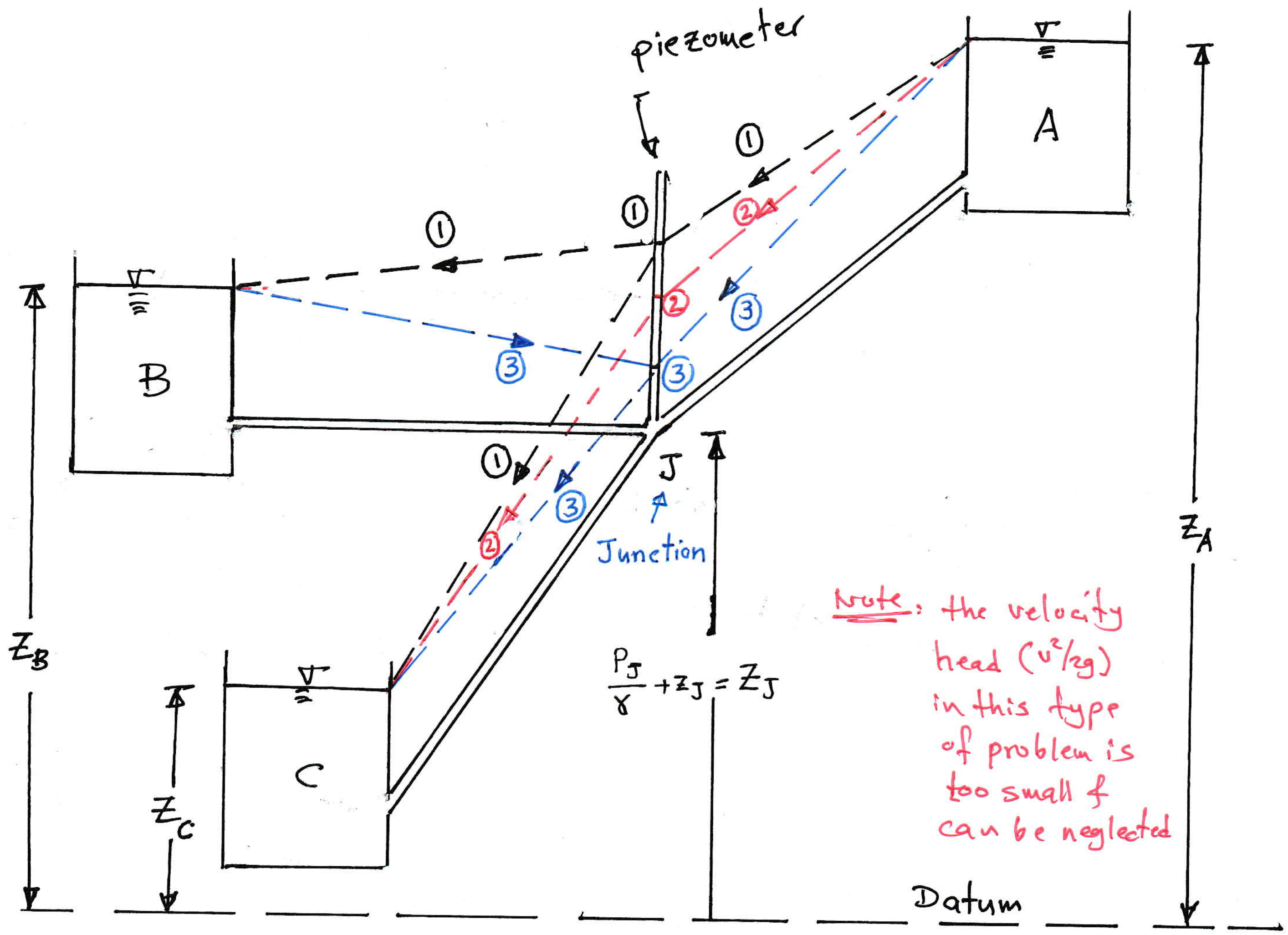
# Branching Pipes




**Case ①** Flow from A to B & C when $Z_J > Z_B$ & $Z_J > Z_c$

$$\Rightarrow (Q_A = Q_B + Q_c)$$

**Case ②** Flow from A to C & no flow in or out from B when

$Z_J = Z_B$ & $Z_J > Z_c \Rightarrow (Q_A = Q_c \text{ \& } Q_B = 0.)$

**Case ③** Flow from A & B to C when $Z_J < Z_B$ & $Z_J > Z_c$

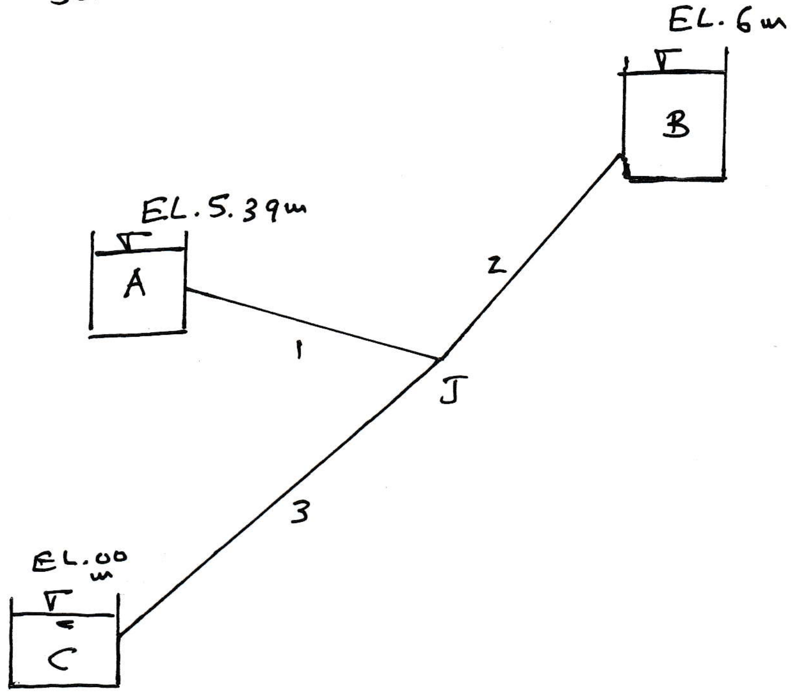
$$\Rightarrow (Q_c = Q_A + Q_B)$$

To check $\boxed{Q_{\text{into } J} = Q_{\text{out from } J} \text{ OR } \sum_J Q = 0.}$

**Ex.1**: As shown in figure below, find the discharges of water through each pipe. The pipe characteristics are as follows:

| pipe | L (m) | D (mm) | $C_{H.W.}$ |
|---|---|---|---|
| 1 | 100 | 250 | 100 |
| 2 | 300 | 300 | 80 |
| 3 | 1500 | 300 | 120 |

**Note**: neglect minor losses.




**Sol.**: let $Z_J = EL_A = 5.39\,m \Rightarrow Q_1 = 0. \Rightarrow Q_2 = Q_3$ (must be checked)

**To check**: By applying Energy Eq. between B & J: $EL_B = Z_J + h_{L_2}$

$$h_{L_2} = EL_B - Z_J = 6 - 5.39 = 0.61\,m$$

$$\therefore Q_2 = 0.2785\,(80)\,(0.3)^{2.63}\left(\frac{0.61}{300}\right)^{0.54} = 0.033\ m^3/s.$$

By applying Energy Eq. between J & C: $Z_J = EL_C + h_{L_3}$

$$h_{L_3} = Z_J - EL_C = 5.39 - 0 = 5.39\,m$$

$$\therefore Q_3 = 0.2785\,(120)\,(0.3)^{2.63}\left(\frac{5.39}{1500}\right)^{0.54} = 0.067\ m^3/s.$$

since, $Q_3 > Q_2 \Rightarrow$ there is a flow at pipe 1 from A to J

$\Rightarrow Z_J < 5.39$ m & $Z_J > 0$.

