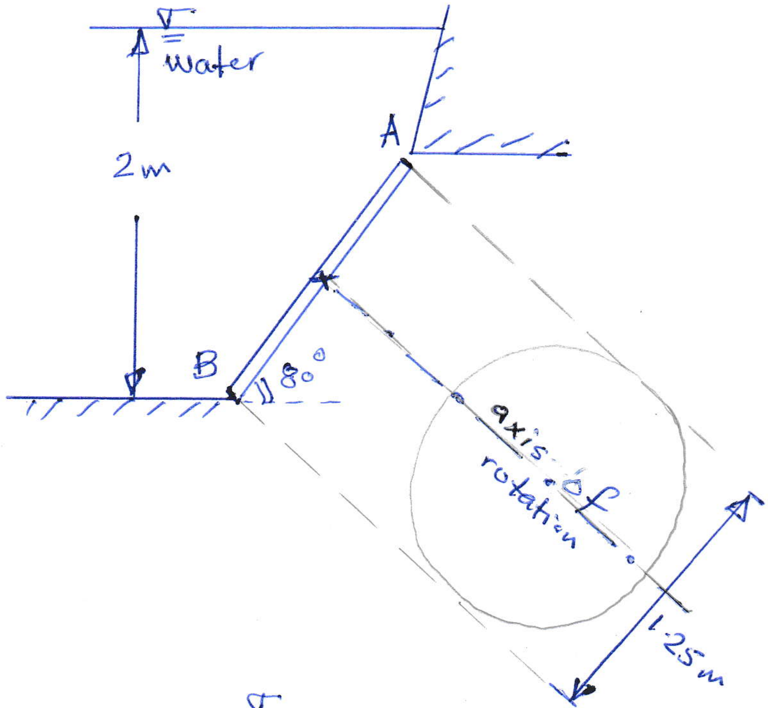
Ex.1: For the gate (AB) shown in the figure below, calculate:

1- Hydrostatic force on the gate.

2- Turning moment about the axis of rotation.



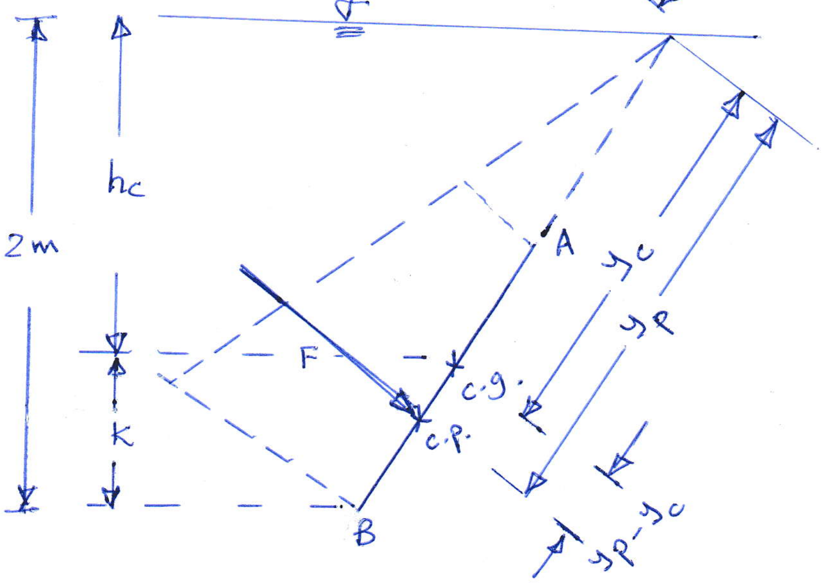
Solution:

Since $F = \gamma h_c A$

$\gamma = \gamma_w = 9810\ N/m^3$

$= 9.81\ KN/m^3$

From the sketch:



$h_c = 2 - K$

$= 2 - \frac{1.25}{2} \sin 80^\circ$

$= 1.384\ m$

$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (1.25)^2$

$= 1.227\ m^2$

$F = 9.81 * 1.384 * 1.227$

$= 16.66\ KN$

Since $y_p = y_c + \frac{I_c}{y_c A}$

$\therefore y_p - y_c = \frac{I_c}{y_c A} = \frac{\frac{\pi}{64} D^4}{\frac{h_c}{\sin 80^\circ} * A}$

$y_p - y_c = \frac{\frac{\pi}{64}(1.25)^4}{\frac{1.384}{\sin 80^\circ} * 1.227} = 0.07 m$

$\therefore$ Turning moment about the axis of rotation $= F(y_p - y_c) = 16.66 * 0.07 = 1.17\ KN.m$

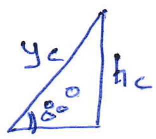
$\sin 80^\circ = \frac{K}{\frac{1.25}{2}}$

$\therefore K = \frac{1.25}{2} \sin 80^\circ$

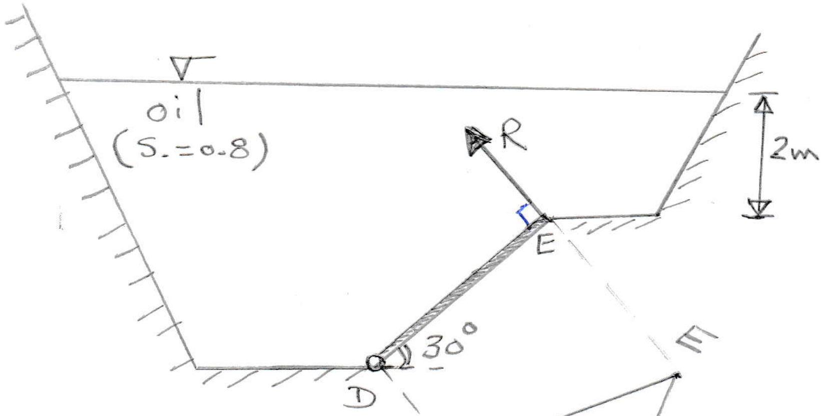
$K = 0.616\ m$

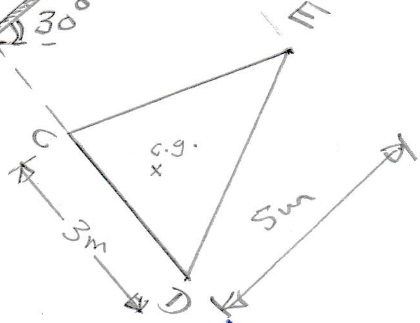
$\sin 80^\circ = \frac{h_c}{y_c}$

$\therefore y_c = \frac{h_c}{\sin 80^\circ}$



Ex.2 : The triangular gate CDE is hinged along CD and is opened by a normal force R applied at E. It holds oil (s. = 0.8) above it and is open to atmosphere on its lower side. The gate weighs 20 kN. Find (a) the magnitude of the hydrostatic force, (b) the location of pressure center, & (c) the force R needed to open the gate.

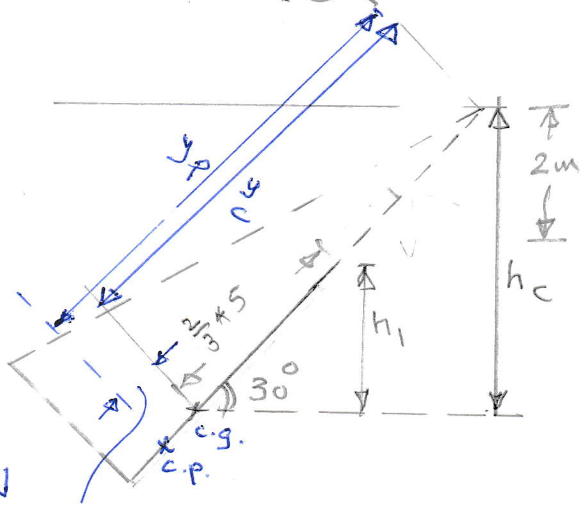




Solution :

(a) since $F = \gamma h_c A$

$\gamma_{oil} = 0.8 * \gamma_{water} = 0.8 * 9810$
$= 7848 \ N/m^3$
$= 7.848 \ KN/m^3$



$h_c = h_1 + 2$

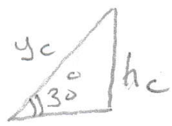
$h_c = \frac{2}{3} * 5 * \sin 30° + 2 = 1.667 + 2$
$= 3.667 \ m$

$A = \frac{1}{2} bh = \frac{1}{2} * 3 * 5 = 7.5 \ m^2$

$\therefore F = 7.848 * 3.667 * 7.5 = 215.84 \ KN$

$y_p - y_c = \frac{I_c}{y_c . A}$

(b) $y_p = y_c + \frac{I_c}{y_c . A}$

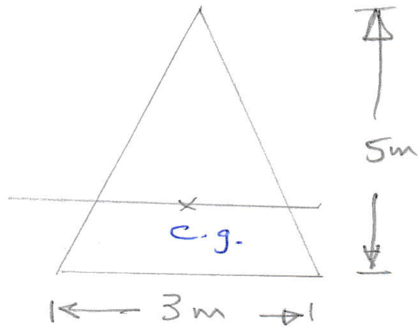


$y_c = \frac{h_c}{\sin 30°} = \frac{3.667}{\sin 30°}$

$\sin 30° = \frac{h_c}{y_c}$

$\therefore y_c = 7.334 \ m$

$$I_c = \frac{bh^3}{36}$$



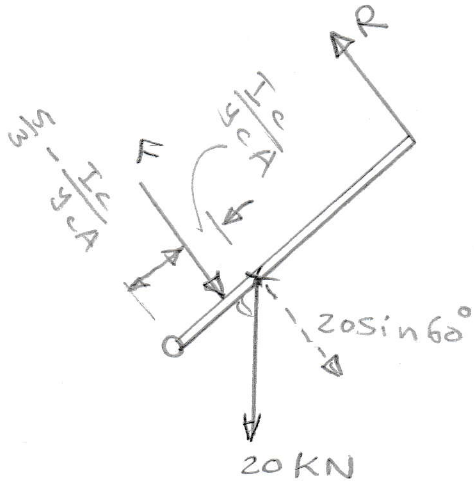
$$\therefore I_c = \frac{3(5)^3}{36} = 10.417 \ m^4$$

$$\therefore \frac{I_c}{y_c A} = \frac{10.417}{7.334 * 7.5} = 0.189 \ m$$

$$\therefore y_p = 7.334 + \qquad = 7.523 m$$

ⓒ $\sum M_{CD} = 0.$

$$R * 5 = F * \left(\frac{5}{3} - \frac{I_c}{y_c A} \right) + 20 \sin 60^\circ * \frac{5}{3}$$



