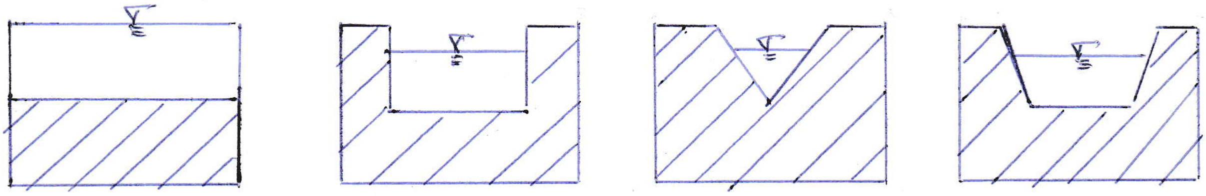
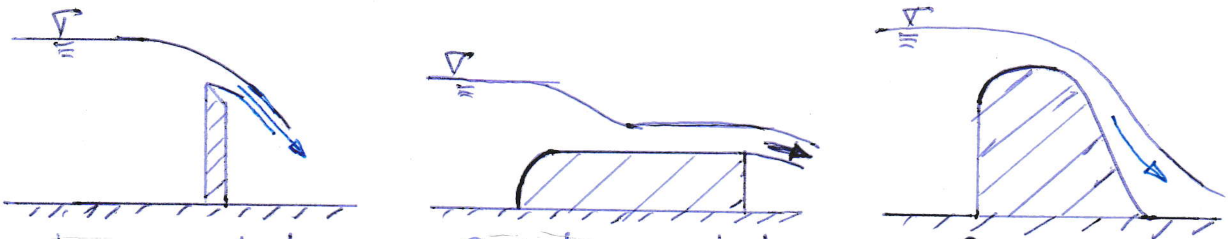
2- Weirs

Types of Weirs



Suppressed rectangular weir | Contracted rectangular weir | Triangular (V-notch) weir | Trapezoidal weir



Sharp-crested weir | Broad-crested weir | Ogee Weir

a- Suppressed Rectangular sharp-Crested Weir

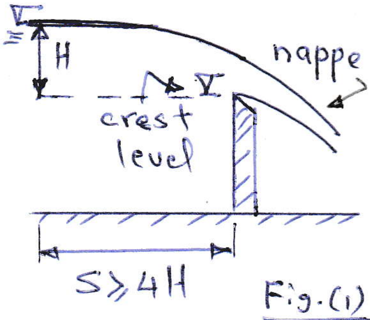


Fig.(1)

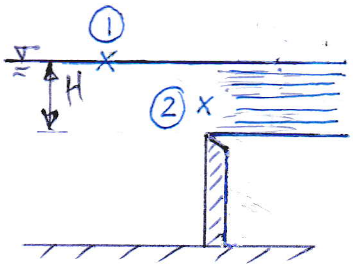
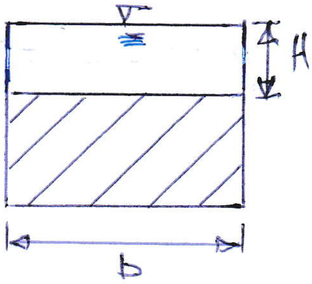


Fig.(2)



The nappe is contracted at the top & bottom as shown in Fig.(1). An equation for discharge can be derived if the contractions are neglected. Without contraction the flow appear as in Fig.(2). The nappe has parallel streamlines with atmospheric pressure throughout.

For ideal Flow

By applying Bern. Eq. between points ① & ② & neglect the approaching velocity (V_1);
(because $\frac{V_1^2}{2g}$ is very small compared with H)

$$Q_{act.} = \frac{2}{3} C_d \cdot b \cdot \sqrt{2g} \; H^{3/2}$$

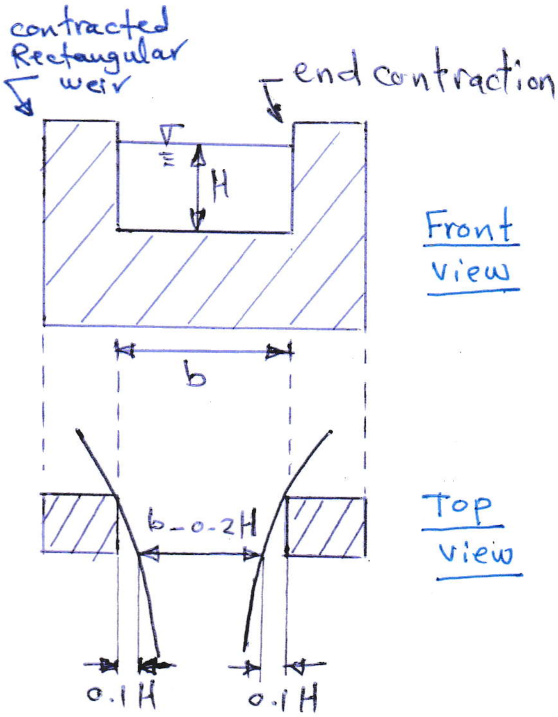
where;

C_d = coeff. of discharge $(0.6 \rightarrow 0.65) \simeq 0.62$

b = width of weir. (m).

H = depth of liquid above crest level (m).

Effect of End Contraction



In general;

$$Q_{act.} = \frac{2}{3} C_d \,(b - n * 0.1 H)\, \sqrt{2g} \; H^{3/2}$$

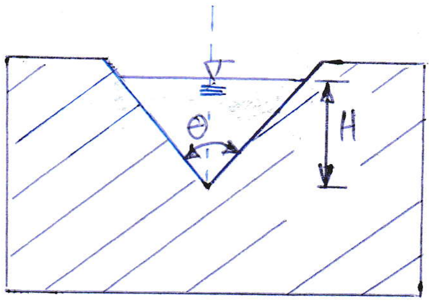
Francis Formula

where;

n : No. of end contraction

b- Triangular (V-notch) Weir

For the same assumptions as in case (a);



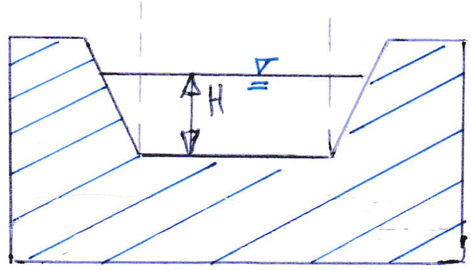
$$Q_{act.} = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \; H^{5/2}$$

In practice; $\theta \approx 30^\circ$ to 90°

& $C_d = 0.6$

C- Trapezoidal Weir

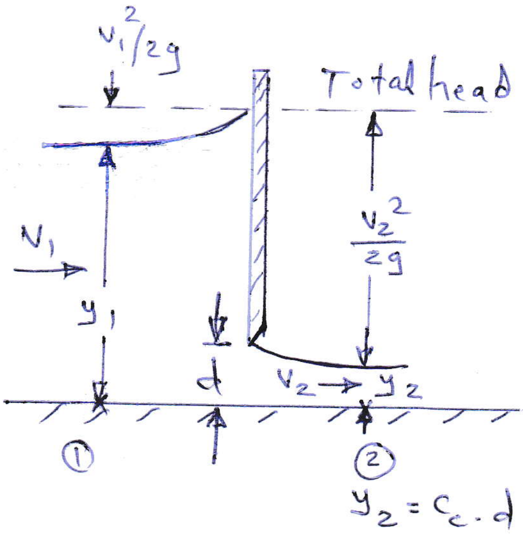
$$\boxed{Q_{act.} = C_d \left[Q_{\substack{theo.\\ rectangular}} + Q_{\substack{theo.\\ triangular}} \right]}$$



3- Sluice Gate

By applying Bern. Eq. & continuity Eq. between points ① & ②:

$$\boxed{Q_{act.} = C_d \cdot A \cdot \sqrt{\frac{2g(y_1 - y_2)}{1 - \left(\frac{y_2}{y_1}\right)^2}}}$$



where, A = Area of gate opening = b.d
b = width " " " ; d = depth of the opening

4- Pitot Tube

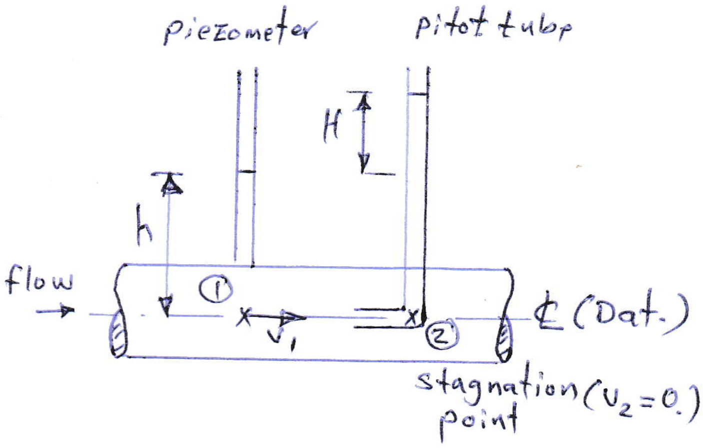
In closed conduit

Bern. Eq. between ① & ②:

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$\cancel{h} + 0. + \frac{V_1^2}{2g} = (H + \cancel{h}) + 0. + 0.$$

$$\therefore V_1 = \sqrt{2gH}$$

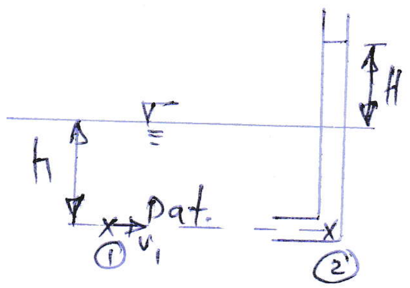


In Open Conduit

Bern. Eq. between ① & ②:

$$\cancel{h} + 0. + \frac{V_1^2}{2g} = (\cancel{h} + H) + 0. + 0.$$

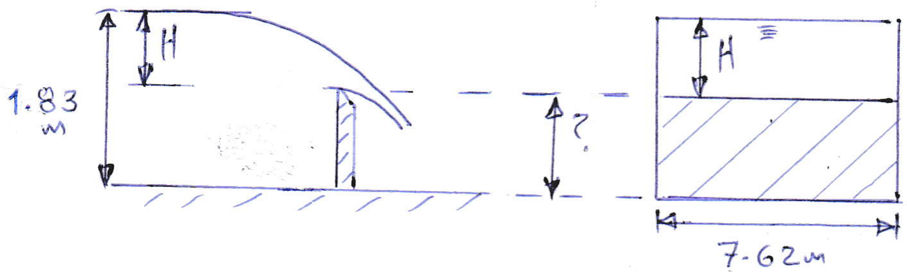
$$\therefore V_1 = \sqrt{2gH}$$



Examples

1- A suppressed rectangular weir, (7.62m) wide, is to discharge (10.6 m^3/s) into a channel. The coeff. of discharge equals (0.626). To what height may the weir be built, if the water behined the weir must not exceed (1.83m) in depth? (Neglect the approaching velocity).