So, $$Q_3 = Q_1 + Q_2 \quad \text{—— } (1)$$

since, $$Q = 0.2785\, C_{H.W.}\, D^{2.63} \left(\frac{h_L}{L}\right)^{0.54}$$

$$\therefore h_L = \frac{10.6\, L}{C_{H.W.}^{1.85}\, D^{4.87}}\, Q^{1.85}$$

$$\therefore h_{L_1} = 180.86\, Q_1^{1.85}$$

$$h_{L_2} = 337.39\, Q_2^{1.85}$$

$$h_{L_3} = 796.77\, Q_3^{1.85}$$

By applying Energy Eq. between B & C $\Rightarrow h_{L_2} + h_{L_3} = 6$

$$\therefore 337.39\, Q_2^{1.85} + 796.77\, Q_3^{1.85} = 6$$

$$\therefore Q_2 = \left[\frac{6 - 796.77\, Q_3^{1.85}}{337.39}\right]^{0.54} \quad \text{—— } (2)$$

By applying Energy Eq. between A & C $\Rightarrow h_{L_1} + h_{L_3} = 5.39$

$$\therefore 180.86\, Q_1^{1.85} + 796.77\, Q_3^{1.85} = 5.39$$

$$\therefore \quad Q_1 = \left[ \frac{5.39 - 796.77\, Q_3^{1.85}}{180.86} \right]^{0.54} \quad \text{——} \quad (3)$$

Subs. eqs. (2) & (3) into eq. (1):

$$Q_3 = \left[ \frac{5.39 - 796.77\, Q_3^{1.85}}{180.86} \right]^{0.54} + \left[ \frac{6 - 796.77\, Q_3^{1.85}}{337.39} \right]^{0.54}$$

By trial & error; $Q_3 = 0.0656\ m^3/s.$

$\Rightarrow Q_2 = 0.0393\ m^3/s.$

& $Q_1 = 0.0273\ m^3/s.$

Another solution for EX.1 : Using trials Solution

Assume $Z_J = 3$

So, $Q_1$ & $Q_2$ flow into J $\Rightarrow$ +ve sign

& $Q_3$ flow out from J $\Rightarrow$ −ve sign

1st trial

| pipe | $h_L$ (m) | C.H.W. | D (m) | L (m) | $S^{0.54}$ | Q ($m^3/s.$) |
|---|---|---|---|---|---|---|
| 1 | 5.39−3 = 2.39 | 100 | 0.25 | 100 | 0.133 | +0.0967 |
| 2 | 6 − 3 = 3 | 80 | 0.3 | 300 | 0.083 | +0.078 |
| 3 | 3 − 0 = 3 | 120 | 0.3 | 1500 | 0.035 | −0.0493 |
| | | | | | | $\Sigma$ = +0.1254 |

$\sum_J Q$ must be = 0.

since $\Sigma Q$ is +ve $\Rightarrow$ this means that the $Q_1$ & $Q_2$ values are greater than $Q_3$

To reduce the difference between ($Q_1$ & $Q_2$) & $Q_3$

$\Rightarrow$ $Z_J$ must be increased in order to decrease the values of $Q_1$ & $Q_2$ and increase $Q_3$ value.

2nd trial : Assume $Z_J = 5$ m

| pipe | $h_L$ (m) | CH.W. | D (m) | L (m) | $S^{0.54}$ | Q ($m^3$/s.) |
|---|---|---|---|---|---|---|
| 1 | 5.39 − 5 = 0.39 | 100 | 0.25 | 100 | 0.05 | +0.0363 |
| 2 | 6 − 5 = 1.0 | 80 | 0.3 | 300 | 0.046 | +0.043 |
| 3 | 5 − 0 = 1.0 | 120 | 0.3 | 1500 | 0.046 | −0.065 |
| | | | | | | $\Sigma$ = +0.0143 |

3rd trial : Assume $Z_J = 5.16$ m

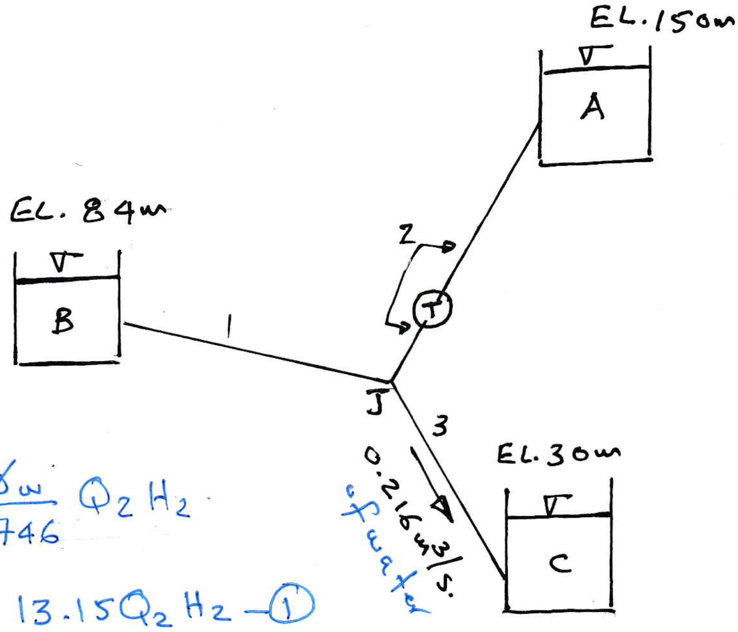
| pipe | $h_L$ (m) | CH.W. | D (m) | L (m) | $S^{0.54}$ | Q ($m^3$/s.) |
|---|---|---|---|---|---|---|
| 1 | 0.23 | 100 | 0.25 | 100 | 0.0376 | +0.0273 |
| 2 | 0.84 | 80 | 0.3 | 300 | 0.042 | +0.0393 |
| 3 | 5.16 | 120 | 0.3 | 1500 | 0.047 | −0.0656 |
| | | | | | | $\Sigma$ = 0.0 |

$\therefore$ O.K

So, $Q_1 = 0.0273$ $m^3/s$ ; $Q_2 = 0.0393$ $m^3/s$ & $Q_3 = 0.0656$ $m^3/s$

**Ex.2**: A turbine is located in the 375 mm pipe line, as shown in figure below. What is the turbine horsepower? The pipe characteristics are as follows:

| pipe | L (m) | D (mm) | f |
|---|---|---|---|
| 1 | 600 | 150 | 0.024 |
| 2 | 1200 | 375 | 0.018 |
| 3 | 2400 | 300 | 0.02 |




**Sol.**

since: power $= \gamma Q H$

$\therefore$ turbine horsepower $= \dfrac{\gamma_w}{746} Q_2 H_2$

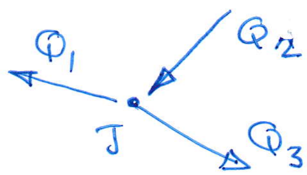
$\therefore$ turbine horsepower $= 13.15 Q_2 H_2$ —— (1)

for pipe no. 3: $V_3 = \dfrac{Q_3}{A_3} = 3.056$ m/s.

$$\therefore h_{L_3} = 0.02 \, \frac{2400}{0.3} \, \frac{(3.056)^2}{2g} = 76.15 \text{m}$$

applying Energy eq. between J & C: $Z_J = EL_c + h_{L_3}$

$\therefore Z_J = 30 + 76.15 = 106.15$ m




So, $Q_2 = Q_1 + Q_3$

$\therefore Q_2 = Q_1 + 0.216$ —— (2)

Energy Eq. between J & B: $Z_J = EL_B + h_{L_1}$

$\therefore h_{L_1} = Z_J - EL_B = 22.15\,m$

for pipe no. 1: $22.15 = 0.024 \dfrac{600}{0.15} \dfrac{v_1^2}{2g} \Rightarrow v_1 = 2.1276\,m/s.$

$\therefore Q_1 = 0.0376\,m^3/s.$

from eq. ② $\Rightarrow Q_2 = 0.2536\,m^3/s.$

$\therefore v_2 = 2.133\,m/s.$

for pipe no. 2: $h_{L_2} = 0.018 \dfrac{1200}{0.375} \dfrac{(2.133)^2}{2g} = 13.356\,m$

By applying Energy eq. between A & J:

