therefore \quad \boxed{dQ.dt = -A.dh}$$

* For Constant cross-sectional area of the tank w.r.t. variable orifice head (Constant strip cross-sectional area).

$$dQ.dt = -A.dh$$

$$\therefore \quad C_d.a.\sqrt{2gh} \times dt = -A.dh$$

$$\therefore \int_0^t dt = \frac{-A}{C_d.a.\sqrt{2g}} \int_{H_1}^{H_2} h^{-1/2}\, dh$$

$$\therefore \quad \boxed{t = \frac{2A}{a.C_d.\sqrt{2g}} \left(\sqrt{H_1} - \sqrt{H_2}\right)}$$

when the tank is completely empty ($H_2 = 0$)

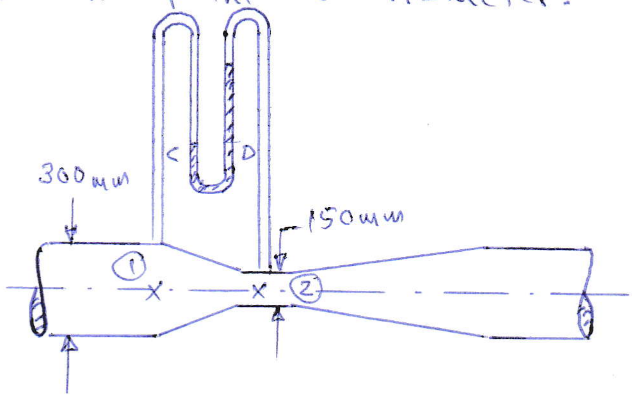
$$\therefore \quad t = \frac{2A}{a.C_d.\sqrt{2g}} \sqrt{H_1}$$

* For variable cross-sectional area of the tank w.r.t. variable orifice head $\Rightarrow A = F(h)$

$$\therefore \quad dQ.dt = -A.dh$$

Examples :-

1- Water flows through a (300 mm X 150 mm) Venturi-meter at the rate (0.04 m^3/sec.) & the differential gage is deflected (1 m) as shown in the figure below. The specific gravity of the gage liquid is (1.25). Determine the coefficient of the Venturi-meter.



Solution

Since, $Q_{act.} = C_d \cdot a_1 \sqrt{\dfrac{2gH}{m^2 - 1}}$

From the differential gage;

$$H = h\left(\frac{\rho_m}{\rho_f} - 1\right) = 1\left(\frac{1.25\rho_w}{\rho_w} - 1\right) = 0.25\,m$$

or $\quad P_C = P_D$

$$\left[P_1 = P_2 - \gamma_w(1) + 1.25\,\gamma_w(1)\right] * \frac{1}{\gamma_w}$$

$$\therefore \frac{P_1 - P_2}{\gamma_w} = 1.25 - 1 \Rightarrow H = 0.25\,m$$

$$a_1 = \frac{\pi}{4}(0.3)^2 = 0.0707\,m^2$$

$$m = \frac{a_1}{a_2} = \frac{\pi/4\,(0.3)^2}{\pi/4\,(0.15)^2} = 4$$

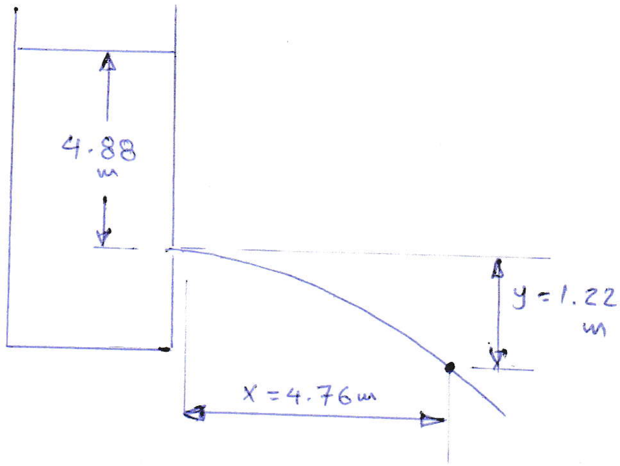
$$\therefore 0.04 = C_d * 0.0707 * \sqrt{\frac{19.62 * 0.25}{(4)^2 - 1}}$$

$$\therefore C_d = 0.99$$

2- A (75 mm) diameter orifice under a head of (4.88 m) discharge (8900 N) of water in (32.6 sec.). The trajectory was determined by measuring (X = 4.76 m) for drop of (1.22 m). Determine C_v, C_d, C_c & the head loss in the orifice flow