Solution



since, $Q_{act.} = \frac{2}{3} C_d . b . \sqrt{2g} \; H^{3/2}$

$$\therefore \quad 10.6 = \frac{2}{3} (0.626)(7.62)\sqrt{19.62} \; H^{3/2}$$

$$\therefore \quad H = 0.827 m$$

therefore, height of weir = (1.83 − 0.827) = 1.003m

2- A reservoir having a surface area of (50 m^2) is emptied by a (0.5m) wide rectangular weir. How long should it take to empty the reservoir from a height (3.2m) to (0.1 m) above the crest? (Take C_d = 0.65 & neglect the approaching velocity & the effect of end contraction)

Solution: since, $Q_{act.} = \frac{2}{3} C_d . b . \sqrt{2g} \; H^{3/2}$

$$\because \quad dQ . dt = - A . dh$$

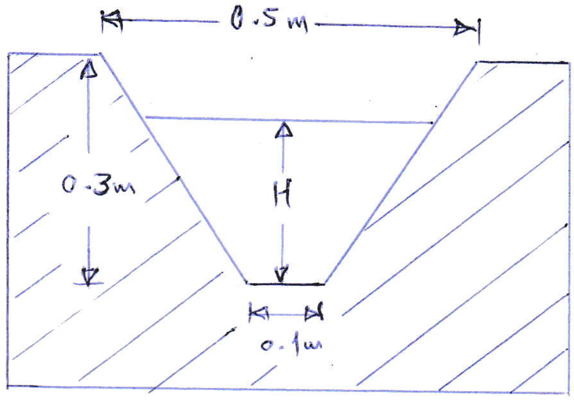
$$\frac{2}{3} (0.65)(0.5) \sqrt{2g} \; h^{3/2} * dt = -50 \, dh$$

$$\therefore \int_0^t dt = -52.1 \int_{3.2}^{0.1} h^{-3/2} dh \Rightarrow t = +52.1 \left(-2 h^{-1/2}\right)\Big|_{0.1}^{3.2}$$

$$\therefore \quad t = 271.2 \text{ sec.}$$

3- A trapezoidal weir with a base of (0.1 m) wide, top (0.5 m) wide & the depth is (0.3 m). Calculate the height of the water level above the base of the trapezoidal weir if the discharge is ($0.043\ m^3/s$). Take $C_d = 0.6$ & neglect the approaching velocity.

Solution



since;

$$Q_{act.} = C_d \left[Q_{\substack{theo.\\rect.}} + Q_{\substack{theo.\\tria.}} \right]$$

$$Q_{\substack{theo.\\rect.}} = \frac{2}{3}\, b \sqrt{2g}\; H^{3/2}$$

$$= \frac{2}{3}(0.1)\sqrt{2g}\; H^{3/2}$$

$$= 0.295\, H^{3/2}$$

$$Q_{\substack{theo.\\tria.}} = \frac{8}{15}\sqrt{2g}\,\tan\frac{\theta}{2}\; H^{5/2} = \frac{8}{15}\sqrt{2g}\left(\frac{0.2}{0.3}\right) H^{5/2} = 1.575\, H^{5/2}$$

$$\therefore\ 0.043 = 0.6\left[0.295\, H^{3/2} + 1.575\, H^{5/2}\right]$$

By trial & error $\Rightarrow H = 0.23\ m$

4- A suppressed rectangular weir ($C_d = 0.62$) under a constant head of (91.44 mm) feeds a tank containing a (76.2 mm) dia. orifice. The weir, which is (609.6 mm) wide, & (823 mm) high, is located in a rectangular channel. The lost head through the orifice is (609.6 mm) & ($C_c = 0.65$). Determine the head to which the water will rise in the tank & the coeff. of velocity for the orifice.

Solution

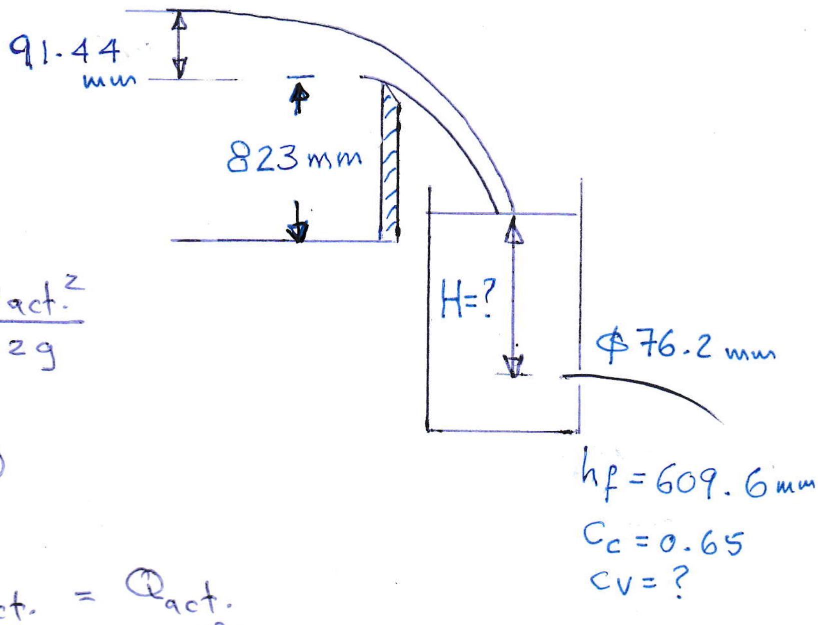
For the orifice;

$\therefore h_f = H(1 - C_v^2)$

$\therefore h_f = H\left(1 - \frac{V_{act.}^2}{V_{theo.}^2}\right)$

$$h_f = H\left(1 - \frac{V_{act.}^2}{2gH}\right) = H - \frac{V_{act.}^2}{2g}$$

$$\therefore H = h_f + \frac{V_{act.}^2}{2g} \quad \text{—— (1)}$$



By continuity requirements; $Q_{act.\,weir} = Q_{act.\,orifice}$

For the weir;

$$Q_{act.} = \frac{2}{3} C_d \cdot b \sqrt{2g}\, H^{3/2} = \frac{2}{3}(0.62)(0.6096)\sqrt{2g}\,(0.09144)^{3/2}$$

$$\therefore Q_{act.} = 0.0308\ m^3/s$$

$$\therefore Q_{act.\,orifice} = 0.0308\ m^3/s$$

For the orifice, $Q_{act.} = A_{act.} * V_{act.} = C_c \cdot A_{theo.} * V_{act.}$

$$\therefore Q_{act.} = C_c * a * V_{act.}$$

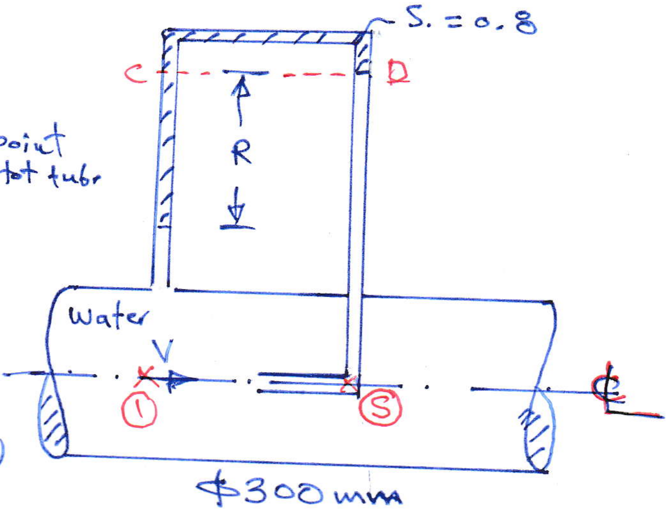
$$\therefore 0.0308 = 0.65 * \frac{\pi}{4}(0.0762)^2 * V_{act.}$$

$$\therefore V_{act.} = 10.39\ m/s$$

From eq. (1) $\Rightarrow$ $H = 0.6096 + \frac{(10.39)^2}{2g} = 6.11\ m$

since, $C_v = \frac{V_{act.}}{V_{theo.}} = \frac{10.39}{\sqrt{2g(6.11)}} = 0.95$

5- In figure below, determine (V) for R = 300 mm



Sol.:

Bern. Eq. between ① & Ⓢ: (Take ℄ as a datum).

$$\frac{P_1}{\gamma_w} + \overset{=0}{Z_1} + \frac{V_1^2}{2g} = \frac{P_s}{\gamma_w} + \overset{=0}{Z_s} + \frac{V_s^2}{2g}$$

$$\frac{P_1}{\gamma} + \frac{V^2}{2g} = \frac{P_s}{\gamma} \Rightarrow \frac{P_s}{\gamma} - \frac{P_1}{\gamma} = \frac{V^2}{2g} \quad -①$$

From the differential gauge (manometer).

$$P_C = P_D$$

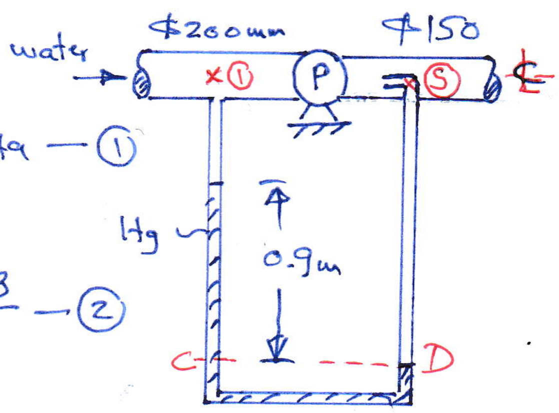
$$\left[P_1 - S\gamma_w R = P_s - \gamma_w R \right] \div \gamma_w$$

$$\frac{P_1}{\gamma} - 0.8R = \frac{P_s}{\gamma} - R \Rightarrow \frac{P_s}{\gamma} - \frac{P_1}{\gamma} = R - 0.8R = 0.2R = 0.2(0.3)$$

$$\therefore \frac{P_s}{\gamma} - \frac{P_1}{\gamma} = 0.06\,m$$

From Eq. ① $\Rightarrow$ $0.06 = \frac{V^2}{2g} \Rightarrow V = 1.085$ m/s.