$EL_A - H_T = Z_J + h_{L_2}$

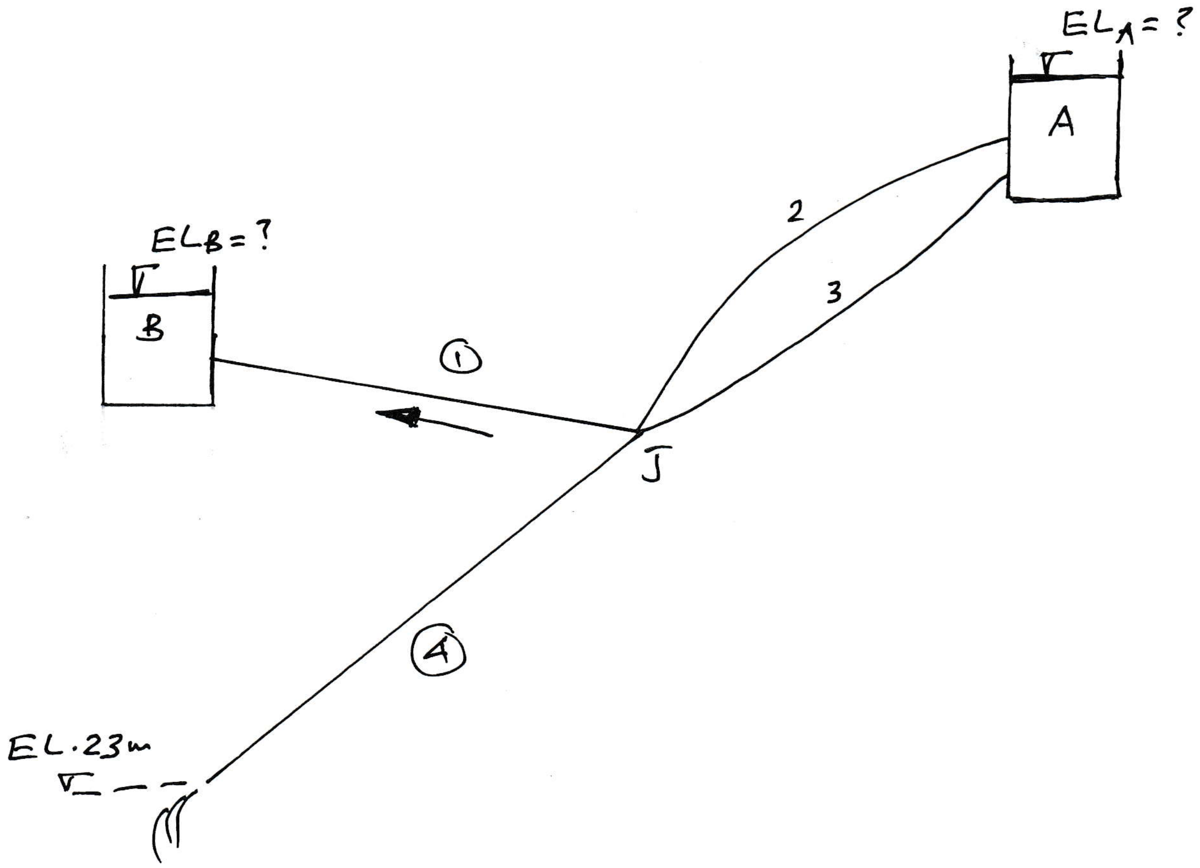
$\therefore H_T = 150 - 106.15 + 13.356 = 30.5\,m$

from eq. ①: turbine horsepower $= 101.714\,hp$

Ex.3 : For the system shown in figure below, when $Q_4 = 2Q_1$ & total flow of water from A is $0.4\ m^3/s.$, determine $EL_B$ & $EL_A$. The pipe characteristics are as follows:

| pipe | L (m) | D (mm) |
|---|---|---|
| 1 | 1500 | 400 |
| 2 | 1000 | 400 |
| 3 | 1800 | 350 |
| 4 | 3000 | 500 |

$C_{H.W.} = 100$ for all pipes




Solution:

$$Q_{\text{from } A} = 0.4\ m^3/s. = Q_2 + Q_3 = Q_1 + Q_4$$

$$\therefore \quad 0.4 = Q_2 + Q_3 = Q_1 + Q_4 \quad \text{— (1)}$$

since $Q_4 = 2Q_1$

from eq. ① : $0.4 = Q_1 + 2Q_1$

$\therefore Q_1 = 0.134 \ m^3/s.$

$\therefore Q_4 = 0.266 \ m^3/s.$

for pipe no. 4 :

$$0.266 = 0.2785(100)(0.5)^{2.63}\left(\frac{h_{L4}}{3000}\right)^{0.54}$$

$\therefore h_{L_4} = 15.94 \ m$

Energy eq. between J & free jet : $Z_J = 23 + h_{L_4}$

$\therefore Z_J = 38.94 \ m$

for pipe no. 1 : $0.134 = 0.2785(100)(0.4)^{2.63}\left(\frac{h_{L_1}}{1500}\right)^{0.54}$

$\therefore h_{L_1} = 6.64 \ m$

Energy eq. between J & B : $Z_J = EL_B + h_{L_1}$

$\therefore EL_B = 38.94 - 6.64 = 32.3 \ m$

Due to parallel connection $\Rightarrow h_{L_2} = h_{L_3}$

from eq. ① : $0.4 = Q_2 + Q_3$

$$\therefore 0.4 = 0.2785(100)(0.4)^{2.63}\left(\frac{h_{L_2}}{1000}\right)^{0.54} + 0.2785(100)(0.35)^{2.63}\left(\frac{h_{L_3}}{1800}\right)^{0.54}$$

$h_{L_2}$

$\therefore h_{L_2} = 15.6 \ m$

Energy eq. between A & J along pipe no. 2 : $EL_A = Z_J + h_{L_2}$

$\therefore EL_A = 54.54 \ m$

# Networks of Pipes

Interconnected pipes through which the flow to a given outlet may come from several circuits are called a network of pipes. Problems on these in general are complicated & require trial solutions in which the elementary circuits are balanced in turn until all conditions for the flow are satisfied.

The following conditions must be satisfied in a network of pipes:

1- $\sum h_f$ around each circuit (loop) $= 0$.

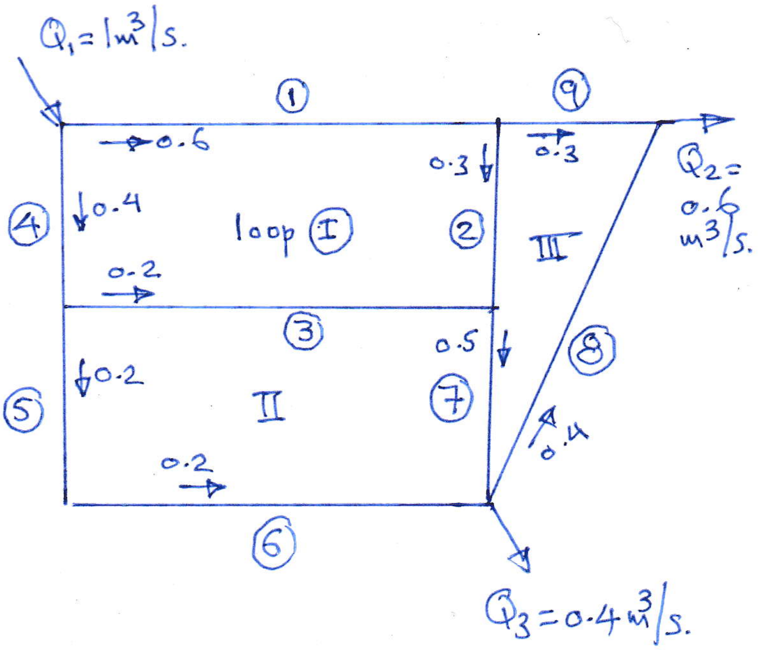
2- $\sum Q_{\text{into } J} = \sum Q_{\text{out from } J}$.

3- The Darcy-Weisbach eq., or equivalent exponential friction formula, must be satisfied for each pipe.

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2gA^2}Q^2$$

$$\therefore h_f = KQ^2$$

or in general $\boxed{h_f = KQ^n}$



The procedure of successive approximation (Hardy-Cross Method) to balance the head is as follows:

1- Assume any reasonable distribution of flow (Q) to Satisfy $\Sigma Q_J = 0$.

2- Calculate (K) and the ($h_f$) for each pipe.

3- Compute $\Sigma h_f$ in any loop (+ve losses for clockwise flow & -ve losses for anti-clockwise flow).

