مشتقة الدوال المثلثية العكسية

$$1)\ \frac{d\sin^{-1}u}{dx}=\frac{1}{\sqrt{1-u^2}}\times\frac{du}{dx}$$

$$2)\ \frac{d\cos^{-1}u}{dx}=\frac{-1}{\sqrt{1-u^2}}\times\frac{du}{dx}$$

$$3)\ \frac{d\tan^{-1}u}{dx}=\frac{1}{1+u^2}\times\frac{du}{dx}$$

$$4)\ \frac{d\cot^{-1}u}{dx}=\frac{-1}{1+u^2}\times\frac{du}{dx}$$

$$5)\ \frac{d\sec^{-1}u}{dx}=\frac{1}{|u|\sqrt{u^2-1}}\times\frac{du}{dx}$$

$$6)\ \frac{d\csc^{-1}u}{dx}=\frac{-1}{|u|\sqrt{u^2-1}}\times\frac{du}{dx}$$

Ex

$$y=\tan^{-1}(2x)\quad find\quad \frac{dy}{dx}$$

Sol

مشتقة الدالة العكسية تحسب من القانون التالي

$$\frac{d(\tan^{-1}u)}{dx}=\frac{1}{1+u^2}\times\frac{du}{dx}$$

$$u=2x\quad\Rightarrow\frac{du}{dx}=2$$

الان نشتق الدالة الاصلية في السؤال

$$\frac{dy}{dx}=\frac{1}{1+(2x)^2}(2)=\frac{2}{1+4x^2}$$

Ex **If** $y=\sec^{-1}\sqrt{x}+\sec^{-1}\frac{a}{x}\quad (a\quad is\ cons\tan t)\quad Find\ \frac{dy}{dx}$

Sol

$$\frac{d\sec^{-1}u}{dx}=\frac{1}{|u|\sqrt{u^2-1}}\times\frac{du}{dx}$$ مشتقة $\sec^{-1}$ تعطى بالقانون الاتي

when $u=\sqrt{x} \Rightarrow \frac{du}{dx}=\frac{1}{2\sqrt{x}}$

when $u=\frac{a}{x} \Rightarrow \frac{du}{dx}=\frac{-1}{x^2}$

الان نشتق الدالة في السؤال a

$$\frac{dy}{dx}=\frac{1}{\sqrt{x}\sqrt{(\sqrt{x})^2-1}}\frac{1}{2\sqrt{x}}+\frac{1}{\frac{a}{x}\sqrt{(\frac{a}{x})^2-1}}\frac{-a}{x^2}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{x}\sqrt{x-1}}\frac{1}{2\sqrt{x}}+\frac{1}{\frac{a}{x}\sqrt{\frac{a^2}{x^2}-1}}\frac{-a}{x^2}$$

$$\frac{dy}{dx}=\frac{1}{2x\sqrt{x-1}}-\frac{1}{x\sqrt{\frac{a^2-x^2}{x^2}}}$$

$$\frac{dy}{dx}=\frac{1}{2x\sqrt{x-1}}-\frac{x}{x\sqrt{a^2-x^2}}=\frac{1}{2x\sqrt{x-1}}-\frac{1}{\sqrt{a^2-x^2}}$$

<u>Ex</u> If $y^2\sin x+y=\cot^{-1}x$ **Find** $\frac{dy}{dx}$

<u>Sol</u>

$$\frac{d\cot^{-1}u}{dx}=\frac{-1}{1+u^2}\frac{du}{dx}$$ مشتقة دالة $\cot^{-1}$ تعطى بالقانون الاتي

$u=x \Rightarrow \frac{du}{dx}=1$

الان نشتق المعالة اعلاه

$$y^2\cos x+\sin x(2y)\frac{dy}{dx}=\frac{-1}{1+x^2}$$
$$y^2\cos x+2y\sin x\frac{dy}{dx}=\frac{-1}{1+x^2}$$
$$2y\sin x\frac{dy}{dx}=\frac{-1}{1+x^2}-y^2\cos x$$