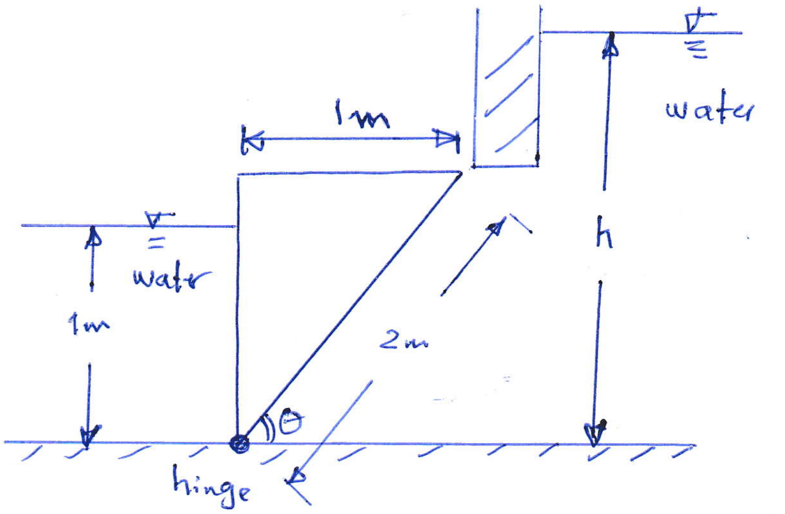
F.B.D of the gate

$$5R = 215.84 \left(\frac{5}{3} - 0.189 \right) + 28.87$$

$$5R = 318.94 + 28.87$$

$$\therefore R = 69.56 \ KN$$

Ex.3: How long will the water on the right (h) has to rise to open the gate shown below. The gate is 2m wide, and is constructed of material with S. = 4.5.



Solution:

For F1

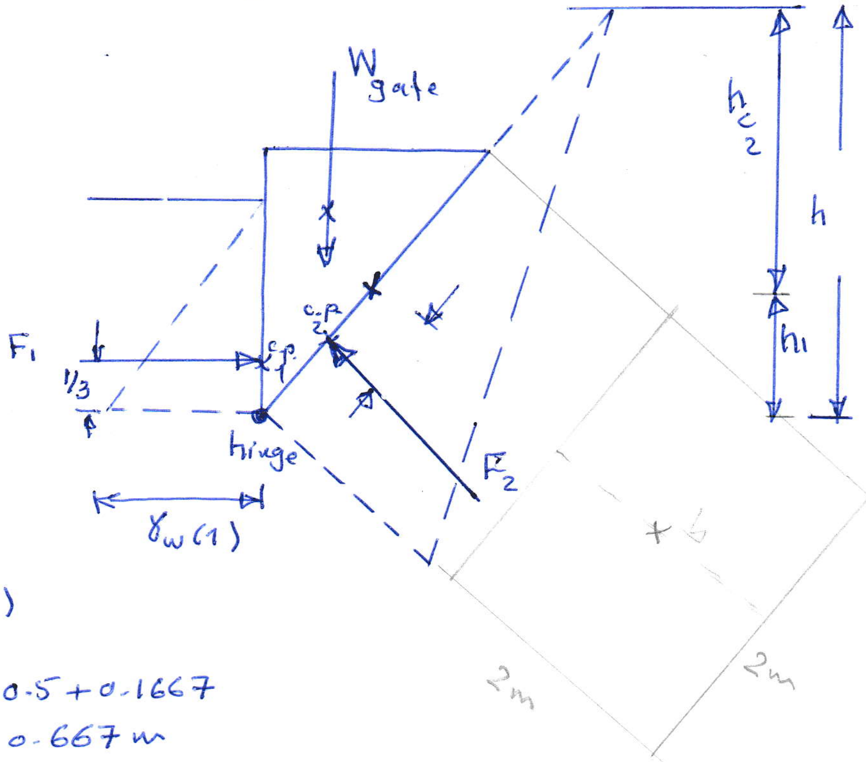
By using press. dist. diagram

$F_1 = \frac{1}{2}(\text{base}) * (\text{height}) * b$

$\text{base} = \gamma_w(1) = 9.81\ KN$

$F_1 = \frac{1}{2} * 9.81 * 1 * 2 = 9.81\ KN$

$y_p = \frac{2}{3} * 1 = 0.667m$



H.W. use $F_1 = \gamma h_{c_1} A_1$

$h_{c_1} = \frac{1}{2}$

$A_1 = 1 * 2$

$F_1 = 9.81 * \frac{1}{2} * 2 = 9.81\ KN$

$$y_{p_1} = y_{c_1} + \frac{I_{c1}}{y_c A_1} = 0.5 + \frac{\frac{2 * 1^3}{12}}{0.5(1*2)}$$

$$= 0.5 + \frac{2}{12} = 0.5 + 0.1667 = 0.667m$$

For F2: $F_2 = \gamma_w h_{c_2} A_2$

$F_2 = 9.81 * h_{c_2} * (2*2) = 39.24\, h_{c_2}$ (KN)

$y_{p_2} = y_{c_2} + \frac{I_{c2}}{y_{c_2} A_2} \Rightarrow$

$\therefore \frac{I_{c2}}{y_{c2} A_2} = \frac{2(2)^3/12}{1.155 h_{c_2}(2*2)} = \frac{0.288}{h_{c_2}}$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

$\sin 60^\circ = \frac{h_1}{1}$

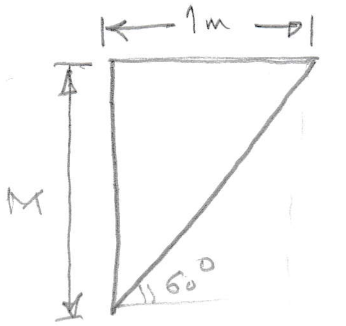
$\therefore h_1 = 0.886\ m$

$\sin 60^\circ = \frac{h_{c2}}{y_{c2}}$

$\therefore y_{c_2} = \frac{h_{c2}}{\sin 60^\circ}$

$\therefore y_{c_2} = 1.155\, h_{c_2}$

$W = m.g$; $\rho = \frac{m}{\forall} \Rightarrow m = \rho \forall$



$\tan 60^\circ = \frac{M}{1}$

$M = \tan 60^\circ$

$M = 1.732 m$

$\therefore W = \rho g \forall = \gamma \forall$

$W_{gate} = S\gamma_w * \forall = 4.5 * 9.81 * \forall$

$\forall = \frac{1}{2} M * 1 * 2 = 1.732 \ m^3$

$W_{gate} = 4.5 * 9.81 * 1.732 = 76.46 \ KN$

$\Sigma M_{hinge} = 0.$

$F_2 * \left[1 - (y_{P_2} - y_{c_2})\right] = F_1 * \frac{1}{3} + W_{gate} * \frac{1}{3}$

$39.24 h_{c_2} \left[1 - \frac{0.288}{h_{c_2}}\right] = \frac{9.81}{3} + \frac{76.46}{3}$

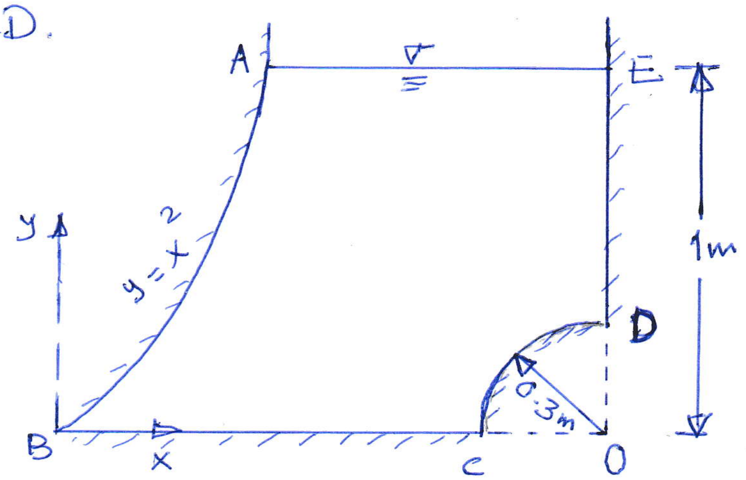
$39.24 h_{c_2} - 11.3 = 28.756$

$\therefore h_{c_2} = 1.021 \ m$

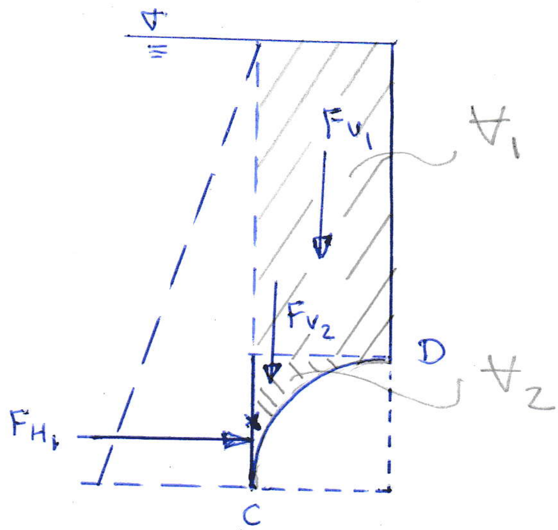
since $h = h_1 + h_{c_2}$

$\therefore h = 0.886 + 1.021 = 1.89 \ m$

Ex.4: A tank ABCDE contains water upto a depth of 1m and is 2m wide. The curve AB is defined by $y = x^2$ and curve CD is a quadrant of a circle of radius 0.3m. Calculate the forces on surfaces AB & CD.