4- If $\Sigma h_f$ (for any loop) $\neq 0$. $\Rightarrow$ adjust the flow by $\Delta Q$.

$$\therefore Q_{new} = Q_{old} + \Delta Q$$

$$\therefore h_f = K(Q + \Delta Q)^n$$

where $$\boxed{\Delta Q = \frac{-\Sigma h_f}{n \Sigma \frac{h_f}{Q}}}$$

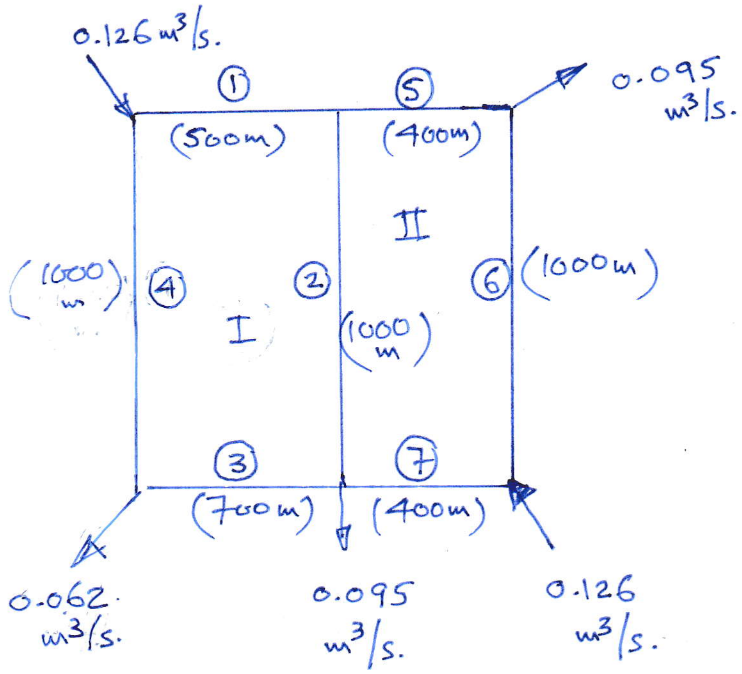
Note: * for Darcy-Weisbach eq.;

$$n = 2 \; ; \quad K = f \frac{L}{D} \frac{1}{2gA^2}$$

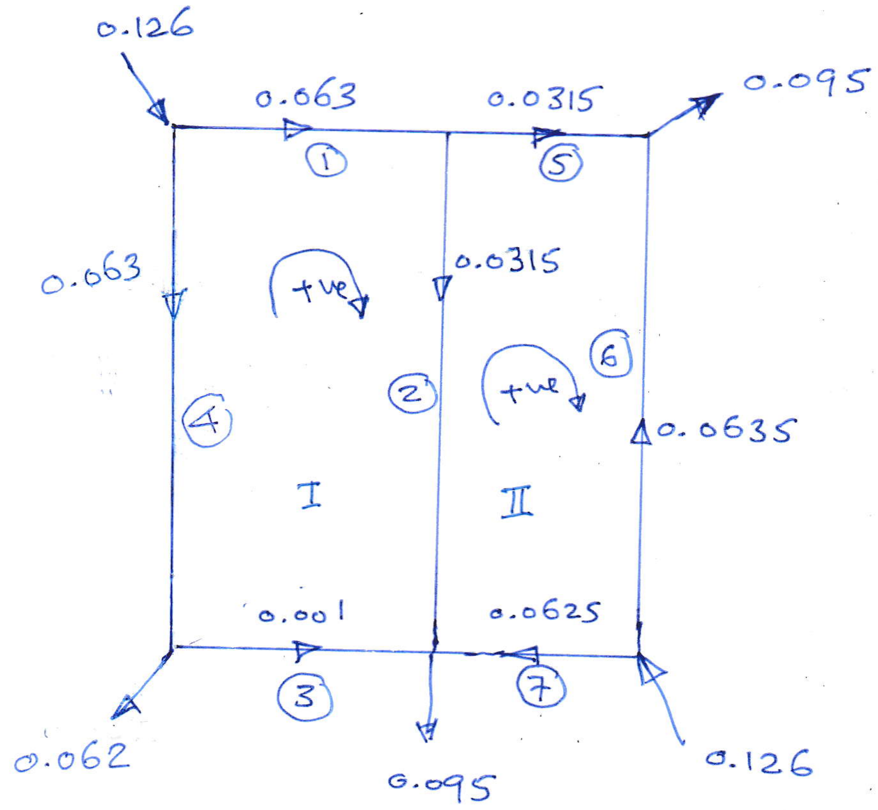
* for Hazen-William's formula;

$$n = 1.85 \; ; \quad K = \frac{10.6\, L}{C_{H.W.}^{1.85} \, D^{4.87}}$$

**Ex:** for the network shown below, determine the flow in each pipe. All pipes have a diameter of 500mm & $C_{H.W.}$ equals 100.




**Sol.**




$n = 1.85$

$$K = \frac{10.6\,L}{C_{H.W.}^{1.85}\,D^{4.87}} = \frac{10.6\,L}{171.389} = 0.062\,L$$

$1^{st}$ - iteration

| loop | pipe | K | Q | hf $(KQ^{1.5})$ | $\frac{hf}{Q}$ always (+ve) | ΔQ |
| --- | --- | --- | --- | --- | --- | --- |
| I | 1 | 31 | +0.063 | +0.186 | 2.95 | $= \frac{-(-0.084)}{1.85(12.26)}$ |
| | * 2 | 62 | +0.0315 | +0.103 | 3.27 | $= 0.0037$ |
| | 3 | 43.4 | -0.001 | $-1.223*10^{-4}$ | 0.1223 | |
| | 4 | 62 | -0.063 | -0.373 | 5.92 | |
| | | | | Σ = -0.084 | Σ = 12.26 | |
| II | 5 | 24.8 | +0.0315 | +0.041 | 1.3 | $= \frac{-(-0.293)}{1.85(12.872)}$ |
| | 6 | 62 | -0.0635 | -0.378 | 5.95 | $= 0.0123$ |
| | 7 | 24.8 | +0.0625 | +0.147 | 2.352 | |
| | * 2 | 62 | -0.0315 | -0.103 | 3.27 | |
| | | | | Σ = -0.293 | Σ = 12.872 | |

Correction: ① for single pipe ; $Q = Q + \Delta Q$

② for common pipe ; $Q = Q + \underset{\text{Samp loop}}{\Delta Q} - \underset{\text{another loop}}{\Delta Q}$

for example : for pipe ① loop I ⇒ $Q = 0.063 + 0.0037 = 0.0667$

for pipe ⑥ loop II ⇒ $Q = -0.0635 + 0.0123 = -0.0512$

for pipe ② loop (I) ⇒ $Q = 0.0315 + 0.0037 - 0.0123 = 0.0229$

for pipe ② loop (II) ⇒ $Q = -0.0315 + 0.0123 - 0.0037 = -0.0229$

## 2nd iteration

| loop | pipe | K | Q | $h_f$ ($KQ^{1.85}$) | $\frac{h_f}{Q}$ | $\Delta Q$ |
|---|---|---|---|---|---|---|
| | 1 | 31 | +0.0667 | +0.207 | 3.1 | |
| I | * 2 | 62 | +0.0229 | +0.057 | 2.49 | $= \frac{-(-0.0683)}{1.85(11.47)}$ |
| | 3 | 43.4 | +0.0027 | $+7*10^{-4}$ | 0.26 | = 0.0032 |
| | 4 | 62 | -0.0593 | -0.333 | 5.62 | |
| | | | | $\Sigma = -0.0683$ | $\Sigma = 11.47$ | |
| | 5 | 24.8 | +0.0438 | +0.076 | 1.74 | |
| | 6 | 62 | -0.0512 | -0.254 | 4.96 | $= \frac{-(-0.03)}{1.85(11.93)}$ |
| II | 7 | 24.8 | +0.0748 | +0.205 | 2.74 | = 0.0014 |
| | * 2 | 62 | -0.0229 | -0.057 | 2.49 | |
| | | | | $\Sigma = -0.03$ | $\Sigma = 11.93$ | |

# Open Channel Flow

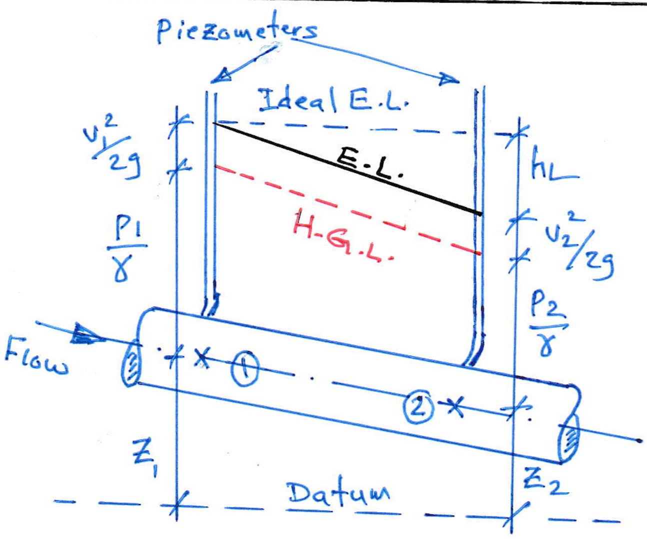
An open channel is a passage through which the water flows under atmospheric pressure.