بالقسمة على معامل المشتقة(2y sinx)

$$\frac{dy}{dx}=(\frac{-1-y^2(1+x^2)\cos x}{1+x^2})/2y\sin x$$
$$\frac{dy}{dx}=\frac{-1-y^2(1+x^2)\cos x}{2y(1+x^2)\sin x}$$

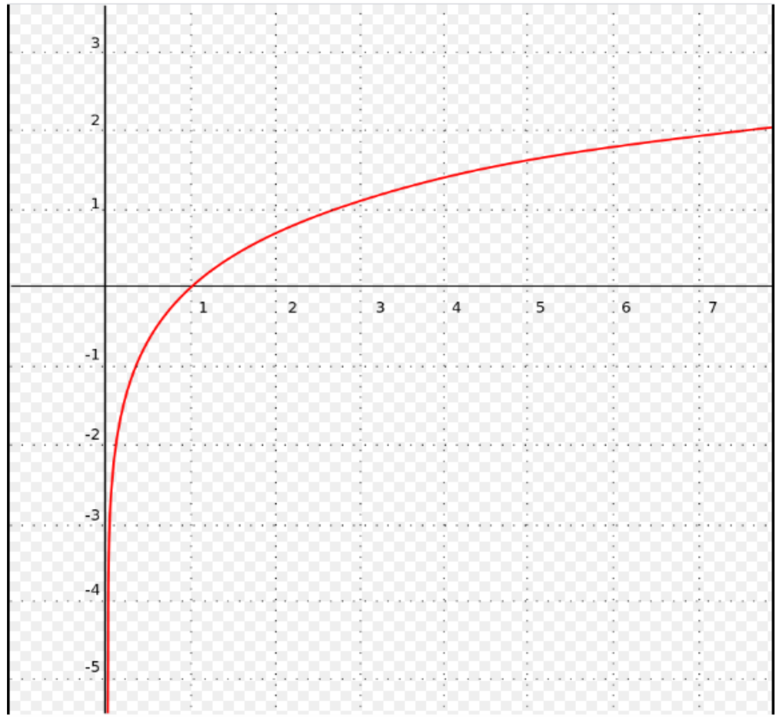
<u>Exercises</u> Find (dy/dx)for the following:

1) $y=c.\sec^{-1}\sqrt{c^2-x^2} \quad , c\ is\ cons\tan t$
2) $y=\sin^{-1}\frac{2x}{1-x^2}-\tan\frac{2x}{1-x^2}$
3) $y=\sin^{-1}\frac{x-1}{x+1}$
4) $y=\tan^{-1}\frac{x-1}{x+1}$
5) $y=\cot^{-1}\frac{2}{x}+\tan^{-1}\frac{x}{2}$
6) $y=x\cos^{-1}(2x)-\frac{1}{2}\sqrt{1-4x^2}$
7) $y=\frac{1}{ab}\tan^{-1}(\frac{b}{a}\tan x)$

<u>The Natural Logarithm</u>

The Natural Logarithm of x which indicated by (ln x) for positive x is

$$\ln x=\log_e^x \quad where \quad (e=2.7182818...)$$



رسم دالة ln x

Properties of (ln x)