Solution: Forces on surface CD:



$F_{H_1} = \gamma_w \, h_{c_1} \, A_{V_1}$

$h_{c_1} = 1 - \frac{0.3}{2} = 0.85 \, m$

$A_{V_1} = 0.3(2) = 0.6 \, m^2$

$\therefore F_{H_1} = 9.81\,(0.85)(0.6) = 5 \, KN \rightarrow$

$F_V = \gamma_w \forall$

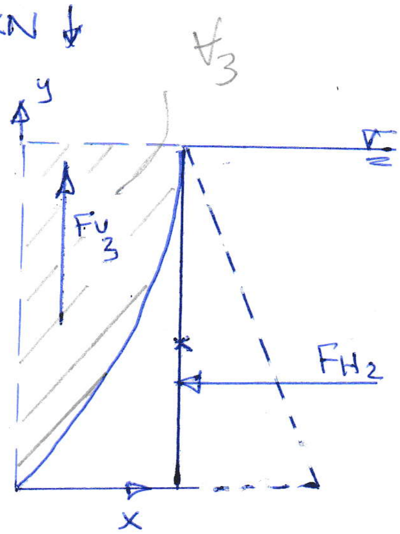
$F_{V_1} = \gamma_w \, \forall_1 = 9.81(0.3 * 0.7 * 2)$

$= 4.12 \, KN \downarrow$

$F_{V_2} = \gamma_w \forall_2 = 9.81 \left[(0.3)^2 - \frac{\pi}{4}(0.3)^2 \right] * 2 = 0.389 \, KN \downarrow$

$\therefore F_V = F_{V_1} + F_{V_2} = 4.5 \, KN \downarrow$

Forces of surface AB



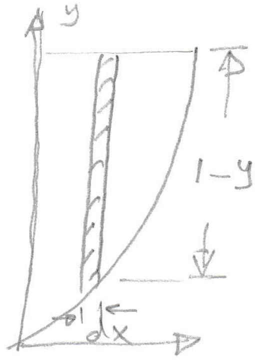
$F_{H_2} = \gamma_w \, h_{c_2} \, A_{V_2} = 9.81 * 0.5 * (1 * 2)$

$= 9.81 \, KN \leftarrow$

$F_{V_3} = \gamma_w \, \forall_3$

$\forall_3 = A_3 * 2$

$$A_3 = \int_a^b (1-y)\,dx$$



at $y=0 \Rightarrow x=0$.

at $y=1 \Rightarrow x=\pm 1 \Rightarrow x=1$ only according to the sketch

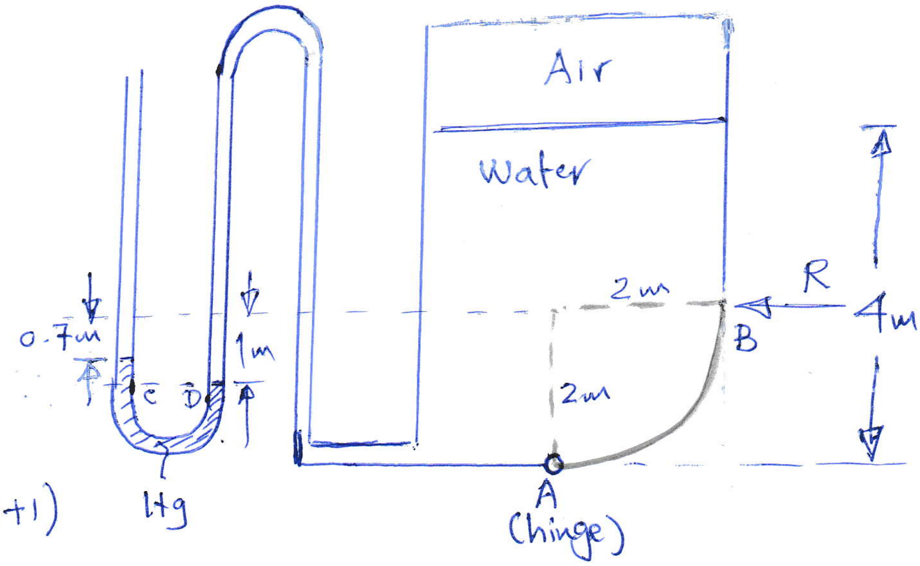
$$A_3 = \int_0^1 (1-x^2)\,dx = x\Big|_0^1 - \frac{1}{3}x^3\Big|_0^1 = 1-\left(\frac{1}{3}\right) = \frac{2}{3}\ m^2$$

$$\therefore \forall_3 = \frac{2}{3} * 2 = \frac{4}{3}\ m^3$$

$$\therefore F_{v_3} = 9.81 * \frac{4}{3} = 13.1\ KN \uparrow$$

<u>H.W.</u>: prove that the resultant of forces acting on surface CD must pass through point O.

Ex.5: Calculate the force R required to hold the gate AB in a closed position. The gate width is 3m. Neglect the weight of the gate.



Solution:

From the manometer;

$$P_C = P_D$$

$$13.6\gamma_w(0.3) = P_{air} + \gamma_w(2+1)$$

$$\therefore P_{air} = 1.08\gamma_w = h_{w*} * \gamma_w$$

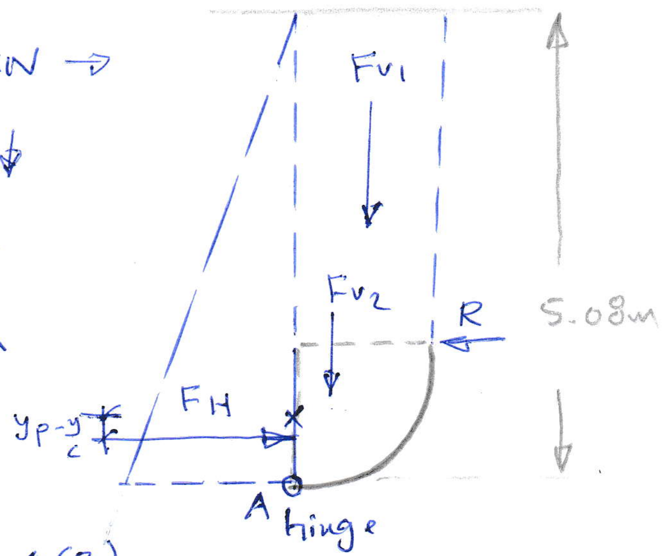
$$\therefore h_w = 1.08\,m$$

$$F_H = \gamma_w(5.08-1)(2*3) = 240.15\ KN \rightarrow$$

$$F_{v_1} = \gamma_w(2*3.08*3) = 181.28\ KN \downarrow$$

$$F_{v_2} = \gamma_w\left(\frac{\pi}{4}(2)^2 * 3\right) = 92.46\ KN \downarrow$$

$$y_p - y_c = \frac{I_c}{y_c A} = \frac{3(2)^3/12}{4.08(2*3)} = 0.082\,m$$



$$\Sigma M_{hinge} = 0.$$

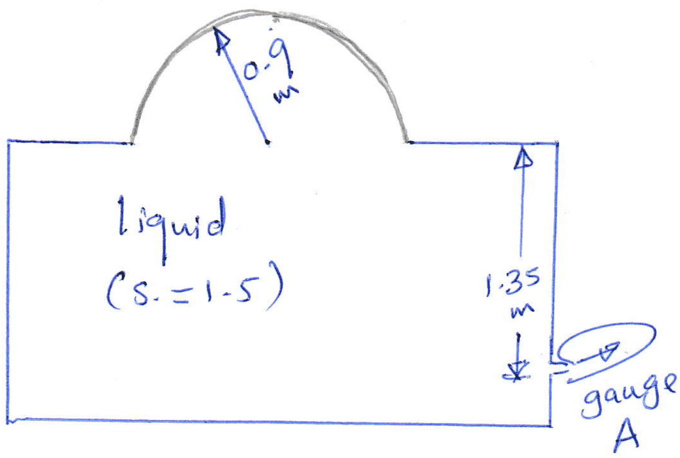
$$F_H * (1-0.082) + F_{v_1} * 1 + F_{v_2} * \frac{4(2)}{3\pi} - R * 2 = 0.$$

$$240.15(0.918) + 181.28*1 + 92.46 * 0.85 = 2R$$

$$\therefore R = 240.16\ KN$$

Ex. 6: Find the Vertical component of force in the metal spherical dome shown in figure below, when gauge A reads 69 kPa. Assume the dome weight 4500N

Note: The volume of sphere $= \frac{\pi D^3}{6}$



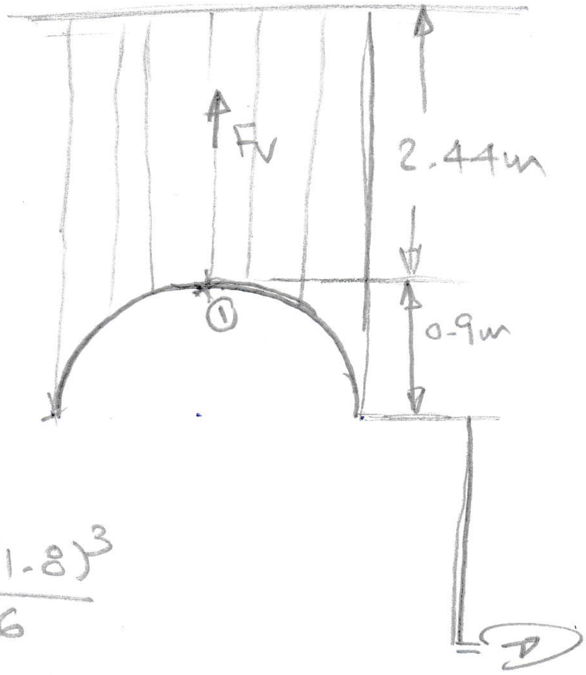
Solution:

$$P_{gauge\,A} = P_1 + 1.5\,\gamma_w\,(0.9 + 1.35)$$

$$69 = P_1 + 1.5(9.81)(2.25)$$

$$\therefore P_1 = 35.89 \text{ KN}$$

$$h_{liquid} = \frac{35.89}{1.5(9.81)} = 2.44\text{m}$$



$$F_V = \gamma_{liquid}\,\forall$$

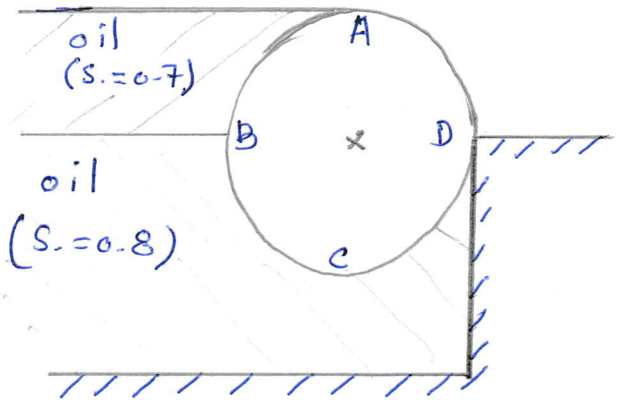
$$\forall = \frac{\pi}{4}(1.8)^2 * (0.9 + 2.44) - \frac{1}{2}\frac{\pi(1.8)^3}{6}$$

$$= 6.97\text{m}^3$$

$$F_V = 1.5(9.81)(6.97) = 102.56 \text{ KN}$$

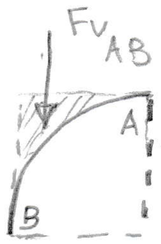
$$F_{V\,net} = F_V - W = 102.56 - 4.5 = 98.06 \text{ KN}$$

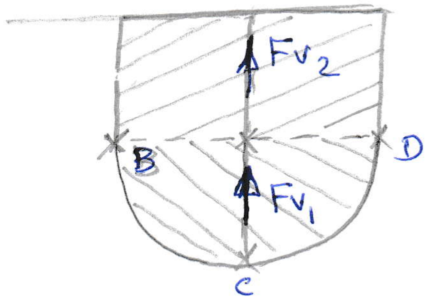
Ex. 7 A cylinder of 1m dia. & 2m length stays in equilibrium, as shown in figure below. Calculate the specific gravity of the material of the cylinder.