## Types of Open Channel

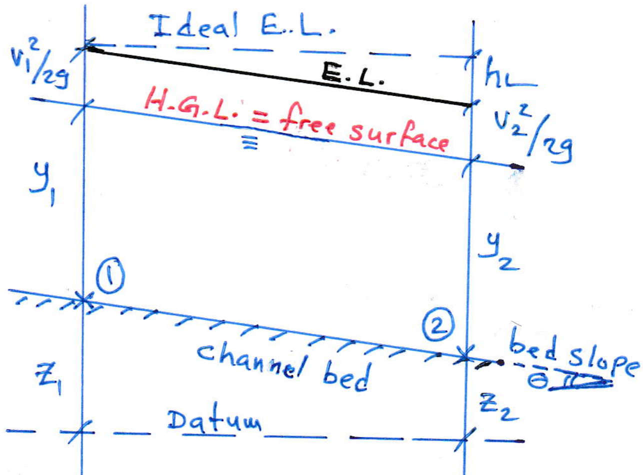
A- Natural Open Channel (Non-prismatic) such as stream, river, .... (Non-uniform cross-section)

B- Artificial Open Channel, such as canal, sewer (Uniform cross-section, constant bed slope)

## Comparison between Flow in Open Channel and Flow in Pipe




Flow in pipe
(closed conduit)




Flow in Open Channel
(Open Conduit)

- Flow due to pressure difference.
- there is no free surface
- the pressure is not atmospheric.

- Flow due to gravity (bed slope).
- there is a free surface (H.G.L.).
- the pressure is atmospheric

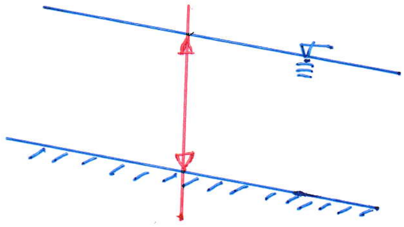
# Types of Flow

## 1- Steady and Unsteady Flows

Steadiness with respect to time.

for example; $\frac{\partial y}{\partial t} = 0. \Rightarrow$ Steady flow

$\frac{\partial y}{\partial t} \neq 0. \Rightarrow$ unsteady flow

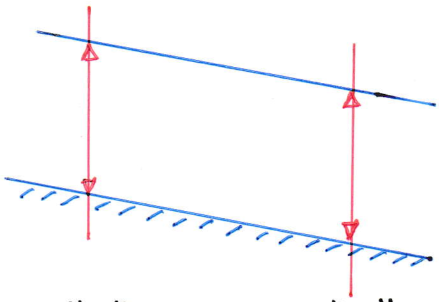


at $t = t_1 \Rightarrow y = y_1$

at $t = t_2 \Rightarrow y = y_2$

## 2- Uniform and Non-uniform Flows

Flow in open channel is said to be uniform if the depth, dischage, cross-section, & velocity remain constant over a given length of the channel, obviously in the artificial open channel uniform flow implies (e.g. $\frac{\partial y}{\partial s} = 0.$). " Uniformity with respect to distance"



at $t = t_1$ $y = y_1$ $y = y_1$

at $t = t_2$ $y = y_1$ $y = y_1$

(Steady - Uniform flow)

### Non-Uniform (Varied) Flow

**Gradually Varied Flow (G.V.F)**

change in depth of flow takes place gradually along channel reach.

**Rapidly Varied Flow (R.V.F.)**

Change in depth of flow occurs obruptly in a short channel reach.

## 3- Laminar, Transition, and Turbulent Flows

$$R_E = \frac{V.D}{\nu} = \frac{V(4R_h)}{\nu}$$

where, $R_h$ = hydraulic-radius = $\dfrac{\text{Area}(A)}{\text{Wetted perimeter}(W.P.)}$

IF; $R_E < 2000$ (Laminar flow)

$2000 < R_E < 3000$ (Transition flow)

$R_E > 3000$ (Turbulent flow)

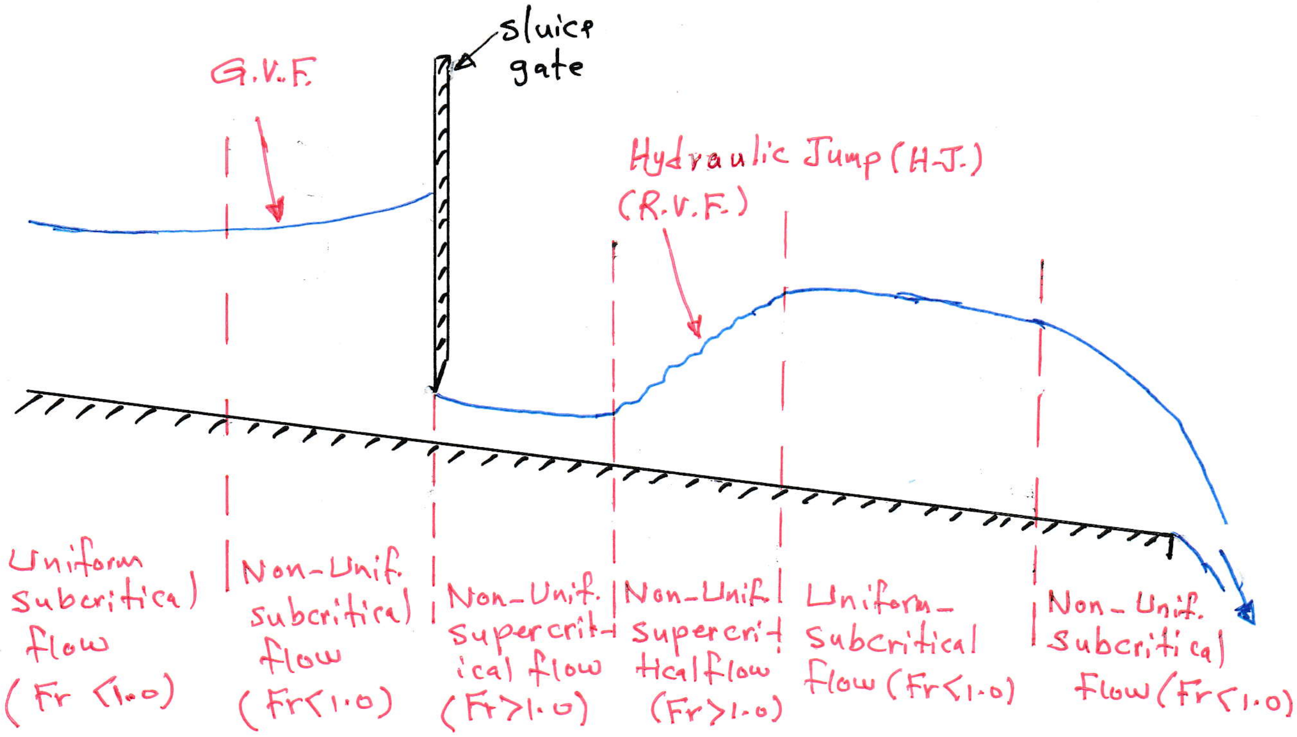
## 4- Subcritical, Critical, and Super critical Flows

IF; $Fr < 1.0$ (Subcritical flow)

$Fr = 1.0$ (Critical flow)

$Fr > 1.0$ (Super critical flow)

where; $Fr$ = Froude Number




Types of flow in open channel with mild slope

## Froude Number (Fr)

$$Fr = \frac{\text{Inertia force } (m \cdot a)}{\text{gravity force } (m \cdot g)}$$

By dimensional analysis;

$$\boxed{Fr = \frac{V}{\sqrt{gL}} = \frac{V}{\sqrt{gD}}}$$

where; $Fr$ = Froude No. (dimensionless)

$V$ = average velocity (m/s.)

$L$ = gravity acceleration ($m/s^2$)

$L$ = Characteristics length = hydraulic depth ($D$)

$$\boxed{D = \frac{A}{B_f}}$$

where; $A$: cross-sectional area ($m^2$)

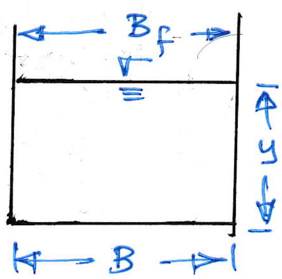
$B_F$: width of the free surface (m).