6- If the pump in figure below developes (5hp) on the flow, What is the flow rate ?



Sol.: Bern. between ① & Ⓢ: (℄ as a datum).

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + H_a = \frac{P_s}{\gamma} \Rightarrow \frac{P_s}{\gamma} - \frac{P_1}{\gamma} = \frac{V_1^2}{2g} + H_a \quad -①$$

$$P = \gamma Q H_a \Rightarrow H_a = \frac{Power}{\gamma_w Q}$$

$$H_a = \frac{5 * 746}{9810 * \frac{\pi}{4}(0.2)^2 * V_1} \Rightarrow H_a = \frac{12.103}{V_1} \quad -②$$

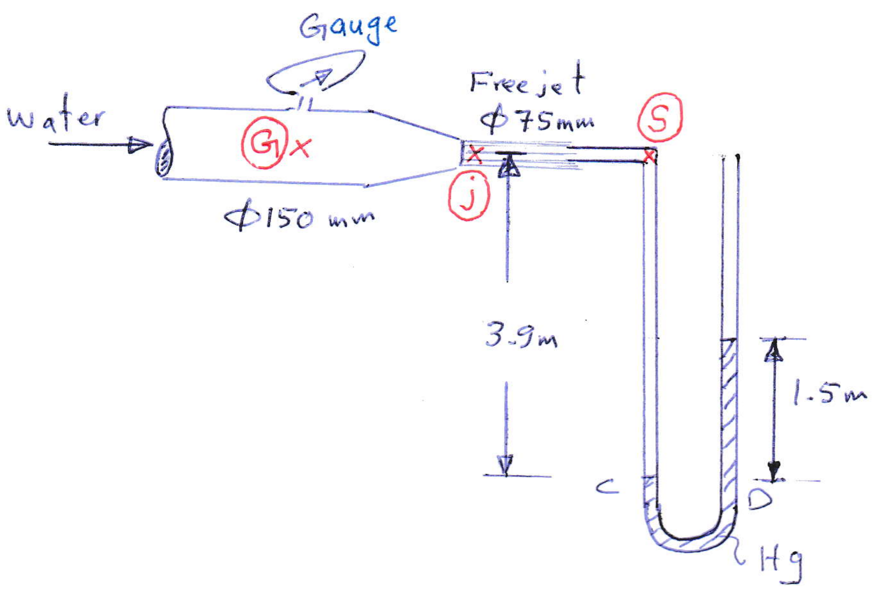
From the manometer; $P_C = P_D$

$$P_1 + 13.6\gamma_w(0.9) = P_s + \gamma_w(0.9) \Rightarrow \frac{P_s}{\gamma_w} - \frac{P_1}{\gamma_w} = 12.6(0.9) = 11.34\,m \quad -③$$

Subs. eqs. ② & ③ into eq. ①

$$11.34 = \frac{V_1^2}{2g} + \frac{12.103}{V_1} \Rightarrow V_1 = 1.067\ m/s. \Rightarrow Q = 0.0335\ m^3/s.$$

7- For the figure shown below, calculate the Gauge reading in meter of mercury.



Solution

By applying Bern. Eq. between points G & Free jet j ; (Take dat. at ℄ of the pipe)

$$\frac{P_G}{\gamma_w} + \overset{0.}{Z_G} + \frac{V_G^2}{2g} = \overset{0.}{\frac{P_j}{\gamma_w}} + \overset{0.}{Z_j} + \frac{V_j^2}{2g}$$

$$\therefore \frac{P_G}{\gamma_w} + \frac{V_G^2}{2g} = \frac{V_j^2}{2g} \quad \text{--- (1)}$$

By continuity eq. ; $Q_G = Q_j \Rightarrow A_G \cdot V_G = A_j V_j$

$$\therefore \frac{\pi}{4}(0.15)^2 * V_G = \frac{\pi}{4}(0.075)^2 V_j$$

$$\therefore V_G = 0.25 V_j \quad \text{--- (2)}$$

Subs. eq. (1) into eq. (2) $\Rightarrow \dfrac{P_G}{\gamma_w} = \dfrac{V_j^2 - (0.25 V_j)^2}{2g}$

$$\therefore P_G = \gamma_w * \frac{(0.9375 V_j^2)}{2g} \quad \text{--- (3)}$$

Bern. Eq. between free jet (j) & stagnation point at the pitot tube (S);

$$0. + 0. + \frac{V_j^2}{2g} = \frac{P_S}{\gamma_w} + 0. + 0. \quad \text{--- (4)}$$

For the gage ; $P_c = P_D$

$$\left[P_s + \gamma_w (3.9) = 13.6\,\gamma_w (1.5)\right] * \frac{1}{\gamma_w}$$

$$\therefore \frac{P_s}{\gamma_w} = 13.6 (1.5) - 3.9 = 16.5\,m$$

From eq. 4 $\Rightarrow$ $V_j^2 = 2g\,(16.5)$ $\Rightarrow$ $V_j = 18$ m/sec.

From eq. 3 $\Rightarrow$ $P_G = \frac{9810}{19.62}\left(0.9375\,(18)^2\right)$

$$\therefore P_G = 151875 \; N/m^2 \; (Pa.)$$

$$\therefore P_{Gauge} = 151875 = 13.6 * 9810 * h_{Hg}$$

$$\therefore h_{Hg} = 1.14 \text{ m of } Hg$$

Flow in Pipes

There are two main types of fluid motion in pipeline:

1- Laminar flow

2- Turbulent Flow

since ; $\tau = \mu \dfrac{du}{dy}$ (Newton's friction Law)

→ For laminar flow ; $\tau = \mu \dfrac{du}{dy}$

→ For turbulent flow ; $\tau = (\mu + \epsilon) \dfrac{du}{dy}$

where; μ = dynamic (laminar) viscosity & it depends on fluid

ϵ = eddy (turbulent) viscosity & it depends on flow property.

→ In between laminar & turbulent flows, there is Transition flow.

Reynold's Number (R_E)

$$R_E = \frac{\text{Inertia force } (m \cdot a)}{\text{Viscous force } (\tau \cdot A)}$$

By dimensional analysis;

$$R_E = \frac{\rho \cdot V \cdot L}{\mu} = \frac{V \cdot L}{\nu}$$

where; R_E = Reynold's No. (dimensionless)

ρ = fluid density (Kg/m^3)

V = average velocity ($m/s.$)

L = linear dimension (m).

μ = dynamic viscosity ($Pa.s.$)

ν = Kinematic viscosity ($m^2/s.$) $= \frac{\mu}{\rho}$

By experiments, Reynold's found that $L = D$; where D is the diameter of circular cross-sectional pipe;

$$\therefore \boxed{R_E = \frac{\rho V D}{\mu} = \frac{VD}{\nu}}$$

For circular pipe;

IF $R_E < 2000$ (laminar flow)

$2000 < R_E < 4000$ (transition flow)

$R_E > 4000$ (turbulent flow)

Major Losses in Pipe Flow

A- Darcy – Weisbach Equation

$$\boxed{h_L = f \frac{L}{D} \frac{V^2}{2g}}$$

where; h_L = head loss (m)

f = friction coefficient (dimensionless)

L = pipe length (m).

D = pipe diameter (m).

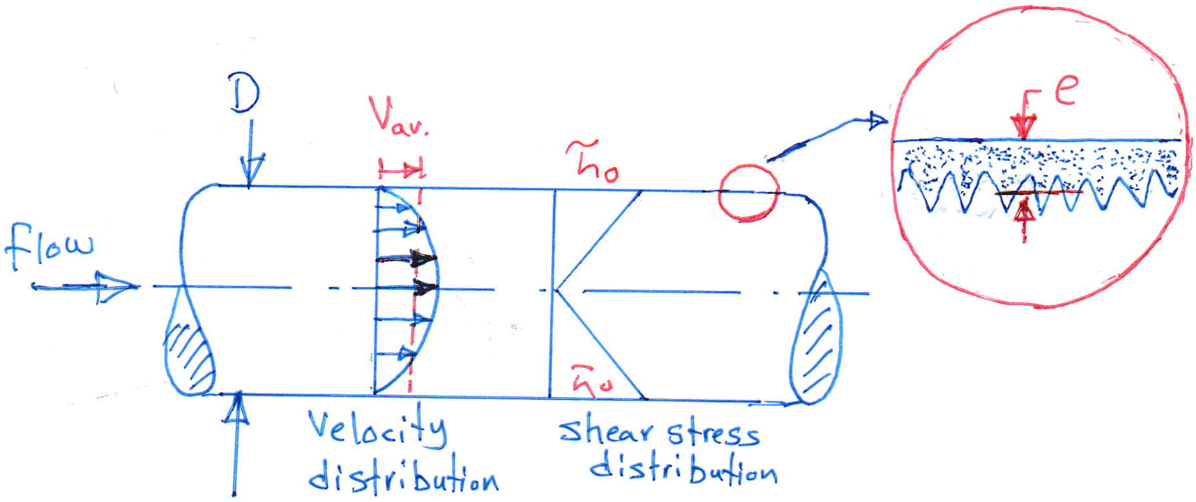
V = average velocity (m/s.).

→ f is a function of R_E & $\frac{e}{D}$ and equals $\left(\frac{8\tau_o}{\rho V^2}\right)$.

where; e = pipe roughness (mm).