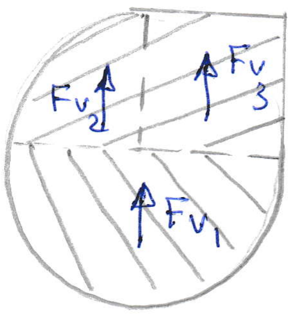
Solution:

For surface AB.





⇒ Net Vertical forces



$$Fv_1 = 0.8\gamma_w * \frac{\pi}{2}(0.5)^2 * 2$$

$$= 0.628\,\gamma_w$$

$$Fv_2 = 0.7\,\gamma_w * \frac{\pi}{4}(0.5)^2 * 2$$

$$= 0.275\,\gamma_w$$

$$Fv_3 = 0.7\,\gamma_w\,(0.5 * 0.5 * 2)$$

$$= 0.35\,\gamma_w$$

$$\therefore\ Fv = Fv_1 + Fv_2 + Fv_3 = 1.253\,\gamma_w$$

$$\Sigma F_y = 0. \Rightarrow N = Fv = 1.253\,\gamma_w = \gamma \forall = S.\,\gamma_w * \forall$$

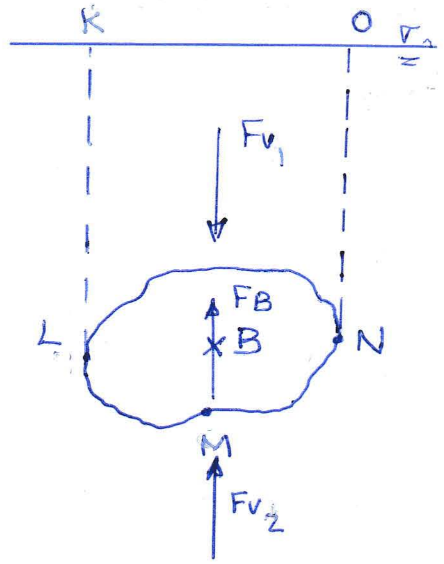
$$1.253\,\gamma_w = S.\,\gamma_w * \frac{\pi}{4}(1)^2 * 2 \Rightarrow S. = 0.8$$

Bouyancy

Theory: Archimedes' principles states that the buoyant force has a magnitude equal to the weight of fluid displaced by the body and is directed vertically upwards.

The buoyant force (F_B) passes through the center of buoyancy (B).

Submerged Body



$F_{v_2} > F_{v_1}$ because pressure increase with depth

$F_{v_2} = \gamma \forall_2$

$F_{v_1} = \gamma \forall_1$

where, γ = specific weight of the fluid.

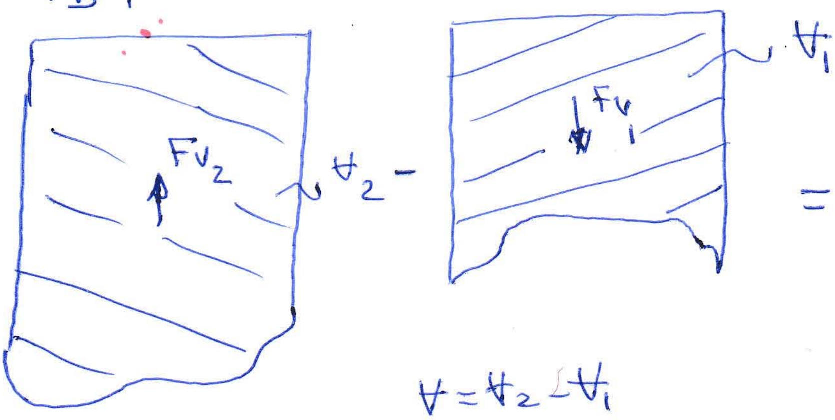
$\forall_1 = \forall_{KLNOK}$

$\forall_2 = \forall_{KLMNOK}$

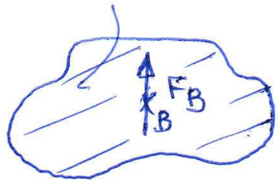
$\forall_2 - \forall_1$ = volume of displaced fluid = volume of submerged body $= \forall$

$\therefore F_{v_2} - F_{v_1} = \gamma \forall = F_B \uparrow$

$\therefore \boxed{F_B = \gamma \forall} \uparrow$



$\forall = \forall_2 - \forall_1$

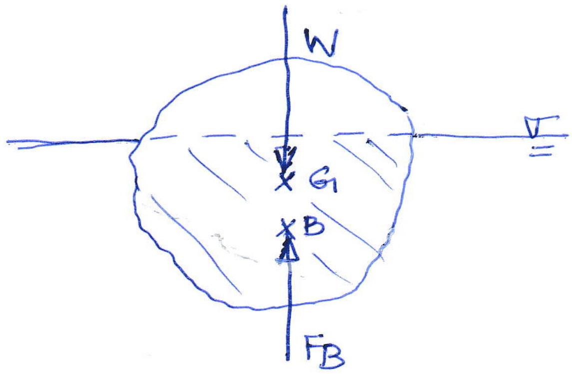


$F_B = \gamma \forall$

Floating Body

$F_B = W = \gamma \forall$

where, W = weight of the body



G: center of gravity

الاستقرارية

Stability of Submerged & Floating Bodies

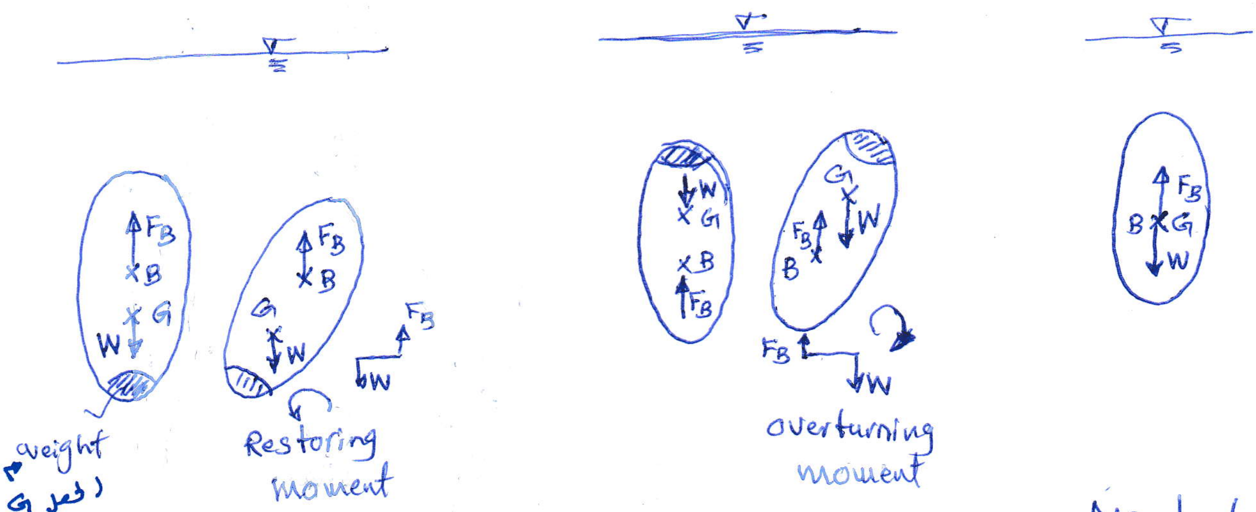
1- Submerged Body

توازن مستقر

- Stable equilibrium : when the submerged body returns to its equilibrium condition. (G below B).

- Unstable equilibrium : The submerged body doesn't return to its equilibrium condition (G above B).

- Neutral Equilibrium : G, coincide with B.



(لجعل G لاينطبق ع B)

Stable

Unstable

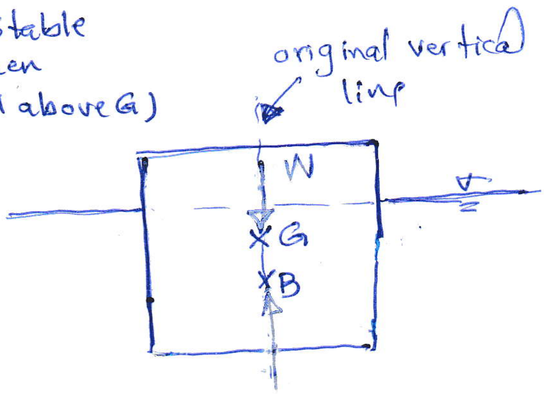
Neutral

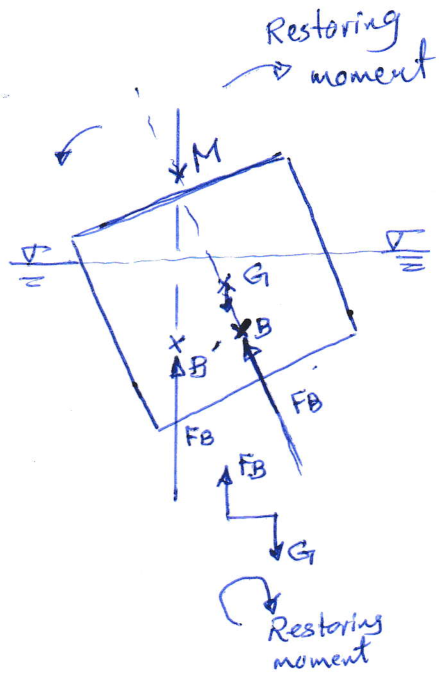
2- Floating Body

a- G below B always stable

b- When G above B :

- Stable when (M above G)

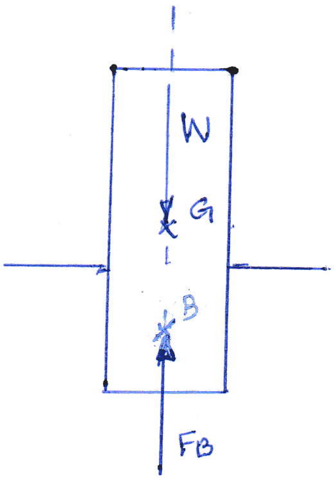


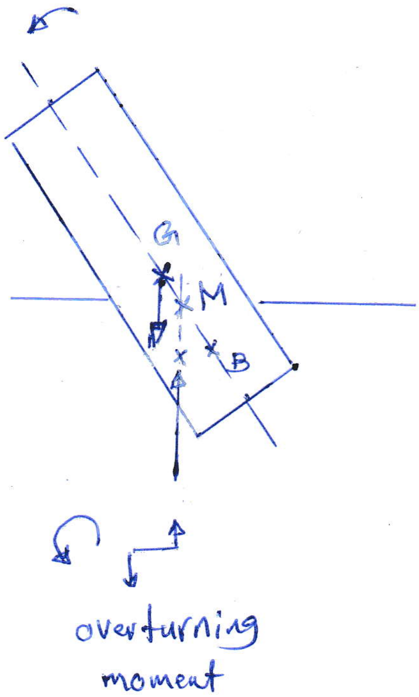


M = Metacenter

: The point at which the line of action of the buoyant force intersects the original vertical line through G.