1) $\ln(x_1 x_2) = \ln x_1 + \ln x_2$

2) $\ln \frac{x_1}{x_2} = \ln x_1 - \ln x_2$

3) $\ln x^n = n \ln x$

4) $\ln \frac{1}{x} = -\ln x$

5) $\ln 1 = 0 \quad , \quad \ln e = 1$

6) $\ln x > 0 \; when \quad x > 1 \quad , \ln x < 0 \; where \; 0 < x < 1$

The Derivative of ln x

1) $\frac{d(\ln x)}{dx} = \frac{1}{x}$

2) If u is function of x then

$$\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx}$$

<u>Ex</u> Let $y = \ln(3x^2 + 4) \quad find \quad y'$

<u>sol</u> $y' = \frac{1}{3x^2 + 4}(6x) = \frac{6x}{3x^2 + 4}$

<u>Ex</u> If $y = \ln(\sin x) \quad find \quad y'$

<u>Sol</u>

$$y' = \frac{1}{\sin x}(\cos x) = \frac{\cos x}{\sin x}$$
$$y' = \cot x$$

<u>Ex</u> Let $y = \ln(5x^3 - 2x)^{3/2}$

<u>Sol</u> $y = \frac{3}{2}\ln(5x^3 - 2x)$

$$y' = \frac{3}{2} \times \frac{15x^2 - 2}{5x^3 - 2x}$$

<u>Ex</u> If $y = x^x \quad find \quad y'$

<u>Sol</u> **بأخذ ln الطرفين ثم نشتق حاصل ضرب دالتين**

$$\ln y = x \ln x$$
$$\frac{1}{y}y' = x\frac{1}{x} + \ln x = 1 + \ln x$$
$$y' = y(1 + \ln x)$$
$$y' = x^x(1 + \ln x)$$

<u>Ex</u> Let $y = \ln\frac{x^2 - 2}{x^3 + 2x} \quad find \quad y'$

<u>Sol</u>

$y = \ln(x^2 - 2) - \ln(x^3 + 2x)$ **باستخدام خواص ال ln حاصل قسمة دالتين**

$$y' = \frac{2x}{x^2-2} - \frac{3x^2+2}{x^3+2x}$$
$$y' = \frac{2x(x^3+2x)-(x^2-2)(3x^2+2)}{(x^2-2)(x^3+2x)}$$
$$y' = \frac{2x^4+4x^2-3x^4-2x^2+6x^2+4}{(x^2-2)(x^3+2x)}$$
$$y' = \frac{-x^4+8x^2+4}{(x^2-2)(x^3+2x)}$$

Ex **If** $y=\sqrt{\ln(2x+1)}$ *find* y'

Sol

$$y = (\ln(2x+1))^{1/2}$$
$$y' = \frac{1}{2}(\ln(2x+1))^{-1/2}\frac{2}{2x+1}$$
$$y' = \frac{1}{\sqrt{\ln(2x+1)}} \times \frac{1}{2x+1}$$

Exercises **Find** $\frac{dy}{dx}$

1) $y = \ln(x^3+6x^2)$
2) $y = \ln(x+3)(2x-7)$
3) $y = \ln(x^2+4)$
4) $y = \ln\frac{\cos x}{\sin x - 2}$
5) $y = x(\ln x - 1)$
6) $y = x^{\tan x}$

The Exponential Function

The Exponential function denoted by $y=e^x$ **which means** $\ln y = x$

$$y = e^x \quad \Leftrightarrow \quad x = \ln y$$

Properties of e^x

$1)e^{x_1} \times e^{x_2} = e^{x_1+x_2}$

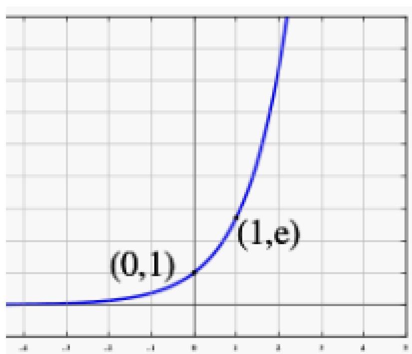
$2)\dfrac{e^{x_1}}{e^{x_2}} = e^{x_1-x_2}$

$3)\dfrac{1}{e^{x}} = e^{-x}$

$4)\left(e^{x_1}\right)^{x_2} = e^{x_1 x_2}$

$5)e^{0} = 1 \quad , \quad e^{1} = e$

$6)e^{\ln x} = x$



رسم الدالة الاسية

<u>The Derivative of e^x</u>

1) $\dfrac{de^{x}}{dx} = e^{x}$

2) If u is a function of x and $y=e^u$ then

$$\frac{de^{u}}{dx} = e^{u}\frac{du}{dx}$$

<u>Ex</u> Let $y = e^{-x^2}$

<u>Sol</u>

$$y' = e^{-x^2}(-2x)$$
$$y' = -2x\, e^{-x^2}$$

<u>Ex</u> If $y = e^{\frac{1}{x}}$ *find* y'

Sol $y' = e^{\frac{1}{x}}(\frac{-1}{x^2}) \Rightarrow \quad y' = \frac{-1}{x^2} \times e^{\frac{1}{x}}$

