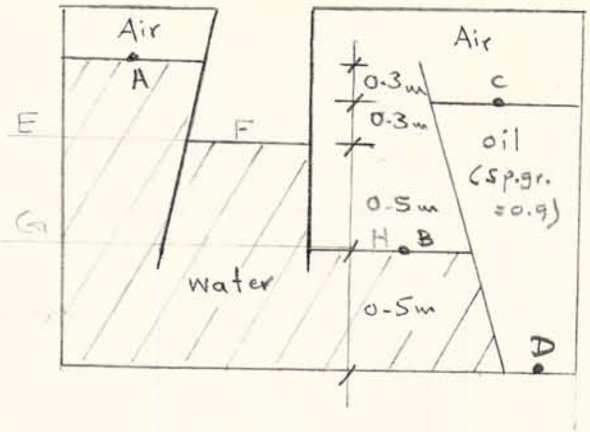
Ex① Calculate the gage pressure at A, B, C & D.



Solution: $P_E = P_F$

$$P_A + \rho_w g (0.6) = \overset{F.O.}{P_{atm.}}$$

$$\therefore P_A = -1000 * 9.81 * 0.6$$

$$= -5886 \ N/m^2$$

$$= 5886 \ Pa. \ (\text{Vacuum})$$

$$P_G = P_H \Rightarrow \overset{F.O.}{P_{atm.}} + \rho_w g (0.5) = P_B$$

$$\therefore P_B = 4905 \ N/m^2$$

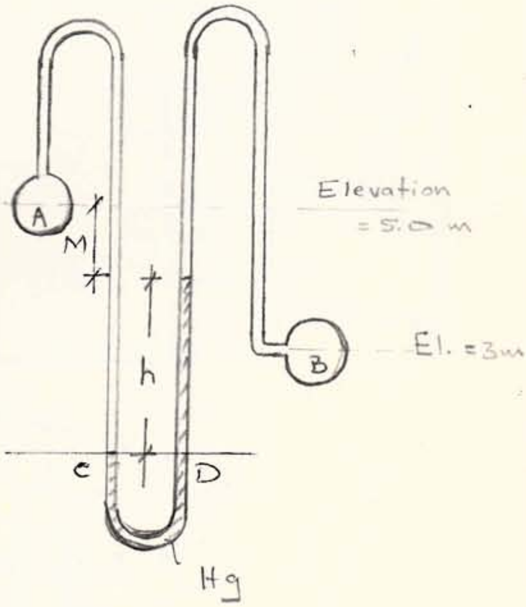
$$P_C = P_B = 4905 \ N/m^2$$

$$P_D = P_C + \rho_{oil} \cdot g \cdot (1.3) = 4905 + 0.9 * 1000 * 9.81 * (1.3)$$

$$\therefore P_D = 16383 \ N/m^2$$

Ex.②: Vessels A & B contain water under pressures of 2.76 bar, 1.38 bar respectively. What is the deflection of the mercury in the differential gage (manometer)?

Note: $1 bar = 10^5 \ N/m^2 = 10^5 \ Pa.$



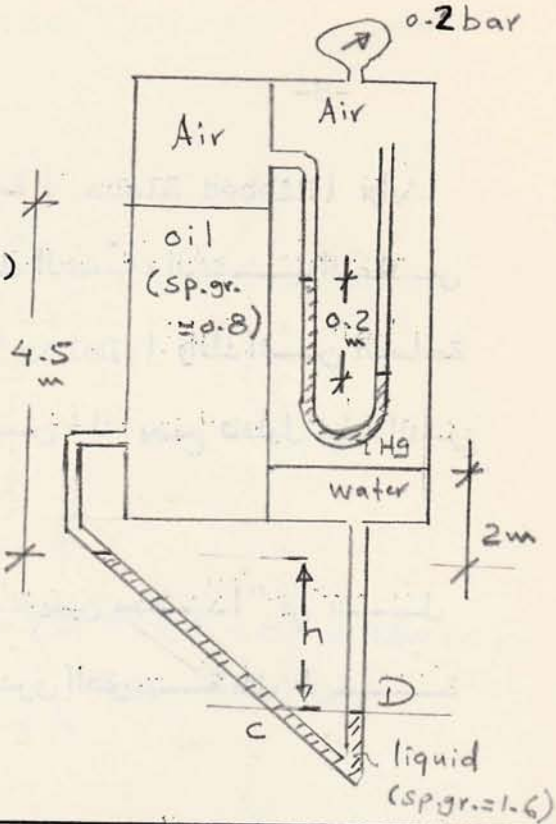
Solution: $P_C = P_D$

$$P_A + \cancel{\rho_w g M} + \rho_w g h = P_B - \rho_w g (z) + \cancel{\rho_w g M} + \rho_{Hg} \cdot g \, h$$

$$2.76 * 10^5 + 9810 h = 1.38 * 10^5 - 9810 (z) + 13.6 * 10^3 * 9.81 * h$$

$$\therefore h = 1.275 m$$

Ex. ③ : Calculate the difference head (h) in the manometer.



Solution:

$$P_C = P_D$$

$$\cancel{0.2*10^5} - \rho_{Hg} \cdot g\,(0.2) + \rho_{oil} \cdot g\,(4.5) + \rho_{liquid} \cdot g\,(h) = \cancel{0.2*10^5} + \rho_w \cdot g(2) + \rho_w\, g(h)$$

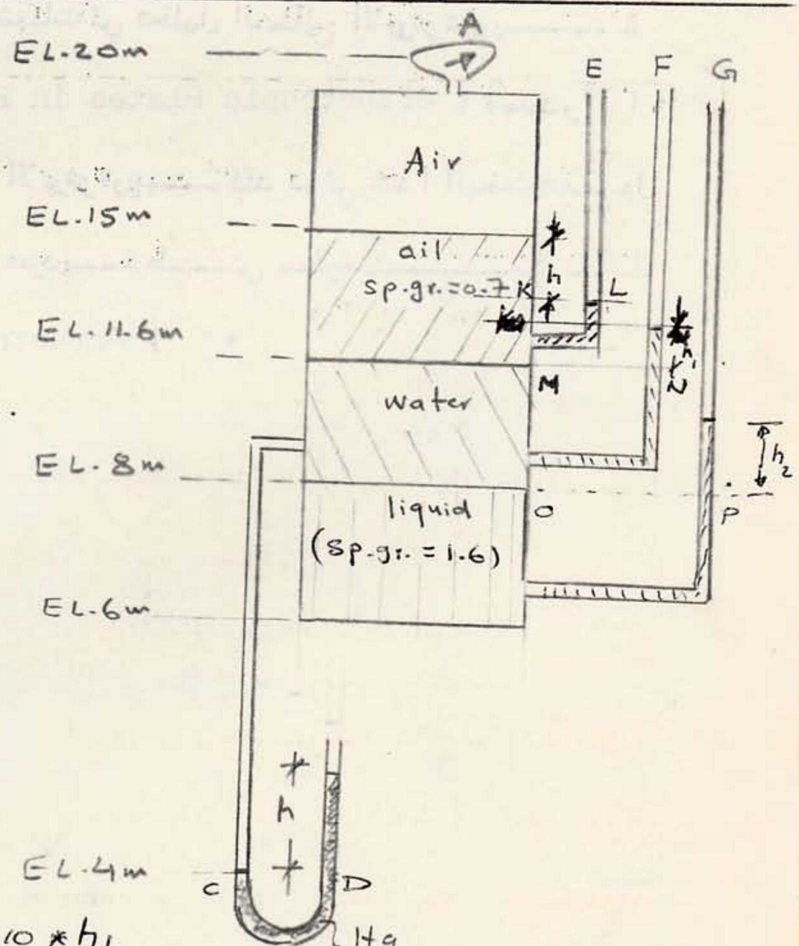
$$-13.6*9810*0.2 + 0.8*9810(4.5) + 1.6*9810\,h = 9810(2) + 9810\,h$$

$$\therefore h = 1.866\,m$$

Ex. ④ : For a gage reading at A of (17200 Pa) Vacuum, Determine:

a - the elevation of the liquids in open piezometer Columns E, F & G

b - the deflection of the mercury in the U-tube gage.



Solution: ⓐ $P_K = P_L$

$$P_A + \rho_{oil}\, g\,(h) = P_{atm}$$

$$\therefore h = \frac{17200}{0.7*9810} = 2.5\,m$$

$$\therefore EL_E = 15 - 2.5 = 12.5\,m$$

$$P_M = P_N$$

$$-17200 + 0.7*9810*(15 - 11.6) = 9810 * h_1$$

$$\therefore h_1 = 0.63\,m$$

$$\therefore EL_F = 11.6 + 0.63 = 12.23\,m$$

$$P_O = P_P$$

$$-17200 + 0.7 * 9810\,(15-11.6) + 9810\,(11.6-8)$$

$$= 1.6 * 9810 * h_2$$

$$\therefore h_2 = 2.64\,m$$

$$\therefore EL_G = 8 + 2.64 = 10.64\,m$$

b - $P_C = P_D$

$$-17200 + 0.7 * 9810\,(15-11.6) + 9810\,(11.6-8) + 9810\,(4)$$

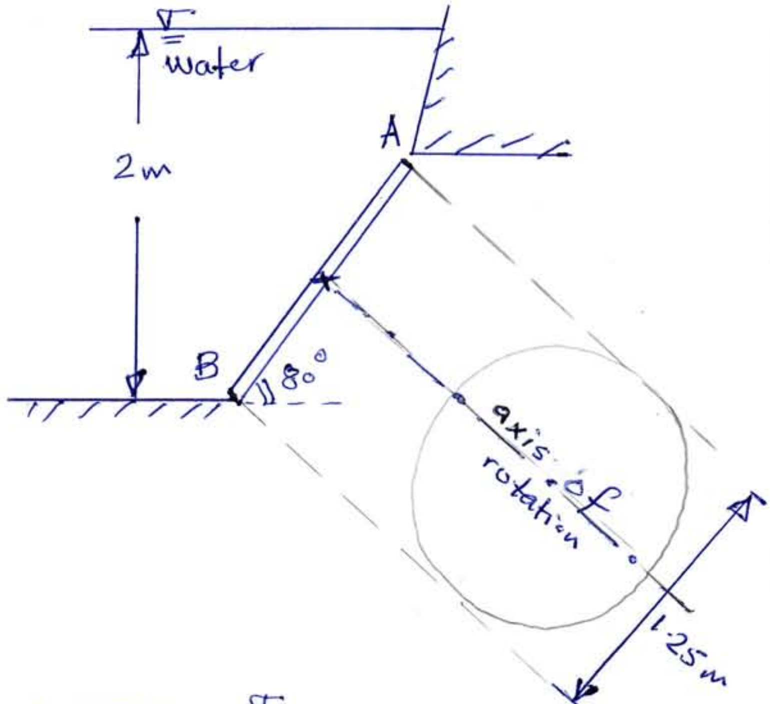
$$= 13.6 * 9810 * h$$

$$\therefore h = 0.605\,m$$

Ex.1: For the gate (AB) shown in the figure below, calculate:

1- Hydrostatic force on the gate.

2- Turning moment about the axis of rotation.