- Unstable when (M below G)



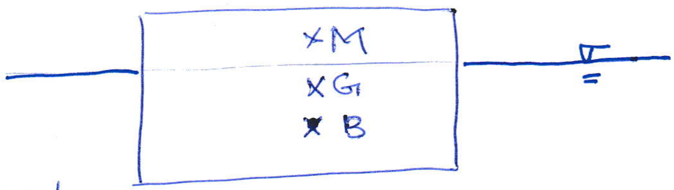


- Neutral when (M coincide with G)

To determine the position of the metacenter point (M) relative to center of buoyancy (B),

$$\overline{BM} = \frac{I}{\forall_{immersed}}$$

where, I = the smallest moment of interia of the object plane at the liquid free surface.



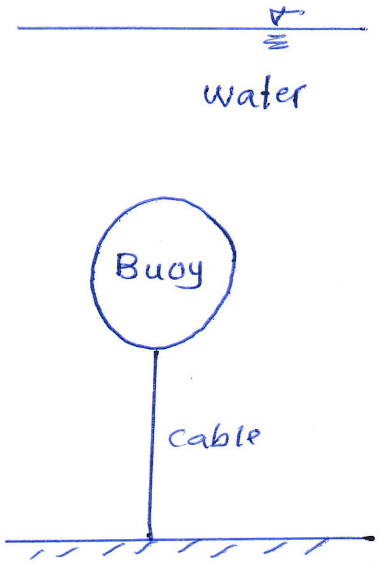
So, The object is stable when;

$\overline{BM} > \overline{BG}$,

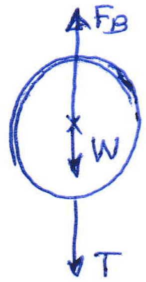
- Unstable when $\overline{BM} < \overline{BG}$
- Neutral when $\overline{BM} = \overline{BG}$.

Ex.1: A spherical buoy has a dia. of (1.5m), weighs 8.5 KN, and is attached as shown in figure below with a cable. Determine the tension force at the cable.

Note: volume of sphere $= \frac{\pi D^3}{6}$



Solution



F.B.D of the bouy

$\Sigma F_y = 0.$

$F_B - W - T = 0.$

$T = F_B - W = \gamma_w \forall_{sphere} - W$

$= 9.81 * \frac{\pi (1.5)^3}{6} - 8.5$

$= 17.34 - 8.5 = 8.84 \text{ KN}$

Ex.2: A rectangular box of dimension 7.6m x 3m x 4m deep floats in water. If the box weighs 40 ton, determine:

1- the deep it will sink

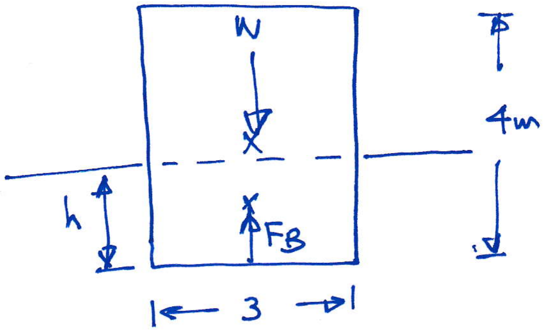
2- the mass of stone placed on the box to sink it 4m depth.

Solution :

$b = 7.6\,m$

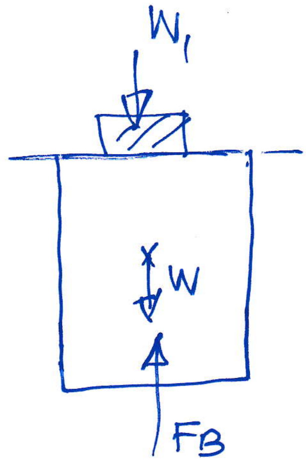
1- $F_B = W$

$$\gamma_{water} \forall_{sink} = m \cdot g$$



$$9810 * 3(7.6)h = 40 * 10^3 * 9.81$$

$$h = \frac{40}{22.8} = 1.754\,m$$



2- $\sum F_y = 0.$

$$W_1 + W = F_B$$

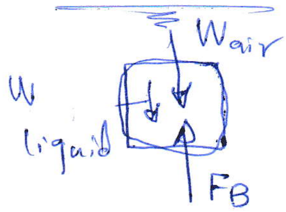
$$W_1 + 40 * 9810 = 9810\,(3)(4)(7.6)$$

$$W_1 = 502272\ N$$

$$\therefore m_1 \cdot g = 502272$$

$$\therefore m_1 = \frac{502272}{9.81} = 51200\,kg = 51.2\ ton$$

Ex3: An object weighs 3N in water and 4N in oil ($S = 0.83$). Determine its volume & specific gravity ($S.$).



$$W_{liquid} = W_{air} - F_B$$

Sol.

$$W_{water} = W_{air} - \gamma_w \forall \quad \text{—} \; (1)$$

$$W_{oil} = W_{air} - \gamma_{oil} \forall \quad \text{—} \; (2)$$

From Eq (1)

$$\therefore \; 3 = W_{air} - 9810 \forall \Rightarrow W_{air} = 3 + 9810 \forall \quad \text{—} \; (3)$$

subs. Eq. (3) into eq. (2)

$$4 = 3 + 9810 \forall - 0.83 \, (9810)(\forall)$$

$$\cancel{4-3} \quad 1 = 0.17 (9810) \forall$$

$$\therefore \forall = 6 * 10^{-4} \; m^3$$

From Eq. (3) $\Rightarrow$ $W_{air} = 3 + 9810 * 6*10^{-4}$

$$W_{air} = 8.886 \; N$$

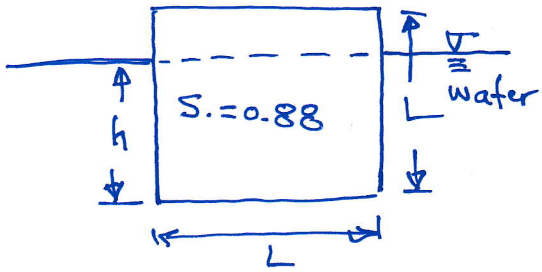
Since $W = \gamma_{object} \forall = \rho_{object} * g * \forall$

$$8.886 = \rho_{object} * 9.81 * 6 * 10^{-4}$$

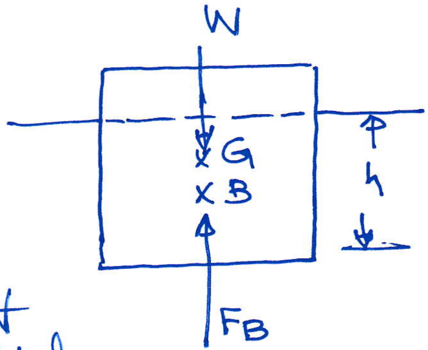
$$\rho_{obj} = 1509.7 \; Kg/m^3$$

$$S. = \underline{\underline{1.51}}$$

Ex.4: For the figure shown below, a cube of wood of side length (L) is float in water. If the specific gravity of the wood is 0.88. Determine if this cube is stable or not.



Sol.



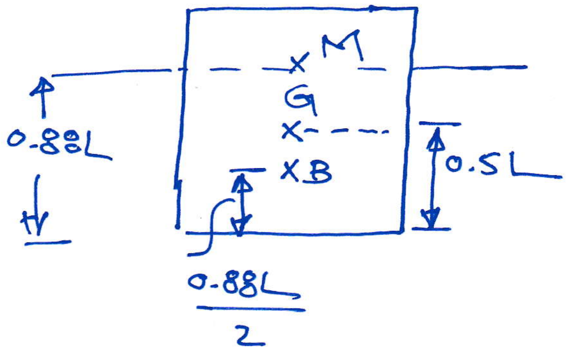
$\Sigma F_y = 0.$

$F_B = W$

$\gamma_w \forall_{immersed} = \gamma_{wood} * \forall_{total}$

$\cancel{\gamma_w}\,(\cancel{L})(\cancel{L})(h) = 0.88\,\cancel{\gamma_w}\,L^{\cancel{3}\,2}$

$\therefore h = 0.88L$

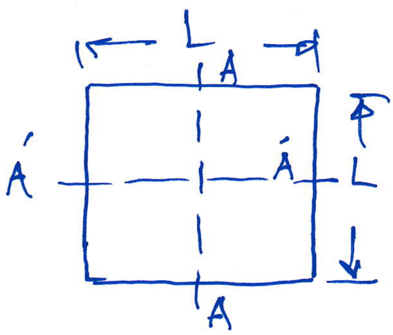


$\therefore \overline{GB} = 0.5L - \dfrac{0.88L}{2} = 0.06L$

Since ; $\overline{BM} = \dfrac{I}{\forall_{immersed}}$

For the cube $I_{AA} = I_{A\acute{A}}$

So, we don't need to find the least $I \Rightarrow I = \dfrac{L.L^3}{12} = \dfrac{L^4}{12}$



Top view of the cube at the water line

$\forall_{immersed} = (L)(L)(0.88L) = 0.88L^3$

$\therefore \overline{BM} = \dfrac{L^4}{12(0.88L^3)} \Rightarrow \overline{BM} = 0.095L$