Ex Let $y = e^{\ln x} \quad find \quad y'$

Sol

$$y' = e^{\ln x}(\frac{1}{x})$$
$$y' = \frac{e^{\ln x}}{x}$$

Ex If $y = e^{3x} \sin 2x \quad find \quad y'$

Sol

نشتق حاصل ضرب دالتين

$$y' = e^{3x}\cos(2x)(2) + 3e^{3x}\sin 2x$$

$$y' = 2e^{3x}\cos 2x + 3e^{3x}\sin 2x$$

Ex Let

Sol

$$let\ u = e^{x} \quad \Rightarrow \frac{du}{dx} = e^{x}$$
$$y = e^{u}$$
$$y' = e^{u}\frac{du}{dx} \quad \Rightarrow \quad y' = e^{e^{x}} \times e^{x} = e^{e^{x}+x}$$

Exercises Find y' **for the following:**

$1) y = e^{x^2} \times 2x \qquad (2) y = e^{\cos x}$

$3) \ln(x+y) = e^x \qquad (4) y = \frac{1}{2}(e^{3x} + e^{-3x})$

$5) \sin(x+y) = ae^{x+y} + b \quad (a, b \quad constant)$

$6) y = x - \ln(e^x - 1) \qquad (7) \tan y = e^x + \ln x$

$8) e^{2x} = \sin(x+3y)$

General Exponential Function(a^x)

If a is any positive real number (a>0) then ($a^x = e^{x \ln a}$)

Properties of a^x

$1) a^{x_1} \times a^{x_2} = a^{x_1 + x_2}$

$2) \frac{a^{x_1}}{a^{x_2}} = a^{x_1 - x_2}$

$3) \left(a^{x_1}\right)^{x_2} = a^{x_1 x_2}$

$4) a^{-x} = \frac{1}{a^x}$

$5) a^0 = 1 \quad , a^1 = a$

Derivative of a^x

$1) \frac{da^x}{dx} = a^x \ln a$

<u>*proof*</u>

$$\frac{da^x}{dx} = \frac{de^{x \ln a}}{dx}$$

$$= e^{x \ln a} \times \ln a = a^x \times \ln a$$

2) *If u isa differentiable function with respect to x then* $\frac{da^u}{dx} = a^u (\ln a) \frac{du}{dx}$

<u>Proof</u>

$$let\ \ y = a^{u} = e^{u\ln a}$$

$$\frac{dy}{dx} = \ln a \times e^{u\ln a} \times \frac{du}{dx}$$

$$\frac{da^{u}}{dx} = \ln a \times a^{u} \times \frac{du}{dx}$$

<u>Ex</u> Let $y = 3^{2x}$ $\quad find \quad \frac{dy}{dx}$

<u>Sol</u>

$$\frac{dy}{dx} = 3^{2x} \times \ln 3 \times (2)$$

<u>Ex</u> If $y = 5^{\sin 3x}$ $\quad find \quad \frac{dy}{dx}$

$$\frac{dy}{dx} = 5^{\sin 3x} \times \ln 5 \times 3\cos 3x$$

<u>Ex</u> let $y = 7^{\ln x^{3}}$ $\quad find \ \ y'$

<u>Sol</u>

$$y' = 7^{\ln x^{3}} \times \ln 7 \times \frac{3x^{2}}{x^{3}} = 7^{\ln x^{3}} \times \ln 7 \times \frac{3}{x}$$

<u>Ex</u> If $y = 9^{\cot^{-1} 2x}$ $\quad find \quad y'$

<u>Sol</u>

$$y' = 9^{\cot^{-1} 2x} \times \ln 9 \times \frac{-1}{1+4x^{2}}$$

<u>Exercises</u> find y' **for the following :**

$1) y = 2^{-x^{3}}$ $\qquad (2) y = 3^{\tan x}$

$3) y = 3^{x} - 5^{2x}$ $\qquad (4) y = 2^{\sec x}$

$5) y = 5^{2x-2}$ $\qquad (6) y = 4^{\ln \frac{2}{x^{2}}}$