Solution: $H(4.88\,m) > 5(0.075) \Rightarrow$ small orifice.

$$C_v = \frac{V_{act.}}{V_{theo.}} = \frac{\sqrt{\frac{g X^2}{2y}}}{\sqrt{2gH}}$$



$$\therefore C_v = \frac{X}{2\sqrt{yH}}$$

$$\therefore C_v = \frac{4.76}{2\sqrt{1.22(4.88)}} = 0.975$$

$$C_d = \frac{Q_{act.}}{Q_{theo.}}$$

since, $W = \gamma \forall \Rightarrow \forall = \frac{W}{\gamma_w} = \frac{8900}{9810} = 0.907\ m^3$

since, $Q = \frac{\forall}{t} \Rightarrow Q_{act.} = \frac{\forall}{t} = \frac{0.907}{32.6} = 0.0278\ m^3/sec.$

$$Q_{theo.} = A_{theo.} * V_{theo.} = a.\sqrt{2gH} = \frac{\pi}{4}(0.075)^2 * \sqrt{2g(4.88)}$$

$$\therefore Q_{theo.} = 0.0432\ m^3/sec.$$

$$\therefore C_d = \frac{0.0278}{0.0432} = 0.643$$

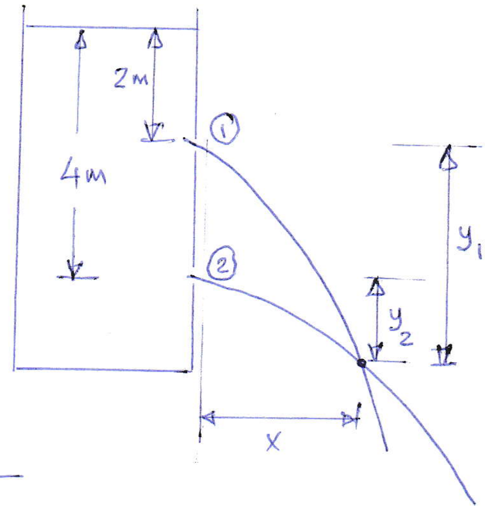
$$C_c = \frac{C_d}{C_v} = \frac{0.643}{0.975} = 0.659$$

$$\because h_f = H(1 - C_v^2) \Rightarrow h_f = 4.88(1 - 0.975^2) = 0.241\ m$$

3- A tank has two identical orifices with (60 mm) dia. in one of its vertical sides. The depths of the two orifices are (2m) & (4m), below the water surface. Find the discharge from the tank & the point at which the two jets will intersect. (Take $C_v = 0.9$ & $C_d = 0.6$).

<u>Solution</u>: since, $H\,(2m \text{ or } 4m) > 5\,(0.06)$

$\therefore$ small orifice



$\therefore \; Q_{act.} = C_d \cdot a \cdot \sqrt{2gH}$

$$Q_{act.\,total} = Q_{act.\,①} + Q_{act.\,②}$$

$$Q_{act.\,total} = C_{d_1} \cdot a_1 \sqrt{2gH_1} + C_{d_2} \cdot a_2 \cdot \sqrt{2gH_2}$$

For identical orifice; $Q_{act.\,total} = C_d \cdot a \cdot \sqrt{2g}\left(\sqrt{H_1} + \sqrt{H_2}\right)$

$$\therefore Q_{act.\,total} = 0.6 \times \frac{\pi}{4}(0.06)^2 \sqrt{19.62}\left(\sqrt{2} + \sqrt{4}\right)$$

$$= 0.0256 \; m^3/sec.$$

since, $C_v = \dfrac{V_{act.}}{V_{theo.}} = \dfrac{X}{2\sqrt{yH}}$

For orifice No. ① : $\quad 0.9 = \dfrac{X}{2\sqrt{y_1(2)}} \quad \Rightarrow X = 2.5456\, y_1^{1/2}$ — ①

For orifice NO. ② : $\quad 0.9 = \dfrac{X}{2\sqrt{y_2(4)}} \quad \Rightarrow X = 3.6\, y_2^{1/2}$ — ②