Since $\overline{BM} > \overline{BG} \Rightarrow$ the cube is stable

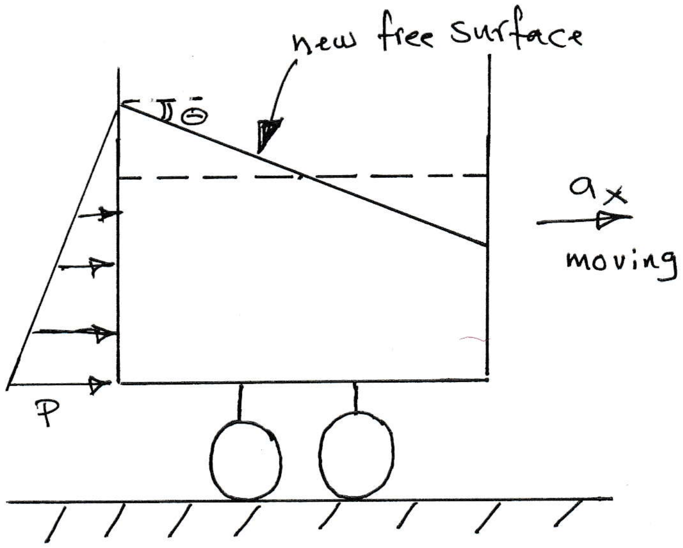
Fluid Masses Subjected to A Constant Acceleration

Liquid in a Container Subjected to A constant Linear Acceleration

A liquid, contained in a vessel, may be subjected to a constant linear acceleration without any relative movement being created between different elements of the liquid in the vessel. The liquid must, of course, orient itself once and for all to stay in that position for the given constant acceleration. The liquid is then said to be in a state of relative rest. In the absence of relative motion between different fluid elements, there are no rates of strain & shear stresses in the liquid. The hydrostatic law is, therefore, applicable for the evolution of hydrostatic forces.

a- Horizontal Acceleration

$$\boxed{\tan\theta = \frac{a_x}{g}} \qquad \boxed{P = \gamma h}$$



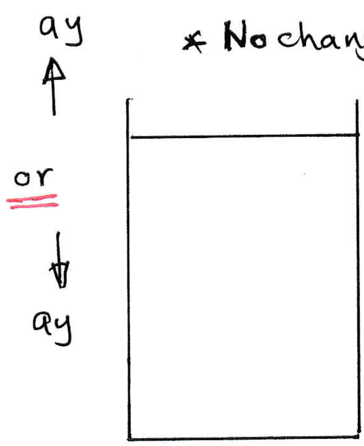
where;

θ = Inclination angle of the new free surface with the horizontal (degree).

a_x = constant horizontal acceleration (m/s^2).

P = pressure value (Pa).

b- Vertical Acceleration



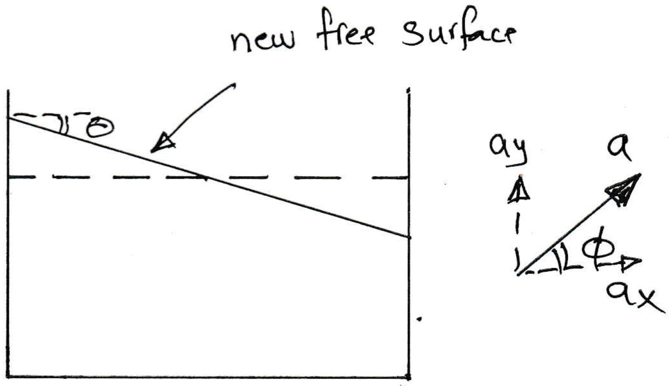
* No change in free surface

$$\boxed{P = \gamma h \left(1 + \frac{a_y}{g}\right)}$$ moving upwards ↑

&

$$\boxed{P = \gamma h \left(1 - \frac{a_y}{g}\right)}$$ moving downwards ↓

c- Inclined Acceleration



$$\boxed{\tan\theta = \frac{a \cdot \cos\phi}{a\sin\phi + g} = \frac{a_x}{a_y + g}}$$

moving upwards ↗

$$\boxed{\tan\theta = \frac{a \cos\phi}{a\sin\phi - g} = \frac{a_x}{a_y - g}}$$

moving downwards ↙

$$\boxed{P = \gamma h}$$

where; θ = Inclination angle of the new free surface (degree).

a : constant inclined acceleration (m/s^2).

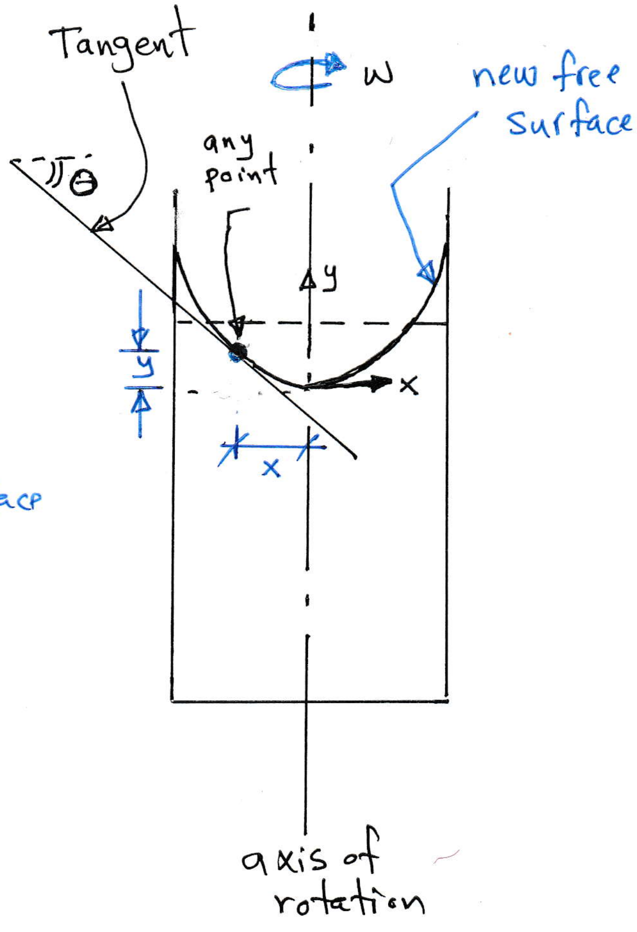
Liquid in A Container Subjected to A Constant Rotation

A liquid, contained in a vessel, may be rotated at a constant rotational velocity (w) without any relative movement being created between different elements of the liquid in the vessel. The liquid reorients itself once & for all to stay in that position with respect to the axis of rotation.

$$\boxed{\tan\theta = \frac{w^2 \cdot x}{g}}$$

$$\boxed{y = \frac{w^2 \cdot x^2}{2g}}$$

Parabola Equation
(new free surface eq.)



where;

θ = Inclination angle of tangent of any point located along new free surface (degree).

x, y = x & y -values for any point located along new free surface.

ω : constant angular velocity (rad./s.) $= \dfrac{2\pi N}{60}$

N : constant angular velocity (r.p.m).

Noting that ; $V = \omega . x$

$a = \omega^2 . x$

where; V = velocity vector (m/s.)

a = acceleration (m/s²).

Ex.1: An open rectangular tank (5 m X 4 m X 3 m high) containing water upto a height of (2m) is accelerated at (3 m/s²)

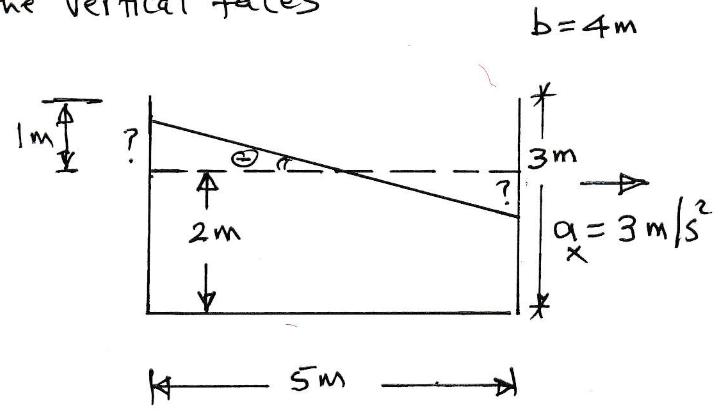
a - horizontally along the longer side.

b - Vertically upwards.

c - " downwards &

d - upwards in a direction inclined 30° to the horizontal along the longer side.

Calculate, in each case, the total force on the base of the tank as well as on the vertical faces



Sol.: a-

$$\tan\theta = \frac{a_x}{g} = \frac{3}{9.81} = \frac{?}{2.5}$$

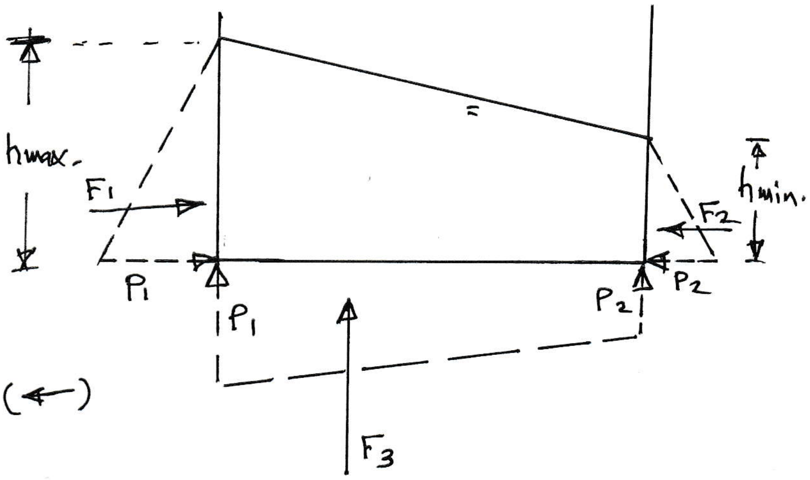
$\therefore$? = 0.764m < 1m

The water does not spilt over

$\therefore h_{max.} = 2 + 0.764 = 2.764\,m$

$h_{min.} = 2 - 0.764 = 1.236\,m$

$b = 4m$



$P_1 = \gamma_w\, h_{max.} = \frac{9810}{1000}(2.764)$

$\therefore P_1 = 27\ KPa$

$F_1 = \frac{1}{2} P_1 \cdot h_{max.} \cdot b = 149.2\ KN\ (\leftarrow)$

$P_2 = \gamma_w\, h_{min.} = 9.81\,(1.236)$

$= 12.125\ KPa$

$\therefore F_2 = \frac{1}{2} P_2 \cdot h_{min.} \cdot b = 30KN\ (\rightarrow)$

$F_3 = \frac{P_1 + P_2}{2}\,(5)(4) = 391.25\ KN\ (\downarrow)$

<u>Note</u>: All the forces above are reaction forces to the action forces.