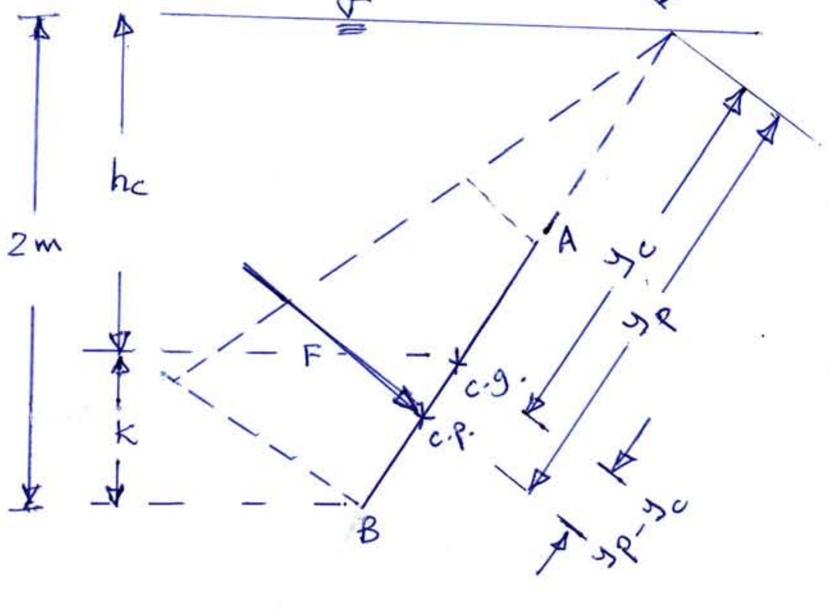
Solution:

Since $F = \gamma h_c A$

$\gamma = \gamma_w = 9810 \ N/m^3$

$= 9.81 \ KN/m^3$

From the sketch:



$h_c = 2 - K$

$= 2 - \frac{1.25}{2} \sin 80^\circ$

$= 1.384 \ m$

$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (1.25)^2$

$= 1.227 \ m^2$

$F = 9.81 * 1.384 * 1.227$

$= 16.66 \ KN$

Since $y_p = y_c + \frac{I_c}{y_c A}$

$\therefore \ y_p - y_c = \frac{I_c}{y_c A} = \frac{\frac{\pi}{64} D^4}{\frac{h_c}{\sin 80^\circ} * A}$

$y_p - y_c = \frac{\frac{\pi}{64} (1.25)^4}{\frac{1.384}{\sin 80^\circ} * 1.227} = 0.07 m$

$\therefore$ Turning moment about the axis of rotation $= F(y_p - y_c) = 16.66 * 0.07 = 1.17 \ KN.m$

$\sin 80^\circ = \frac{K}{\frac{1.25}{2}}$

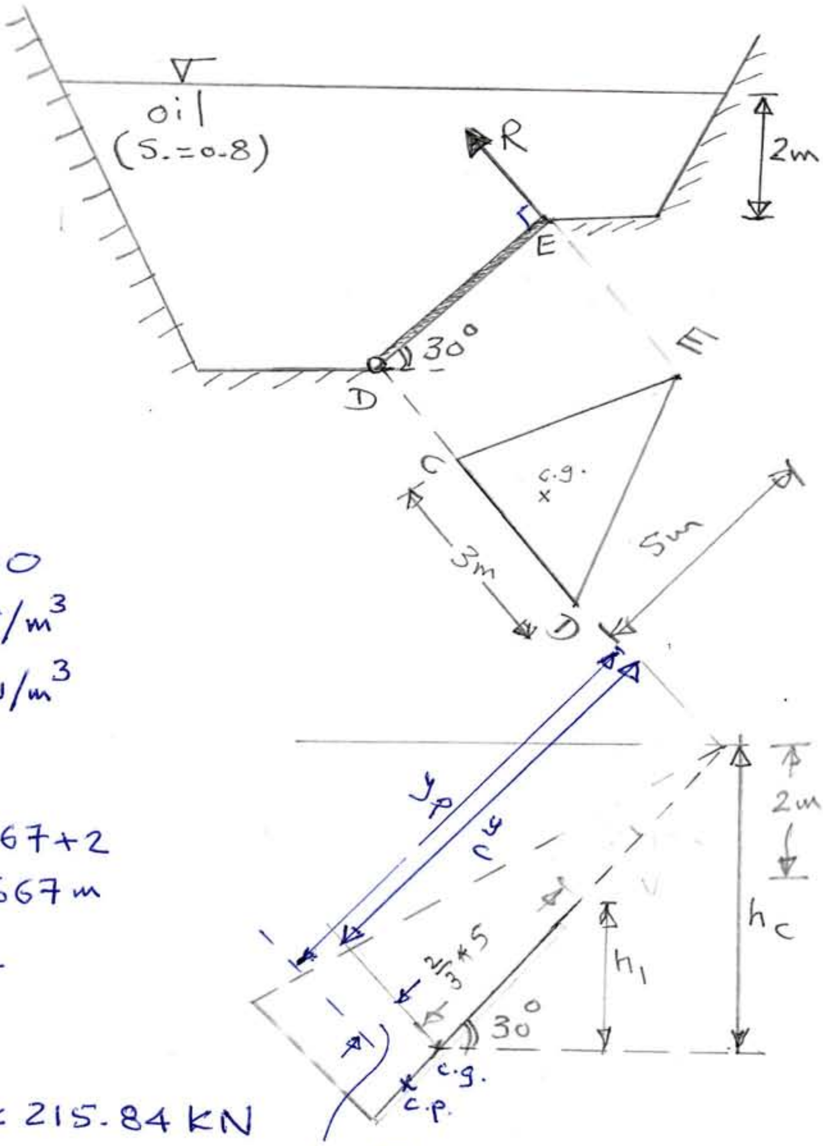
$\therefore K = \frac{1.25}{2} \sin 80^\circ$

$K = 0.616 m$

$\sin 80^\circ = \frac{h_c}{y_c}$

$\therefore y_c = \frac{h_c}{\sin 80^\circ}$

Ex.2:- The triangular gate CDE is hinged along CD and is opened by a normal force R applied at E. It holds oil (s. = 0.8) above it and is open to atmosphere on its lower side. The gate weighs 20 kN. Find (a) the magnitude of the hydrostatic force, (b) the location of pressure center, & (c) the force R needed to open the gate.



Solution:

(a) since $F = \gamma h_c A$

$\gamma_{oil} = 0.8 * \gamma_{water} = 0.8 * 9810$
$= 7848\ N/m^3$
$= 7.848\ KN/m^3$

$h_c = h_1 + 2$

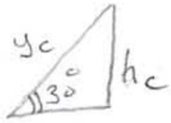
$h_c = \frac{2}{3} * 5 * \sin 30^\circ + 2 = 1.667 + 2$
$= 3.667\ m$

$A = \frac{1}{2} bh = \frac{1}{2} * 3 * 5 = 7.5\ m^2$

$\therefore F = 7.848 * 3.667 * 7.5 = 215.84\ KN$

$y_p - y_c = \dfrac{I_c}{y_c \cdot A}$

(b) $y_p = y_c + \dfrac{I_c}{y_c \cdot A}$

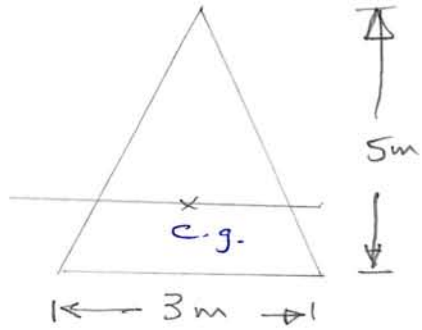


$y_c = \dfrac{h_c}{\sin 30^\circ} = \dfrac{3.667}{\sin 30^\circ}$

$\sin 30^\circ = \dfrac{h_c}{y_c}$

$\therefore y_c = 7.334\ m$

$$I_c = \frac{bh^3}{36}$$



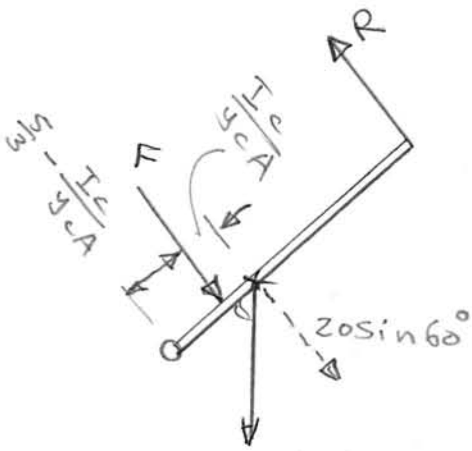
$$\therefore I_c = \frac{3(5)^3}{36} = 10.417 \text{ m}^4$$

$$\therefore \frac{I_c}{y_c A} = \frac{10.417}{7.334 * 7.5} = 0.189 \text{ m}$$

$$\therefore y_p = 7.334 + \qquad = 7.523 \text{ m}$$

ⓒ $\Sigma M_{CD} = 0.$

$$R * 5 = F * \left(\frac{5}{3} - \frac{I_c}{y_c A}\right) + 20 \sin 60^\circ * \frac{5}{3}$$



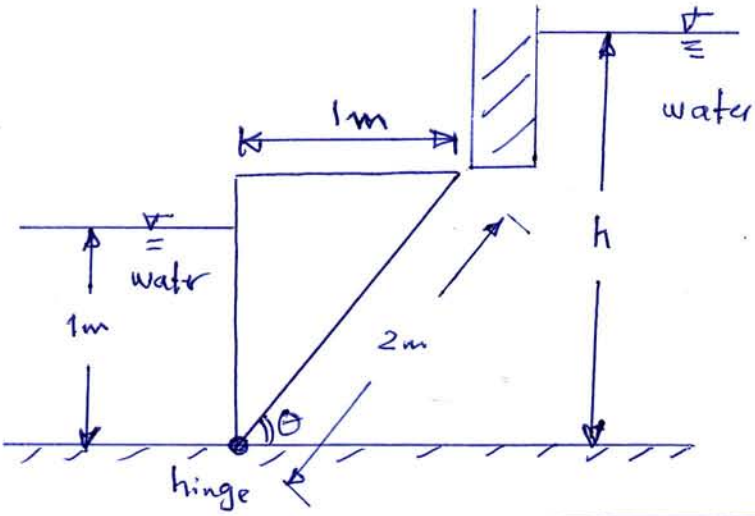
$$5R = 215.84\left(\frac{5}{3} - 0.189\right) + 28.87$$

$$5R = 318.94 + 28.87$$

$$\therefore R = 69.56 \text{ KN}$$

F.B.D of the gate

Ex.3: How long will the water on the right (h) has to rise to open the gate shown below. The gate is 2m wide, and is constructed of material with S. = 4.5.



Solution:

For F1

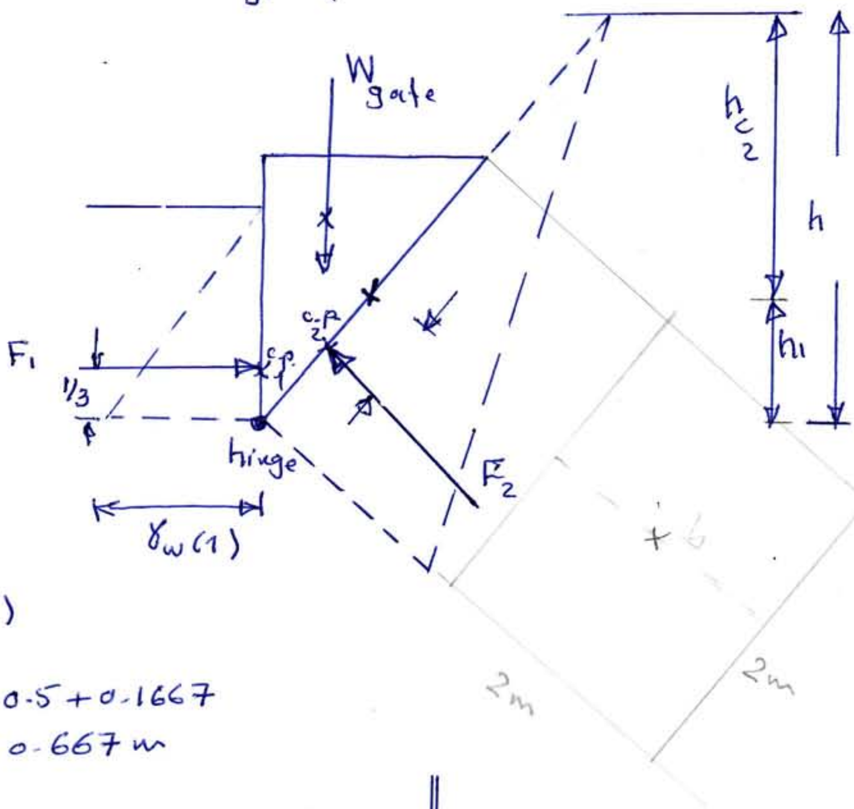
By using press. dist. diagram

$F_1 = \frac{1}{2}(\text{base}) * (\text{height}) * b$

$\text{base} = \gamma_w(1) = 9.81 \text{ KN}$

$F_1 = \frac{1}{2} * 9.81 * 1 * 2 = 9.81 \text{ KN}$

$y_p = \frac{2}{3} * 1 = 0.667 \text{ m}$



H.W. use $F_1 = \gamma h_{c_1} A_1$

$h_{c_1} = \frac{1}{2}$

$A_1 = 1 * 2$

$F_1 = 9.81 * \frac{1}{2} * 2 = 9.81 \text{ KN}$

$y_{p_1} = y_{c_1} + \frac{I_{c_1}}{y_{c_1} A_1} = 0.5 + \frac{\frac{2 * 1^3}{12}}{0.5(1*2)}$