### for Rectangular channel

$A = By$ ; $B_f = B$

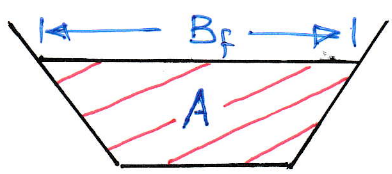
$\therefore D = \frac{By}{B} \Rightarrow D = y$

$\therefore Fr = \frac{V}{\sqrt{gy}}$




### for Trapezoidal channel

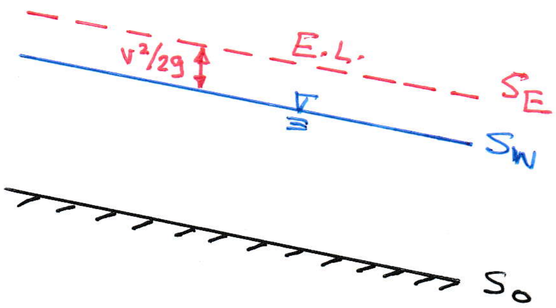
$$Fr = \frac{V}{\sqrt{g \frac{A}{B_f}}}$$



# Flow Resistance

Steady State

- Uniform Flow ⇒ The discharge, flow area, flowdepth, channel slope, & mean velocity remain constant at every section along the channel reach
  ⇒ Bed slope ($S_o$) = water surface slope ($S_w$) = Energy line slope ($S_E$)




- Non-Uniform Flow ⇒ velocity, for example, is changing along the channel reach

## Chezy formula

$$\boxed{V = C\sqrt{R_h \cdot S}}$$

## Manning formula

$$\boxed{V = \frac{1}{n} R_h^{2/3} S^{1/2}}$$

where; $V$ = average velocity (m/s.)

$C$ = Chezy coefficient $= \sqrt{\frac{8g}{f}}$

$f$ = Darcy-Weisbach friction coeff.

$R_h$ = hydraulic radius $= \frac{A}{W.P.}$ (m).

$S =$ bed slope

$n =$ Manning coefficient

## Relationship between Chezy and Manning formulas

since; $V = C\sqrt{R_h \cdot S}$ (Chezy formula)

& $V = \frac{1}{n} R_h^{2/3} S^{1/2}$ (Manning formula)

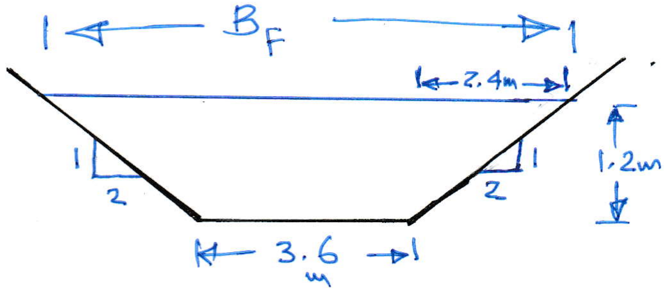
from the two-equations;

$$C\sqrt{R_h \cdot S} = \frac{1}{n} R_h^{2/3} S^{1/2}$$

$$\therefore \boxed{C = \frac{1}{n} R_h^{1/6}}$$

**Ex.1:** Determine the discharge & the type of flow for a trapezoidal channel with a bottom width (3.6m) & side slopes (1V : 2H). The water depth is (1.2m) & bottom (bed) slope is (0.0009). Take C = 50.

**Sol.:**




$\because V = C\sqrt{R_h \cdot S}$

$R_h = \dfrac{A}{W.P.}$

$A = (3.6 + 2.4)(1.2) = 7.2\ m^2$

$W.P. = 3.6 + 2\sqrt{1.2^2 + 2.4^2} = 8.96m$

$\therefore R_h = 0.804m$

$\because Q = A.V \Rightarrow Q = C A\sqrt{R_h \cdot S} = 50(7.2)\sqrt{0.804(0.0009)} = 9.684\ m^3/s$

$\therefore V = \dfrac{Q}{A} = 1.345\ m/s.$

$\because Fr = \dfrac{V}{\sqrt{g\dfrac{A}{B_f}}}$

$B_f = 3.6 + 2(2.4) = 8.4m$

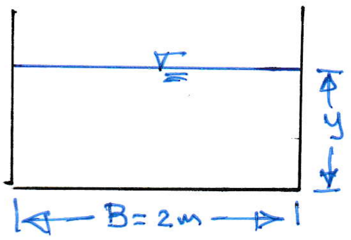
$\therefore Fr = 0.464 < 1.0$

$\therefore$ subcritical flow.

**Ex.2**: What depth is required for ($4\ m^3/s.$) flow in a rectangular planed-wood channel ($n = 0.012$) of wide ($2m$) with a bed slope of ($0.002$)?

**Sol.**:

$$\because\ V = \frac{1}{n} R_h^{2/3} S^{1/2}$$




$$\therefore\ Q = \frac{1}{n} A\, R_h^{2/3}\, S^{1/2}$$

$A = 2y$ ; $W.P. = 2 + 2y = 2(1+y)$

$$\therefore\ R_h = \frac{2y}{2(1+y)} = \frac{y}{1+y}$$

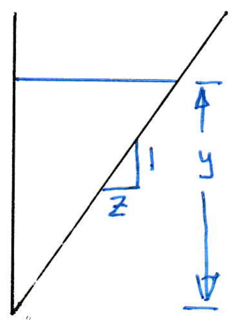
$$\therefore\ 4 = \frac{1}{0.012}\,(2y)\left(\frac{y}{1+y}\right)^{2/3}(0.002)^{1/2}$$

$$\therefore\ 0.536 = y\left(\frac{y}{1+y}\right)^{2/3}$$

By trial & error $\Rightarrow$ $y = 0.89m$

**H.W.**: Prove the following equation for the discharge in a triangular highway gutter having one side vertical & the other side slope ($z(H) : 1(V)$)

$$Q = \frac{0.316}{n} F(z)\,(y)^{8/3}\,(S)^{1/2}$$




where;

$$F(z) = \frac{z^{5/3}}{\left(1+\sqrt{1+z^2}\right)^{2/3}}$$

Also, compute the discharge if $z = 12$, $n = 0.015$, $y = 0.1m$ & $S = 0.03$

# Best Hydraulic Section (B.H.S) OR Most Economical channel Section

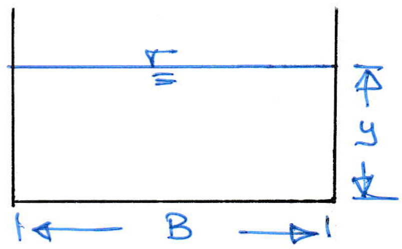
$$\because \quad V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

$$\therefore \quad Q = \frac{1}{n} A \left(\frac{A}{W.P.}\right)^{2/3} S^{1/2}$$

with S, n, & A are fixed (constant), a minimum (W.P.) will represent the B.H.S. as it conveys the maximum discharge

## A- B.H.S. for Rectangular Channel

$$\boxed{B = 2y}$$




since; $R_h = \frac{A}{W.P.}$

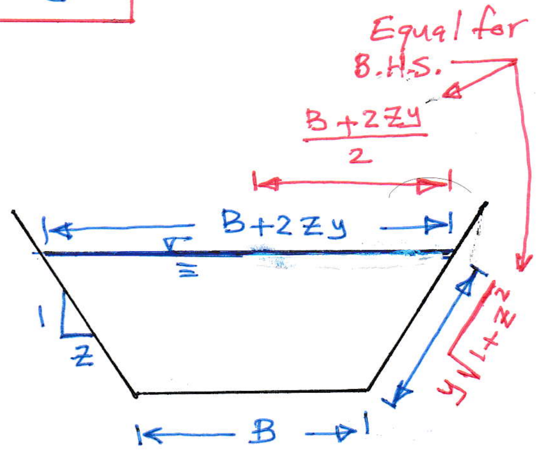
$$\therefore \quad A = By = 2y^2$$

$$W.P. = B + 2y = 2y + 2y = 4y$$

$$\therefore \quad R_h = \frac{2y^2}{4y} \Rightarrow \boxed{R_h = \frac{y}{2}}$$

## B- B.H.S. for Trapezoidal Channel

$$\boxed{\frac{B+2Zy}{2} = y\sqrt{1+Z^2}}$$




Since; $R_h = \frac{A}{W.P.}$

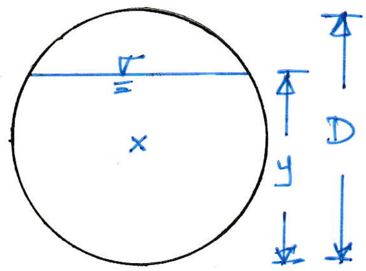
$$A = (B+Zy)y \; ; \quad W.P. = B + 2y\sqrt{1+Z^2}$$
$$= B + (B+2Zy) = 2(B+Zy)$$

$$\therefore \quad R_h = \frac{(B+Zy)y}{2(B+Zy)} \Rightarrow \boxed{R_h = \frac{y}{2}}$$