From eqs. ① & ② $\Rightarrow$ $2.5456\, y_1^{1/2} = 3.6\, y_2^{1/2}$

$\therefore y_1 = 2\, y_2$ — ③

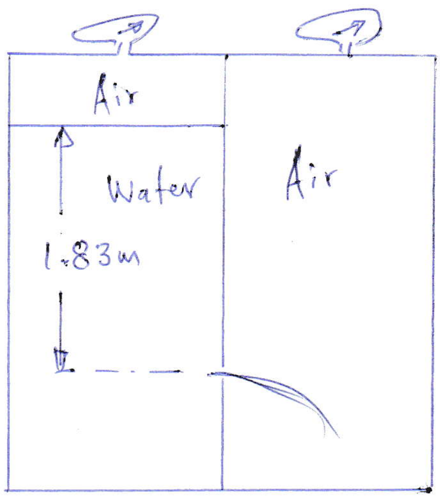
From the figure; $y_1 = y_2 + 2$ — ④

From eqs. ③ & ④ $\Rightarrow$ $y_2 + 2 = 2y_2 \Rightarrow y_2 = 2\,m$

From eq. ③ $\Rightarrow y_1 = 4\,m$

From eq. ① $\Rightarrow X = 5.1\,m$

4- What will be the real discharge from (50mm) circular orifice shown in figure below. Also determine the head loss in the orifice flow. (Take Cd = 0.6 & Cv = 0.9).



Solution:

$$H = 1.83 + \frac{830*10^3}{9810} + \frac{30*10^3}{9810}$$

$$= 89.5\ m$$

since, $H(89.5) > 5(0.05)$

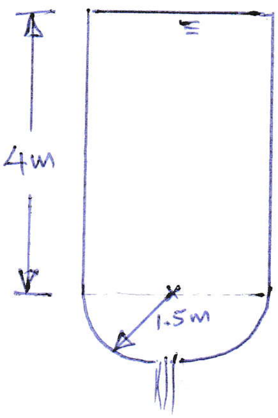
$\therefore$ small orifice.

$$\therefore Q_{act.} = Cd.\ a.\ \sqrt{2gH} = 0.6 * \frac{\pi}{4}(0.05)^2 \sqrt{19.62 * 89.5}$$

$$\therefore Q_{act.} = 0.0493\ m^3/sec.$$

$$\because h_f = H(1 - C_v^2) = 89.5(1 - 0.9^2) = 17\ m$$

5- The tank has an upper cylindrical portion of (3m) dia. & (4m) height with a hemispherical base. The cylinder is full of water. Determine the time taken to empty it through an orifice of (100mm) dia. its bottom. Take Cd = 0.62.



Solution

$T = T_1 + T_2$

where; T_1 = time required to empty the cylinder.

T_2 = " " " " " hemisphere.

$$T_1 = \frac{2A}{a.\ cd.\sqrt{2g}}\left(\sqrt{H_1} - \sqrt{H_2}\right)$$ { constant tank area w.r.t. variable orifice head }

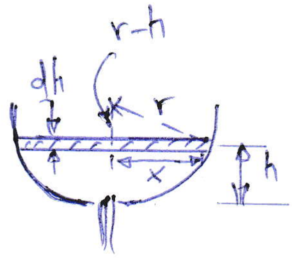
$$T_1 = \frac{2 * \frac{\pi}{4}(3)^2}{0.62 * \frac{\pi}{4}(0.1)^2\sqrt{19.62}}\left(\sqrt{5.5} - \sqrt{1.5}\right) = 734\ sec.$$

To find T_2 ; $\quad dQ.dt = -A.dh$ { variable tank area. w.r.t. variable orifice head }

$$A = F(h)$$

$$\therefore c_d . a . \sqrt{2gh} . dt = - A . dh \quad ;$$

From the figure;



$$r^2 = x^2 + (r-h)^2$$

$$\therefore 1.5^2 = x^2 + (1.5-h)^2$$

$$\therefore x^2 = 1.5^2 - (1.5^2 - 3h + h^2)$$

$$\therefore x^2 = 3h - h^2$$

$$A = \pi x^2 = \pi (3h - h^2)$$

$$\therefore 0.62 * \frac{\pi}{4} (0.1)^2 \sqrt{19.62h} . dt = -\pi (3h - h^2) dh$$

$$\therefore \int_0^{T_2} dt = -145.65 \int_{1.5}^{0} (3h^{1/2} - h^{3/2}) dh$$

$$\therefore T_2 = 374 \text{ sec.}$$

$$\therefore T = 734 + 374 = 1108 \text{ sec.}$$