$= 0.5 + \frac{2}{12} = 0.5 + 0.1667$
$= 0.667 \text{ m}$

For F2: $F_2 = \gamma_w h_{c_2} A_2$

$F_2 = 9.81 * h_{c_2} * (2*2) = 39.24 h_{c_2}$ (KN)

$y_{p_2} = y_{c_2} + \frac{I_{c_2}}{y_{c_2} A_2} \Rightarrow$

$\therefore \frac{I_{c_2}}{y_{c_2} A_2} = \frac{2(2)^3/12}{1.155 h_{c_2}(2*2)} = \frac{0.288}{h_{c_2}}$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

$\sin 60^\circ = \frac{h_1}{1}$

$\therefore h_1 = 0.886 \text{ m}$

$\sin 60^\circ = \frac{h_{c_2}}{y_{c_2}}$

$\therefore y_{c_2} = \frac{h_{c_2}}{\sin 60^\circ}$

$\therefore y_{c_2} = 1.155 h_{c_2}$

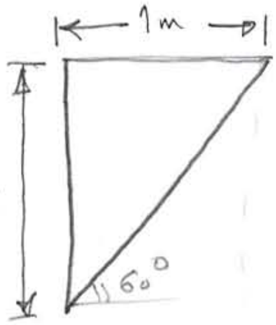
$W = m.g \; ; \quad \rho = \frac{m}{\forall} \Rightarrow m = \rho \forall$

$\therefore \; W = \rho g \forall = \gamma \forall$

$W_{gate} = S \gamma_w * \forall = 4.5 * 9.81 * \forall$

$\forall = \frac{1}{2} M * 1 * 2 = 1.732 \; m^3$

$W_{gate} = 4.5 * 9.81 * 1.732 = 76.46 \; KN$



$\tan 60^\circ = \frac{M}{1}$

$M = \tan 60^\circ$

$M = 1.732 m$

$\Sigma M_{hinge} = 0.$

$F_2 * \left[1 - (y_{p_2} - y_{c_2})\right] = F_1 * \frac{1}{3} + W_{gate} * \frac{1}{3}$

$39.24 h_{c_2} \left[1 - \frac{0.288}{h_{c_2}}\right] = \frac{9.81}{3} + \frac{76.46}{3}$

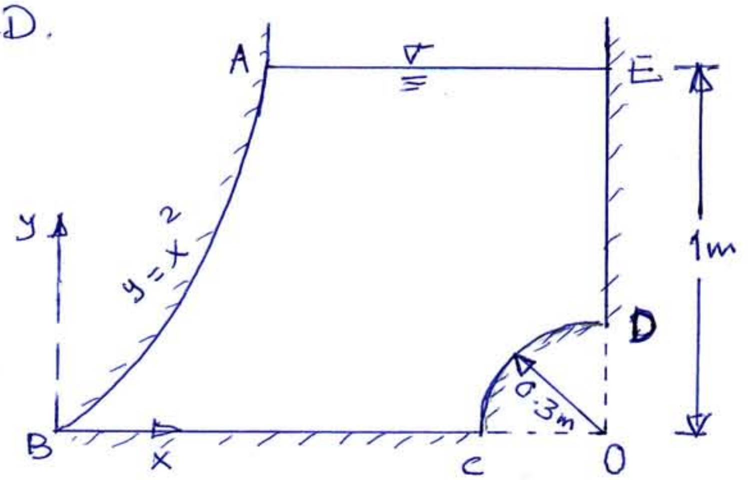
$39.24 h_{c_2} - 11.3 = 28.756$

$\therefore h_{c_2} = 1.021 \; m$

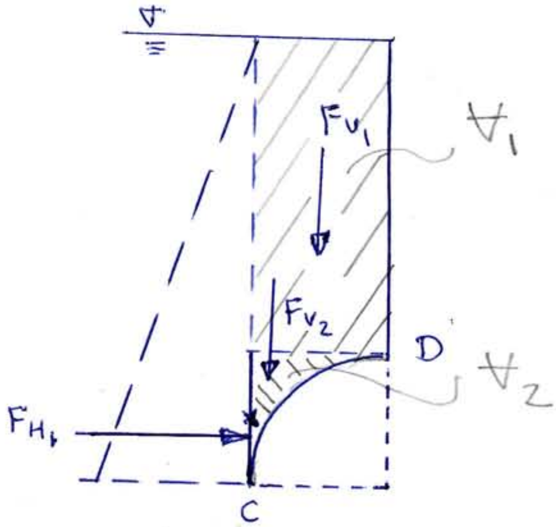
since $h = h_1 + h_{c_2}$

$\therefore \; h = 0.886 + 1.021 = 1.89 \; m$

Ex.4: A tank ABCDE contains water upto a depth of 1m and is 2m wide. The curve AB is defined by $y = x^2$ and curve CD is a quadrant of a circle of radius 0.3m. Calculate the forces on surfaces AB & CD.



Solution: Forces on surface CD:



$F_{H_1} = \gamma_w h_{c_1} A_{v_1}$

$h_{c_1} = 1 - \frac{0.3}{2} = 0.85m$

$A_{v_1} = 0.3(2) = 0.6 m^2$

$\therefore F_{H_1} = 9.81 (0.85)(0.6) = 5 KN \rightarrow$

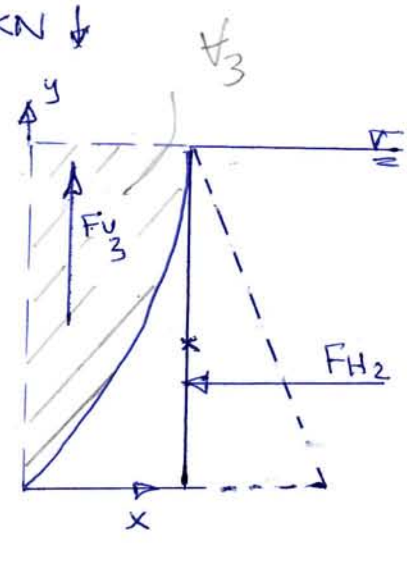
$F_v = \gamma_w \forall$

$F_{v_1} = \gamma_w \forall_1 = 9.81(0.3 * 0.7 * 2)$
$= 4.12 KN \downarrow$

$F_{v_2} = \gamma_w \forall_2 = 9.81 \left[(0.3)^2 - \frac{\pi}{4}(0.3)^2\right] * 2 = 0.38 KN \downarrow$

$\therefore F_V = F_{v_1} + F_{v_2} = 4.5 KN \downarrow$

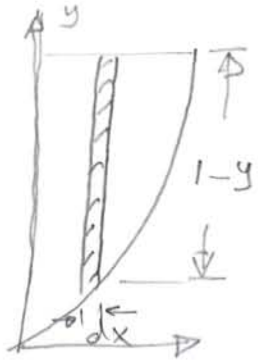
Forces of Surface AB



$F_{H_2} = \gamma_w h_{c_2} A_{v_2} = 9.81 * 0.5 * (1 * 2)$
$= 9.81 KN \leftarrow$

$F_{v_3} = \gamma_w \forall_3$

$\forall_3 = A_3 * 2$



$$A_3 = \int_a^b (1-y)\,dx$$

at $y=0 \Rightarrow x=0$.

at $y=1 \Rightarrow x=\pm 1 \Rightarrow x=1$ only according to the sketch

$$A_3 = \int_0^1 (1-x^2)\,dx = x\Big|_0^1 - \frac{1}{3}x^3\Big|_0^1 = 1-\left(\frac{1}{3}\right) = \frac{2}{3}\ m^2$$

$$\therefore \forall_3 = \frac{2}{3}*2 = \frac{4}{3}\ m^3$$

$$\therefore F_{v_3} = 9.81 * \frac{4}{3} = 13.1\ KN \uparrow$$

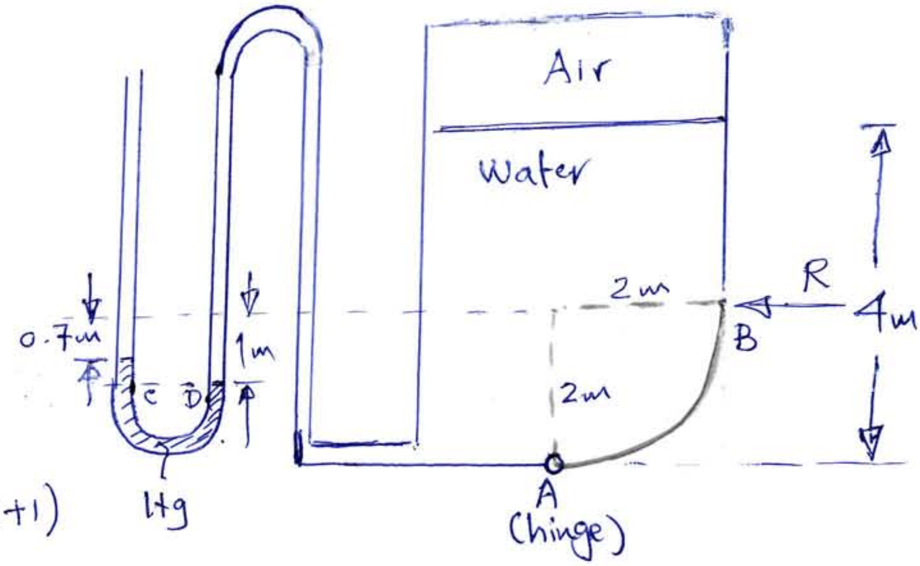
H.W.: prove that the resultant of forces acting on surface CD must pass through point O.

Ex.5: Calculate the force R required to hold the gate AB in a closed position. The gate width is 3 m. Neglect the weight of the gate.