## C- B.H.S. for Circular Channel

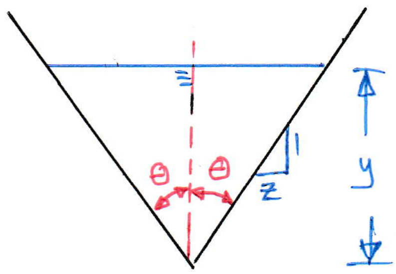
1- Condition of $Q_{max.}$ $\Rightarrow$ $\boxed{y = 0.95D}$

2- Condition of $V_{max.}$ $\Rightarrow$ $\boxed{y = 0.81D}$




## D- B.H.S. for Triangular Channel

$$\boxed{\theta = 45^\circ \text{ or } Z = 1.0}$$




$\therefore$ $A = \frac{1}{2}(2Zy)\,y = y^2$

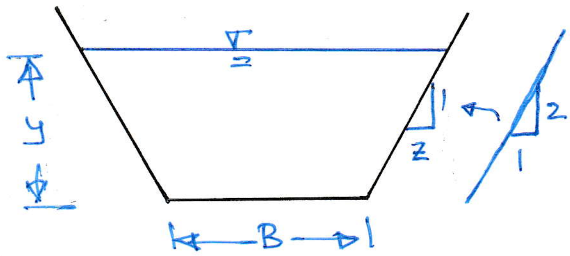
$W.P. = 2y\sqrt{1+Z^2} = 2\sqrt{2}\;y$

$$\therefore \boxed{R_h = \frac{y}{2\sqrt{2}}}$$

**Ex.1:** Design a trapezoidal channel of B.H.S. with side slopes (1H:2V) to allow a discharge of ($5.6\ m^3/s.$) at a velocity of ($1.5\ m/s.$). Find the saving in (hp) per (Km) length of channel in using the above section in comparison with a rectangular section of (1.2m) water depth & (3m) width. Take $c = 55$.

**Sol.:** side slope 1H:2V

so; $\frac{1}{Z} = \frac{2}{1} \Rightarrow Z = \frac{1}{2}$




since, $A = (B+Zy)y = (B+\frac{y}{2})y$

$A = \frac{Q}{V} \Rightarrow (B+\frac{y}{2})y = \frac{5.6}{1.5}$

$$\therefore \quad \left(B+\frac{y}{2}\right)y = 3.73 \quad \text{—} \quad ①$$

For B.H.S. ; $\qquad \dfrac{B+2Zy}{2} = y\sqrt{1+Z^2}$

for $Z=\frac{1}{2}$ ; $\qquad B+y = 2y\sqrt{1+\frac{1}{4}} = \sqrt{5}\, y$

$$\therefore \quad B+y = \sqrt{5}\, y$$

$$\therefore \quad B = 1.24\, y \quad \text{—} \quad ②$$

Subs. eq. ② into eq. ① : $\quad \left(1.24y+\frac{y}{2}\right)y = 3.73$

$$\therefore \quad y = 1.46\,m$$

$$B = 1.81\,m$$

$$\because \quad V = C\sqrt{R_h \cdot S}$$

$$R_h = \frac{y}{2}$$

$$\therefore \quad 1.5 = 55\sqrt{\frac{1.46}{2}\, S} \Rightarrow S = \frac{1}{978} = \frac{h_L}{L \leftarrow 1000\,m}$$

When Using Rectangular section:

$$V = \frac{Q}{A} = \frac{5.6}{1.2(3)} = 1.556\ m/s.$$

$$R_h = \frac{A}{W.P.} = \frac{1.2(3)}{2(1.2)+3} = 0.667\,m$$

From Chezy formula ; $\quad 1.556 = 55\sqrt{0.667\, S} \Rightarrow S = \frac{1}{838} = \frac{h_L}{L \leftarrow 1000\,m}$

$\therefore$ dissipation horsepower in trapezoidal section $= \dfrac{9810}{746} * 5.6 * \dfrac{1000}{978} = 75.29\ hp$

& dissipation horsepower in rectangular section $= \dfrac{9810}{746}(5.6)\left(\dfrac{1000}{838}\right) = 87.87\ hp$

So, the saving in (hp) due to adopting trapezoidal section is:

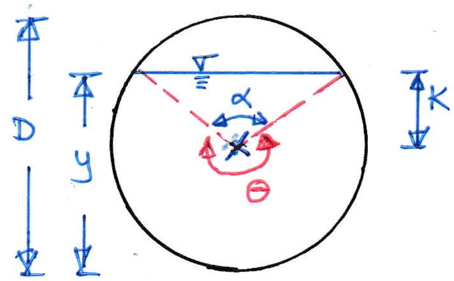
$$87.87 - 75.29 = 12.58 \text{ hp}$$

**EX.2**: At what slope should a circular pipe of (0.8m) diameter be laid to maintain:

a- a max. velocity when the discharge equals ($0.21 \; m^3/s.$);

b- " " discharge " " velocity " ($0.48 \; m/s.$);

(Take $C = 70$).




**Sol.**: a- $y = 0.81D = 0.648m$

$$K = y - \frac{D}{2} = 0.248m$$

$$\cos\frac{\alpha}{2} = \frac{K}{D/2} = \frac{0.248}{0.4}$$

$$\therefore \frac{\alpha}{2} = 51.684^\circ \Rightarrow \alpha = 103.368^\circ$$

$$\theta = 360 - \alpha = 256.632^\circ \Rightarrow \theta(\text{rad.}) = \frac{256.632 * \pi}{180} = 4.479 \text{ rad.}$$

$$A = \frac{r^2}{2}(\theta - \sin\theta) = \frac{0.4^2}{2}(4.479 - \sin(256.632^\circ)) = 0.436 \; m^2$$

$$W.P. = r \cdot \theta = 0.4(4.479) = 1.79m$$

$$\therefore R_h = 0.2435$$

since; $Q = C \cdot A \cdot \sqrt{R_h \cdot S}$

$$\therefore 0.21 = 70(0.436)\sqrt{0.2435 \cdot S} \Rightarrow S = 1.944 * 10^{-4} \; m/m \text{ length}$$

**H.W.** b- $y = 0.95D$ & use the same procedure above

## Specific Energy and Critical Depth

Since; $H = \frac{P}{\gamma} + z + \frac{V^2}{2g} = y + z + \frac{V^2}{2g}$ (Total Energy)

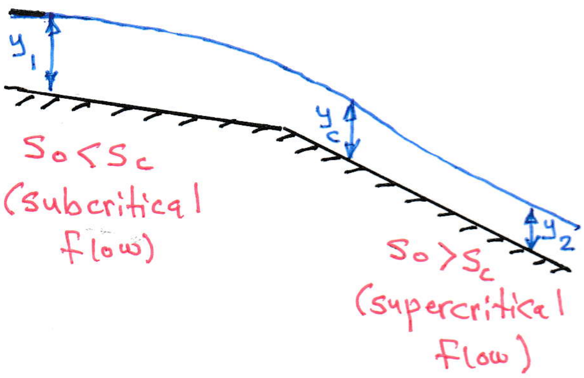
$E$ = specific Energy $= y + \frac{V^2}{2g}$

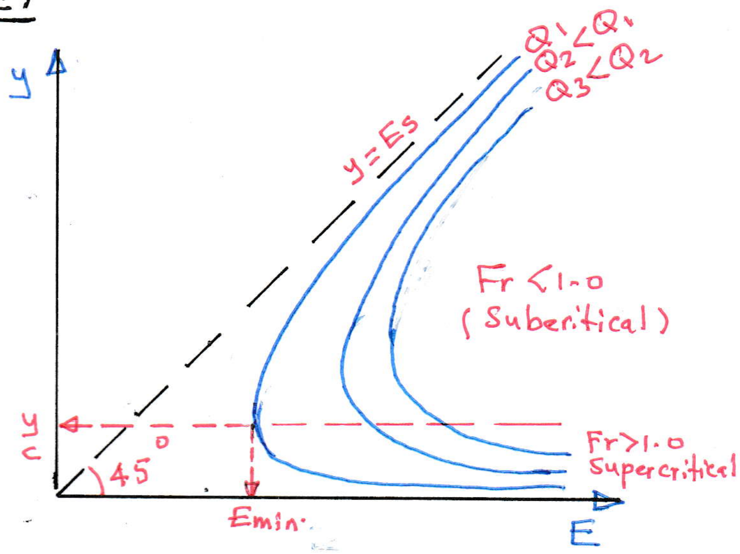
where; $y$ : static Energy $= E_s$

$\frac{V^2}{2g}$ : Kinetic Energy $= E_K$

$\therefore \quad E = y + \frac{V^2}{2g} = E_s + E_K$

## Occurrence of Critical Depth ($y_c$)

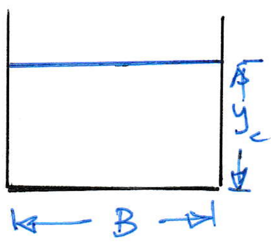



* Critical depth occurs when $E$ is minimum ($E = E_{min.}$) $\Rightarrow \frac{dE}{dy} = 0$.