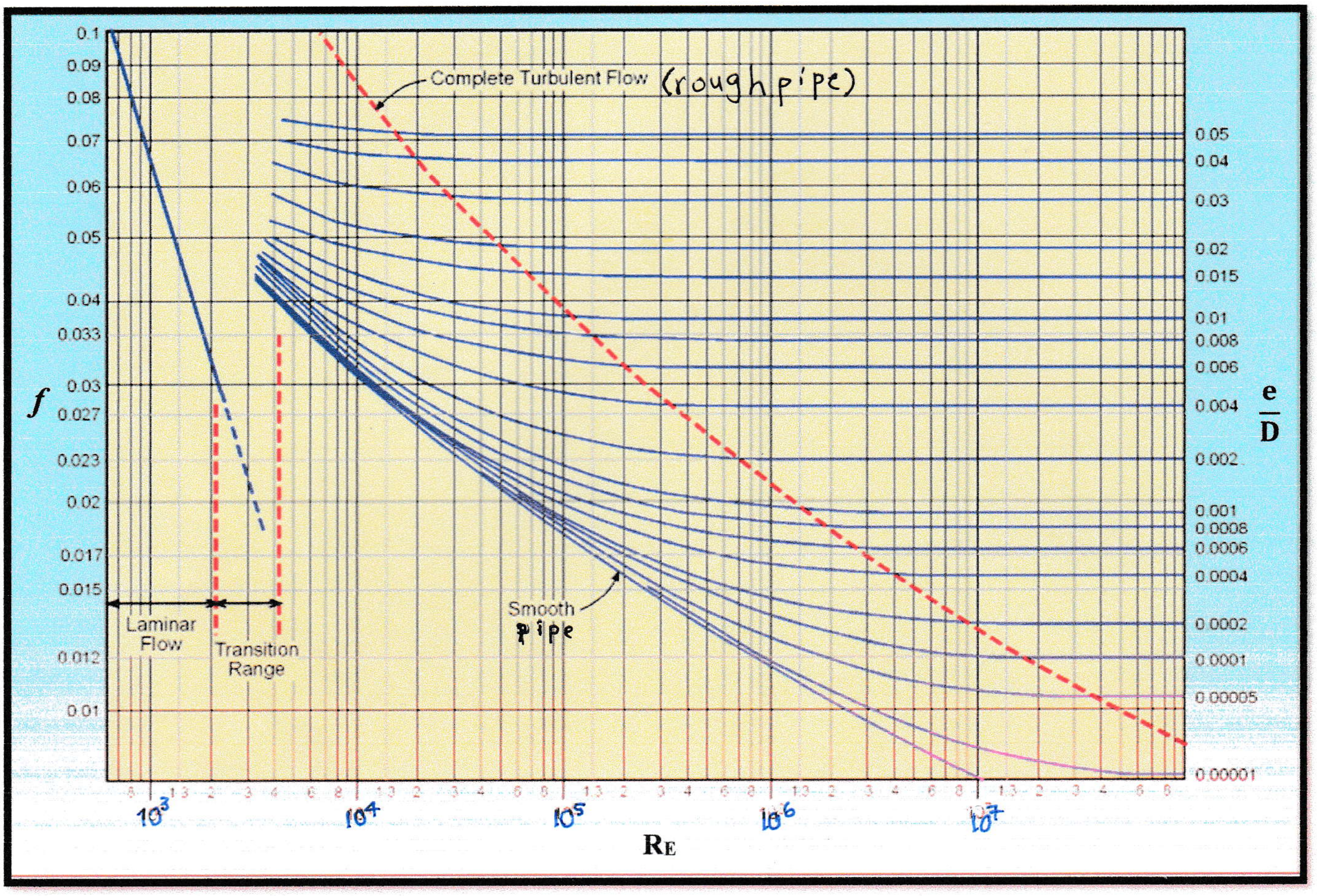
$\frac{e}{D}$ = relative roughness (dimensionless).

τ_o = shear stress at pipe wall (max. τ).



Moody Diagram

It is a family of curves that relate friction coeff. (f) to Reynolds number (R_E), & the relative roughness of a pipe $(\frac{e}{D})$.



- For Laminar flow; $f = F(R_E)$ only $\Rightarrow f = \frac{64}{R_E}$
- For Turbulent flow;
  - * In smooth pipe; $f = F(R_E)$ only
  - * In rough pipe; $f = F(\frac{e}{D})$ only
  - * In transition region between smooth to rough pipes; $f = F(R_E, \frac{e}{D})$

→ Empirical Equations to find f in smooth pipe & turbulent flow;

Blasius formula;

$$f = \frac{0.3164}{R_E^{1/4}} \qquad 3000 \leq R_E \leq 10^5$$

Colebrook-White formula;

$$\frac{1}{\sqrt{f}} = 2 \log_{10} \left(R_E \cdot \sqrt{f} \right) - 0.8 \qquad R_E > 10^5$$

Lee formula;

$$f = 0.0072 + \frac{0.611}{R_E^{0.35}} \qquad 4000 \leq R_E \leq 4 * 10^5$$

B- Hazen-Williams Formula

$$\boxed{Q = 0.2785\, C_{H.W.}\, D^{2.63}\, S^{0.54}}$$ Empirical formula

where; Q = discharge ($m^3/s.$)

$C_{H.W.}$ = Hazen-Williams coefficient (dimensionless)

D = pipe diameter (m)

S = Slope of the Energy line $= \frac{h_L}{L}$ (m/m length)

h_L = head loss (m)

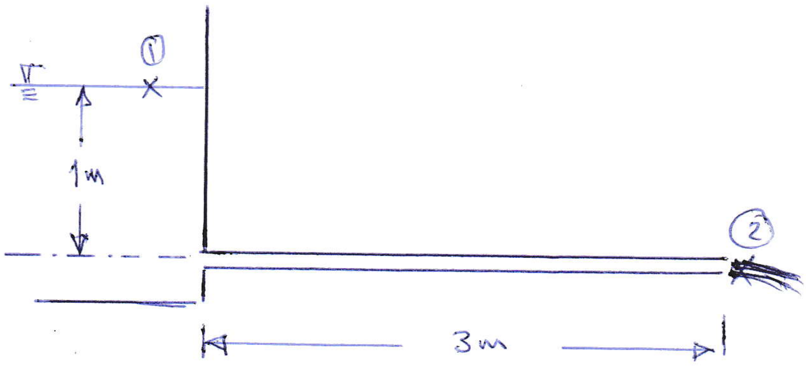
L = pipe length (m)

Examples

1- As shown in figure below, given round pipe (ϕ 20 mm), ($Q = 0.5\ l/s.$);

a- show that the flow is laminar.

b- Determine the kinematic viscosity of the liquid.



Solution:

a- By applying the Energy Eq. between points ① & ②: Datum at point ②

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} + h_L$$

$$\therefore 0 + 1 + 0 = 0 + 0 + \frac{V_2^2}{2g} + h_L \quad \text{—} \ ①$$

$$\because Q = A \cdot V \Rightarrow V = \frac{Q}{A} \Rightarrow V_2 = \frac{0.5 * 10^{-3}}{\frac{\pi}{4}(0.02)^2} = 1.59\ m$$

from eq. ① $\Rightarrow h_L = 0.87\ m$; $\quad \because h_L = f \frac{L}{D} \frac{V^2}{2g}$

Assume laminar flow $\Rightarrow \therefore h_L = \frac{64}{R_E} \cdot \frac{L}{D} \frac{V^2}{2g}$

$$\therefore 0.87 = \frac{64}{R_E} * \frac{3}{0.02} * \frac{(1.59)^2}{2g}$$

$$\therefore R_E = 1422$$

$\because R_E < 2000 \Rightarrow$ the assumption correct

$\therefore$ the flow is Laminar

b- $\because R_E = \frac{V \cdot D}{\nu} \Rightarrow \nu = \frac{V \cdot D}{R_E}$

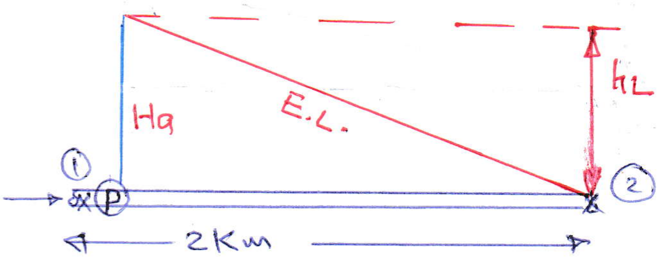
$$\therefore \nu = \frac{1.59 * 0.02}{1422} = 2.236 * 10^{-6}\ m^2/sec.$$

2- Given horizontal smooth pipe (dia. = 150 mm), (Q = 30 l/s.), (s. = 0.8) & ($\nu = 4.4 * 10^{-6}$ m²/s.), calculate:

a- Power required to maintain the flow upto (2 km).

b- shear stress at the pipe wall.

<u>Solution</u> (a) Energy Eq. between points ① & ②:



$$\cancel{\frac{P_1}{\gamma}} + \cancel{Z_1} + \cancel{\frac{V_1^2}{2g}} + Ha = \cancel{\frac{P_2}{\gamma}} + \cancel{Z_2} + \cancel{\frac{V_2^2}{2g}} + h_L$$

$\therefore Ha = h_L$

$\therefore$ power $= \gamma Q Ha \Rightarrow$ power $= \gamma Q h_L$ — ①

$\because h_L = f \dfrac{L}{D} \dfrac{v^2}{2g}$

$v = \dfrac{Q}{A} = 1.7$ m/s $\Rightarrow R_E = \dfrac{1.7 * 0.15}{4.4 * 10^{-6}} = 5.8 * 10^4$

$\because R_E < 10^5$ & smooth pipe $\Rightarrow f = \dfrac{0.3164}{R_E^{1/4}}$

$\therefore f = 0.0204$

<u>or</u> by using Moody Diagram

$\therefore h_L = 0.0204 * \dfrac{2000}{0.15} \dfrac{(1.7)^2}{2g} = 40.1$ m

$\therefore$ from eq. ① $\Rightarrow$ power $= 9.44$ KW

(b) since; $f = \dfrac{8 \tau_o}{\rho v^2}$

$\therefore \tau_o = 5.89$ N/m²

3- Given cast-iron pipe ($e = 0.25$ mm), ($L = 120$ m), ($h_L = 5$ m)

($\nu = 10^{-5}$ m^2/s.) of water & ($D = 100$ mm) find:

a- Discharge.

b- shear stress at the pipe wall.

Solution:	Given	Required
	h_L, L, D, ν & e	Q

follow the procedure;

1- Assume f.

2- determine (v) from Darcy-Weisbach Eq.

3- " R_E & $\frac{e}{D}$.

4- Calculate (f_{new}) from Moody diagram & compare its value with (f_{old}). continue with the above procedure until (f_{new}) converged to (f_{old}). & satisfy the ccuracy criteria shown below;

$$\text{Accuracy}(\%) = \left| \frac{f_{new} - f_{old}}{f_{old}} \right| * 100 \leqslant 5\%$$

1st-iteration

1- Assume $f = 0.026$.

2- $5 = 0.026 \frac{120}{0.1} \frac{v^2}{2g} \Rightarrow v = 1.773$ m/s

3- $R_E = \frac{V.D}{\nu} = 1.773 * 10^4$, $\frac{e}{D} = \frac{0.25}{100} = 0.0025$

4- From Moody diagram $\Rightarrow f_{new} = 0.0316$

accuracy $= 21.5\% > 5\%$ NOT o.k

2nd-iteration

1- let $f = 0.0316$

2- $v = 1.608$ m/s.