Solution:

From the manometer;

$$P_C = P_D$$



$$13.6\gamma_w(0.3) = P_{air} + \gamma_w(2+1)$$

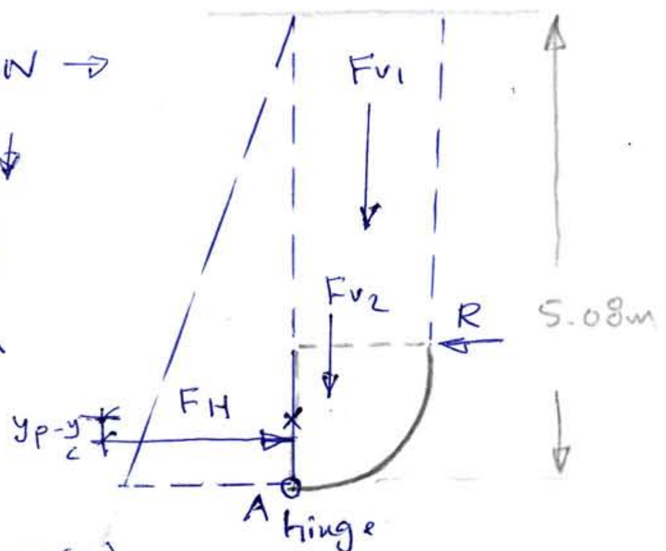
$$\therefore P_{air} = 1.08\gamma_w = h_{w*} * \gamma_w$$

$$\therefore h_w = 1.08\,m$$

$$F_H = \gamma_w(5.08-1)(2*3) = 240.15\ KN \rightarrow$$

$$F_{v_1} = \gamma_w(2*3.08*3) = 181.28\ KN \downarrow$$

$$F_{v_2} = \gamma_w\left(\frac{\pi}{4}(2)^2*3\right) = 92.46\ KN \downarrow$$



$$y_p - y_c = \frac{I_c}{y_c A} = \frac{3(2)^3/12}{4.08(2*3)} = 0.082\,m$$

$$\Sigma M_{hinge} = 0.$$

$$F_H*(1-0.082) + F_{v_1}*1 + F_{v_2}*\frac{4(2)}{3\pi}$$

$$-R*2 = 0.$$

$$240.15(0.918) + 181.28*1 + 92.46*0.85 = 2R$$

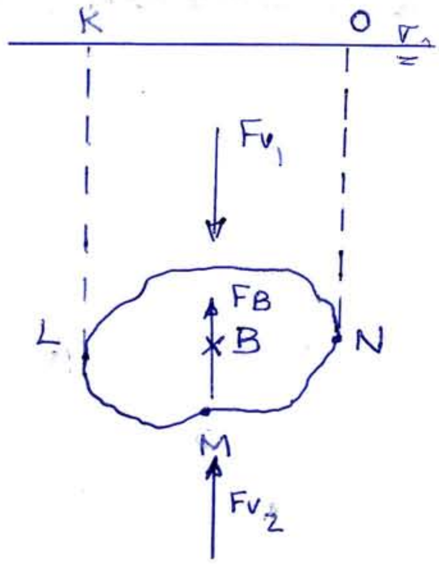
$$\therefore R = 240.16\ KN$$

Bouyancy

Theory: Archimedes's principles states that the buoyant force has a magnitude equal to the weight of fluid displaced by the body and is directed vertically upwards.

The buoyant force (F_B) passes through the center of buoyancy (B).

Submerged Body.



$F_{v_2} > F_{v_1}$ because pressure increase with depth

$F_{v_2} = \gamma \forall_2$

$F_{v_1} = \gamma \forall_1$

where, γ = specific weight of the fluid.

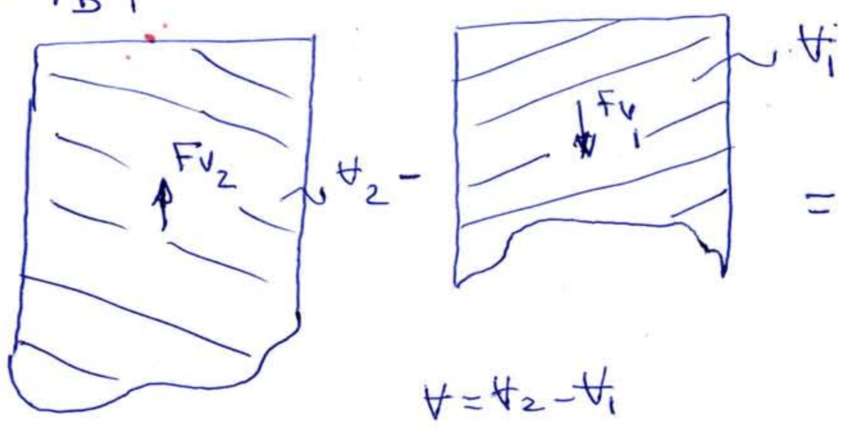
$\forall_1 = \forall_{KLMNOK}$

$\forall_2 = \forall_{KLNOK}$

$\forall_2 - \forall_1$ = volume of displaced fluid = volume of submerged body $= \forall$

$\therefore F_{v_2} - F_{v_1} = \gamma \forall = F_B \uparrow$

$\therefore \boxed{F_B = \gamma \forall} \uparrow$



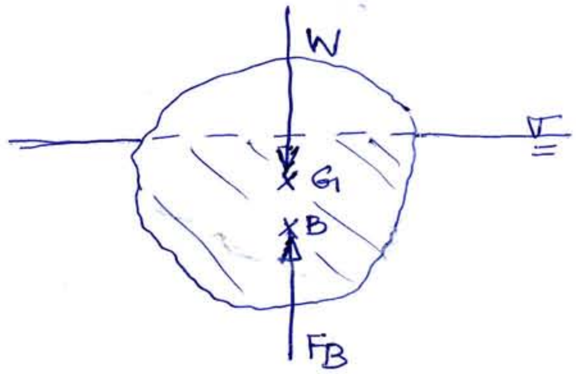
$\forall = \forall_2 - \forall_1$

$F_B = \gamma \forall$

Floating Body

$F_B = W = \gamma \forall$

where, W = weight of the body

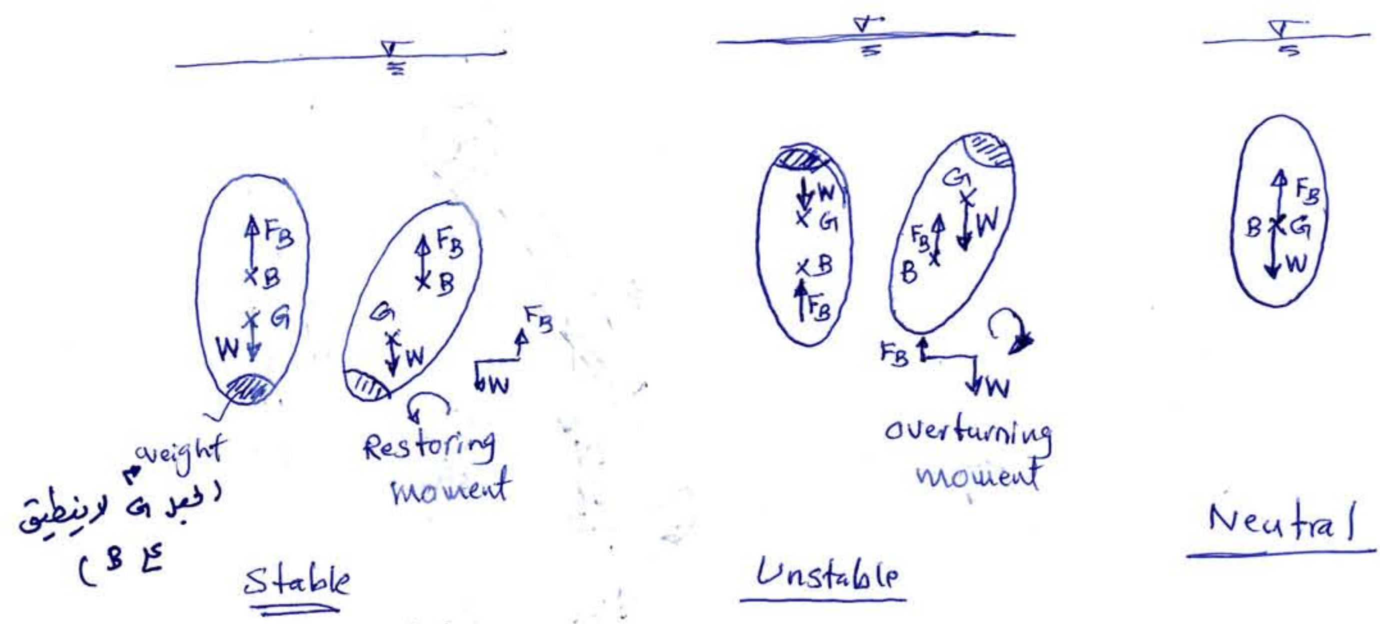


G: center of gravity

الاستقرارية
Stability of Submerged & Floating Bodies

1- Submerged Body

توازن مستقر
- Stable equilibrium : when the submerged body returns to its equilibrium condition. (G below B).
- Unstable equilibrium : The submerged body doesn't return to its equilibrium condition (G above B).
- Neutral Equilibrium : G coincide with B.

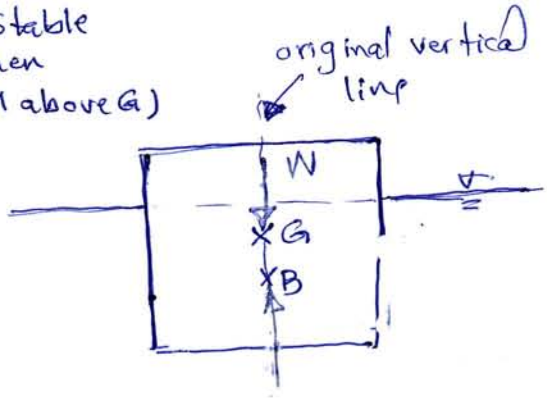


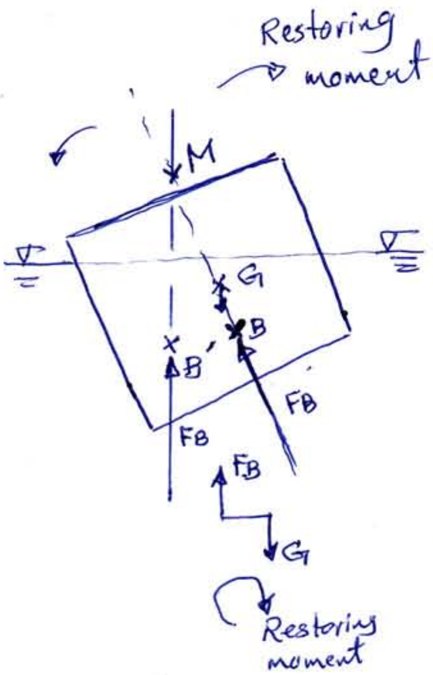
2- Floating Body

a- G below B always stable

b- When G above B :

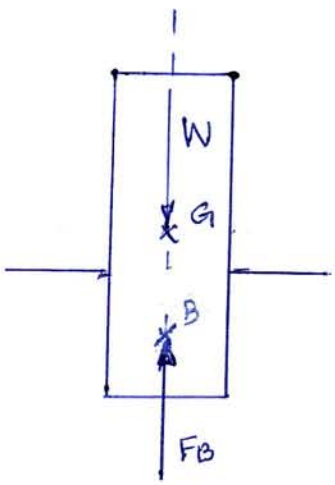
- Stable when (M above G)

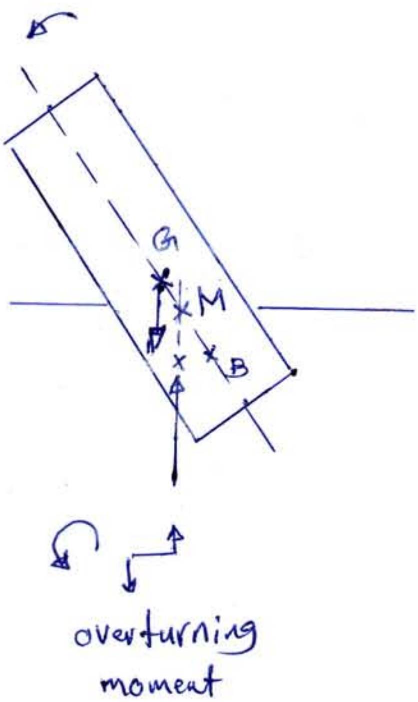




M = Metacenter
: The point at which the line of action of the buoyant force intersects the original vertical line through G.

- Unstable when (M below G)



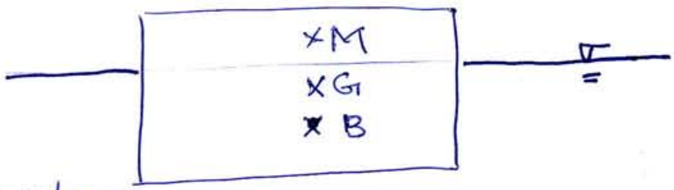


- Neutral when (M coincide with G)

To determine the position of the metacenter point (M) relative to center of buoyancy (B),

$$\overline{BM} = \frac{I}{\forall_{immersed}}$$

where, I = the smallest moment of interia of the object plane at the liquid free surface.