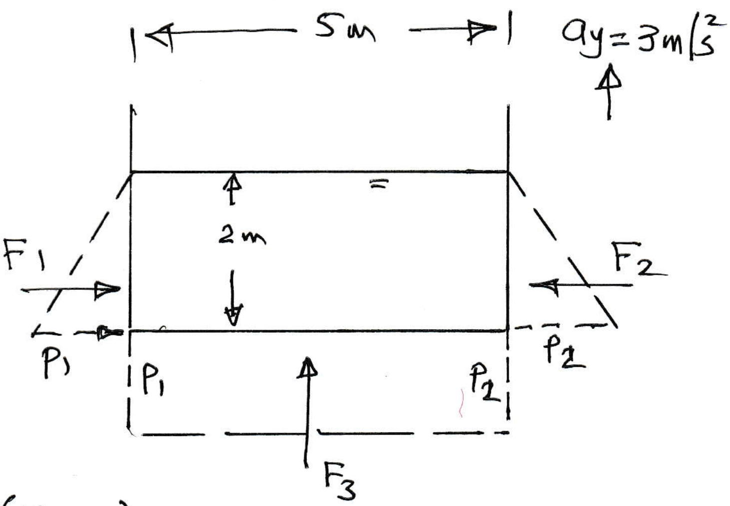
b-

$P_1 = \gamma h\,(1 + \frac{a_y}{g})$

$= 9.81(2)\left(1 + \frac{3}{9.81}\right)$

$= 25.62\ K.Pa$

$P_2 = P_1$



$\therefore F_1 = F_2 = \frac{1}{2} P_1\,(2)(4)$

$= 102.48\ KN\ (F_1 \leftarrow);(F_2 \rightarrow)$

$F_3 = P_1(5)(4) = 512\ KN\ (\downarrow)$

c- $P_1 = P_2 = \gamma h\left(1 - \frac{a_y}{g}\right) = 13.62$ KPa

$F_1 = F_2 = \frac{1}{2} P_1 (2)(4) = 54.48$ KN

$F_3 = P_1 (5)(4) = 272.4$ KN

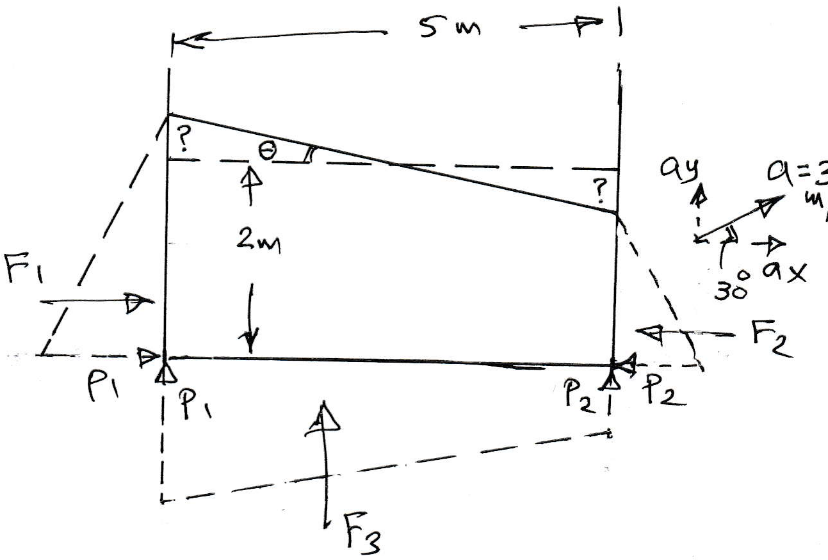
d-

$b = 4m$

$\tan\theta = \frac{a_x}{a_y + g}$

$a_x = 3\cos 30^\circ = 2.6 \, m/s^2$

$a_y = 3\sin 30^\circ = 1.5 \, m/s^2$



$\therefore \frac{2.6}{1.5 + 9.81} = \frac{?}{2.5}$

$\therefore ? = 0.575m$

$P_1 = \gamma_w h_{max.} \left(1 + \frac{a_y}{g}\right) = 29.123$ KPa

$P_2 = \gamma_w h_{min.} \left(1 + \frac{a_y}{g}\right) = 16.116$ KPa

$\therefore F_1 = \frac{1}{2} P_1 \cdot h_{max.} (4) = 150$ KN

$F_2 = 45.93$ KN

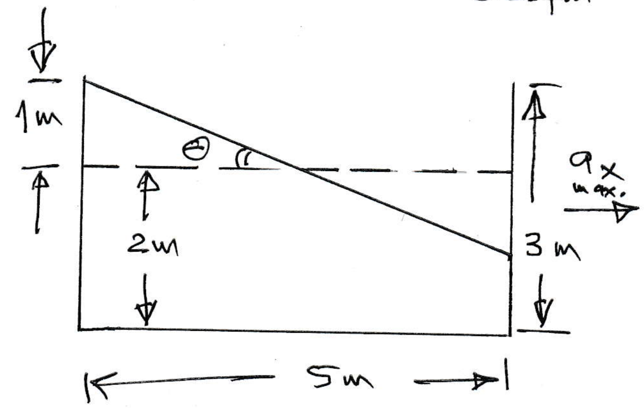
$F_3 = \frac{P_1 + P_2}{2} (5)(4) = 452.4$ KN

Ex. 2 : If the tank in Ex. 1 is accelerated horizontally along the longer side, determine the maximum acceleration that can be given without spilling the water. Also, calculate the percentage of water spilt if this max acceleration is increased by 20 %.

Sol. :

$$\tan\theta = \frac{a_{x_{max.}}}{g} = \frac{1}{2.5}$$

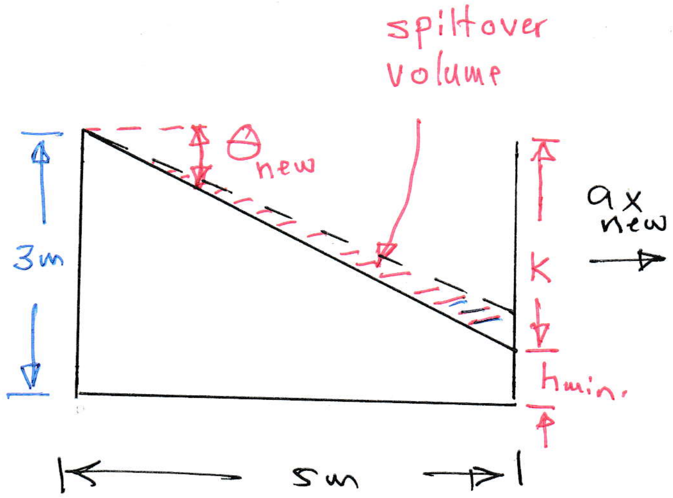
$$\therefore a_{x_{max.}} = 3.92 \ m/s^2$$



$$a_{x_{new}} = 1.2\,(a_{x_{max.}}) = 4.7 \ m/s^2$$

$$\tan\theta_{new} = \frac{a_{x_{new}}}{g} = \frac{K}{\ }$$

$$\therefore \frac{4.7}{9.81} = \frac{K}{5}$$



$$\therefore K = 2.4m$$

$$\therefore h_{min.} = 3 - K = 0.6m$$

$$h_{max.} = 3m$$

$$\therefore \text{Volume of water that kept in the tank} = \frac{3 + h_{min.}}{2}(5)(4)$$

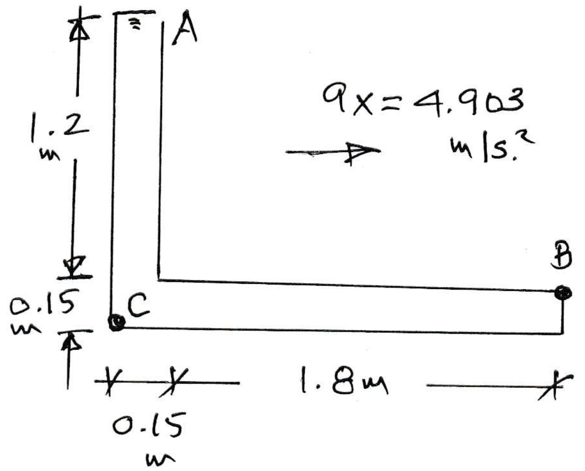
$$= 36 \ m^3$$

$\therefore$ The volume of water that spilt over $= \forall_{\text{original}} - \forall_{\substack{\text{Kept in}\\ \text{the tank}}}$

$$= 4\,m^3$$

$\therefore$ % of water spilt over $= \dfrac{\forall_{spilt}}{\forall_{original}} * 100 = 10\%$

Ex.3: The tank in figure is filled with oil (s.=0.8) & acceleration as shown. There is a small opening in the tank at A. Determine the pressure at B & C; and the acceleration (a_x) required to make the pressure at B equals (7 KPa "Vacuum").