## Critical Flow in Rectangular Channels

$$\boxed{y_c = \frac{q^2}{g} \quad \& \quad E_{min.} = \frac{3}{2} y_c}$$




where; $y_c$ : critical depth

$q$ = discharge per unit width ($m^3/s./m$ width) or ($m^2/s.$)

$= \frac{Q}{B}$

$Q$ : total discharge ($m^3/s.$)

$B$ : channel width ($m$).

$g$ : gravity acceleration ($m/s^2.$)

$E_{min.}$ : min. specific energy

Maximum discharge occurs when $y = y_c$ & $E = E_{min.}$

since; $E = y + \frac{v^2}{2g}$

$$v = \frac{Q}{A} = \frac{Q}{B.y} = \frac{Q/B}{y} = \frac{q}{y}$$

$$\therefore \quad E = y + \frac{q^2}{2gy^2}$$

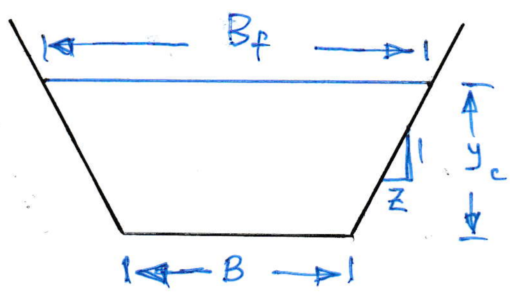
for $y = y_c$ & $E = E_{min.}$ $\Rightarrow$ $E =$

$$\frac{3}{2} y_c = y_c + \frac{q^2_{max.}}{2g\, y_c^2}$$

$$\boxed{q_{max.} = \sqrt{g\, y_c^3}}$$

## Critical flow in Trapezoidal channels

$$\boxed{\frac{Q^2}{g} = \frac{A_c^3}{B_f}}$$




where;

$A_c$ = critical cross-sectional ($m^2$)
$= (B + Zy_c)\, y_c$

$B_f$ = free surface width ($m$) $= B + 2Zy_c$

$Q$ = Total discharge ($m^3/s.$) ; $g$ = gravity acc. ($m/s^2$)

EX.1 : Given, rectangular channel, $Q = 5.66\ m^3/s.$, $B = 3.66m$, find:

a- Critical depth ($y_c$) & critical velocity ($V_c$).

b- Critical slope ($S_c$) of the channel bed when $n = 0.02$.

c- type of flow for bed slope ($S = 0.0005$)

Sol.: a- since; $y_c = \left[\frac{q^2}{g}\right]^{1/3}$

$$q = \frac{Q}{B} = \frac{5.66}{3.66} = 1.546\ m^2/s.$$

$$\therefore\ y_c = \left[\frac{1.546^2}{9.81}\right]^{1/3} = 0.625\ m$$

$$V_c = \frac{q}{y_c} = 2.476\ m/s.$$

b- since; $V = \frac{1}{n} R_h^{2/3} S^{1/2} \Rightarrow V_c = \frac{1}{n} R_{h_c}^{2/3} S_c^{1/2}$

$$\therefore\ 2.476 = \frac{1}{0.02}\left(\frac{3.66 * 0.625}{2(0.625)+3.66}\right)^{2/3} S_c^{1/2}$$

$$\therefore\ S_c = 0.0068\ m/m\ \text{length}$$

c- since; bed slope ($S = 0.0005$) $< S_c$

$\therefore$ the flow is subcritical

Ex.2: A rectangular channel is laid on a bed slope of (0.0064) and carries a discharge of ($20\ m^3/s.$). Determine the width of the channel when the flow is in critical condition. (Take Manning's $n = 0.015$).

Sol.: since; $y_c = \left[\frac{q^2}{g}\right]^{1/3}$

$$\therefore\ y_c = \left[\frac{(\frac{Q}{B})^2}{g}\right]^{1/3} = \left[\frac{20^2}{B^2 \cdot g}\right]^{1/3}$$

$$\therefore\ y_c = \frac{3.442}{B^{2/3}} \quad \text{—— (1)}$$

Since; $V = \frac{1}{n} R_h^{2/3} S^{1/2} \Rightarrow Q = \frac{1}{n} A\, R_h^{2/3} S^{1/2}$

for critical flow; $Q = \frac{1}{n} A_c\, R_{h_c}^{2/3} S_c^{1/2} = \frac{1}{n} \frac{A_c^{5/3}}{W.P._c^{2/3}} S_c^{1/2}$

$$\therefore\ A_c = B\, y_c = B * \frac{3.442}{B^{2/3}} = 3.442\, B^{1/3}$$

$$W.P._c = B + 2y_c = B + \frac{2(3.442)}{B^{2/3}} = \frac{B^{5/3} + 6.884}{B^{2/3}}$$

$$\therefore\ 0.33 = \frac{1}{0.015} \frac{(3.442\, B^{1/3})^{5/3}}{\left(\frac{B^{5/3} + 6.884}{B^{2/3}}\right)^{2/3}} (0.0064)^{1/2}$$

By re-arrangement

$$0.478 = \frac{B}{(B^{5/3} + 6.884)^{2/3}}$$

By trial & error. $\Rightarrow$ $B = 2.41\,m$

**EX.3**: Given, trapezoidal channel, side slope (1V:2H), $Q = 16.7\ m^3/s.$, bottom width $(B) = 3.66m$, calculate:

a- critical depth ($y_c$).

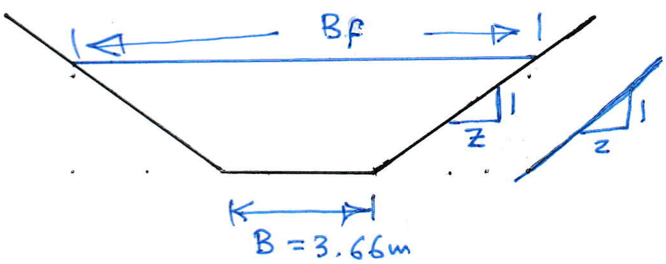
b- 〃 velocity ($V_c$).

c- Min. Specific Energy ($E_{min.}$)

d- type of flow when $n = 0.015$ & $S = 0.01$.

**Sol.**:

a- $\frac{1}{Z} = \frac{1}{2} \Rightarrow Z = 2$




$$A_c = (B + Zy_c)\,y_c = (3.66 + 2y_c)\,y_c$$

$$B_f = B + 2Zy_c = 3.66 + 4y_c$$

since; $$\frac{Q^2}{g} = \frac{A_c^3}{B_f}$$

$$\therefore \frac{(16.7)^2}{9.81} = \frac{(3.66 + 2y_c)^3\,y_c^3}{3.66 + 4y_c}$$

By trial & error $\Rightarrow y_c = 1.05m$

b- $$V_c = \frac{Q}{A_c}$$

$A_c = 6.048\ m^2 \Rightarrow V_c = 2.76\ m/s.$

c- $$E_{min.} = y_c + \frac{V_c^2}{2g} = 1.438m$$

d- since; $$V_c = \frac{1}{n} R_{h_c}^{2/3} S_c^{1/2}$$

$$2.76 = \frac{1}{0.015}\left(\frac{6.048}{3.66 + 2\sqrt{5}\,(1.05)}\right)^{2/3} S_c^{1/2}$$

$\therefore S_c = 0.0026$ m/m length

since; $S = 0.01 > S_c \Rightarrow$ the flow is supercritical