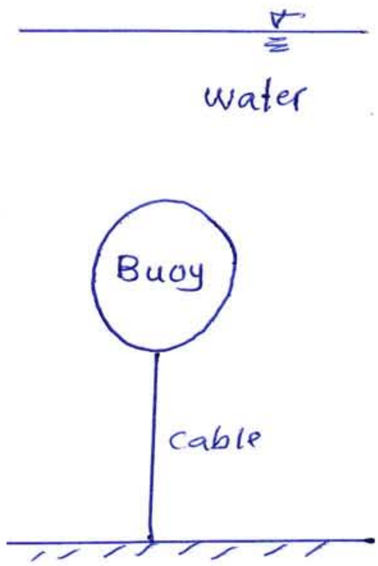
So, The object is stable when:

$\overline{BM} > \overline{BG}$,

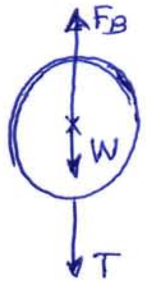
- Unstable when $\overline{BM} < \overline{BG}$
- Neutral when $\overline{BM} = \overline{BG}$.

Ex.1: A spherical buoy has a dia. of (1.5m), weighs 8.5 KN, and is attached as shown in figure below with a cable. Determine the tension force at the cable.

Note: volume of sphere $= \frac{\pi D^3}{6}$



Solution



F.B.D of the bouy

$\Sigma F_y = 0.$

$F_B - W - T = 0.$

$$T = F_B - W = \gamma_w \forall_{sphere} - W$$

$$= 9.81 * \frac{\pi (1.5)^3}{6} - 8.5$$

$$= 17.34 - 8.5 = 8.84 \text{ KN}$$

Ex.2: A rectangular box of dimension 7.6m x 3m x 4m deep floats in water. If the box weighs 40ton, determine:

1- the deep it will sink

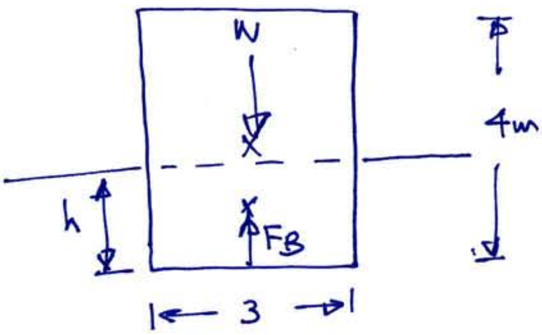
2- the mass of stone placed on the box to sink it 4m depth.

Solution :

$b = 7.6m$

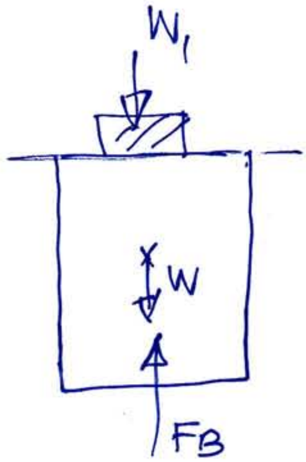
1- $F_B = W$

$$\gamma_{water} \forall_{sink} = m \cdot g$$



$$98\cancel{10} * 3(7.6)h = 40 * \cancel{10}^{3} * \cancel{9.81}$$

$$h = \frac{40}{22.8} = 1.754m$$



2- $\Sigma F_y = 0.$

$$W_1 + W = F_B$$

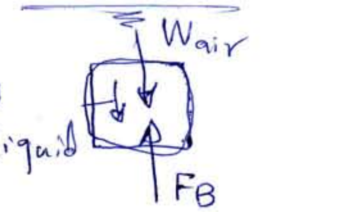
$$W_1 + 40 * 9810 = 9810(3)(4)(7.6)$$

$$W_1 = 502272 \text{ N}$$

$$\therefore m_1 \cdot g = 502272$$

$$\therefore m_1 = \frac{502272}{9.81} = 51200 \text{ kg} = 51.2 \text{ ton}$$

Ex3: An object weighs 3N in water and 4N in oil (s = 0.83). Determine its volume & specific gravity (s.).



Sol.

$$W_{water} = W_{air} - \gamma_w \forall \quad — (1)$$

$$W_{oil} = W_{air} - \gamma_{oil} \forall \quad — (2)$$

$$W_{Liquid} = W_{air} - F_B$$

From Eq (1)

$$\therefore 3 = W_{air} - 9810\forall \Rightarrow W_{air} = 3 + 9810\forall \quad — (3)$$

subs. Eq. (3) into eq. (2)

$$4 = 3 + 9810\forall - 0.83\,(9810)(\forall)$$

$$\cancel{4-3} \quad 1 = 0.17(9810)\forall$$

$$\therefore \forall = 6 * 10^{-4}\ m^3$$

From Eq. (3) $\Rightarrow W_{air} = 3 + 9810 * 6 \times 10^{-4}$

$$W_{air} = 8.886\ N$$

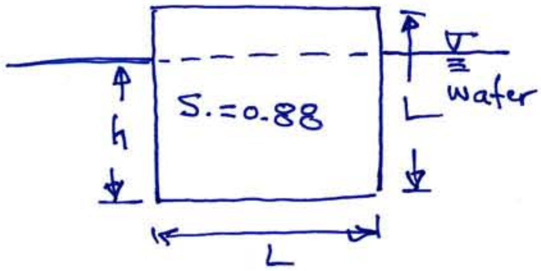
since $W = \gamma_{object} \forall = \rho_{object} * g * \forall$

$$8.886 = \rho_{object} * 9.81 * 6 * 10^{-4}$$

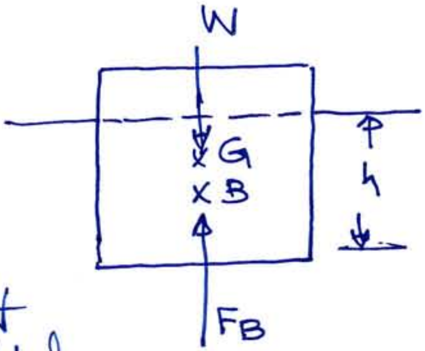
$$\rho_{obj} = 1509.7\ Kg/m^3$$

$$S. = \underline{1.51}$$

Ex.4: For the figure shown below, a cube of wood of side length (L) is float in water. If the specific gravity of the wood is 0.88. Determine if this cube is stable or not.



Sol.



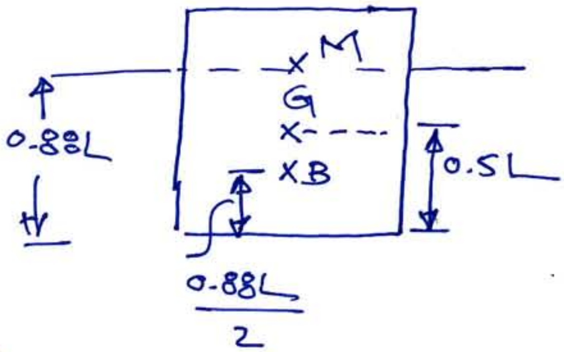
$\Sigma F_y = 0.$

$F_B = W$

$\gamma_w \, \forall_{immersed} = \gamma_{wood} * \forall_{total}$

$\cancel{\gamma_w} (\cancel{L})(\cancel{L})(h) = 0.88 \, \cancel{\gamma_w} \, L^{\cancel{3}\,2}$

$\therefore h = 0.88L$

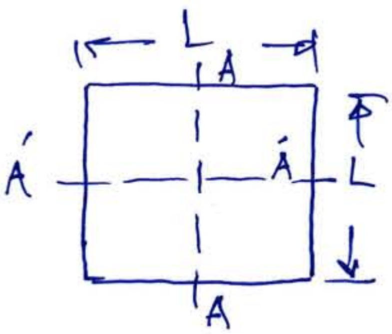


$\therefore \overline{GB} = 0.5L - \frac{0.88L}{2} = 0.06L$

since ; $\overline{BM} = \frac{I}{\forall_{immersed}}$

For the cube $I_{AA} = I_{AA'}$

So, we don't need to find the least $I \Rightarrow I = \frac{L \cdot L^3}{12} = \frac{L^4}{12}$



Top view of the cube at the water line

$\forall_{immersed} = (L)(L)(0.88L) = 0.88L^3$

$\therefore \overline{BM} = \frac{L^4}{12(0.88L^3)} \Rightarrow \overline{BM} = 0.095L$

Since $\overline{BM} > \overline{BG} \Rightarrow$ the cube is stable

Fluid Dynamics

Fluid dynamics is a study of fluids in motion, the parts of which move at different velocities are subjected to various & changing accelerations - both +ve & -ve. These acceleration occur both in the direction of motion & in the direction normal to the direction of motion.

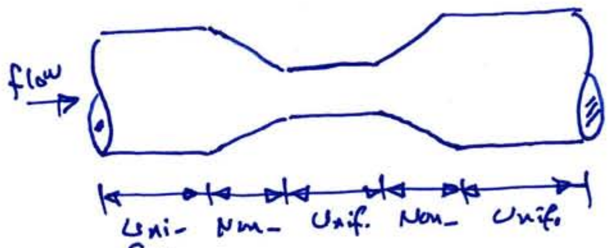
Types of flow:

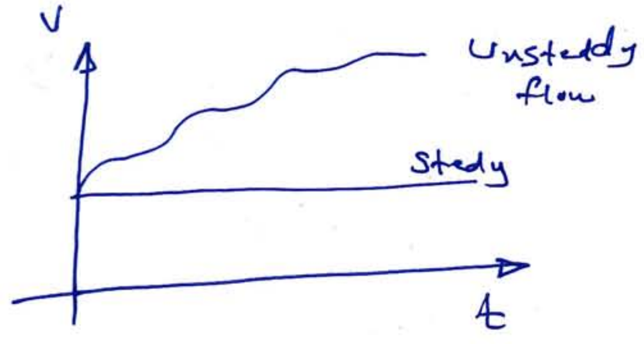
1- Steddy flow: exists if any variable at a point remains constant w.r.t. time (i.e. $\frac{\partial v}{\partial t} = 0$.).

2- Unsteady flow: exists if any variable at a point changes either in magnitude or in direction w.r.t. time (i.e. $\frac{\partial v}{\partial t} \neq 0$)

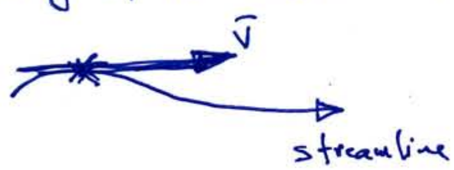
3- Uniform Flow: exists if the variable in the flow remains constant w.r.t. distance (i.e. $\frac{\partial v}{\partial s} = 0$.).

4- Non-Uniform Flow: exists if any variable in the flow changes either in magnitude or in direction w.r.t. distance (i.e. $\frac{\partial v}{\partial s} \neq 0$.)

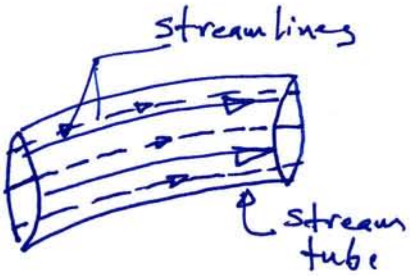




Stream line: Is an imaginary line within the flow for which the tangent at any point is the time average of the direction of motion at that point.



Stream Tube: Is an element of fluid bounded by a special group of stream lines which enclose or confine the flow



Types of Fluid:

There are two types of fluid
- → Real (viscous) fluid → $\eta \neq 0$. → viscosity $\neq 0$.
- → Perfect (Ideal) fluid → viscosity $= 0$. → $\eta = 0$.