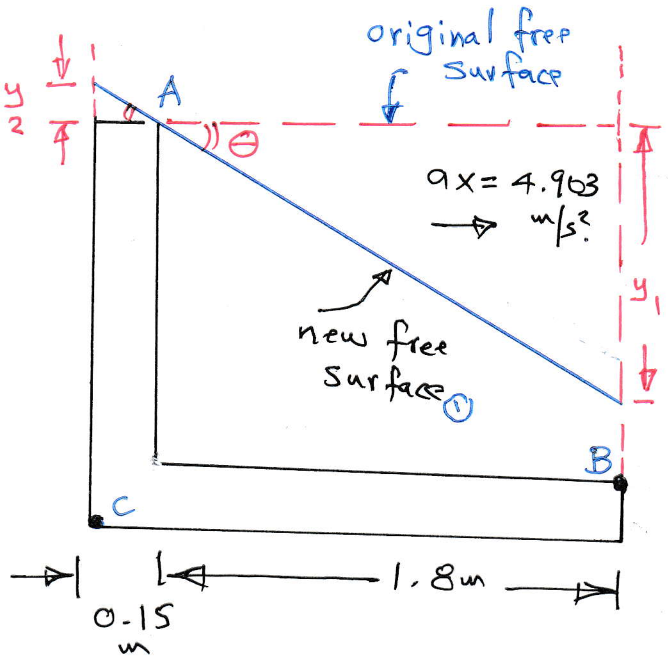
Sol.:

$$\tan\theta = \frac{a_x}{g}$$

$$\frac{a_x}{g} = \frac{y_1}{1.8} = \frac{y_2}{0.15}$$

$\therefore y_1 = 0.9\,m$

$y_2 = 0.075\,m$



$$P_B = \gamma_{oil}\,(1.2 - y_1) = 2.35\ KPa$$

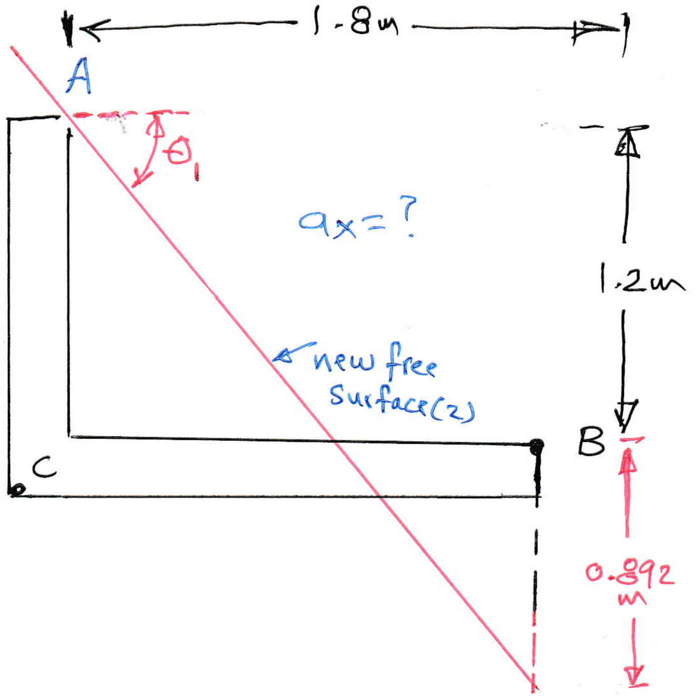
$$P_C = \gamma_{oil}\,(1.2 + 0.15 + y_2) = 11.18\ KPa.$$

when $P_B = -7$ KPa

convert P_B to oil depth

$$-7 \times 10^3 = \gamma_{oil}\, h_{oil}$$

$$\therefore h_{oil} = -\ 0.892\,m$$



$$\tan\theta_2 = \frac{a_x}{g} = \frac{1.2 + 0.892}{1.8}$$

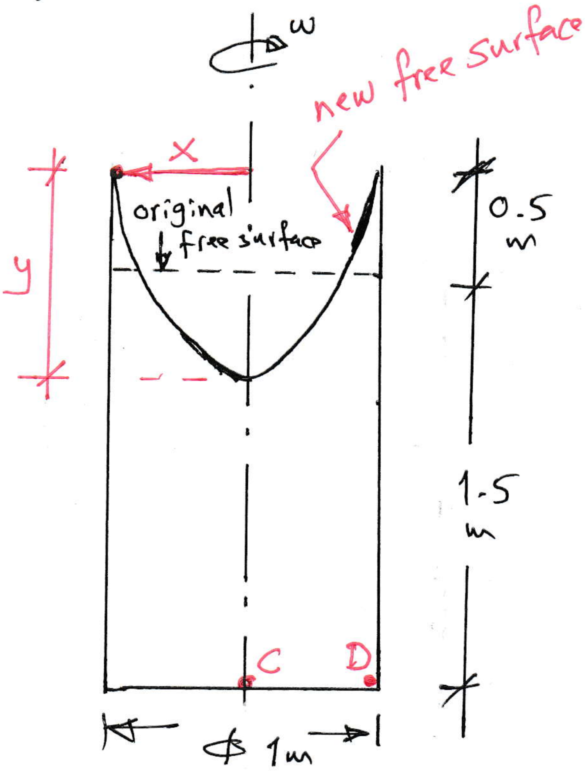
$$\therefore a_x = 11.4\ m/s^2$$

Ex. 4 : An open cylindrical tank (2m) high & (1m) in diameter contains (1.5m) of water. If the cylinder rotates about its geometric axis,

(a) hat constant angular velocity in (r.p.m) can be attained without spilling any water?

(b) what is the pressure at the tank bottom (at C & D points) when $\omega = 6$ rad/s. ?

Sol.: (a)



Volume of paraboloid of revolution = $\frac{1}{2}$ (volume of circumscribed cylinder)

If no liquid is spilled, this volume equals the volume above the original free surface

Volume of paraboloid of revolution $= \frac{1}{2}(\pi x^2 y)$

Volume above the original free surface $= \pi (0.5)^2 (0.5)$

since there is no spilling of water $\Rightarrow$

$$\therefore \frac{1}{2}\left(\pi (0.5)^2 y\right) = \pi (0.5)^2 (0.5)$$

$$\therefore y = 1 \text{ m}$$

$$\text{Since ; } y = \frac{\omega^2 \cdot x^2}{2g}$$

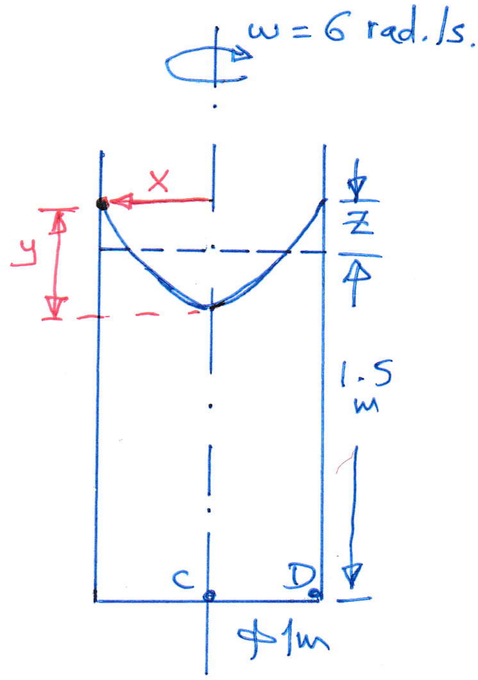
$$\therefore 1 = \frac{\omega^2 (0.5)^2}{2g}$$

$$\therefore \omega = 8.86 \text{ rad./s.}$$

$$\text{since } \omega = \frac{2\pi N}{60}$$

$$\therefore N = 84.6 \text{ r.p.m}$$

(b) Since ω < the above ω $\Rightarrow$ this means that the water does not reach the tank top edge & does not spilt over.



volume of paraboloid of revolution = volume above the original free surface

$$\frac{1}{2}\left(\pi (0.5)^2 y\right) = \pi (0.5)^2 * Z \quad \text{—(1)}$$

$$\text{since; } y = \frac{\omega^2 \cdot x^2}{2g}$$

$$\therefore y = \frac{(6)^2 (0.5)^2}{2g} = 0.46$$

From eq. (1) $\Rightarrow Z = 0.23\,m$

$\therefore \; P_C = \gamma_w (1.5 - (y - z)) = 12.46 \; KPa$

$P_D = \gamma_w (1.5 + z) = 16.97 \; KPa$

Ex.5: Consider the tank in Ex.4 closed with air space subjected to a pressure of (1.07 bar). When the angular velocity is (12 rad./s.), what are the pressures in (bar) at points C & D?

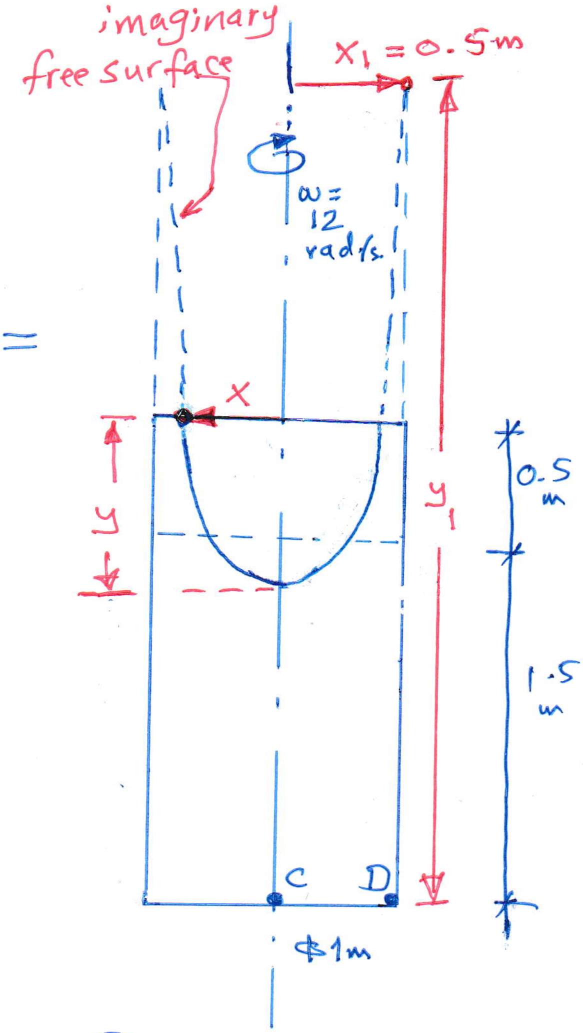
Sol.: From Ex.4, the ω-value that makes the water reach the tank top edge is 8.86 rad./s.

Since $\omega = 12 \text{ rad./s.} > 8.86 \text{ rad./s.}$

for closed tank $\Rightarrow$ new free surface has an imaginary part above the tank top edge

Since there is no water spilt over;

volume of paraboloid of revolution = volume above the original free surface



$$\frac{1}{2}(\pi x^2 y) = \pi (0.5)^2 (0.5)$$

$$y = \frac{1}{4x^2} \quad \text{— (1)}$$

Since; $$y = \frac{w^2 \cdot x^2}{2g}$$

$$\therefore y = \frac{(12)^2 x^2}{2g} = 7.34\, x^2 \quad \text{— (2)}$$

From eqs. (1) & (2):

$$\frac{1}{4x^2} = 7.34\, x^2 \Rightarrow x = 0.43\,m$$

From eq. (2) $\Rightarrow y = 1.357\,m$

$$\therefore \quad h_c = 2 - y = 0.643\,m$$

$$\therefore P_c = 1.07 + \frac{\gamma_w}{10^5} h_c = 1.13 \text{ bar}$$

For point D: $$P_D = 1.07 + \frac{\gamma_w}{10^5} h_D$$

$h_D = y_1$

since ; $y = \frac{\omega^2 \cdot x^2}{2g}$

$\therefore y_1 = \frac{(12)^2 (0.5)^2}{2g} = 1.83 \text{ m}$

$\therefore P_D = 1.31$ bar