3- $R_E = 1.608 * 10^4$ & $\frac{e}{D} = 0.0025$

4- $f_{new} = 0.032$ $\Rightarrow$ accuracy $= 1.26\% < 5\%$ $\therefore$ o.k

$\therefore v = 1.608$ m/s. $\Rightarrow Q = 0.0126$ m^3/s.

4- Determine the diameter of a galvanized iron pipe ($e = 0.09$ mm) & the average velocity in the pipe if it must carry ($0.27\ m^3/s.$) of water ($\nu = 10^{-5}\ m^2/s.$) with a head loss not exceed (6m) per (300m).

Solution:

Given	Required
h_L, Q, L, ν, e	D

1- Assume f.
2- Determine (D) from Darcy-Weisbach Eq.
3- From D & Q, determine RE.
4- Determine ($\frac{e}{D}$).
5- Item (4) from the previous example.

since, $h_L = f \frac{L}{D} \frac{v^2}{2g} \Rightarrow h_L = f \frac{L}{D} \cdot \frac{1}{2g} \left(\frac{4Q}{\pi D^2}\right)^2$

$$\therefore h_L = \frac{8 L Q^2}{\pi^2 g D^5} f$$

$$\therefore D = \left[\frac{8 L Q^2}{\pi^2 g h_L} f\right]^{1/5} \quad \text{——} \quad (1)$$

since, $RE = \frac{V.D}{\nu} = \frac{4Q}{\pi D^2} \cdot \frac{D}{\nu}$

$$\therefore RE = \frac{4Q}{\pi D \nu} \quad \text{——} \quad (2)$$

1st-iteration:

1- Assume $f = 0.026$.
2- From eq. (1) $\Rightarrow D = 0.379$ m
3- " eq. (2) $\Rightarrow RE = 9.07 * 10^4$
4- $\frac{e}{D} = 0.00024$
5- from Moody diagram $\Rightarrow f_{new} = 0.0195$

$\therefore$ accuracy = 25% > 5% NOT O.K

2nd-iteration

1- let $f = 0.0195$.

2- $D = 0.358\,m$

3- $R_E = 9.6 * 10^4$

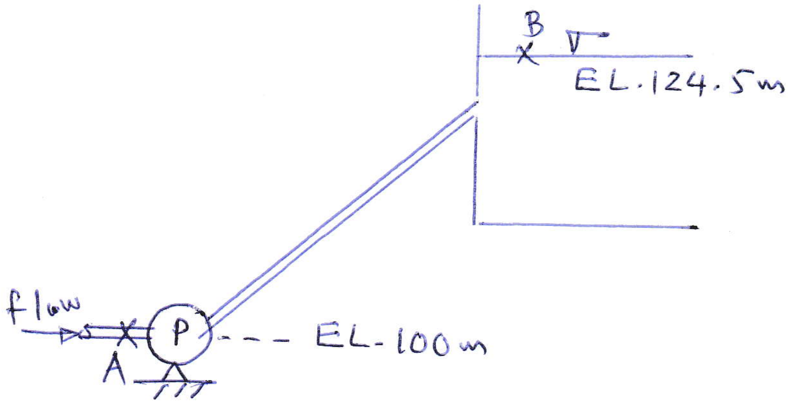
4- $\frac{e}{D} = 0.00025$

5- From Moody diagram $\Rightarrow f_{new} = 0.0193$

accuracy $= 1\% < 5\%$ $\therefore$ O.K

$D = 0.358\,m$
$V = 2.68\,m/s$.

5- As shown in figure below, steel pipe ($e = 2\,mm$), ($D = 400\,mm$) & ($L = 1850\,m$) carries ($200\,l/s.$) of oil ($s. = 0.86$) & ($\nu = 5.16 * 10^{-6}\,m^2/s$). Calculate the pump power when pressure at (A) equals (1.5 bar).



Solution :

$$Power = \gamma Q\, Ha = 0.86(9810)(0.2)(Ha)$$

$$\therefore Power = 1687.32\, Ha \quad — (1)$$

Energy Eq. between (A) & (B) : Datum at A.

$$\frac{1.5 * 10^5}{0.86 * 9810} + 0 + \frac{V_A^2}{2g} + Ha = 0 + 24.5 + 0 + h_L \quad — (2)$$

$$V_A = \frac{Q}{A} = 1.59 \text{ m/s.}$$

$$R_E = \frac{V.D}{\nu} = 1.23 * 10^5$$

$$\frac{e}{D} = \frac{2}{400} = 0.005$$

from Moody diagram $\Rightarrow f = 0.031$

$$\therefore h_L = 0.031 \frac{1850}{0.4} \frac{(1.59)^2}{2g} = 18.74 \text{m}$$

from eq. (2) ; $H_a = 25$m

from eq. (1) ; power = 42183 Watt

Fluid Friction in Non-Circular pipes

To find the friction in circular pipes, the hydraulic-radius concept can be used;

$$\boxed{R_h = \frac{A}{W.P.}}$$

where; R_h = hydraulic-radius (m).

A = pipe cross-sectional area (m^2).

W.P. = Wetted perimeter (m).

$$R_h(\text{circular pipe}) = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$$

$$\therefore \boxed{D = 4 R_h}$$

So, to find the head loss (using Darcy-Weisbach Eq.) & R_E in Non-circular pipe; $h_L = f \frac{L}{4R_h} \frac{V^2}{2g}$

$$R_E = \frac{\rho V (4R_h)}{\nu} = \frac{V (4R_h)}{\nu}$$

Also to use the Moody Diagram, $\left(\frac{e}{D}\right)$ is replaced by $\left(\frac{e}{4R_h}\right)$.

Minor Losses in Pipe Flow

Losses due to local disturbances of the flow in conduits such as, changes in cross section, elbows, values & similar items are called minor losses.

In general; $\boxed{h_L = K \frac{v^2}{2g}}$

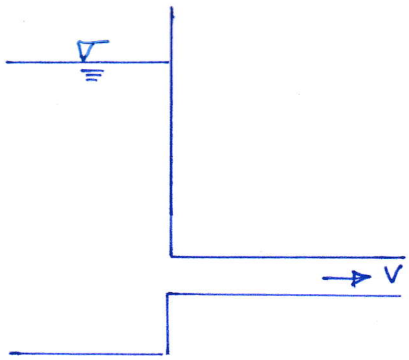
where; h_L = minor losses (m).

K = loss coefficient (depends on type of pipe fittings)

v = average velocity (m/sec.).

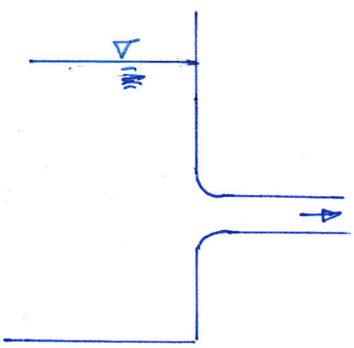
1- Loss of head at Entrance

$$\boxed{h_L = K \frac{v^2}{2g}}$$



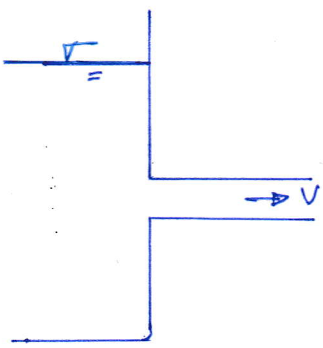
where;

K = loss coefficient depends on type of entrance



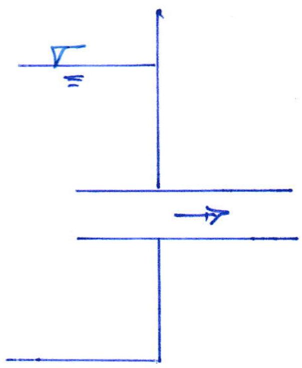
Bell-mouth $K = 0.04$

round orifice $K = 0.05$



square-edged

$K = 0.5$

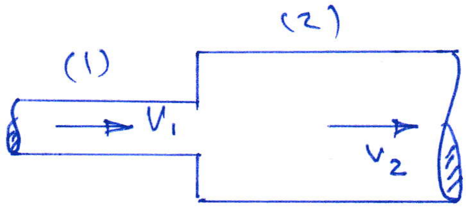


Re-entrant

$K \simeq 0.8$

2- Loss Due to Expansion

Sudden Expansion



$$\boxed{h_L = \frac{(V_1 - V_2)^2}{2g}}$$

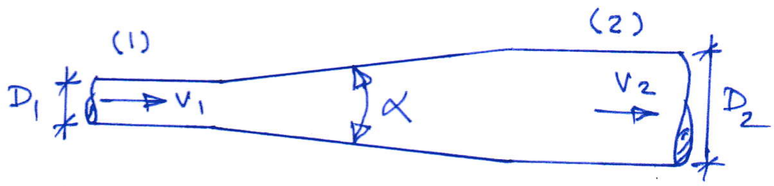
or

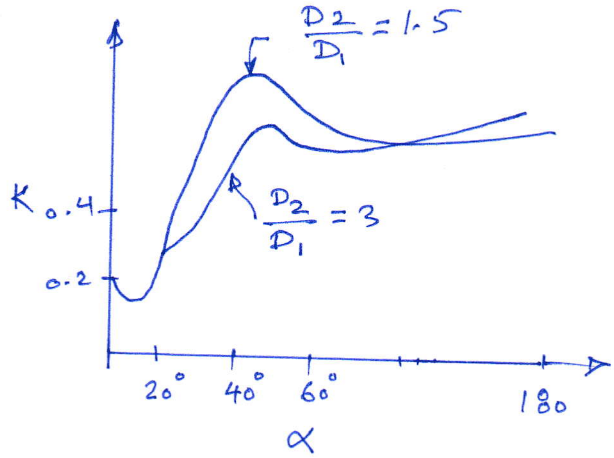
$$\boxed{h_L = K\frac{V_1^2}{2g} \qquad K = \left(1 - \frac{A_1}{A_2}\right)^2}$$

where; A_1 & A_2 : cross-sectional area of (1) & (2), respectively

Gradual Expansion

$$\boxed{h_L = K\frac{(V_1 - V_2)^2}{2g}}$$



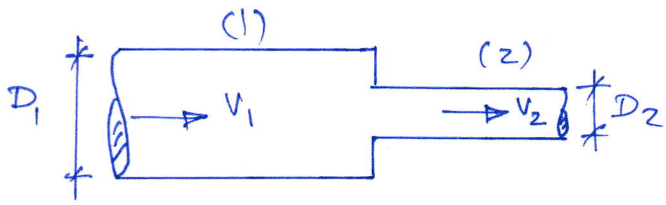


3- Loss Due to Contraction

Sudden Contraction

$$\boxed{h_L = K\frac{V_2^2}{2g}}$$

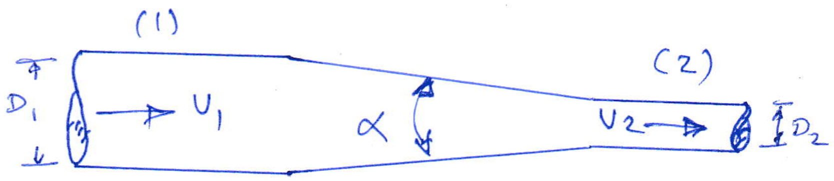
K- depends on $\frac{D_2}{D_1}$



$\frac{D_2}{D_1}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K	0.5	0.45	0.42	0.39	0.36	0.33	0.28	0.22	0.15	0.06	0.0