Velocity & Acceleration:

In cartesian coordinates, $x, y \; \& \; z \Rightarrow V_x = u$, $V_y = v$, $V_z = w$

where, $u = u(x, y, z, t)$ & $w = w(x, y, z, t)$
$v = v(x, y, z, t)$

since $a = \dfrac{DV}{Dt} \Rightarrow$ there are a_x, a_y, & a_z

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t}\frac{dt}{dt} + \frac{\partial u}{\partial x}\overbrace{\frac{dx}{dt}}^{u} + \frac{\partial u}{\partial y}\overbrace{\frac{dy}{dt}}^{v} + \frac{\partial u}{\partial z}\overbrace{\frac{dz}{dt}}^{w}$$

$$\therefore a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \underbrace{\frac{\partial w}{\partial t}}_{\text{local acc.}} + \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{\text{convective acc.}}$$

In general or

position along the stream line

$$V = V(s, t)$$

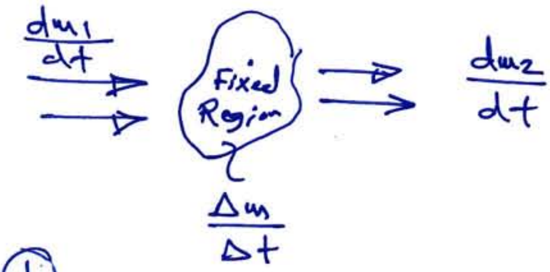
since, $a = \dfrac{DV}{Dt} \Rightarrow a = \dfrac{\partial V}{\partial s}\dfrac{ds}{dt} + \dfrac{\partial V}{\partial t}\dfrac{dt}{dt}$

$$\therefore a = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}$$

Conservation of Mass : The continuity Equation

Conservation of Mass : Rate of change of accumulating materials inside the control volume

$$\frac{dm_1}{dt} - \frac{dm_2}{dt} = \pm \frac{\Delta m}{\Delta t}$$



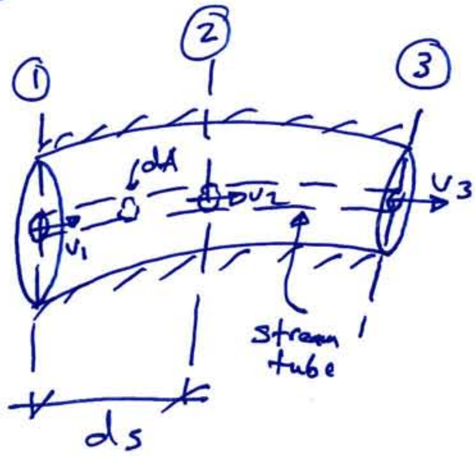
for steady flow $\Rightarrow \frac{\Delta m}{\Delta t} = 0.$

$$\therefore \frac{dm_1}{dm} = \frac{dm_2}{dt} = \frac{dm_3}{dt} = \text{constant} \quad \text{--- (1)}$$

since $$\frac{dm}{dt} = \frac{\rho\, d\forall}{dt} = \frac{\rho\, ds\, dA}{dt} = \rho\, v\, dA \quad \text{--- (2)}$$

subs. eq. (2) into (1):

$$\rho_1 v_1 dA_1 = \rho_2 v_2 dA_2 = \rho_3 v_3 dA_3 = \rho\, v\, dA \quad \text{--- (3)}$$



- For Incompressible fluid $\Rightarrow \rho_1 = \rho_2 = \rho_3 = \rho = \text{const.}$

From Eq. (3) $\Rightarrow v_1 d\ \ _1 = v_2 dA_2 = v_3 dA_3 = v\, dA = dQ$ --- (4)

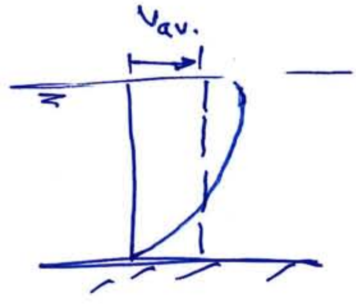
by Integration $Q = \int v \cdot dA = V \cdot A$

$$\therefore \boxed{v_1 A_1 = v_2 \qquad = v_3 A_3 = Q} \quad \text{continuity Eq.}$$

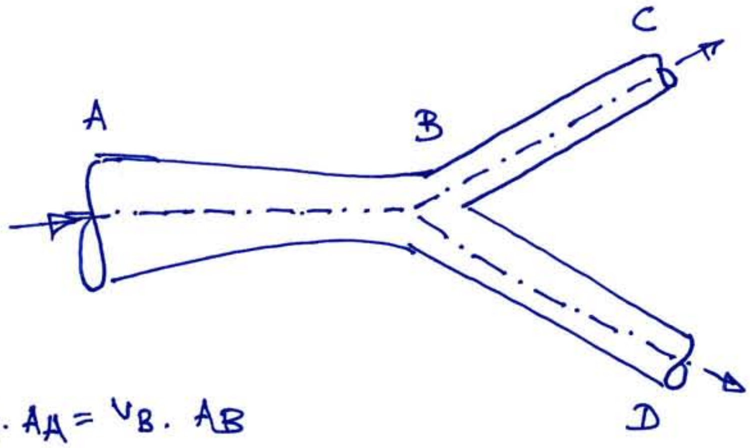
where; Q = Flowrate (discharge) $(\ \ /T^3)$

A = cross_ sectional area (L^2)

V = average velocity (L/T)



Ex: For the system shown below, If $\phi_A = 450mm$, $\phi_B = 300mm$, $\phi_c = 150mm$, $\phi_D = 225mm$, $V_A = 1.8 m/s.$, & $V_D = 3.6 m/sec.$, determine V_B & V_c.



Sol. By continuity

$$Q_A = Q_B = Q_c + Q_D$$

$$Q_A = Q_B \Rightarrow V_A \cdot A_A = V_B \cdot A_B$$

$$1.8 * \frac{\pi}{4}(0.45)^2 = V_B * \frac{\pi}{4}(0.3)^2$$

$$\therefore V_B = 4.05 \ m/s.$$

$$Q_B = Q_C + Q_D$$

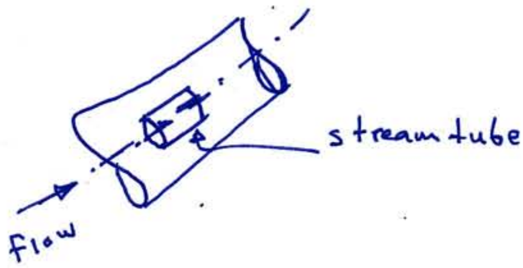
$$V_B A_B = V_c A_c + V_D A_D$$

$$4.05 * \frac{\pi}{4}(0.3)^2 = V_c * \frac{\pi}{4}(0.15)^2 + 3.6 * \frac{\pi}{4}(0.225)^2$$

$$\therefore V_c = 8.09 \ m/s$$

Equations of Fluid Motion

Euler's Equation

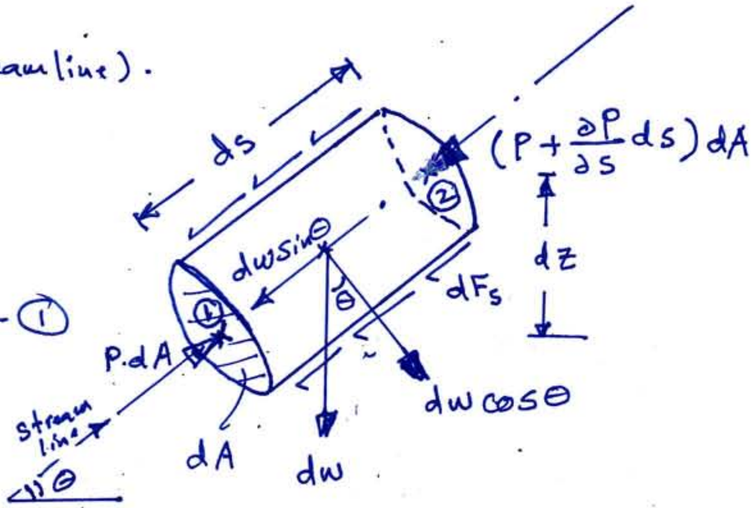


For the forces acting on the stream tube shown below & by applying Newton's second law:

$\sum \vec{F} = m \cdot \vec{a}$ (along the streamline).

$\therefore \; P \cdot dA - \left(P + \frac{\partial P}{\partial s} ds\right) dA$

$- dw \sin\theta - dF_s = \rho \, ds \, dA \, \vec{a}$ — (1)



where;

dF_s = shear force (N)

dw = weight of stream tube $= \gamma V = \rho g V$

dA = cross-sectional area of stream tube.

ds = length of stream tube.

For one-dimensional (1D) & steady flow ($\frac{\partial(\text{any variable})}{\partial t} = 0.$).

1D flow: $\partial P = dP$; $\partial s = ds$ (any $\partial \to d$) — (2)

Steady flow: since $a = \frac{dv}{dt}$; $v = v(\underset{\text{space}}{s}, \underset{\text{time}}{t})$

in general ; $dv = \frac{\partial v}{\partial s} ds + \underbrace{\frac{\partial v}{\partial t}}_{\substack{\text{steady flow} \\ =0}} dt \Rightarrow dv = \frac{\partial v}{\partial s} ds \overset{1D}{\Rightarrow} dv = \frac{dv}{ds} ds$

$\therefore \; a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v \cdot \frac{dv}{ds}$ — (3)

By substituting eqs. (2) & (3) into eq. (1):

$$-dP\,dA - \rho g\,dA\,ds\cdot\sin\theta - dF_s = \rho\,ds\,dA\,V\cdot\frac{dV}{ds} \quad — (4)$$

Dividing eq. (4) by $\rho g\,dA$:

$$-\frac{dP}{\rho g} - ds\,\sin\theta - \frac{dF_s}{\rho g\,dA} = \frac{V\cdot dV}{g}$$

$$\therefore \frac{dP}{\rho g} + dZ + \frac{V\cdot dV}{g} + \frac{dF_s}{\rho g\,dA} = 0. \quad — (5)$$

Term; $\dfrac{dF_s}{\rho g\,dA} = \dfrac{\tau\,.\,dP_w\,.\,ds}{\rho g\,dA} = dh_L$

where; τ = shear stress

dP_w = wetted perimeter = $2\pi r$

dh_L = head loss

$\therefore$ Eq. (5) becomes;

$$\boxed{\frac{dP}{\rho g} + dZ + \frac{V\cdot dV}{g} + dh_L = 0.}$$

Euler's Equation.

- For incompressible fluid $\Rightarrow$ ρ const.

$$\therefore \int_{P_1}^{P_2}\frac{dP}{\rho g} + \int_{Z_1}^{Z_2} dZ + \int_{V_1}^{V_2}\frac{V\,dV}{g} + \int_1^2 dh_L = 0.$$

$$\therefore \left(\frac{P_2}{\rho g} - \frac{P_1}{\rho g}\right) + (Z_2 - Z_1) + \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) + (h_{L_2} - h_{L_1}) = 0. \quad — (6)$$

let $h_{L_2} - h_{L_1} = H_L$

From eq. ⑥ :

$$\boxed{\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + H_L}$$ Energy Eq.

- For Ideal Fluid (Non-viscous) $\Rightarrow \eta = 0 \Rightarrow HL = 0$.

From Energy Eq.

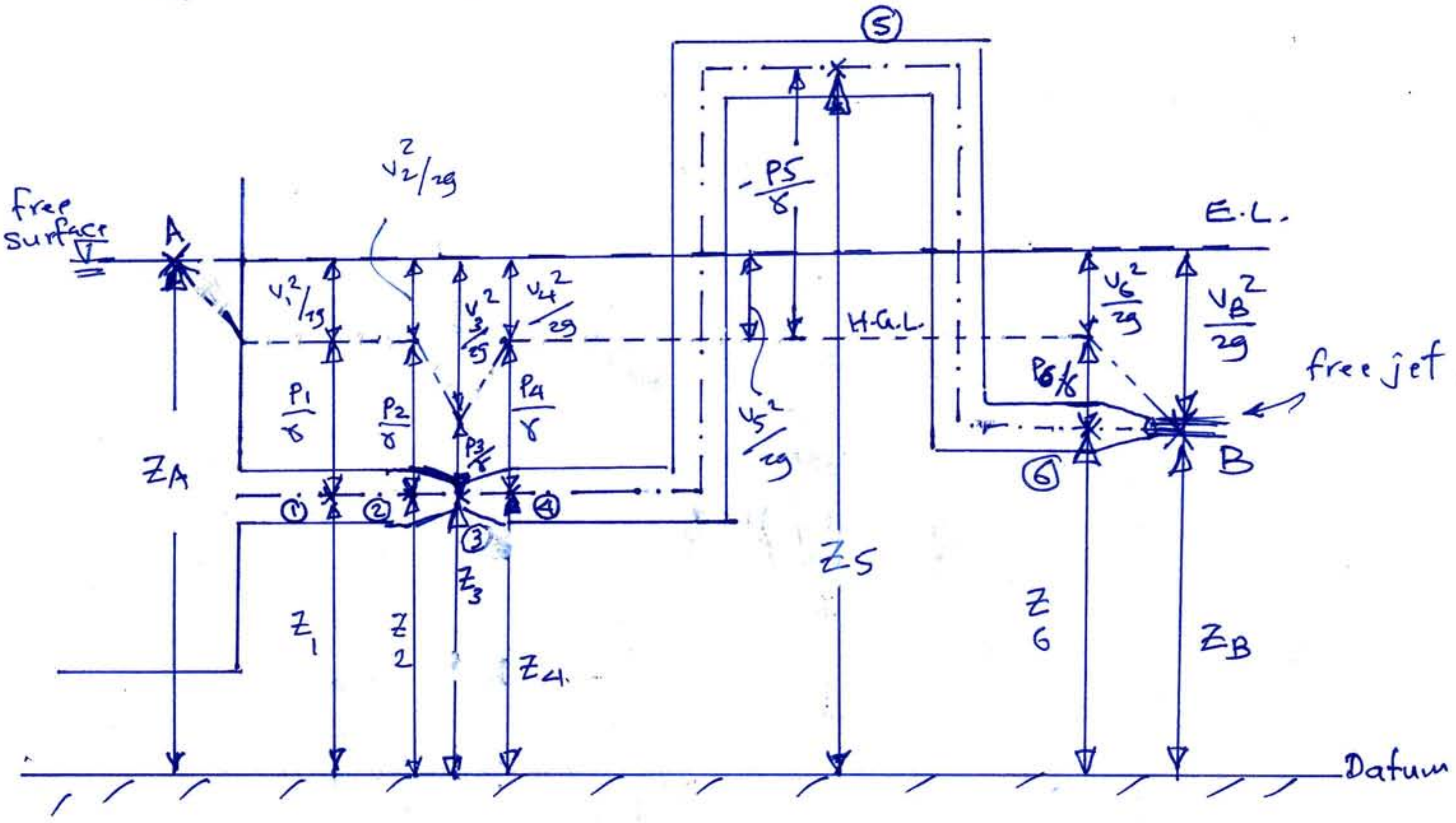
$$\therefore \boxed{\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} = H = \text{constant}}$$

Bernoulli's Eq.

<u>Energy Line (E.L.) & Hydraulic Grade Line (H.G.L.)</u>

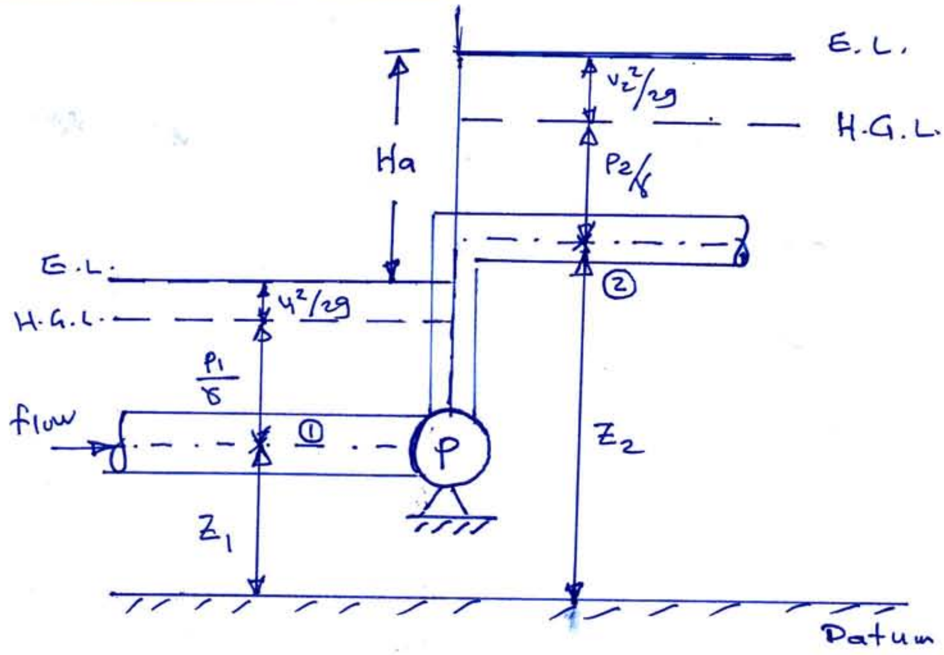
$E.L. = \frac{P}{\gamma} + Z + \frac{V^2}{2g}$; H.G.L. or piezometric line $= \frac{P}{\gamma} + Z$

(Total Energy) (potential Energy)



Hydraulic Systems with Pump & Turbine

with Pump



For Ideal fluid;

By applying Bern. Eq. between points ① & ②:

$$\frac{P_1}{\gamma} + Z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

For Real fluid; By applying Energy Eq. between points ① & ②:

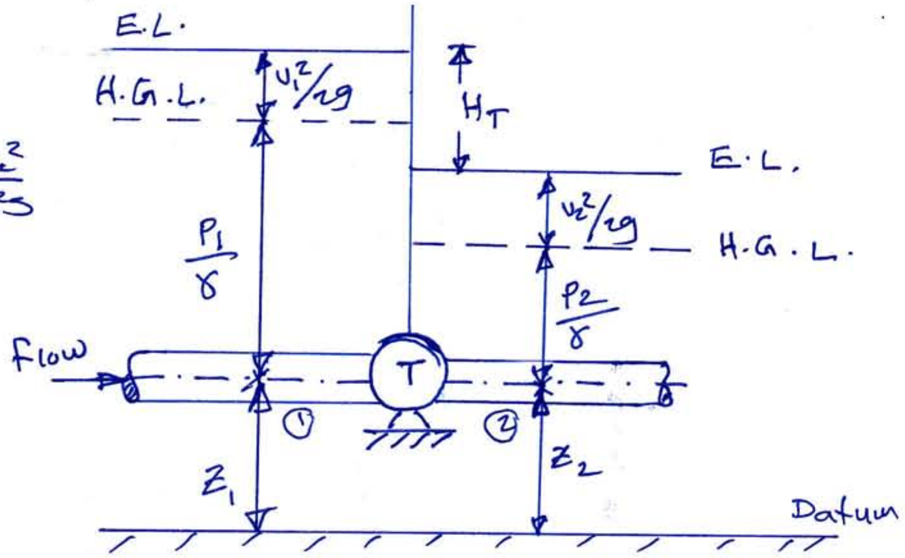
$$\frac{P_1}{\gamma} + Z_1 + \frac{v_1^2}{2g} + Ha = \frac{P_2}{\gamma} + Z_2 + \frac{v_2^2}{2g} + h_{L_{1-2}}$$

with Turbine

For Ideal flow

By applying Bern. Eq. between ① & ②:

$$\frac{P_1}{\gamma} + Z_1 + \frac{v_1^2}{2g} - H_T = \frac{P_2}{\gamma} + Z_2 + \frac{v_2^2}{2g}$$



For Real flow:

By applying Energy Eq. between ① & ②:

$$\frac{P_1}{\gamma} + Z_1 + \frac{v_1^2}{2g} - H_T = \frac{P_2}{\gamma} + Z_2 + \frac{v_2^2}{2g} + h_{L_{1-2}}$$

Power:

Power (P) = Energy per unit time

$$P = \frac{E}{T} \quad -(1) \quad (J/s.) \text{ or Watt or horsepower (hp)}$$

Head (H) = Energy per unit weight

$$H = \frac{E}{W} \quad -(2)$$

From eq. (2) $\Rightarrow E = H.W \quad -(3)$

subs. eq. (3) into eq. (1):

$$P = H.\frac{W}{T} \quad -(4)$$

Since $W = \gamma \forall \Rightarrow \forall = \frac{W}{\gamma} \quad -(5)$

& $Q = \frac{\forall}{T} \Rightarrow \forall = QT \quad -(6)$

From eqs. (5) & (6) $\Rightarrow \frac{W}{\gamma} = QT \Rightarrow \frac{W}{T} = \gamma Q \quad -(7)$

subs. eq. (7) into eq. (4)

$$\therefore \boxed{P = \gamma Q H}$$

where P = power $(\frac{J}{s.})$ or Watt or horsepower (hp)

Note: 1 hp = 746 Watt

Q = flowrate ($m^3/s.$)

γ = specific weight of the liquid (N/m^3)

H = head (m).

power of press. $\Rightarrow \gamma Q \frac{P}{\gamma} = QP$

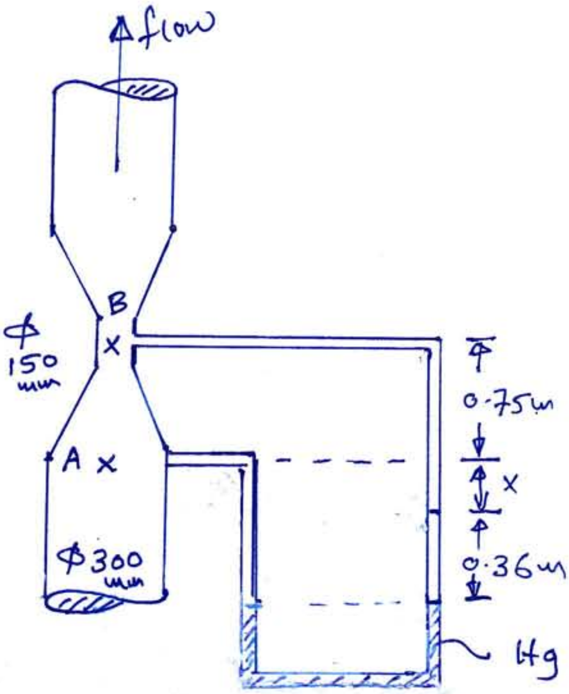
" " velocity $\Rightarrow \gamma Q \frac{v^2}{2g} = Q \frac{\rho v^2}{2}$

" " elevation $\Rightarrow \gamma Q Z$

power lost due to friction (dissipation power) $= \gamma Q h_L$

Examples

1- For the Venturi meter shown in figure below, the deflection of the mercury in the differential gauge is 0.36m. Determine the flow of water through the meter if no energy is lost between A & B.



since there is no energy

Sol.: By applying Bern. Eq. between A & B: (Take datum at A).

$$\frac{P_A}{\gamma_w} + \overset{=0}{Z_A} + \frac{V_A^2}{2g} = \frac{P_B}{\gamma_w} + \overset{=0.75}{Z_B} + \frac{v_B^2}{2g} \quad -(1)$$

From the manometer (or differential gauge):

$P_C = P_D$

$$P_A + \cancel{\gamma_w X} + \gamma_w (0.36) = P_B + \gamma_w (0.75) + \cancel{\gamma_w X} + 13.6\,\gamma_w (0.36) \quad -(2)$$