Gradual Contraction

$$\boxed{h_L = K \frac{V_2^2}{2g}}$$



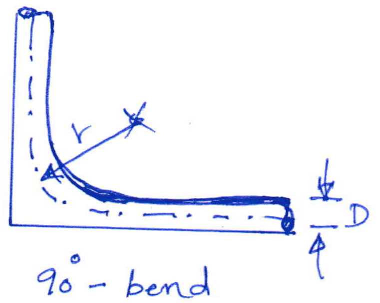
D_2/D_1	0.2	0.4	0.6	0.8
K	0.3	0.25	0.15	0.1

- For smooth curved transition; $K = 0.05$
- For conical reducers; $K = 0.1$ $(\alpha = 20° - 40°)$

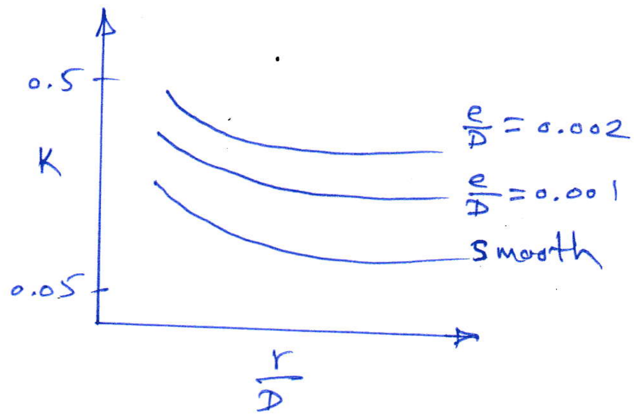
4- Losses at the Bend & Elbow

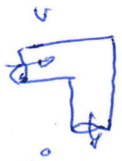
$$\boxed{h_L = K \frac{V^2}{2g}}$$

Bend



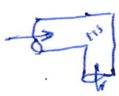
90° - bend



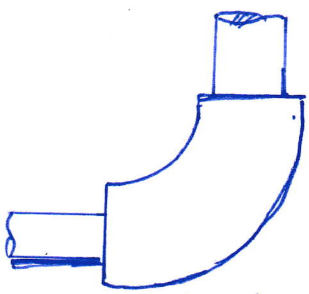


90° - mitter bend

K = 0.1


90° - mitter bend with vane (K = 0.2)

Elbow


standard elbow

K = 0.3

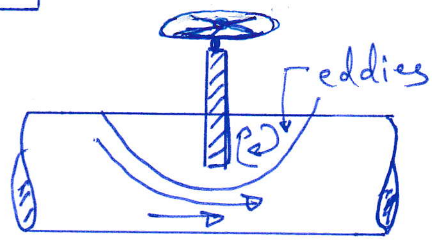
- Long sweep elbow

K = 0.2

5- Losses at the Valves

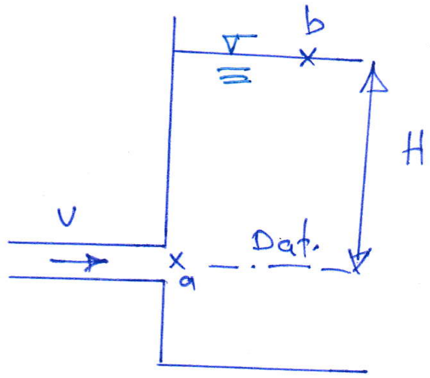
$$\boxed{h_L = k\frac{v^2}{2g}}$$

Gate Valve



Valve	K
- Gate valve (fully open)	0.15
- " " ($\frac{1}{4}$ closed)	0.26
- " " ($\frac{1}{2}$ closed)	2.1
- " " ($\frac{3}{4}$ closed)	17
- Globe valve (fully open)	10
- Angle " (" ")	2
- Swing check valve (fully open)	2

6- Losses at the Exit



By applying the Energy Eq. between points ⓐ & ⓑ

$$\frac{P_a}{\gamma} + Z_a + \frac{V_a^2}{2g} = \frac{P_b}{\gamma} + Z_b + \frac{V_b^2}{2g} + h_L$$

$$\cancel{H} + 0. + \frac{V^2}{2g} = 0. + \cancel{H} + 0. + h_L$$

$$\therefore \boxed{h_L = \frac{v^2}{2g}} \quad ; \quad K = 1.0$$

The Equivalent length (Le)

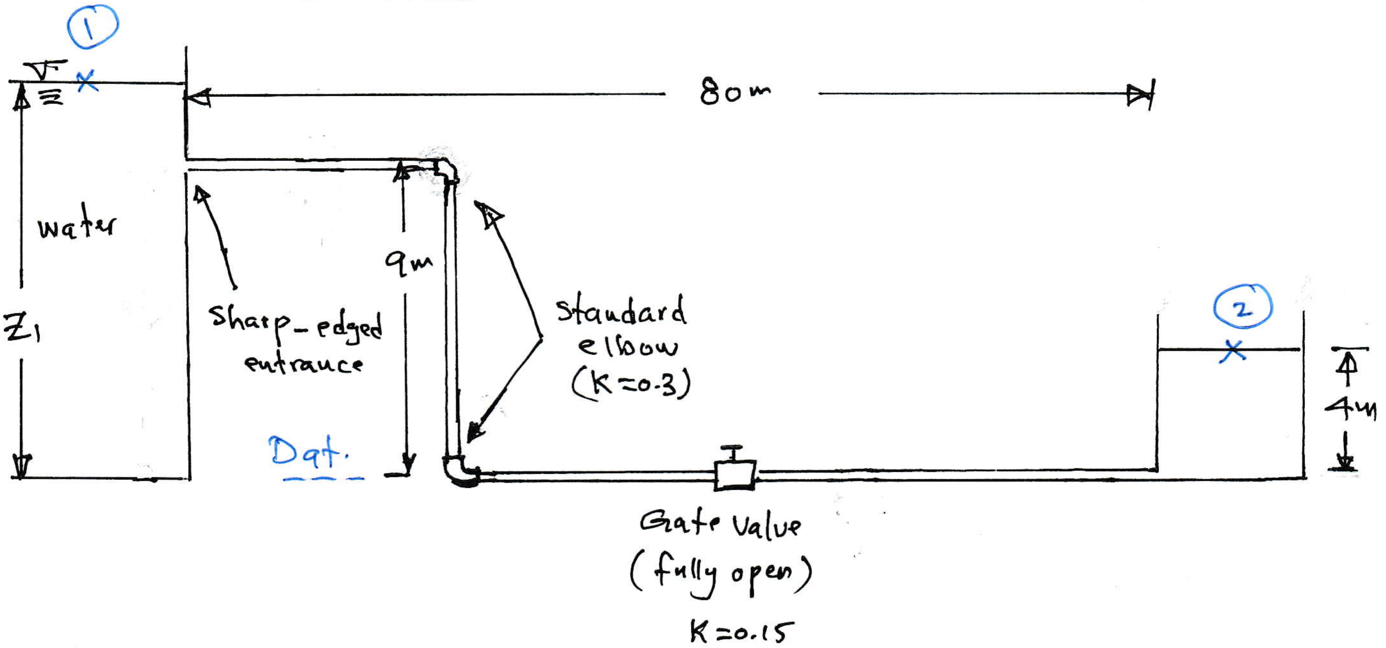
Minor losses may be expressed in terms of the equivalent length (Le) of pipe that has the same head loss for the same discharge;

$$f\frac{Le}{De}\frac{v^2}{2g} = K\frac{v^2}{2g}$$

in which K may refer to one minor loss or the sum of several losses.

$$\therefore Le = \frac{K\,De}{f}$$

Ex.1 : Water flows from a large reservoir to a small one through a (5 cm) diameter cast iron pipe system, as shown in figure below. Determine the elevation z_1 for a flow rate of (6 l/s.) when $\mu = 1.307 * 10^{-3}$, $e = 0.26$ mm



Sol.: By applying the Energy eq. between points 1 & 2:

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_{L\,major\,(1-2)} + h_{L\,minor\,(1-2)}$$

$$0 + z_1 + 0 = 0 + 4 + 0 + h_{L\,major} + h_{L\,minor}$$

$$\therefore z_1 = 4 + h_{L\,major} + h_{L\,minor} \quad \text{—} \ (1)$$

$h_{L\,major}$:

$$h_L = f \frac{L}{D} \frac{v^2}{2g} \quad ; \quad v = \frac{Q}{A} = \frac{6*10^{-3}}{\frac{\pi}{4}(0.05)^2} = 3.06 \text{ m/s.}$$

$$R_E = \frac{\rho V D}{\mu} = \frac{10^3 (3.06)(0.05)}{1.307*10^{-3}} = 1.17*10^5$$

$$\frac{e}{D} = \frac{0.26}{50} = 0.0052$$

From Moody diagram $\Rightarrow f = 0.032$

$$\therefore \quad h_{L_{major}} = 0.032 * \frac{(80+9)}{0.05} \frac{(3.06)^2}{2g} = 27.18 \text{ m}$$

$h_{L_{minor}}$:

$$h_{L_{minor}} = h_{L_{entrance}} + 2 h_{L_{elbow}} + h_{L_{gate\,value}} + h_{L_{exit}}$$