Divided Eq. (2) by γ_w

$$\frac{P_A}{\gamma_w} + 0.36 = \frac{P_B}{\gamma_w} + 0.75 + 13.6(0.36)$$

$$\therefore \frac{P_A}{\gamma_w} = \frac{P_B}{\gamma_w} + 5.286 \quad — (3)$$

By continuity eq. $\Rightarrow Q_A = Q_B$

$$A_A \cdot V_A = A_B \cdot V_B$$

$$\therefore V_A = \frac{A_B \cdot V_B}{A_A} = \frac{\pi/4\,(0.15)^2\,V_B}{\pi/4\,(0.3)^2}$$

$$\therefore V_A = 0.25\,V_B \quad - (4)$$

Subs. eqs. (3) & (4) into eq. (1):

$$\cancel{\frac{P_B}{\gamma_w}} + 5.286 + \frac{(0.25\,V_B)^2}{2g} = \cancel{\frac{P_B}{\gamma_w}} + 0.75 + \frac{V_B^2}{2g}$$

$$\therefore V_B = 9.74 \text{ m/s.}$$

$$\therefore Q_B = Q_A = \frac{\pi}{4}(0.15)^2\,(9.74) = 0.17 \text{ m}^3/\text{s.}$$

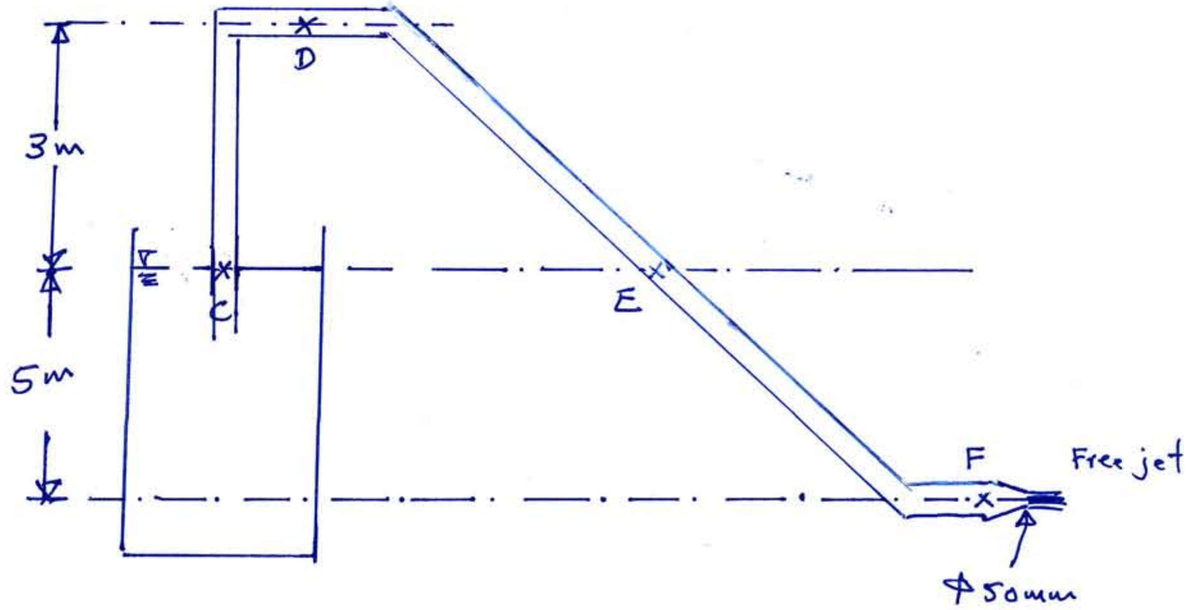
2 - For the siphone shown in figure below, if its diameter is 100mm, determine:

i - the outlet flow.

ii - the pressures at points C, D, E, & F.

iii - plot H.G.L. & E.L.

<u>Note</u>: Assume no energy lost.



Sol. : (1) For no energy lost, Applying Bern. eq. between points (A) & (G) : (Take datum at G).

$$\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_G}{\gamma} + Z_G + \frac{V_G^2}{2g}$$

$$0 + 5 + 0 = 0 + 0 + \frac{V_G^2}{2g}$$

$\therefore V_G = 9.9$ m/s.

$\therefore Q = \frac{\pi}{4}(0.05)^2(9.9) = 0.019 \ m^3/s$

(2) By continuity eq. $\Rightarrow Q_C = Q_D = Q_E = Q_F = Q = 0.019 m^3/s$.

for const. cross-sectional area at C, D, E, & F

$\therefore V_C = V_D = V_E = V_F = \frac{0.019}{\frac{\pi}{4}(0.1)^2} = 2.42$ m/s.

Bern. between (A) & (C): $\frac{P_A}{\gamma} + Z_A + \frac{V_A^2}{2g} = \frac{P_c}{\gamma} + Z_c + \frac{V_c^2}{2g}$

Take datum at (A)

$$0 + 0 + 0 = \frac{P_c}{\gamma} + 0 + \frac{(2.42)^2}{2g}$$

$\therefore \frac{P_c}{\gamma} = -0.298 \Rightarrow P_C = -2928.2$ Pa. (vacuum).

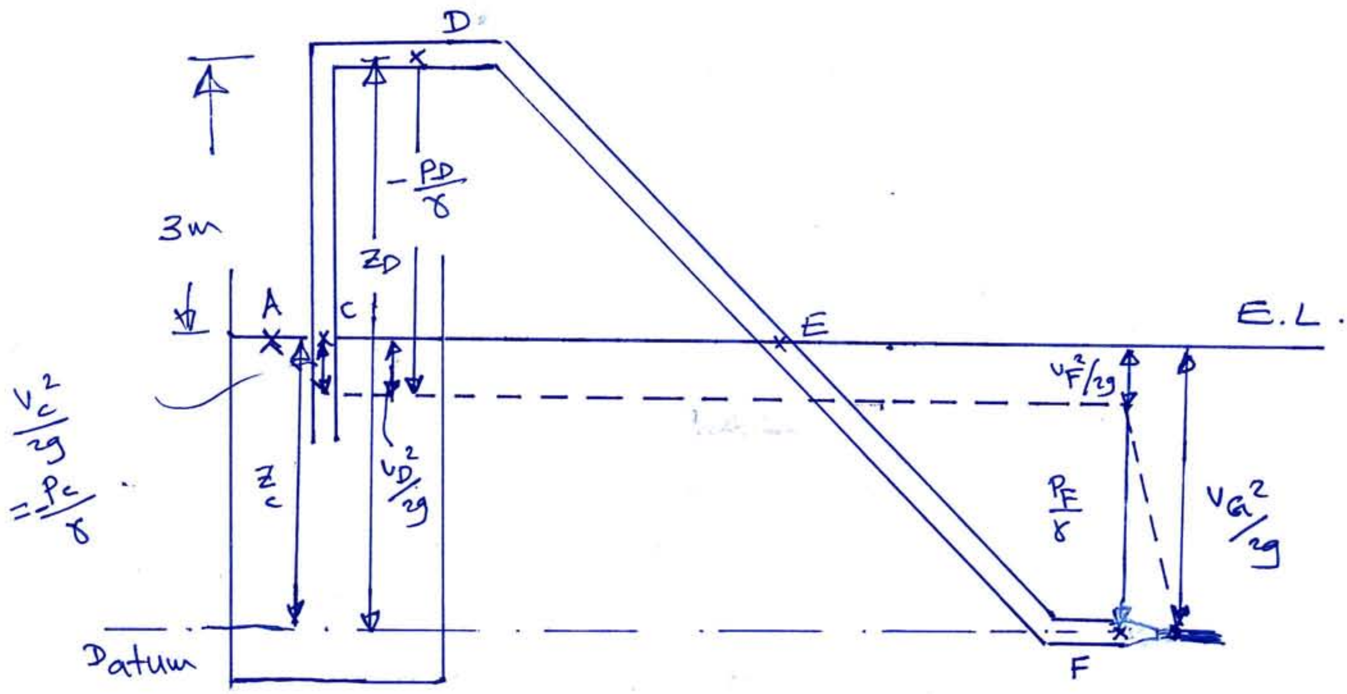
or $\frac{V_c^2}{2g} = -\frac{P_c}{\gamma}$

H.W.

Similarily $\Rightarrow$ $P_D = -32358$ Pa

$P_E = -2928$ Pa.

$P_F = 46122$ Pa.

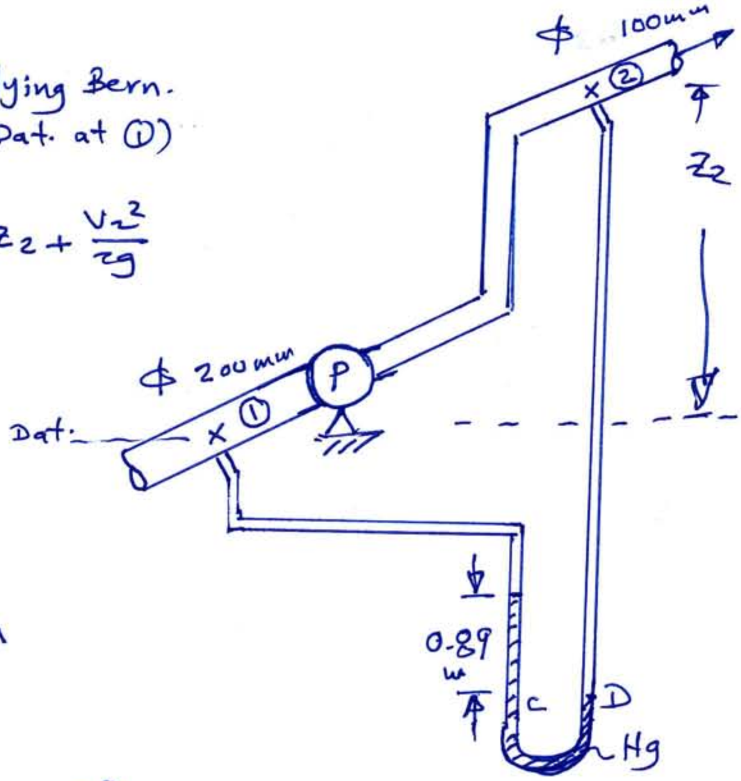


From the sketch $\frac{V_c^2}{2g} = -\frac{P_c}{\gamma}$

$$-\frac{P_D}{\gamma} = 3 + \frac{V_D^2}{2g}$$

$$\frac{P_F}{\gamma} = 5 - \frac{V_F^2}{2g}$$

3 _ For the figure shown below, Calculate the flowrate of water delivered by a pump which add 12hp. Assume frictionless flow.



<u>Sol.</u>: For frictionless flow, applying Bern. Eq. between ① & ②: (Dat. at ①)

$$\frac{P_1}{\gamma} + \overset{0}{Z_1} + \frac{V_1^2}{2g} + H_a = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$\therefore \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + H_a = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} \qquad — ①$$

<u>for pump</u>

since $P = \gamma Q H_a$

$$12 * 746 = 9810\ Q\ H_a$$

$$\therefore\ H_a = \frac{0.912}{Q} \quad — \quad (2)$$

Since $Q_1 = Q_2 = Q \Rightarrow$ $$v_1 = \frac{Q}{\frac{\pi}{4}(0.2)^2} = 31.83\,Q \quad — \quad (3)$$

$$v_2 = \frac{Q}{\frac{\pi}{4}(0.1)^2} = 127.32\,Q \quad — \quad (4)$$

From the manometer; $P_C = P_D$

$$P_1 + 13.6\,\gamma_w(0.89) = P_2 + \gamma_w Z_2 + \gamma_w(0.89)$$

Divided by γ_w

$$\frac{P_1}{\gamma_w} + 13.6(0.89) = \frac{P_2}{\gamma_w} + Z_2 + 0.89$$

$$\therefore\ \frac{P_2}{\gamma_w} + Z_2 = \frac{P_1}{\gamma_w} + 11.214 \quad — \quad (5)$$

Subs. eqs. (2), (3), (4), & (5) into eq. (1)

$$\cancel{\frac{P_1}{\gamma}} + \frac{(31.83Q)^2}{2g} + \frac{0.912}{Q} = \cancel{\frac{P_1}{\gamma}} + 11.214 + \frac{(127.32Q)^2}{2g}$$

$$\therefore\ 826.217\,Q^3 - 51.64\,Q^3 + 11.214\,Q - 0.912 = 0.$$

$$774.577\,Q^3 + 11.214\,Q - 0.912 = 0.$$

By trial & error $\Rightarrow Q = 0.0814\ m^3/s.$