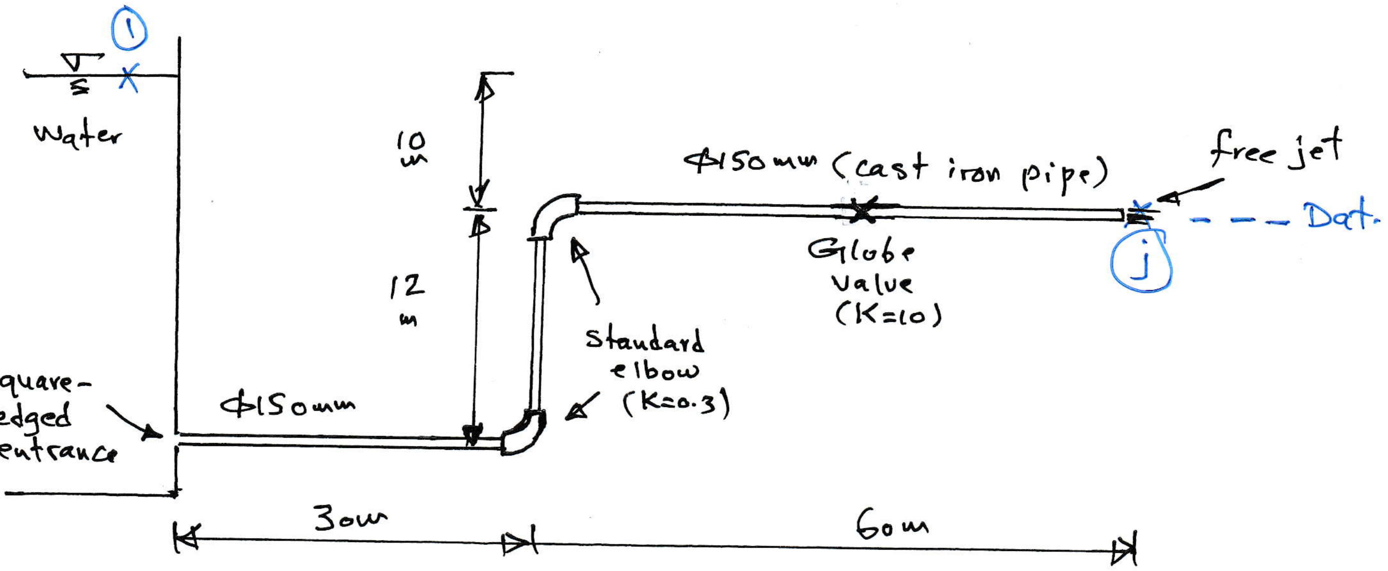
$$h_{L_{minor}} = 0.5 \frac{V^2}{2g} + 2 * 0.3 \frac{V^2}{2g} + 0.15 \frac{V^2}{2g} + \frac{V^2}{2g}$$

$$h_{L_{minor}} = (0.5 + 0.6 + 0.15 + 1) \frac{(3.06)^2}{2g} = 1.074 \text{m}$$

From eq. (1) :

$$Z_1 = 4 + 27.18 + 1.074 = 32.254 \text{ m}$$

Ex.2 : Find the discharge through the pipeline, shown in figure below. ($\nu = 1.01 * 10^{-6}$ m²/s. , $e = 0.255$ mm).



Sol. : By applying the Energy Eq. between points (1) & (j) : Take Datum at (j)

$$0 + 10 + 0 = 0 + 0 + \frac{V_j^2}{2g} + h_{L_{major}} + h_{L_{minor}} \quad \text{---} (1)$$

$\underline{h_{L_{major}}}$:

$$h_{L_{major}} = f \frac{L}{D} \frac{V^2}{2g}$$

$$\phi_{pipe} = \phi_{jet} \Rightarrow V_{pipe} = V_j$$

$$\therefore h_{L_{major}} = f \frac{(30+60+12)}{0.15} \frac{V_j^2}{2g} = 680 f \frac{V_j^2}{2g} \quad — (2)$$

$$h_{L_{minor}} = h_{L_{entrance}} + 2h_{L_{elbow}} + h_{L_{value}} = 0.5 \frac{V_j^2}{2g} + 2 * 0.3 \frac{V_j^2}{2g} + 10 \frac{V_j^2}{2g}$$

$$\therefore h_{L_{minor}} = 11.1 \frac{V_j^2}{2g} \quad — (3)$$

subs. eqs. (2) & (3) into eq. (1):

$$10 = \frac{V_j^2}{2g} + 680 f \frac{V_j^2}{2g} + 11.1 \frac{V_j^2}{2g}$$

$$10 = \frac{V_j^2}{2g}(12.1 + 680 f) \quad — (4)$$

$\underline{1^{st}\text{-iteration}}$:

1- Assume $f = 0.03$

2- From eq. (4) : $V_j = 2.457$ m/s

3- $R_E = \frac{2.457 (0.15)}{1.01 * 10^{-6}} = 3.649 * 10^5$; $\frac{e}{D} = 0.0017$

4- From Moody diagram $\Rightarrow f_{new} = 0.023$

Accuracy(%) = 23.33 % > 5 % NOT o.k

2^{nd} iteration :

1- let $f = 0.023$

2- From eq. 4 : $V_j = 2.659$ m/s.

3- $R_E = 3.949 * 10^5$; $\frac{e}{D} = 0.0017$

4- from Moody diagram $\Rightarrow f_{new} = 0.0229$

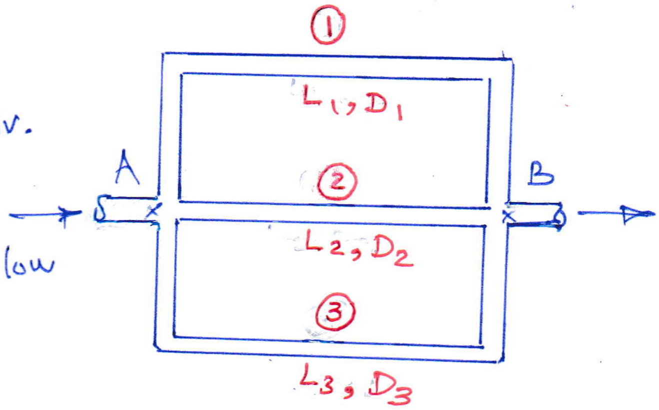
Accuracy(%) = 0.435% < 5% ∴ o.k

$\therefore V_j = 2.659$ m/s.

$\therefore Q = A_j * V_j = 0.047\ m^3/s.$

Parallel Pipes, Series Pipes & Equivalent Pipe

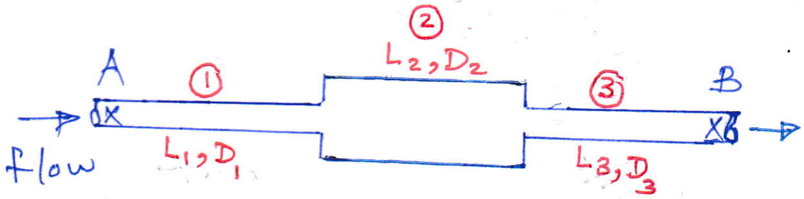
Parallel pipes



$$Q_A = Q_B = Q_1 + Q_2 + Q_3 = Q_{equiv.}$$

$$h_{L_{AB}} = h_{L_1} = h_{L_2} = h_{L_3} = h_{L_{equiv.}}$$

Series Pipes



$$Q_A = Q_B = Q_1 = Q_2 = Q_3 = Q_{equiv.}$$

$$h_{L_{AB}} = h_{L_1} + h_{L_2} + h_{L_3} = h_{L_{equiv.}}$$

Equivalent pipe

It is a single pipe which is used to replace the actual piping system. The equivalent pipe gives the same flow rate (discharge) & head loss for the actual piping system.

Ex.1: As shown in figure below. Given: $L_1 = 100\,m$; $D_1 = 50\,mm$, $L_2 = 100\,m$; $D_2 = 100\,mm$; f for all pipes $= 0.032$. Find:

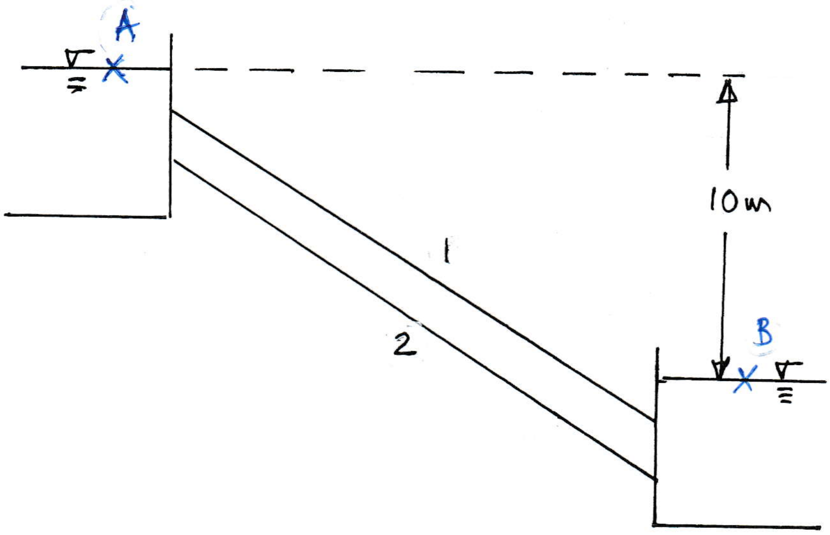
a- discharge at each pipe.

b- equivalent diameter for a single pipe of (100 m) length

Note: If $L_{pipe} \geq 1000\, D_{pipe} \Rightarrow$ neglect minor losses
In general

$L_{pipe} < 1000\, D_{pipe} \Rightarrow$ take the minor losses into consideration

OR: If the question asked to take the minor losses into consideration, the above limitations are not applicable.



Sol.: $L_1 > 1000\, D_1$ & $L_2 = 1000\, D_2 \Rightarrow$ neglect minor losses

a- Due to parallel connection;

$$h_{L_1} = h_{L_2} \quad \text{— } (1)$$

By applying the Energy eq. between points (A) & (B) & along pipe no. 1 : (Take datum at B)

$$0 + 10 + 0 = 0 + 0 + 0 + h_{L_1}$$

$$\therefore \; h_{L_1} = 10\,m$$

for pipe no. 1:

$$h_{L_1} = f_1 \frac{L_1}{D_1} \cdot \frac{V_1^2}{2g}$$

$$\therefore \; 10 = 0.032 \, \frac{100}{0.05} \, \frac{V_1^2}{2g}$$

$$\therefore \; V_1 = 1.75 \; m/s.$$

$$\therefore \; Q_1 = 3.436 * 10^{-3} \; m^3/s.$$

from eq. 1

$$h_{L_1} = h_{L_2} = 10\,m$$

for pipe no. 2:

$$h_{L_2} = f_2 \frac{L_2}{D_2} \, \frac{V_2^2}{2g}$$

$$10 = 0.032 \, \frac{100}{0.1} \, \frac{V_2^2}{2g}$$

$$\therefore \; V_2 = 2.476 \; m/s.$$

$$Q_2 = 0.0194 \; m^3/s$$

b-

$$Q_{equiv.} = Q_1 + Q_2 = 0.0228 \; m^3/s.$$

$$V_{equiv.} = \frac{Q_{equiv.}}{A_{equiv.}} \Rightarrow V_{equiv.} = \frac{0.029}{D_{equiv.}^2}$$

$$h_{L_{equiv.}} = h_{L_1} = h_{L_2} = 10\,m$$

So;

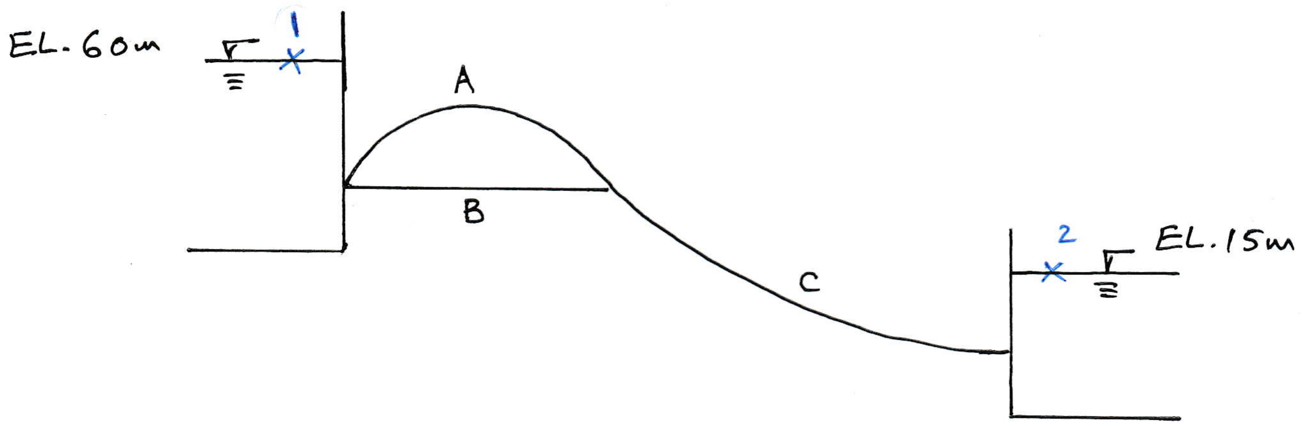
$$10 = 0.032 \, \frac{100}{D_{equiv.}} \cdot \frac{1}{2g} \left(\frac{0.029}{D_{equiv.}^2} \right)^2$$

$$\therefore \; D_{equiv.} = 0.106 \; m$$

Ex.2: Three pipes A, B, & C are interconnected as show in figure below. The pipe characteristics are as follows:

Pipe	D (mm)	L (m)	f
A	150	600	0.02
B	100	480	0.032
C	200	1200	0.024

Find the rate at which water will flow in each pipe.



Sol.: since $L_A > 1000\,D_A$, & $L_B > 1000\,D_B$, & $L_c > 1000\,D_2$ } Neglect minor losses

By applying the Energy eq. between points 1 & 2 & along pipes A & C: (Take datum at 2)

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} + h_{L_A} + h_{L_c}$$

$$0 + 45 + 0 = 0 + 0 + 0 + h_{L_A} + h_{L_c}$$

$$\therefore\ 45 = 0.02\,\frac{600}{0.15}\,\frac{V_A^2}{2g} + 0.024\,\frac{1200}{0.2}\,\frac{V_c^2}{2g}$$

$$45 = 80 \frac{V_A^2}{2g} + 144 \frac{V_c^2}{2g} \quad — \quad (1)$$

- Due to parallel connection between pipes A & B :

$$\therefore \; h_{L_A} = h_{L_B} \quad — \quad (2)$$

- Due to a series connection between pipes (A & B) & C :

$$Q_A + Q_B = Q_c \quad — \quad (3)$$

from eq. (2) :

$$0.02 \, \frac{600}{0.15} \, \frac{V_A^2}{2g} = 0.032 \, \frac{480}{0.1} \, \frac{V_B^2}{2g}$$

$$\therefore \; V_B = 0.722 \, V_A \quad — \quad (4)$$

from eq. (3) :

$$A_A V_A + A_B V_B = A_c . V_c$$

$$\therefore \; V_c = 0.56 \, V_A + 0.25 \, V_B \quad — \quad (5)$$

Subs. eq. (4) into eq. (5) :

$$V_c = 0.56 \, V_A + 0.25 \, (0.722 \, V_A)$$

$$\therefore \; V_c = 0.7405 \, V_A \quad — \quad (6)$$

Subs. eq. (6) into eq. (1) :

$$45 = 80 \frac{V_A^2}{2g} + \frac{144}{2g} (0.7405 \, V_A)^2$$

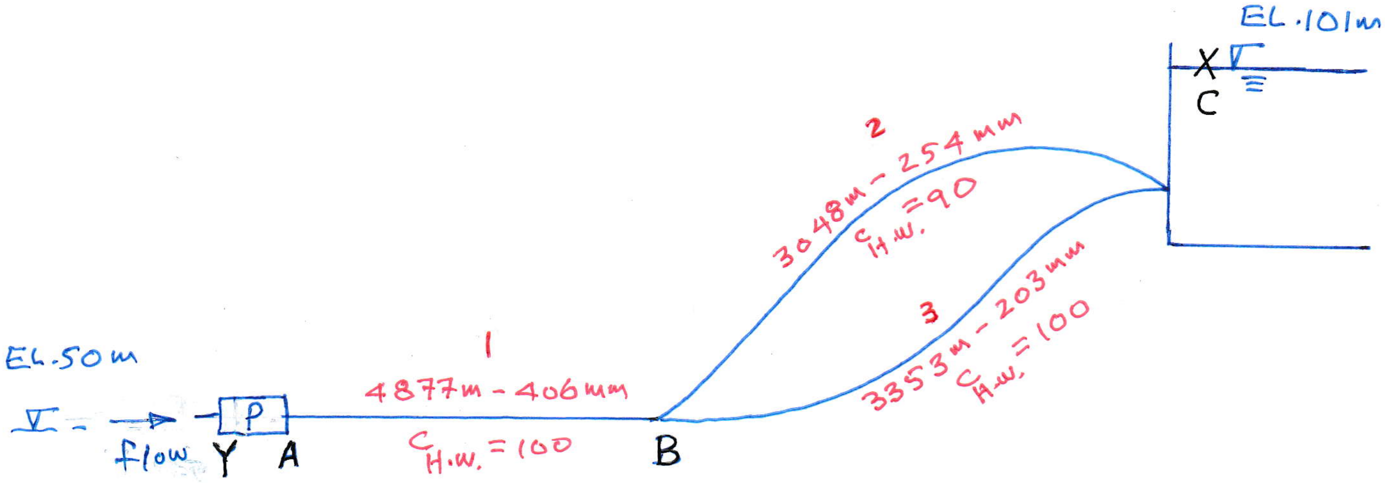
$$\therefore \; V_A = 2.35 \text{ m/s.}$$

$$\therefore \; V_c = 1.75 \text{ m/s.} \;\; \& \;\; V_B = 1.7 \text{ m/s.}$$

$$\therefore \quad Q_A = 0.042 \text{ m}^3\text{/s.} \; ; \; Q_B = 0.013 \text{ m}^3\text{/s.} \; ; \; Q_c = 0.055 \text{ m}^3\text{/s.}$$

To check : $Q_c = Q_A + Q_B$

EX.3: As shown in figure below, when pump (YA) delivers $0.1415\ m^3/s$. Find the pressure heads at A & B.



Sol.: $L > 1000\,D$ for all pipes ⇒ neglect minor losses

By applying the Energy eq. between points A & C along pipes 1 & 2 : (Take datum at A)

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_C}{\gamma} + Z_C + \frac{V_C^2}{2g} + h_{L_1} + h_{L_2}$$

$$\frac{P_A}{\gamma} + 0 + \frac{V_A^2}{2g} = 0 + 51 + 0 + h_{L_1} + h_{L_2}$$

$$\therefore \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = 51 + h_{L_1} + h_{L_2} \quad \text{— (1)}$$

for pipe (1): $Q_1 = 0.1415 = 0.2785\,(100)\,(0.406)^{2.63}\left(\frac{h_{L_1}}{4877}\right)^{0.54}$

$$\therefore h_{L_1} = 22.2\ m$$

– Due to parallel connection between pipes 2 & 3 :

$$\therefore h_{L_2} = h_{L_3} \quad \text{— (2)}$$

- Due to parallel connection between pipes 1 & (2, 3)

$\therefore \quad Q_1 = Q_2 + Q_3$

$$\therefore \quad 0.1415 = 0.2785\,(90)\,(0.254)^{2.63}\left(\frac{h_{L_2}}{3048}\right)^{0.54} + 0.2785\,(100)\,*\,(0.203)^{2.63}\left(\frac{h_{L_3}}{3353}\right)^{0.54} \quad \text{—— (3)}$$

Subs. eq. (2) into eq. (3):

$$\therefore \quad h_{L_2} = 70.56\,m \quad \Rightarrow h_{L_3} = 70.56\,m$$

<u>for pipe (1)</u>

$$Q_1 = A_1 \cdot V_1$$

$$\therefore \quad V_1 = 1.093\,m/s. = V_A$$

Subs. values of V_A, h_{L_1}, & h_{L_2} into eq. (1):

$$\therefore \quad \frac{P_A}{\gamma} = 143.7\,m$$

<u>To find pressure head at B:</u>

Applying the Energy eq. between points B & C along either pipe 2 or pipe 3 : (Take datum at B).

$$\frac{P_B}{\gamma} + Z_B + \frac{V_B^2}{2g} = \frac{P_C}{\gamma} + Z_C + \frac{V_C^2}{2g} + h_{L_2} \text{ or } h_{L_3}$$

$$\frac{P_B}{\gamma} + 0 + \frac{(1.093)^2}{2g} = 0 + 51 + 0 + 70.56$$

$$\therefore \quad \frac{P_B}{\gamma} = 121.5\,m$$